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Optimizing last trains timetable in the urban rail network: social welfare and synchronization

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Optimizing last trains timetable in the urban rail network: social welfare and synchronization

Last train timetable design is to coordinate last train services from different lines in an urban rail network for maximizing the number of transfers. It is a challenging operational research problem to balance the competing demand of two decision agents: that of the government agencies to provide the best social services with minimal government subsidy, and that of the train operating companies to minimize operating costs. A bi-level programming model is formulated for the last-train timetabling problem, in which the upper level is to maximize the social service efficiency, and the lower level is to minimize the revenue loss for the operating companies. To solve this problem, a genetic algorithm combined with an active-set approach is developed. We report the optimization results on real-world cases of the Beijing subway network. The results show that the optimized last-train timetable can significantly improve the transfer coordination.

Keywords: last train; timetable; urban rail network; bi-level programming, government subsidy

1 Introduction

In an urban rail network (URN), it is common for passengers to make one or more interchanges between different lines at transfer stations in order to complete their trips. During the day when there are continuous train services, passengers can usually reach their destinations successfully. For passengers travelling late at night, however, there is a real danger of missing the last train or missing the transfer to the last train, if the last train schedules and connections are not well designed. Last train timetable design in an URN, which is to coordinate last train schedules so as to maximize successful transfers for last trains, is a challenging operational research problem. The problem is becoming increasingly complex as URNs around the world are experiencing rapid expansions to ever greater scales. The problem becomes more prominent in the cases of opening a new line and with multiple companies operating in the same URN.

It is also a problem of competing interests between the system managers (usually the government) and the train operating companies, with the government providing subsidy to the operating companies to lay out the services. In many URNs, train lines are run by different operating companies who design their own service timetables. As the system-wide manager, the government has a role to ensure efficient and coordinated timetables across the whole network. The government wishes to offer the best social service (e.g. maximizing the number of successful transfers, minimizing transfer times and total travel times, etc.) for all passengers with minimal subsidy. To improve successful transfer probability, extending the operation time is one option but then the subsidy will also increase. On the other hand, the operating companies would want to complete their transport tasks with minimum cost, which also means they would plan the operation time as short as possible if without the government's regulation and subsidy. Therefore, determining the out-of-service time of urban subway lines has always been a balancing act between the two sides. To design an optimal last train timetable that achieves good social benefit desired by the government and low economic losses to the urban subway companies, requires the cooperation and constraints from both sides.

Mathematically, such cooperation and constraints for scheduling and optimizing last train timetable can be described as a bi-level decision problem (BLDP), in which the upper level is to maximize the social service efficiency (i.e. the difference between the benefit of successfully-transferred passengers and the subsidy), and the lower level is to minimize the difference between the operation cost and the subsidy. From the management viewpoint, this timetabling problem has an equilibrium status between the two decision agencies. Therefore, this problem can be transferred into a timetable design problem for the last train synchronization to satisfy the transfer requirements of passengers. In the previous studies, the last train timetable problem has been represented as to optimize the coordination quality between different lines. The equilibrium balancing the acts between the government and the operating companies on social welfare has been ignored but is addressed in this article. Therefore, the main contributions of this article are: (1) a bi-level programming model is formulated for the last-train timetabling problem considering the equilibrium status between the two decision agencies, in which the upper level is to maximize the social service efficiency (defined as the difference between the benefit of successfully-transferred passengers and the subsidy), and the lower level is to minimize the revenue loss for the operating companies (defined as the difference between the operation cost and the subsidy); (2) a genetic algorithm combined with an active-set approach is developed to solve the model. We demonstrate the method in a case study of Beijing subway system and the results show that, with a synchronized last train timetable, the number of transfer directions and passengers can be significantly increased.

The remainder of the article is organized as follows. In section 2, we review the literatures on the timetable optimization problem. In section 3, we formulate a bi-level programming model to determine the last train timetable. The genetic algorithm solution procedure combined with an active-set approach is presented in Section 4. In Section 5, some numerical examples are computed to test our model and algorithm. A case study based on the real-world data from the Beijing Rail Netwok is presented in section 6. Finally, conclusions are given in Section 7.

