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## MELNIKOV'S METHOD, STOCHASTIC LAYERS AND NONINTEGRABILITY OF A PERTURBED DUFFING-OSCILLATOR

L.M. PERKO

ABSTRACT. Melnikov's method is used to show that the perturbed Duffing-oscillator with Hamiltonian

$$H_\varepsilon(q, p, x, y) = \frac{p^2 - q^2}{2} + \frac{q^4}{4} + \frac{x^2 + y^2}{2} + \varepsilon q(p - y)$$

has a subharmonic of order  $m$  provided  $\varepsilon \neq 0$  is sufficiently small and  $h_0 > h_m$  where  $h_0 = H_0(q, p, x, y)$  is the total energy of the unperturbed Duffing-oscillator and  $h_m$  is the energy of the resonant periodic orbit of order  $m$  of Duffing's equation; furthermore, for  $\varepsilon \neq 0$  sufficiently small and  $h_0 > 0$ , this system is nonintegrable and there is a primary stochastic layer of width

$$d = \frac{2\pi\varepsilon\sqrt{h_0}\operatorname{sech}(\pi/2)}{\sqrt{p_0^2 + q_0^2(1 - q_0^2)^2}} + O(\varepsilon^2)$$

near a point  $(q_0, p_0) \neq (0, 0)$  on the homoclinic manifold of the unperturbed system as well as resonant stochastic layers whose bandwidths  $d_m = O(\sqrt{\varepsilon/m})$ .

**1. Introduction.** The occurrence of stochastic regions in two-degree-of-freedom Hamiltonian systems has been a topic of mathematical interest for many years, from both a theoretical and an applied point of view. In their 1964 paper on the nonintegrability of galactic motions [6], Henon and Heiles presented a numerical study of a Hamiltonian system with

$$H(q, p, x, y) = \frac{p^2 + q^2}{2} - \frac{q^3}{3} + \frac{y^2 + x^2}{2} + x^2q.$$

Their work clearly showed the appearance and evolution of stochastic regions with increasing energy levels. It also raised the question as to whether such behavior could be described analytically.

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