# CHANGING CLASSROOM PRACTICE THROUGH A RICH MODEL OF NUMERACY ACROSS THE CURRICULUM 

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#### Abstract

This paper reports on a research and development project that helped teachers to plan and implement numeracy strategies across the school curriculum. It presents a rich model of numeracy whose elements comprise mathematical knowledge, dispositions, tools, contexts, and a critical orientation to the use of mathematics. This model is then applied to analyse changes in one teacher's planning, classroom practice, and personal conceptions of numeracy. Numeracy; teacher development; curriculum development.


## INTRODUCTION

The term numeracy is used in many English-speaking countries to describe the capacity to deal with quantitative aspects of life. Numeracy, quantitative literacy and mathematical literacy are similar in that they refer to:
an individual's capacity to identify and understand the role mathematics plays in the world, to make well-founded judgments, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (Organisation for Economic Cooperation and Development, 2004, p. 15)
Steen (2001) maintains that, for numeracy to be useful to students, it must be learned in multiple contexts and in all school subjects, not just mathematics. This is a challenging notion, but a recent review of numeracy education undertaken by the Australian government (Human Capital Working Group, Council of Australian Governments, 2008) concurred, recommending:

That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment. (p. 7)

The study reported here aimed to investigate approaches to help teachers plan and implement numeracy strategies across the Australian school curriculum in Grades 6-9. The purpose of this paper is to introduce a new model of numeracy that was developed to synthesise and extend previous research in this area, and demonstrate how this model was used to analyse changes in teachers' planning, classroom practice, and personal conceptions of numeracy. In
relation to the themes of TSG21, the paper proposes a theoretical perspective and research approach in defining, identifying, assessing, and improving the quality of classroom practice.

## THEORETICAL FRAMEWORK

In Australia, educators and policy makers have embraced a broad interpretation of numeracy similar to the OECD definition of mathematical literacy. The definition proposed by a 1997 national numeracy conference, "To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (Australian Association of Mathematics Teachers, 1997, p. 15), became widely accepted in Australia and formed the basis for much numeracy-related research and curriculum development.


Figure 1. A model for numeracy in the $21^{\text {st }}$ century

Recently, however, Goos (2007) argued that a description of numeracy for new times needs to better acknowledge the rapidly evolving nature of knowledge, work, and technology (e.g., see Noss, Hoyles, \& Pozzi, 2000; Zevenbergen, 2004). She developed the model shown in Figure 1 to represent the multi-faceted nature of numeracy in the twenty-first century. This model was designed to capture the richness of current definitions of numeracy, while introducing a greater emphasis on tools as mediators of mathematical thinking and action (Sfard \& McClain, 2002) and a critical orientation to the ways mathematics is used persuade, manipulate, disadvantage or shape opinions about social or political issues (Jablonka, 2003). The model was intended to be readily accessible to teachers as an instrument for planning and reflection. Its elements are described in Table 1.

Table 1: Description of elements of the numeracy model

| Element of model | Description of element |
| :--- | :--- |
| Mathematical <br> knowledge | Mathematical concepts and skills; problem solving strategies; <br> estimation capacities. |
| Contexts | Capacity to use mathematical knowledge in a range of contexts, both <br> within schools and beyond school settings. |
| Dispositions | Confidence and willingness to use mathematical approaches to <br> engage with life-related tasks; preparedness to make flexible and <br> adaptive use of mathematical knowledge. |
| Tools | Use of material (models, measuring instruments), representational <br> (symbol systems, graphs, maps, diagrams, drawings, tables, ready |
| reckoners) and digital (computers, software, calculators, internet) |  |
| tools to mediate and shape thinking. |  |

## PROFESSIONAL DEVELOPMENT APPROACH

A major feature of our project was its focus on teacher professional learning and development. As Loucks-Horsley, Love, Stiles, Mundry and Hewson (2003) point out, a great deal is now known about designing effective professional development to bring about changes in the way that mathematics is taught in schools. For example, because all learning is contextual, professional development needs to occur in school-based contexts so teachers can try out and validate ideas in their own classrooms. Teachers also need time and opportunities to discuss pedagogical and curricular issues with supportive colleagues as they attempt to implement new practices.

In working with teachers we integrated four professional development strategies recommended by Loucks-Horsley et al. (2003). The first strategy involved formation of collaborative partnerships between participating teachers, university researchers, and curriculum support officers from the state Department of Education, which commissioned the project. Collaborative structures provide opportunities for professional learning around topics negotiated and agreed upon by the group, thus ensuring common goals. The second strategy was to examine teaching and learning using action research. Somekh and Zeichner (2009) claim that action research is an appropriate methodology for supporting educational reform, in our case, the embedding of numeracy throughout the school curriculum. We conducted a series of project meetings and school visits to support teachers through action research cycles of plan-implement-evaluate-reflect in order to replan and continue through the next cycle. Third, we provided teachers with immersion experiences that included numeracy-based
learning opportunities and examples of numeracy investigations and assessment tasks. Fourth, we expected curriculum implementation by requiring teachers to develop and implement units of work that targeted numeracy demands of the diverse curriculum areas from which they were drawn. This contrasts with the approach of short-term workshops that demonstrate innovative materials but provide no support for teachers to trial these ideas in their own classrooms.

