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**Adjustment of Phreatic Line  
in Seepage Analysis by  
Finite Element Method**

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search Report No. CE2  
March, 1979

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ADJUSTMENT OF PHREATIC LINE IN SEEPAGE  
ANALYSIS BY FINITE ELEMENT METHOD

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RESEARCH REPORT NO. CE 2  
Department of Civil Engineering  
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March, 1979

Synopsis

*In the application of finite element methods to analysis of seepage through earth dams, previous researchers have reported various methods for adjusting the phreatic line with differing degrees of success in satisfying the exit boundary conditions.*

*This paper reviews five such methods for phreatic line adjustment, and reports the results of a set of comparison tests for assessing their relative performance in terms of (a) position of the phreatic line, (b) pore pressures, and (c) estimation of seepage flow rates.*

*It is concluded that the method used for the adjustment of the phreatic line makes no significant difference to the results obtained in principal applications and the choice of method depends, therefore, on its suitability for the particular application and the ease with which it can be implemented.*

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## 1. INTRODUCTION

### 1.1 Steady Seepage Through a Dam

Consider the typical problem of flow through an earth dam shown in Fig. 1. The dam is founded on a horizontal, impervious layer DC and retains water to depth H. The free surface meets the downstream face, BC at point B, the exit point. Other exit point conditions are possible. The region of saturated seepage may include zones under the dam which extend upstream and downstream and may be composed of zones of different materials. In general the material in each zone is anisotropic. Because the construction method usually involves the dumping, spreading and compaction of relatively thin horizontal layers of soil, the principal axes of permeability are usually horizontal and vertical, with the permeability in the horizontal direction greater than that in the vertical.

If the principal axes are parallel to the x and y axes, for saturated seepage according to Darcy's Law the governing differential equation is

$$\frac{\partial}{\partial x} (k_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (k_y \frac{\partial h}{\partial y}) = 0 \quad (1)$$

in which

$h = \frac{p}{\gamma} + y =$  piezometric head,

$p =$  pressure relative to atmospheric pressure,

$k =$  permeability coefficient,

$\gamma =$  specific weight of water.

For the dam shown in Fig. 1 the boundary conditions are

- (i)  $h = H$  (the water surface elevation) along AD,
- (ii)  $h = y$  along BC,
- (iii)  $\frac{\partial h}{\partial n} = 0$  along CD, and
- (iv)  $h = y$  and  $\frac{\partial h}{\partial n} = 0$  along AB.

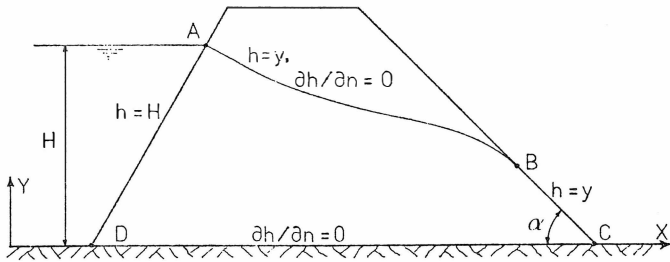


FIGURE 1 : Typical dam cross section

The position of the phreatic line  $AB$  is not known *a priori* and must be found as part of the total solution. The extra condition specified along  $AB$  is sufficient for a solution.

### 1.2 Method of Solution by Finite Element Method

An initial position for the phreatic line,  $AB$ , is assumed and one or other (but *not* both) of the boundary conditions is applied along  $AB$ . The next step is the solution by the finite element method of the field problem within the region defined by the fixed boundaries and the approximate phreatic line. This is a standard finite element solution.

The computed values of either  $h$  or  $\frac{\partial h}{\partial n}$  which were not used for the boundary conditions of the finite element analysis are then compared with the values required on the phreatic line, and the position of the phreatic line is adjusted to improve the agreement between them. Full details are given later for five different methods.

The new position of the phreatic line is used as the boundary for the next finite element analysis and the procedure is repeated until satisfactory convergence is obtained.

### 1.3 Exit Conditions

The angle at which the phreatic line,  $AB$  meets the downstream seepage surface,  $BC$ , depends on the angle of slope,  $\alpha$ , of the face  $BC$  (Fig. 1). For  $\alpha \leq 90^\circ$  the phreatic line is tangent

to the seepage surface and for  $\alpha \geq 90^\circ$  it is vertical when it meets the seepage surface (5, 8, 12). The piezometric head,  $h$ , equals the elevation,  $y$ , on both AB and BC while  $\frac{\partial h}{\partial n}$  is zero on AB but non-zero along BC.