2 Literature review

Timetabling problems have traditionally been investigated using optimization methods to determine the departure and arrival times of trains at each station or block for a given set of lines and service frequencies in order to maximize the service quality of the subway network. The service quality is usually measured in terms of transfer waiting time and travelling time. Passenger transfer waiting time is defined as the minimum waiting time for passengers who transfer from a feeder train to the first coming connecting train, assuming that they can board the first train and that there is no capacity constraint. It has been found that passengers consider their waiting time to be twice of what it actually is (Mohring et al., 1987). There is a substantial body of literature on timetabling which minimize waiting time or transfer waiting time for passengers. Domschke (1989) proposed a model to minimize the waiting time (or cost) of passengers who changed lines at the transfer stations for a periodic timetable optimization problem. Cevallos and Zhao (2006) examined the network-wide bus transfers synchronization problem. They considered the objective of minimizing transfer waiting time in an existing timetable under constrains of strictly fixed headways between lines. Wong et al. (2008) developed a mixed integer programming optimization model which minimizes the total transfer waiting time of all passengers in a railway system. Shafahi and Khani (2010) formulated a mixed integer programming model to minimize the passenger transfer waiting time.

In large cities, service frequencies tend to be high and missing one connection train simply adds a few more waiting minutes to the overall journey (Carrese et al., 2002;

Chakroborty, 2003). However, in low population density areas with less frequent services, missing a connection incurs much longer waiting time and a lack of timetable synchronization may even discourage people from using public transportation mode (Yan et al., 2002). Ceder (2001) and Daduna et al. (1995) developed a mathematical formulation designed to generate timetables with maximum synchronization between trips. Fleurent et al. (2004) proposed the idea of weighing transfers and described concepts that were implemented in the commercial software HASTUS to generate synchronized transit timetables. Wong and Leung (2004) presented a method to synchronize timetable between lines which minimizes transfer waiting time. Wu et al. (2015) proposed a timetable synchronization optimization model to optimize passengers' waiting time while limiting the waiting time equitably over all transfer station in an urban subway network. Guo et al. (2016) proposed a model of timetable coordination of first trains in urban railway networks based on the importance of lines and transfer stations. Guo et al. (2017) focused on the timetable optimization problem in the transitional period (from peak to off-peak hours or vice versa).

There has also been wide interest in designing train timetables that minimizes a general cost, including for example operating cost, waiting time cost, delay cost, energy consumption and so on. Yan and Chen (2002), and Yan et al. (2006) minimized a combined operating cost and waiting cost for an intercity bus routing and scheduling problem. Vansteenwegen and Van Oudheusden (2006) proposed a linear programming model to minimize the generalized waiting cost using discrete event simulation. Gallo et al. (2011) considered a weighted sum of transit user costs, car user costs, operator costs and external costs as the objective function, where transit user costs depend on on-board time, waiting time and access/egress time. Goverde (1998), Chowdhury and Chien (2001), and Meng and Zhou (2011) considered the train delay as an important factor and formulated the timetable optimization model to minimize delay cost caused by the late departure of train at stations. Scheepmaker et al. (2017) gave an extensive literature review on energy-efficient train control and the related topic of energy-efficient train timetabling. Li and Lo (2014a, 2014b) proposed an integrated energy-efficient operation model to jointly optimize the timetable and speed profile with minimum net energy consumption. Yang et al. (2013, 2015, 2016, 2017 and 2018) proposed an optimization method for train scheduling with minimum energy consumption and travel time in metro rail systems. Yang et al. (2017, 2018) proposed a bi-objective nonlinear programming model with minimum energy consumption and passenger waiting time for metro systems. Lai and Leung (2017) considered to maximize the route frequencies and mileage to provide good passenger service and simultaneously minimize crew overtime and meal-break delays. Jiang et al. (2017) studied the problem of scheduling passenger trains in a highly congested railway double-track line with the aim of increasing the number of scheduled trains. Robenek et al. (2016) studied the Passenger Centric Train Timetabling Problem as a Mixed Integer Linear Programming (MILP) problem with an objective of maximizing the train operating company's profit while maintaining ε level of passenger satisfaction.