## RESEARCH METHODOLOGY

Twenty teachers were recruited on the basis of their interest in cross-curricular numeracy education. They came from ten schools spread across urban, regional and rural locations in Australia: four primary schools (Kindergarten-Grade 7), one secondary school (Grades 8-12), four smaller schools in rural areas (Grades 1-12), and one school that combined middle and secondary grades (Grades 6-12). The focus on teaching numeracy across the whole curriculum meant that it was important to include teachers with varying subject specialisations. Thus participants included generalist primary school teachers as well as secondary teachers qualified to teach particular subjects (mathematics, English, science, social education, health and physical education, design studies).

The project was conducted from January-November 2009. The research design allowed for two action research cycles in which the research team worked with teachers to plan, implement, evaluate, and reflect on the numeracy curriculum units they had taught. The project began by conducting a curriculum audit to identify the numeracy demands inherent in all school subjects (see Goos, Geiger, \& Dole, 2010). Three, whole-day meetings in March, August, and November brought together all participants to investigate the numeracy model and curriculum audit, design numeracy tasks, develop curriculum plans incorporating a numeracy approach, and reflect on their progress. In the time between these meetings (June and October), the teachers implemented their curriculum plans. During this time the research team also conducted two, daylong visits to each school for lesson observations, collection of planning documents, interviews with teachers and students, and feedback to teachers for further development of numeracy teaching strategies. Field notes from lesson observations recorded the extent to which teaching incorporated elements of the numeracy model shown in Figure 1. Interviews with teachers focused on evaluation of numeracy tasks, student learning, and teacher learning. Interviews with students asked about mathematical knowledge, contexts, dispositions, and tools for numeracy learning. In the final project meeting, teachers were asked to map their personal progress in understanding and implementing the numeracy model by annotating a copy of the model (see Geiger, Goos, \& Dole, 2011).
A case study of one teacher is presented in order to illustrate the use of the numeracy model for changing classroom practice.

## CHANGES IN CLASSROOM PRACTICE

Catherine had been teaching for about 20 years. She teaches English, health and physical education, and mathematics to a Grade 8 (lower secondary) class, she described herself as not being a "proper" mathematics teacher because she lacked formal qualifications for teaching
this subject at the secondary school level. Catherine's motivation for joining the project was to improve her students' numeracy dispositions, especially their confidence with using mathematics in different contexts. To do this she knew she would need to change her teaching practice by becoming "less directive". Vignettes from the two lessons we observed, one in mathematics, the other in health and physical education, illustrate this change.
On the first school visit the research team observed a mathematics lesson in which Catherine introduced her students to directed numbers. She began the lesson by asking students to say anything they remembered about directed numbers, including real world examples. Students volunteered information such as "moving to the right is positive and to the left is negative", and temperatures can be above or below zero. Catherine explained that the class would go outdoors to work with a number line that they would draw on the floor with chalk. The aim was to work out how to add and subtract directed numbers. She demonstrated the method via a number line drawn on the blackboard, which was also illustrated on the handout she distributed to students. Students were to stand on the first number listed and face in the positive direction if the operation was to be addition or the negative direction if the operation was to be subtraction. They were then to walk the number of steps indicated by the second number, walking forward if this number was positive and backwards if it was negative. The number at which they arrived via this process was the answer to the problem. The handout provided a systematically developed list of problems involving adding and subtracting positive and negative numbers. Two final questions then asked students to describe any patterns they observed in their walks and to explain some of the rules they discovered while adding and subtracting. Although this lesson was characterised by explicit instruction, we observed that some students seemed uncertain about the purpose of the activity and at the end only one could articulate patterns to explain addition and subtraction rules.

Using the numeracy model, the mathematical knowledge dealt with in this lesson was addition and subtraction of directed numbers. At the start of the lesson the teacher elicited some real life contexts in which directed numbers appeared, but otherwise the context for learning was purely mathematical. She attempted to use number lines as a representational tool to help students create patterns and explain rules concerning these operations on directed numbers. At one point she addressed students' dispositions towards mathematics and the learning activity by asking them how they felt about the lesson. There appeared to be no scope in the lesson for developing a critical orientation to this subject matter.