The finite element solutions should satisfy these boundary conditions, but because of the discretization and approximation inherent in the numerical method, exact satisfaction may not be possible. France et al. (4) made no attempt to satisfy conditions exactly at the exit point. Taylor and Brown (13) referred to an ambiguity in the specification of the boundary conditions at the exit point and used it as an explanation for the lack of convergence of their method near the exit point. Their phreatic line exited into a rockfill toe. Finn (3) mentioned that the phreatic line should be tangent to the downstream face (for  $\alpha \leq 90^\circ$ ). The ambiguity of Taylor and Brown was claimed by Neuman and Witherspoon (10) to result from a basic lack of convergence in their iterative method. The latter authors developed an elaborate method which they successfully demonstrated for a number of different exit conditions. This writer has used the condition of vertical exit for  $\alpha > 90^\circ$  to resolve the problem of locating the exit point into toe drains and underdrains (7). Further details are given later when the various methods are described.

Although it is desirable that the boundary conditions should be satisfied exactly at the exit point, the fact that some researchers have ignored this problem completely indicates that they considered the effects are local and do not significantly influence the overall accuracy. Whether or not such an assumption is valid is one of the questions studied by this writer in a set of comparative tests of five different methods reported herein. These tests are reported following the detailed description of the various methods.

## 2. METHODS FOR ADJUSTING THE PHREATIC LINE

The methods for the analysis of seepage with a free surface proposed by different writers may vary in such details as choice of element, method for the solution of the simultaneous linear equations, and techniques for mesh modification in each iteration. However, the fundamental and essential difference is in the choice of boundary condition for the finite element analysis along the phreatic line and the method used to adjust the position of the phreatic line. The available methods are described and compared. One of these has not been previously published. They are titled Methods A, B, C, D, and E for convenience. The authors are acknowledged in the text.

### 2.1 Method A

The condition  $\frac{\partial h}{\partial n} = 0$  is used as the boundary condition along the phreatic line for the finite element analysis. The value of  $h$  computed in the finite element solution is compared with the  $y$  co-ordinate at each node along the phreatic line, and the node is shifted towards the position where  $y$  would equal the computed value of  $h$ . It is often convenient to let the node move along a vertical line, but in some cases, e.g. a node on the boundary between two soil zones, nodes must be moved in specified directions. The writer has shown (7) how this method may be adapted to obtain the location of the phreatic line in the region near the exit point when the phreatic line is vertical at the exit point. In this case the nodes are moved along inclined lines and the movement is damped. Method A was used also by Taylor and Brown (13), Finn (3) and Volker (15). The last two authors did not use automatic adjustment but modified the phreatic line position manually between iterations.

The boundary condition used at the exit point, B, is  $\frac{\partial h}{\partial n} = 0$  and the value of  $h$  is not specified. The computed value of  $h$  is used to estimate the next  $y$  value for the exit point as the node at the exit point is moved along the downstream face. However, the approximations in the finite element method cause some problems.



Consider the details of the finite element mesh near the exit point B shown in Fig. 2. If a three node triangular element is used, a constant value of  $\frac{\partial h}{\partial n}$  (consistent with the linear variation of  $h$ ) should be used across the segment  $i - j$ .

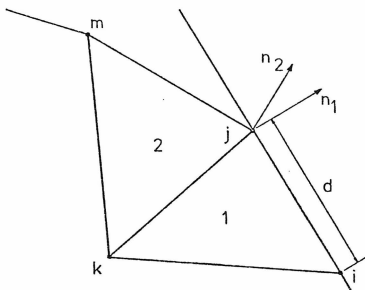


FIGURE 2 : Exit point on downstream face

Even if a linear variation of  $\frac{\partial h}{\partial n}$  is assumed, the equivalent nodal flow calculated for node  $j$  would be  $[2(\frac{\partial h}{\partial n})_j + (\frac{\partial h}{\partial n})_i]d/6$ , and this is zero only if  $(\frac{\partial h}{\partial n})_i$  and  $(\frac{\partial h}{\partial n})_j$  are both zero. The best that could be achieved with this element would be results that gave  $h = y$  at both  $i$  and  $j$ , and  $\frac{\partial h}{\partial n} = 0$  across  $i - j$ . In these circumstances, the calculated exit point would be above the true position.

For the six node element, the equivalent nodal flow at node  $j$  calculated from the assumption of a linear variation of  $\frac{\partial h}{\partial n}$  along  $i - j$  is  $(\frac{\partial h}{\partial n})_j d/6$ . Although the condition  $\frac{\partial h}{\partial n} = 0$  is wanted at node B, the data specification is that the equivalent nodal flow at B is zero. With this element it is theoretically possible to have a linear variation of  $\frac{\partial h}{\partial n}$  from zero at node  $j$  to its value at node  $i$  which is compatible with the zero equivalent flow at node  $j$  and the interpretation of node  $j$  as the exit point.