In practice, simply maximizing (or minimizing) one certain performance index is not sufficient. A good model should also produce robust timetables that are capable of absorbing, as much as possible, delays or disturbances in the network. Taking into account of stochastic passenger demand, Sun et al. (2011) studied the robust optimization for the transit timetable design problem using a stochastic demand robust model. Peterson (2012) proposed on-time performance as the key to evaluate a railway timetable's robustness to disturbances and evaluated the on-time performance for two single train services. Goverde (2007) described a stability theory to analyze a timetable's sensitivity and robustness to delays using a max-plus algebra. Fischetti et al. (2009) computationally analyzed four different methods to improve the robustness of a given train timetabling problem for the periodic timetable. Goerigk et al. (2011, 2014) proposed a conservative heuristic which identifies a large subset of these robust changing activities in polynomial time by dynamic programming and so allows managers to find strictly robust paths efficiently. Odijk and Romeijn (2006) proposed a heuristic sampling method to define a new probability distribution where the probability of each class depends on the robustness of the timetables.

Most of the existing literatures on timetable synchronization have their main objectives as to minimizing transfer wait time or cost, assuming all transfers can be successfully made and passenger numbers in the system are conserved. Such methods are not applicable to the last train timetabling problem, where it is possible that some passengers may fail to transfer and have to seek alternative modes of transport to reach their destinations. Therefore, passenger number conservation is no longer valid in the last train timetabling problem. The problem here not only affects passengers waiting time but also determines whether passengers can successfully transfer to the connecting trains. The last train transfer problem has only recently begun to draw attention in the literatures. Zhou et al. (2013) established a last train coordination model considering multi-point transfers for rail lines with which took the minimum cost of all the coordination relationship as the objective. Zhou et al. (2013) built a coordination optimization model of the last trains' departure time to reduce passengers' transfer waiting time for the last trains and inaccessible passenger volume of all origin-destinations. Kang and colleagues developed a last train scheduling model to maximize the transfer redundant headways in a subway network (Kang, et al., 2015a) and a rescheduling model for last trains with the consideration of train delays caused by incidents that occurred in train operations(Kang, et al., 2015b). Yang et al. (2017) formulated an optimization model for last-train timetabling based on mean-variance (MV) theory that explicitly considered two significant factors including the number of successful transfer passengers and the running time of last trains. Table 1 summarizes the different categories of existing literatures on timetable studies.

Table 1
Literatures for the timetable problem of URN.

Schedule type	Objective	Selected references
Non-last train	Minimize the waiting time and transfer waiting time	Domschke (1989); Cevallos and Zhao (2006); Wong et al. (2008); Shafahi and Khani (2010)
	Maximize synchronization	Ceder (2001); Daduna and Voss (1995); Wong and Leung (2004); Fleurent et al. (2004); Wu et al. (2015); Guo et al. (2016, 2017)
	Minimize energy consumption	Li and Lo (2014a, 2014b); Yang et al. (2013, 2015, 2016, 2017, 2018); Yang et al. (2017, 2018) ; Scheepmaker et al. (2017)
	Minimize the general cost	Yan and Chen (2002); Yan et al. (2006); Gallo et al. (2011); Goverde (1998); Vansteenwegen and Van Oudheusden (2006); Chowdhury and Chien(2001); Meng and Zhou(2011); Lai and Leung (2017); Jiang et al. (2017); Robenek et al. (2016).
	Robust optimization	Sun et al. (2011); Peterson (2012); Goverde (2007); Fischetti et al. (2009); Goerigk et al.(2011,2014); Odijk and Romeijn (2006)
Last train	Maximize transfer redundant headways	Kang et al. (2015a)
	Minimize the running time and the dwell time;	Kang et al. (2015b), Yang et al. (2017)

	Maximize the average transfer redundant time and the network accessibility	
	Minimize the coordination cost	Zhou et al. (2013), Zhou et al. (2013)

Thus far, in the previous studies, the last train timetable problem has been represented as to optimize the coordination quality between different lines. The equilibrium balancing the acts between the government and the operating companies on the social service efficiency, and between the subsidy and operating costs, has been ignored. This article addresses this specific equilibrium problem between the two decision agents, and develops a bi-level programming method to maximize the difference between the benefit of successful transfers and the subsidy, and to minimize the difference between operating costs and the subsidy. The model is useful to both agencies: it helps them to understand the complexity of the last trains timetabling, and to guide the government on how to increase the transfer coordination with a given subsidy and the operating companies on how to schedule last trains with a required coordination. The optimization method is applied to a real-world instance of the Beijing subway system. Computational results show that the transfer coordination is significantly improved and the number of missed transfers reduces.

3 Model Formulation

In this section, we firstly describe the URN with the graph theory. And then, the assumptions are given. Government subsidy, operating cost and the transfer passengers are formulated. Based on the abovementioned issues, the bi-level model of last train timetable optimization (LTTO) is finally proposed.