On the second school visit, Catherine was teaching a health and physical education unit in which students wore a pedometer to record how many steps they took over a week. In previous years students had recorded their own data in a table in their notebooks. Catherine would demonstrate the procedure for converting the number of steps to kilometres, and instruct students to draw a bar graph to show how many kilometres per day they had walked. However, in the lesson that was observed it was obvious that Catherine was now taking a less directive approach. She began the lesson by taking students outside to walk a distance they estimated to be 100 metres. Catherine then measured out 100 metres so students could compare their individual estimates to the actual distance. Next, students walked the measured
distance and counted how many steps they took. On returning to the classroom, Catherine used questioning to elicit a method of converting steps to kilometres via a ratio approach. She used her own pace data, 119 paces walked in 100 metres, to help her students work out the conversion method:
119 paces $=100$ metres
$119 \times 10$ paces $=100 \times 10$ metres
1190 paces $=1000$ metres
1190 paces $=1$ kilometre
Catherine then displayed an Excel spreadsheet summary of students' individual pedometer results that were recorded each day over a week. She asked students how they would convert their results to kilometres. Students found it difficult to begin this task so Catherine entered her own pedometer data into the spreadsheet and encouraged students to help her "discover" a method. After some discussion Catherine, with the help of her students, produced the following result:

## 117,581 paces in a week

From the previous calculation Catherine takes 1190 paces per kilometre.
So 117,581 must be divided by 1190 to find the number of kilometres.
$1190=98.8075 \approx 98.8 \mathrm{~km}$.
Students were amazed by the distance that Catherine walked in a week, with one exclaiming "Wow - that's nearly all the way to Whyalla!" (a neighbouring town). This indicated to us that the student was trying to make sense of the answer in terms of a meaningful local context.
After modelling the procedure, Catherine asked students to complete conversions of their own pace totals to kilometres. Some students were able to proceed with this task independently while others required assistance from Catherine. She supported students through questions and probes rather than simply telling them how to perform the procedure directly. She also suggested that students compare their kilometric distances with each other and to discuss why they were different.

Catherine had introduced digital tools into her teaching for the first time, relying on the few students in the class who were familiar with spreadsheets to help their peers and, often, their teacher. Students entered the daily numbers of steps recorded on their personal pedometers into a class spreadsheet that Catherine displayed via an interactive whiteboard. Students used Excel formulas to calculate the total number of steps per week and, as described above, the number of kilometres this represented for each student in the class. When we spoke to students, they were able to explain why the number of steps was greater on some days than others for particular people (e.g., a girl who played netball on Wednesdays but watched TV on week-ends). Students chose charts from the Excel menu to display their data and make comparisons, for example, between steps taken on different days of the week for one student, or between total steps taken by a male and a female student. If students chose inappropriate
graphs for these purposes (e.g., line graphs), Catherine questioned them to draw out the reasons and lead them to identify better ways of representing their data.

This lesson dealt with a range of mathematical knowledge involving number (ratio), measurement (converting units), and statistics (collecting, organising and representing data). The curriculum context was health and physical education, but students also related their data to personal real life contexts involving physical activity. Teacher and students used physical tools (measuring tape, pedometers) and digital tools (spreadsheets, interactive whiteboard). The teacher explicitly addressed numeracy dispositions by encouraging students' confidence in estimation and flexibility in representing their own and others' data. A critical orientation was beginning to emerge in that some students were observed to evaluate the reasonableness of results, and they posed their own questions of the data (e.g., comparison of distances walked in a week, daily variations in steps walked).

## CONCLUSION

The numeracy model was designed to support improvement in numeracy teaching in all school subject areas, but analysis of Catherine's experience demonstrates that the model can also be used to trace teachers' changing conceptions of numeracy. At the end of the project Catherine reflected on her changing understanding of numeracy by tracing her pathway through the model presented in Figure 1. Her desire to improve students' dispositions marked her entry point to the model, and she attempted to do this by exploring the numeracy demands of different curriculum and real world contexts. This necessitated a change in teaching practice towards a less directive and more inquiry-oriented approach, a "letting go" process that Catherine found difficult but more effective for enriching students’ mathematical knowledge and promoting a critical orientation to evaluating information and answers. Once she began to give students more responsibility for their learning she became more willing to experiment with unfamiliar tools, such as spreadsheets, for problem solving. Catherine confirmed that changes in her teaching approach were guided by the model for numeracy that had now become part of her way of thinking when planning numeracy learning experiences in all the subjects she taught.
There are challenges in working with teachers to plan for and promote numeracy learning across the whole school curriculum. First, teaching in context is difficult because students' ability to understand, recognise and apply mathematical concepts is dependent on the purposes and goals of the activity in which they are encountered: mathematical understanding demonstrated in one context may not easily translate into understanding in a different context. Second, although it is possible to plan for numeracy learning, teachers also need to be alive to serendipitous moments for promoting numeracy as opportunities occur during lessons, for example, by "seeing" the numeracy embedded in current events or students' personal experiences. As Catherine explained, "I'm still learning to address the numeracy as it arises, not to be so driven by content and getting things finished". Finally, sharing the responsibility for teaching numeracy in all curriculum areas is challenging because teachers themselves need to model the kind of numeracy communicated by the model in Figure 1. In Catherine's case, this involved overcoming her lack of confidence in her mathematical knowledge and
ability to teach mathematics, and her lack of experience in using digital tools such as spreadsheets. Her willingness to use unfamiliar technologies in her teaching modelled the dispositions that she wished to develop in her students.

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