Provided the 6 node element or one of higher order is used, the exit point conditions will be satisfied exactly in the limit as the mesh spacing approaches zero. The problems arise from the discretization. The normal direction  $n_1, n_2$  for the segments  $i - j$  and  $j - m$  are different, and the solution cannot

satisfy the condition  $\frac{\partial h}{\partial n} = 0$  for both directions. This is true at all nodes along the phreatic line when the phreatic line is approximated by straight segments.

If it were necessary that the condition  $\frac{\partial h}{\partial n} = 0$  be modelled exactly, it could be achieved with curved elements. Since satisfactory results can be obtained with straight elements, it is not necessary to model  $\frac{\partial h}{\partial n} = 0$  exactly.

Method A has the advantage that it is simple and easily coded into a program. Some of this advantage derives from the fact that the change at each node depends only on the values of  $h$  and  $y$  at the node. The change is not directly related to the movements of other nodes (as in Method D) nor is it necessary to use some form of curve fitting to obtain the new position of the phreatic line (as in Method C). It also yields a good estimate of the two-dimensional discharge calculated from the equivalent nodal discharges at all nodes where  $h$  is specified, and there is no ambiguity in the interpretation of this discharge. Its disadvantage is that it predicts changes to the  $y$  co-ordinate only. Although it can be used where the slope of the phreatic line approaches the vertical, this use is based on an empirical method that requires some experience and judgement by the program user.

## 2.2 Method B

Method B, proposed by Verruijt (14), uses the fact that the equivalent nodal discharges are zero at all nodes along a boundary where  $\frac{\partial h}{\partial n}$  is zero. The boundary condition  $y = h$  is used along the assumed phreatic line for the finite element analysis. If the assembled set of equations before substitution of the fixed values is written

$$[A] \{h\} = \{q\} \quad (2)$$

$q_i$  is the equivalent nodal flow at node  $i$ . In general, these values are calculated when  $\frac{\partial h}{\partial n}$  is specified and non-zero. If the value of  $h_i$  is fixed,  $q_i$  may be found from equation  $i$  of the set in Eq. 2,

$$q_i = \sum a_{ij} h_j \quad (3)$$

If  $q_i$  is not zero, the value of  $h_i$  which would make it zero is

$$h_i = -\frac{1}{a_{ii}} \sum_{j \neq i} a_{ij} h_j \quad (4)$$

Node  $i$  is shifted in the direction which would make  $y_i$  equal to the value of  $h_i$  calculated in Eq. 4.

For an exit point on the downstream face,  $q$  would be made zero and, therefore, the same approximations are present as in Method A.

Like Method A, Method B has the advantage that it is also simple to program, especially if the equations are solved by an iterative method which leaves the coefficients,  $a_{ij}$ , unaltered. The changes are made node by node without any need for curve fitting or integration along the phreatic line. Method B has the additional advantage that the specification of  $h$  along the phreatic line reduces the number of unknowns in the finite element equations. If a significant number of nodes is involved, this markedly improves the convergence rate when an iterative method is used for the solution of the equations. Method B has the same disadvantage as Method A that changes to only the  $y$  co-ordinate are predicted. In the test case described later, this method proved less accurate than Method A in the calculation of discharges (from the equivalent nodal discharges) at nodes where  $h$  is fixed, and there is some ambiguity in the interpretation of these discharges.

### 2.3 Method C

This method was developed by France et al. (4) and Mills (9) for unsteady seepage. The steady state solution is the final result of an unsteady analysis that starts with the assumed phreatic line.

The finite element method is used to calculate  $h$  for the saturated region, using the boundary condition  $h = y$  along the phreatic surface.  $\frac{\partial h}{\partial x}$  and  $\frac{\partial h}{\partial y}$  are calculated at nodes on the free surface, and the average velocity of fluid particles normal to the phreatic surface,  $V_n$ , is found as follows:

$$V_x = -\frac{k_x}{S_y} \frac{\partial h}{\partial x}, \quad V_y = -\frac{k_y}{S_y} \frac{\partial h}{\partial y} \quad (5)$$

$$V_n = V_x \sin \theta + V_y \cos \theta \quad (6)$$

in which

$$S_y = \text{specific yield} = \frac{\text{volume of water drained}}{\text{bulk volume of medium}}$$

$\theta$  = slope of phreatic line.