We represent an URN with $L=\{l \mid l=1,2,\cdots,n\}$ lines. For line l, we only consider the stations including the starting station, transfer stations and the ending station. Let $S(l)=\{s \mid s=1,2,\cdots,m_l\}$ be the set of transfer stations, numbered sequentially from the first ($s=1$) to the last transfer station ($s=m_l$) along line l. Especially, we define $s=0$ as the index of starting station and $s=e$ as the ending station, with an assumption of the starting station and the ending station are non-transfer stations along the line. Figure 1 illustrates schematically a graph representation of an URN.

Here, a line is directional, as such the different directions in the same service line are considered as two separate lines and are denoted as an up direction (U) and a down direction (D), e.g., 1U and 1D as shown in Figure 1. A line can be further divided into blocks. Let $B(l)=\{b \mid b=1,2,\cdots,r_l\}$ denotes the set of blocks along line l, numbered sequentially from the first block ($b=1$) just after the starting station on the line, to the last block (r_l) between the last transfer station and the ending station on line l.

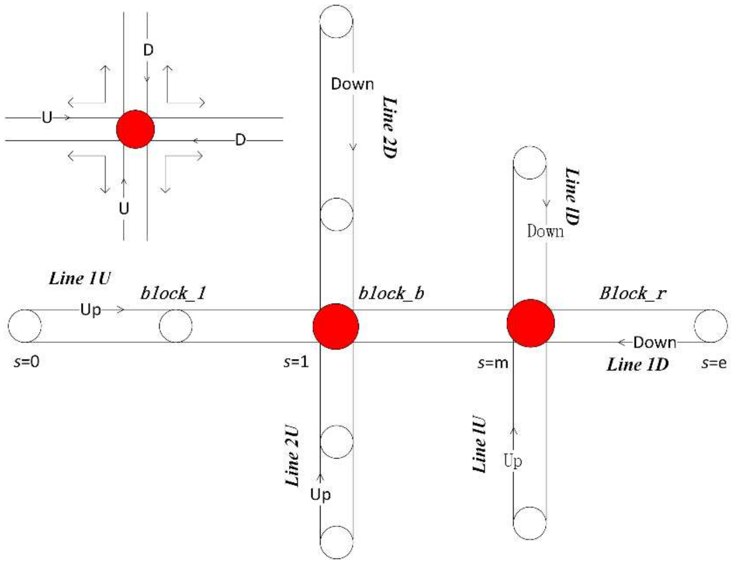

Fig. 1. A simple URN.

3.1 Notations

Notations used throughout this article are listed as follows and all boldface letters denote the corresponding vectors. All variables are assumed to be integer numbers to satisfy the engineering requirements.

(1) Parameters and sets

l: The urban rail line index, $l=1,2,\cdots,n$

n: The last the rail line

L: Set of the rail lines.

s: Station index, $s=1,2,\cdots,m$

m: The last station

m_l: The last stations of line l

$S(l)$: Set of transfer stations along the line l, $S(l)=\{s \mid s=1,2,\ldots,m_l\}$

b: Block index, $b=1,2,\ldots,r$

r: The last block

r_l: The last block of line l

$B(l)$: The set of blocks along line l

t_{lb}^{R}: The running time over each of the block b of line l

$p_{sll'}$: The probability density of passenger arrival at station s from line l wishing to transfer to line l'

$t_{l'}^{H}$: The headway of line l' between the last train and the penultimate train

$\psi_{sll'}$: the degree of priority given to transfers at station s from line l to l'

λ: A parameter to balance the weights between social welfare and subsidy

α: A social benefit parameter which converts the number of successfully transferred passengers to benefits

M: A large positive value

$t_{sll'}^{Tra}$: The passenger walking time between line l to line l' at station s

$t_{l,0}^{max}$, $t_{l,0}^{min}$: the upper and lower bound of departure time of last trains

T_{min}, T_{max}: The upper and lower bounds on the total trip time

t_{ls}^{max}, t_{ls}^{min}: The upper and lower bound of dwell time at the transfer station of last trains

(2) Decision variables

$t_{l,0}^{D}$: The departure time of the last train at the starting station on line l

t_{ls}^{Dw}: The train dwell time at each of the station s of line l

(3) Intermediate variables

$t_{l,e}^{A}$: The arrival time of the last train on line l at the destination station e.