$V_n$  is the velocity at which the phreatic surface is moving, and the distance it moves in the normal direction in the time interval,  $\Delta t$ , is  $V_n \Delta t$ .

The steady state condition is reached when the velocity normal to the free surface is zero. In an exact solution,  $V_n \rightarrow 0$  as  $t \rightarrow \infty$ . Because of this and because of the effects of numerical round-off, the final steady state is assumed when the normal velocity is less than a set tolerance. If this method is used for a steady seepage analysis, there is no need to know  $S_y$ , and  $\Delta t$  does not represent a real time step.  $\Delta t$  is arbitrarily chosen to give the best convergence.

France et al. used a curve-fitting procedure to determine the new position of the free surface. This has the effect of smoothing out any local irregularities.

Since this method aims for  $V_n = 0$  and  $h = y$  at all nodes on the phreatic surface, including the exit point, it aims, ideally, to satisfy the exact conditions at the exit point. However, the problem of the indeterminacy of the direction of  $n$  at the exit point occurs. Furthermore, when the curve fitting procedure used by France et al. is employed, the assumed polynomial for this curve does not, in general, satisfy the slope condition required of the phreatic line at the exit point.

An advantage of this method is that the adjustment is in a direction perpendicular to the phreatic line. This is useful where the phreatic line approaches the vertical. The fact that  $\Delta t$  is an arbitrary parameter in this method is a disadvantage. In the tests described later, rapid convergence was achieved with

Method C, but this occurred because a good value of  $\Delta t$  had previously been determined from numerical results obtained while the program was being debugged. If  $\Delta t$  is too small, convergence is very slow, and if it is too large the solution oscillates about the steady state position and the oscillations may even be divergent. The need for a curve fitting procedure in the method of France et al. is a disadvantage because it restricts the shape of the phreatic line. The other methods do not. This disadvantage would be serious in a general purpose program, which should be able to handle most practical problems including different soil zones. It would probably be necessary to fit different curves (using the same polynomial form) in the different zones.

#### 2.4 Method D

This method has not, to the author's knowledge, been used before for locating the phreatic line in a seepage analysis. It is derived from the method used by Chan, Larock and Herrman (1) for potential flow with a free surface. The boundary condition  $h = y$  is used for the finite element analysis, and the condition  $\frac{\partial h}{\partial n} = 0$  is used to estimate the new position of the phreatic line through the following procedure.

Since the condition  $\frac{\partial h}{\partial n} = 0$  means that there is no flow across the boundary, the velocity vector should be tangent to the phreatic line. Its slope,  $S$ , is given by

$$S = \frac{dy}{dx} = \frac{v}{u} \quad (7)$$

in which

$$u = k_x \frac{\partial h}{\partial x}, \quad v = k_y \frac{\partial h}{\partial y} \quad (8)$$

Hence

$$dy = S dx \quad (9)$$

Integration of Eq. 9 yields

$$y_2 - y_1 = \int_1^2 S dx \quad (10)$$

If  $i - j$  is a straight line segment (e.g. the side of an element) approximating part of the phreatic line, and  $m$  is its midpoint, numerical integration by Simpson's rule yields

$$y_j - y_i = \frac{x_j - x_i}{6} (S_i + 4S_m + S_j) \quad (11)$$

in which  $S_i$ ,  $S_j$ ,  $S_m$  are the values of  $S$  evaluated at  $i$ ,  $j$ ,  $m$ .

Alternatively,

$$x_j - x_i = \frac{y_j - y_i}{6} \left( \frac{1}{S_i} + \frac{4}{S_m} + \frac{1}{S_j} \right) \quad (12)$$

Eqs. 11 and 12 may be used to establish a new profile provided the procedure is used from the known point A (Fig. 1) at the upstream end. In general, if  $|x_j - x_i| \geq |y_j - y_i|$  for an element boundary, Eq. 11 is used. Otherwise Eq. 12 is used.

In a test based on Kozeny's Solution (5) which is an analytical solution the calculated position of the phreatic line was above and outside the exact position. The pore pressures calculated from these results would be slightly larger than the exact values, and the method would therefore be conservative. In the set of comparative tests described later, this method predicted the highest position of the exit point.

Any errors at the exit point in this method arise from the finite mesh spacing, since the conditions at the exit point would be satisfied in the limit as the spacing approaches zero. In the tests which are described later, the local errors in the region of the exit point were found to be of similar magnitude to those from the other methods.

An advantage of method D is that it may be used to compute either a  $\Delta x$  or a  $\Delta y$  increment for any boundary segment. In fact, if local axes are used, the movement can be in any desired direction. A disadvantage is that the change at any one node depends on the line integral between that node and the fixed node.

The reason for the disadvantage is that a node is shifted  $\Sigma \Delta x$ ,  $\Sigma \Delta y$  relative to the fixed point at the upstream end. The summations are taken over all segments between the fixed point

and the node being adjusted. Therefore, the effects of any errors at the upstream end must be propagated downstream. In the test, the node at  $x = 7.5$  nearest the fixed point required a large initial shift. This shift was transmitted to the other nodes on the phreatic line even though their initial positions proved to be close to the final positions. The node nearest the fixed node settled first, followed by the next nearest and so on down the line. Therefore, this method could possibly be improved if the movement of each node were calculated relative to the old positions of the upstream nodes.

## 2.5 Method E

This method was developed by Neuman and Witherspoon (10) and based on a generalized variational principle which had both  $h$  and the evaluation of the free surface as unknowns. The procedure can, however, be explained in terms of the techniques previously described.

The position of the phreatic line is assumed and the boundary condition  $h = y$  is used along the phreatic line, AB, and the seepage face, BC, Fig. 1, for a finite element analysis.  $\frac{\partial h}{\partial n}$  along the seepage face is calculated from the results. In the second step in any one iteration the region is re-analysed with  $\frac{\partial h}{\partial n} = 0$  along the phreatic line and  $\frac{\partial h}{\partial n}$  equal to the value just computed along the seepage face. The new solution yields values of  $h$  that are not equal to the elevation  $y$  and the nodes on the phreatic line are adjusted in the direction that would make  $y$  equal to  $h$ .

Neuman and Witherspoon claimed that the treatment of the seepage face, BC, as a prescribed head boundary tends to retard convergence of the solution along AB in the region of BC. This claim has not been supported by any results obtained by the writer using any of the other methods described all of which use the prescribed head condition along BC. In the tests described later, Method E generally converged more rapidly than the other methods but it solves the finite element equations twice in each iteration and, therefore, costs about twice as much per iteration as the other methods.

As for all the other methods, this method can satisfy the conditions exactly at the exit point in the limit as the mesh spacing approaches zero. The results from the test problem indicate that, for a given mesh arrangement, this method gives results that are not significantly different from those of the other methods in the region of the exit point.

This method suffers from the disadvantage that it is more complicated than any of the others and requires that the finite element equations be solved twice in each iteration. It also predicts changes only in the y direction but can be used with changes made along sloping lines (10).

### 3. COMPARISON TESTS

The problem chosen for the tests was steady seepage flow through an embankment with vertical faces (Fig. 3). The material is isotropic and a permeability coefficient of unity was used. A vertical face was chosen for the downstream side slope because this gave the most severe test of the ability of the finite element method to model the tangency condition at the exit point.

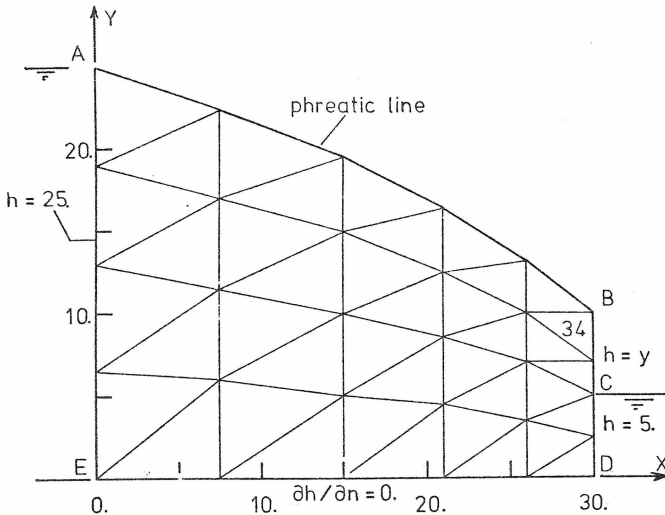


FIGURE 3 : Test problem for comparison of methods of analysis for seepage with a free surface



The initial profile and mesh used for all tests is shown in Fig. 3. In each run ten iterations were done, and the whole mesh was regenerated for each iteration so that there were equal divisions along all internal vertical lines, equal divisions between B and C, while the nodes between A and E and between C and D remained in their original positions.

In Methods A and B the full calculated changes to the y co-ordinates were used and the apex nodes along the phreatic line were moved vertically.

The first approach used in Method C was based on the average nodal velocities at the apex nodes. The problem of the definition of the direction normal to the boundary was encountered, and the direction normal to the element side adjacent to and upstream of the node was used. This appeared to be satisfactory at all nodes except that at the exit point where divergence occurred. The exit point was moved lower and lower in the successive adjustments. This approach was abandoned.