TOC_l: The total operating cost of line l

$CAP_{sl'}$: Cumulative arrival passengers of the last train at station s of line l'

$x_{sll'}$: A binary variable that represents whether passengers can transfer from line l to line l' at station s successfully:

$$x_{sll'} = \begin{cases} 1, & \text{if passengers transfer from } l \text{ to } l' \text{ at } s \text{ successfully;} \\ 0, & \text{otherwise.} \end{cases}$$

3.2 Model assumptions

As a time sequence, a timetable defines, for each train, the departure time from the starting station, the arrival time at ending station, and the arrival and departure time at the intermediate stations (Caprara et al., 2002). For simplicity, some assumptions used in this article are listed.

a) Only the starting stations, transfer stations and ending stations in the URN are considered in the model. It means that the ordinary stations on a certain line are integrated and ignored between two transfer stations.

b) The subsidy is a continuous and differentiable function of time in the range of departure time at the original station of the first train and arrival time at the destination station of the last train. Therefore, the subsidy is related to the operating time including dwell time and running time.

c) Passengers will always ride the first connecting train to reduce the waiting time. Furthermore, as we are concerned with the last train problem, we assume here that the demand for the last trains is well below capacity and all passengers can get on board.

d) The transfer walking time is known and fixed for all passengers between two lines. In addition, the headways are given as a prior knowledge (e.g. from the existing timetables) and are assumed to be uniform in each line.

3.3 Objective measures

We consider government subsidy or the operating cost, and the social benefit as the objective measures in the bi-level programing model.

（1） Government subsidy and operating cost

As the passenger demand is relatively low for late night trains, most of the companies would prefer to stop their services earlier. However, as a social welfare, the government needs the companies to provide late train services and this is usually done through a subsidy policy. Nowadays, URNs in many cities depend on the government subsidy. In Beijing, for example, the government subsidy is based on the total number of passengers transported by the operating companies. However, this policy is not suitable for the last trains, or is not attractive enough to the operating companies to run last trains, because the demand is low. Here, we propose a new subsidy model based on the passengers' arrival times at their destination stations, with the following proposition.

Property 1. If the subsidy function is represented by $f_l(t_{l,e}^{A})$, where $t_{l,e}^{A}$ is the arrival time of the last train on line l at the destination station e, then:

a) $f_l(t_{l,e}^{A})$ is a non-negative monotone increasing function of dwell time and running time;

b) $f_l(t_{l,e}^{A})$ can be described by the departure time of the last train $t_{l,0}^{D}$ at the starting station on line l, the train dwell time t_{ls}^{Dw} at each of the station s of line l and the running time t_{lb}^{R} over each of the block b of line l.

Proof. (a) In practical operations, the government should give more subsidies for the company with a longer operational time. Therefore, the subsidy is always non-negative monotone increasing function and is related to the operating time (e.g., dwell time and running time) according to Assumption (a); (b) For the last train operation, the longer the line operates the more subsidies it gets. Since $\sum_{s\in S(l)} t_{ls}^{Dw}$ is the total dwell time on line l. Then, $t_{l,e}^{A} = t_{l,0}^{D} + \sum_{s\in S(l)} t_{ls}^{Dw} + \sum_{b\in B(l)} t_{lb}^{R}$ is satisfied, and we get $f_l(t_{l,e}^{A}) = f_l(t_{l,0}^{D} + \sum_{s\in S(l)} t_{ls}^{Dw} + \sum_{b\in B(l)} t_{lb}^{R})$. This completes the proof for Property 1.

$f_l(\cdot)$ is a user-defined non-decreasing function of the departure time from the start station, the dwell time at the transfer station and the running time in a block. Then,

$\sum_{l} f_l(\cdot)$ is the total subsidy for the last train. In fact, the total operation time is the sum of running time and dwell time. Assuming that the running time between two stations is a constant, the total operation time can then be represented by the total dwell time of last train at all stations. In addition, for the last trains, the operation cost is related to the departure time of line l. The later the departure time is, the higher the operation cost will be because of the longer usage of devices and staffs. Therefore, we assume that the total operating cost TOC_l of line l can be represented simply by

$$TOC_l = \varphi_l \cdot (t_{l,0}^{D} + \sum_{s \in S(l)} t_{ls}^{Dw} - t_l^{P}) \tag{1}$$

where φ_l is a parameter which transfers the time to the operating cost and t_l^P represents the latest departure time of line l given by the government.