The technique that did work successfully for Method C used the velocity component normal to the element side of the midside node,  $V_n$ , and these nodes were moved a distance  $V_n \Delta t$  along the normal. A parabola passing through the fixed upstream point, A, was fitted using the least squares method, and the apex nodes were then located on this parabola. A time step of  $\Delta t = 4.0$  was used, and this gave a good convergence rate. This value was chosen after some preliminary analyses that were done while the program was being debugged. In general, however, the choice of  $\Delta t$  appears to be arbitrary for steady state analyses, and some method for monitoring the solution and adjusting  $\Delta t$  is desirable.

For Method D, changes to the y co-ordinate were calculated. Only one half the indicated change was used because of the results from the earlier test described previously.

In Method E, after the first solution in each iteration with h specified, the averaged nodal velocities were calculated for the nodes on BC. The equivalent nodal discharges were then calculated from the component normal to the face. This approach differed from that suggested by Neuman and Witherspoon. They

calculated the equivalent nodal discharges with Eq. 3.

Since the value calculated in this way for the node at the exit point, B, also includes a contribution from any flow normal to the element side along AB, they suggested that only half this value should be used at B (provided the elements adjacent to B are sufficiently small).

All the tests were done for ten iterations. For Methods A, B, C and D, the set of finite element equations was solved ten times, while it was solved twenty times for Method E.

### 3.1 Position of Phreatic Line

The final co-ordinates of the apex nodes which defined the phreatic line are given in Table 1. The differences between the profiles would not be significant in most practical applications.

TABLE 1 : Phreatic line profiles after 10 iterations

X	Y Values for Method:					
	A	B	C	D	E	Chapman
0	25.00	25.00	25.00	25.00	25.00	25.00
7.5	22.77	22.50	22.98	23.10	22.77	22.09
15.0	19.59	19.25	19.63	19.65	19.61	18.73
21.0	16.39	16.03	15.99	16.52	16.41	15.53
26.0	13.14	12.65	12.31	13.07	13.23	12.24
30.0	8.51	8.51	8.93	9.09	8.38	8.75

Because of the discretization, the tangency condition at the exit point, B, was not realized, nor was the condition that the phreatic line should be horizontal at point A.

The value of 8.75 for  $y$  at the exit point was calculated by a method given by Polubarinova - Kochina (11), but the relative error could be large because numbers used in the calculation were read from her small diagram. However, the profiles compare favourably with the approximate profile calculated by a method suggested by Chapman (2) when the value of  $y = 8.75$  was used at

the exit point. Some points on this approximate profile are given in Table 1.

The convergence of the phreatic line for each method is demonstrated by the plot of exit point elevation,  $y_B$ , against the iteration number (Fig. 4). Methods C and E converged more rapidly than the others. For Method C, the reason was the good choice of  $\Delta t$  determined in earlier analyses as described above. In Method E, the finite element equations are solved twice in each iteration, so for the same cost of analysis, its performance in the first five iterations should be compared with the performance of the other methods in ten iterations. The convergence in Method D is not smooth, and the reason is that the errors are swept downstream, and convergence of the phreatic line occurs progressively from the upstream end. The convergence of Method D might be improved if the movement of each node were calculated relative to the *previous* positions of the upstream nodes rather than to their *new* positions.

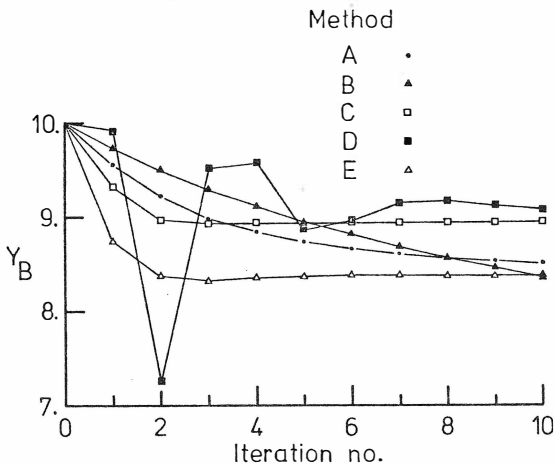


FIGURE 4 : Elevation of exit point as a function of iteration number

### 3.2 Pore Pressures

Generally, in practical applications, the values of pore pressure or of discharge are more important than the location of the phreatic line. Envelopes of computed pore pressure heads along the horizontal line  $y = 0$  and along the vertical line  $x = 26$  are shown in Figs. 5 and 6. The divergence of the upper and lower envelopes as  $p \rightarrow 0$  for  $x = 26$  (Fig. 6) was a direct result of the differences in the location of the phreatic line. As the distance below the phreatic line increased, the differences between the values of pore pressure calculated by the various methods decreased, and for  $y = 0$  these differences are insignificant. These results indicate that the choice of method would have no significant effect on a stability analysis.