（2） Social benefit and transfer passengers

Here, we define the social benefit as the number of passengers transferring to the last trains successfully. Let $p_{sll'}(t)$ be the probability density function of passengers arrival at station s of line l' transferring from line l. Therefore, the cumulative arrival passengers (CAP) of the last train at station s of line l' transferring from line l can be calculated as follows.

$$CAP_{sll'} = \int_{t_{l',s}^{D} - t_{l'}^{H}}^{t_{l',s}^{D}} p_{sll'}(t)dt \tag{2}$$

where $t_{l',s}^{D}$ is the departure time of the last train from s on line l', $t_{l'}^{H}$ is the headway of line l' between the last train and the penultimate train.

3.4 Bi-level model of last train timetable optimization (LTTO)

We formulate the last train timetable synchronization problem as a bi-level programming problem, in which the upper level is to maximize the social benefit with minimum total subsidy, and the lower level is to minimize cost loss for companies. The decision variable in the upper level is the departure time, $t_{l,0}^{D}$ for line l at the starting station 0, whilst, for the lower level, the total dwell time $\sum_{s \in S(l)} t_{ls}^{Dw}$ is the decision variable which affects the finishing time of the last train at its destination station.

（1） Upper level model of LTTO problem

The upper level objective in our model is to maximize the social benefit, with the following formulations:

$$\text{(U) Max } Z = \alpha \sum_{s\in S(l)} \sum_{l\in L} \sum_{l'\in L} \left(\int_{t^D_{l',s}-t^H_{l'}}^{t^D_{l',s}} p_{sll'}(t)dt \cdot \psi_{sll'} \cdot x_{sll'} \right) - \lambda \cdot \sum_{l} f_l \left(t^D_{l,0} + \sum_{s\in S(l)} t^{Dw}_{ls} + \sum_{b\in B(l)} t^R_{lb} \right) \quad (3)$$

$$\text{s.t. } t^A_{l,s} = t^D_{l,0} + \sum_{k=1}^{s-1} t^{Dw}_{lk} + \sum_{b=1}^{s-1} t^R_{lb} \quad (4)$$

$$t^D_{l',s} = t^D_{l',0} + \sum_{k=1}^{s} t^{Dw}_{l'k} + \sum_{b=1}^{s-1} t^R_{l'b} \quad (5)$$

$$M(x_{sll'} - 1) \le t^D_{l',s} - t^A_{l,s} - t^{Tra}_{sll'} < M \cdot x_{sll'} \quad (6)$$

$$t^{min}_{l,0} \le t^D_{l,0} \le t^{max}_{l,0}, \quad \forall l \in L \quad (7)$$

In Eq. (3) $\int_{t^D_{l',s}-t^H_{l'}}^{t^D_{l',s}} p_{sll'}(t)dt$ captures the number of transfer passengers at station s between lines l and l', where $t^H_{l'}$ is the headway of line l'. In practice, different transfer directions may have a different degree of priority or importance value to the government. Here, we denote parameter $\psi_{sll'}$ to represent the degree of priority given to transfers at station s from line l to l'. $x_{sll'}$ is a binary variable that represents whether passengers can transfer from line l to line l' at station s successfully. If $x_{sll'} = 1$, passengers transfer successfully. Otherwise, they fail to transfer. Parameter λ is to balance the weights between social welfare and subsidy. Parameter α is a social benefit parameter which converts the number of successfully transferred passengers to benefits.

Formulations (4)-(7) are train operational constraints. (4) tracks the arrival time of the last train in feeder line l at station s. The total train running time from the original station to station s is $\sum_{b=1}^{s-1} t_{lb}^{R}$, where t_{lb}^{R} represent the running time over the block b (between stations s-1 and s) of line l. $\sum_{k=1}^{s-1} t_{lk}^{Dw}$ calculates the total dwell time of the last train on line l over all stations prior to station s. Constraint (5) indicates the departure time of the last train in connecting line l' at station s. In (6), the M is a large positive value and $t_{sll'}^{Tra}$ is the passenger walking time between line l to line l' at station s. It is clear that $0 \le t_{l',s}^{D} - t_{l,s}^{A} - t_{sll'}^{Tra} < M$ if $x_{sll'}$ equals to 1. Otherwise, we obtain $-M \le t_{l',s}^{D} - t_{l,s}^{A} - t_{sll'}^{Tra} < 0$ which means passengers will fail to transfer. Finally, constraint (7) sets the upper bound $t_{l,0}^{max}$ and the lower bound $t_{l,0}^{min}$ of departure time of last trains.