### 3.3 Estimation of Discharge

The computed two-dimensional discharges are given in Table 2.

TABLE 2 : Comparison of computed two-dimensional discharges at selected points

Boundary segment	Discharge for method:				
	A	B	C	D	E
A-E	9.947	10.602	10.106	9.868	9.929
A-B excluding A	0	-1.659	-0.254	-0.017	-0.145
node B	0	+0.077	+0.340	+0.212	-0.145
B-D	-9.940	-9.500	-9.502	-10.035	-9.920
B-D excluding B	-9.940	-9.577	-9.842	-10.247	-9.775
Total in	9.947	10.691	10.865	10.506	9.929
Total out	9.940	11.325	10.855	10.901	9.920

These discharges were obtained from the equivalent nodal discharges at nodes where the head was specified (Eq. 3) for Methods A, B, C, and D. For Method E the equivalent nodal discharges specified in the second step of the last iteration were added. The exact value is 10.000 (11). Method A yielded the best estimate of the discharge and had the advantage that there was no ambiguity in the interpretation of the equivalent nodal discharges.

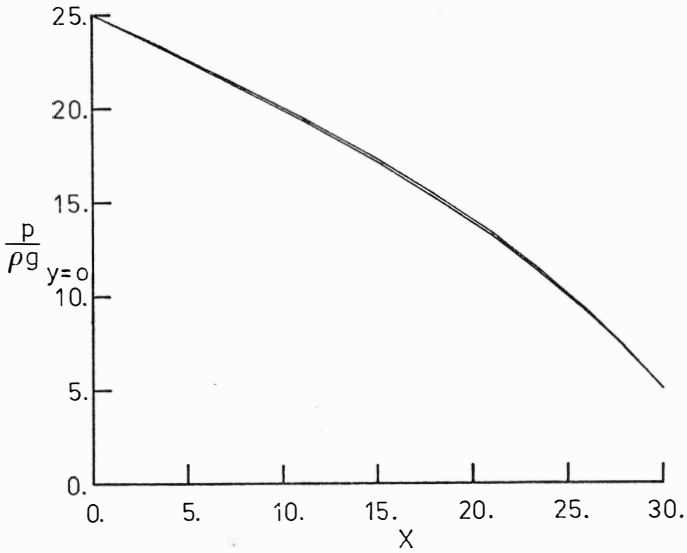


FIGURE 5 : Upper and lower envelopes for pore pressure along base line ( $y = 0$ )

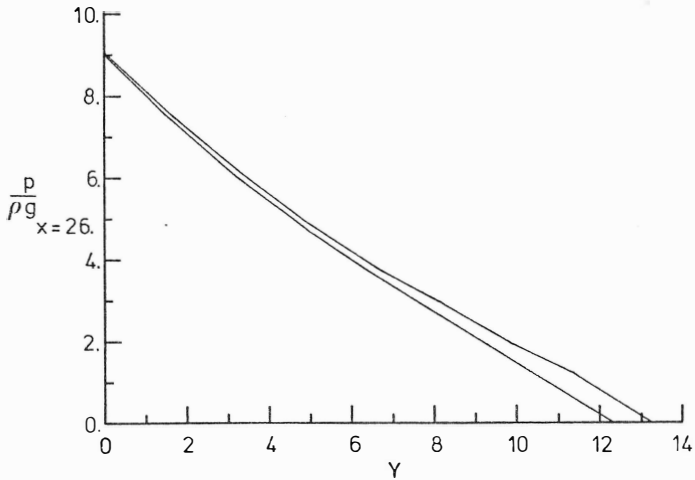


FIGURE 6 : Upper and lower envelopes for pore pressure at  $x = 26$

Since  $h = y$  was used as the boundary condition along AB for the finite element analyses in Methods B, C and D, the non-zero discharges along AB indicated how well the condition  $\frac{\partial h}{\partial n} = 0$  was satisfied in the final solution. The equivalent nodal discharge at node B should, for Methods B, C and D, be included with those on the phreatic line and excluded from those on the seepage face because it has been demonstrated that this discharge should be zero if the tangency condition were satisfied. This interpretation yielded the best results for the Methods B and C while the inclusion of the discharge at node B improved the result for Method D. The discharge at node B should be included for Method E because it was specified as part of the boundary conditions for the final solution in that method. The equivalent nodal discharge at A must be included with the inflow across the boundary AE even though in Methods B, C and D, there may have been a contribution from the element boundary on the phreatic line.