(2) Lower level model of LTTO problem

From the companies' viewpoints, if the profit of running a train service is negative, it is best to cancel the service. With government subsidy to offer a last train service, the companies would like to schedule the last train with minimal loss after subsidy. We formulate this as the lower level model, with the objective function as follows:

$$(L)\,\mathrm{Min}\,C = \sum_{l} TOC_l - \sum_{l} f_l\big(t_{l,0}^{D} + \sum_{s\in S(l)} t_{ls}^{Dw} + \sum_{b\in B(l)} t_{lb}^{R}\big) \tag{8}$$

$$\text{s.t. } T_{min} \le t_{l,0}^{D} + \sum_{s\in S(l)} t_{ls}^{Dw} + \sum_{b\in B(l)} t_{lb}^{R} \le T_{max},\ \forall l \in L \tag{9}$$

$$t_{ls}^{min} \le t_{ls}^{Dw} \le t_{ls}^{max},\ \forall l \in L,\ \forall s \in S(l) \tag{10}$$

Constraint (9) sets the upper and lower bounds on the total trip time (e.g. T_{min} and T_{max}), which ensures the last train running within a trip horizon. The decision variable t_{ls}^{Dw} of the lower level model is restricted with the constraint (10), where t_{ls}^{max} is the upper bound and t_{ls}^{min} is the lower bound of dwell time at the transfer station of last trains; in between these two time boundaries will the passengers get on or off the train.

Theorem 1. The lower level in LTTO has a global optimization solution because the feasible domain is non-empty of lower level model.

Proof. For the decision variables in upper level $\left\{t_{l,0}^{D}, \forall l \in L\right\}$, $t_{l,0}^{min} \leq t_{l,0}^{D} \leq t_{l,0}^{max}$, given T_{min}, T_{max}, t_{ls}^{min} and t_{ls}^{max}, the feasible domain of the lower level is non-empty according to the constraints (9) and (10). Then, the lower level has a minimum solution. The following equation: $T_{min} - T_{max} - t_{sll'}^{Tra} \leq t_{l',s}^{D} - t_{l,s}^{A} - t_{sll'}^{Tra} \leq T_{max} - T_{min} - t_{sll'}^{Tra}$, $\forall l \in L,\ \forall s \in S(l)$ holds if $M \geq \max_{s \in S(l), s \in S(l'), l \neq l'}\{|T_{min} - T_{max} - t_{sll'}^{Tra}|, |T_{max} - T_{min} - t_{sll'}^{Tra}|\}$. Since M is an arbitrary large number and satisfies $M(x_{sll'} - 1) \leq t_{l',s}^{D} - t_{l,s}^{A} - t_{sll'}^{Tra} < M \cdot x_{sll'}$, the feasible domain of LTTO will be non-empty. Therefore, the model has a global optimization solution. This completes the proof for Theorem 1.

3.5 Performance measures of the optimized timetable for last train

In addition to the transfer waiting time of passengers, an important measure for the last train timetable is the “absolute miss”. Wong et al. (2008) introduced a metric of “just miss” to define the type of missed transfers caused by the unreasonable timetable in which the sum of the cross-platform time and the transfer waiting time exceeds the headway of the connecting train. In general, passengers do not like just missing the connecting train by a few seconds. However, for the last train, “just miss” the connecting train means “absolute miss” because it is the last chance to finish the trip. Therefore, an importance purpose of this article is to minimize the number of absolute misses.

4 Solution Algorithm

The bi-level programming is a NP-hard problem (Ben-Ayed ET AL., 1988), thus it is difficult to solve with many of the standard optimization algorithms which require calculation of the gradients of the objective functions. The non-convexity of the problem is another reason that results in the complexity of the solution algorithm:

even if the upper and lower level problems are both convex, it is possible that the whole bi-level problem will be a non-convex one. The nature of non-convexity indicates that, even if a solution for the bi-level problem can be found, it is usually a local optimum not the global optimum.