When  $\frac{\partial h}{\partial n} = 0$  is used as the boundary condition along AB for the finite element analyses, the accuracy of the solution along the phreatic line is measured by the agreement between  $h$  and  $y$ . Table 3 shows, for Methods A and E, the values of  $h$  calculated at the apex nodes in the tenth iteration and compares them with the  $y$  co-ordinates calculated in the ninth iteration and used in the tenth. This table also presents a comparison of  $h$  and  $y$  after five iterations of Method E. The agreement between  $h$  and  $y$  after ten iterations of Method E was very good and was much better than

TABLE 3 : Comparison of elevation ( $y$ ) and seepage head ( $h$ ) values along phreatic line

X	Method A		Method E		Method E	
	$y_9$	$y_{10}$	$y_9$	$h_{10}$	$y_4$	$h_5$
0	25.000	25.000	25.000	25.000	25.000	25.000
7.5	22.767	22.767	22.770	22.770	22.770	22.766
15.0	19.593	19.595	19.606	19.606	19.623	19.597
21.0	16.395	16.395	16.407	16.408	16.501	16.442
26.0	13.122	13.137	13.233	13.234	13.232	13.266
30	8.592	8.505	8.376	8.376	8.350	8.372

that obtained with Method A. The results from Method A were satisfactory for most practical applications and were slightly better than those obtained after five iterations with Method E.

An argument was given above for a zero velocity component normal to the seepage face at the exit point, B, for consistency between the finite element model and the tangency condition. The actual velocity component calculated for the element adjacent to the exit point for each method is shown in Fig. 7.

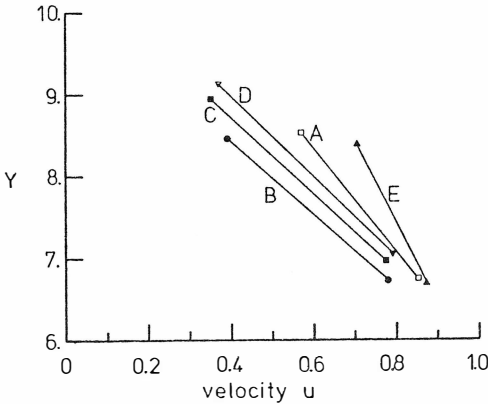


FIGURE 7 : Computed velocities across vertical boundary ( $x = 30$ ) of element 34

The maximum value of the component normal to the seepage face BC was approximately 1.4 and the mean value over the side BCD was approximately  $1.1 \rightarrow 1.2$ . Therefore, the error at the exit point was significant. It resulted from the finite element discretization and better results for the same element could be obtained only through the use of a finer mesh in the region of the exit point. The results from the tests indicated that this local error in the vicinity of the exit point did not significantly affect the accuracy of the solution for the position of the phreatic line, the pore pressures or the total discharge.

#### 4. CONCLUSION

The comparison tests showed that there were no significant differences in the results obtained by the different methods for the location of the phreatic line, the calculation of pore pressure or the estimation of discharge. Therefore the choice of method should be determined by the particular application and the ease with which it can be implemented. The author's choice for a general purpose program for steady state seepage is Method A, because it is easy to apply and gives unambiguous results for the equivalent nodal discharges. Methods B and D have advantages that may make them more suited to some specific applications. Method C is not recommended for steady state analyses, because of its dependence on an arbitrary choice of time step, and because of the use of a restrictive curve fitting procedure for the phreatic line. Method E is not recommended because it is the most complicated of the methods and, despite a theoretical advantage in its treatment of the seepage face, does not show any practical advantages over the other methods.

In the tests reported here, all five methods gave local errors at the exit point, where none satisfied the condition of tangency and zero velocity normal to the seepage face. These errors were caused mainly by the discretization which was essentially the same in all tests. The method adopted for the adjustment of the phreatic line did not significantly alter these local errors.



## APPENDIX A - NOMENCLATURE

<u>Symbol</u>	<u>Meaning</u>
[A]	coefficient matrix of finite element equations
a	a term in [A]
h	$\frac{P}{\gamma} + y =$ piezometric head
k	permeability coefficient
n	direction normal to boundary
p	pressure relative to atmospheric pressure
q	two dimensional discharge
$S_y$	specific yield
S	slope
$\Delta t$	time step
V	velocity
u,v	components of V
x,y	horizontal and vertical cartesian co-ordinates.
$\alpha$	angle from horizontal to downstream face of embankment
$\theta$	slope of phreatic line
$\rho$	density of water

## APPENDIX B - REFERENCES

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