Generally, two classes of solution algorithms are applied to solve a bi-level model. One is the standard optimization algorithm, such as the branch-and-bound method, Lagrange relaxation, dual ascent procedures and support function method (Dimitriou et al., 2007). However, the many assumptions made in these algorithms preclude their application in large and realistic networks. The second class of solution algorithms is the intelligent algorithms, e.g., genetic algorithm (GA) (Holland et al., 1975; Yang et al., 2013, 2016, 2017), particle swarm optimization algorithm and chaotic optimization algorithm. For more introduction of the algorithm, see Wu et al. (2009). These methods make fewer or no assumptions and do not use the gradient of the objective function. It also doesn't need any special conditions of functions and is efficient for the large scale network. In this article, we adopt GA to solve the last train timetable problem.

4.1 Genetic algorithm for last train timetabling

The decision variables in the proposed upper-level model are the departure times of the last trains at their origin stations. Therefore, they are chosen as genes for any chromosome in the GA. A vector $(t^D_{1,0}, t^D_{2,0},...,t^D_{1,0}, ..., t^D_{n,0})$ forms the genes of a chromosome in the algorithm, and population of N chromosomes are generated. In this article, the first chromosome is initialized randomly in the feasible domain specified in (7). The total number of chromosomes $N*n$ consists of the population as illustrated in Figure 2.

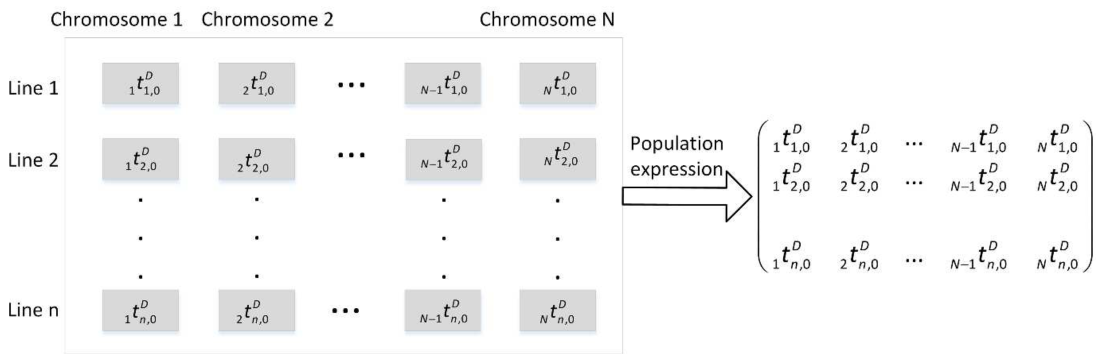

Fig. 2. Encoding and population diagram

The GA is performed based on three important operators: crossover, mutation and selection operators. The crossover operator is to generate new solutions with a given probability of ξ. We adopt a replacing method in the crossover operation in which a gene is replaced by the same gene in another chromosome (Kang et al., 2015), as shown in Figure 3.

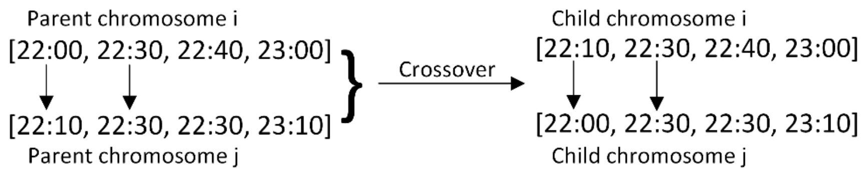

Fig.3. Crossover operation.

The mutation operation helps to escape local optimums because the crossover operation used in two parent chromosomes will result in no conflicts to the bounds of the parent genes with a probability of ζ (see Figure 4). With the mutation operation, the genes of children chromosomes can escape from the limitations of parent chromosomes. The variables in a vector are not binary. Then, for the genes $t_{1,0}^{D}$, random numbers are generated to replace them.

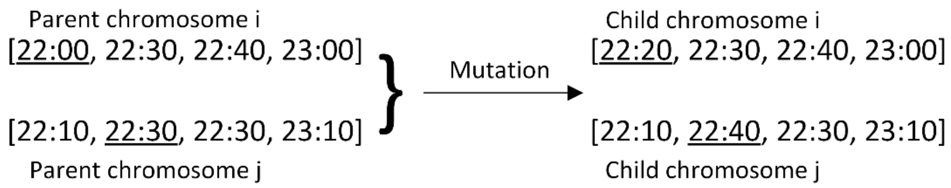

Fig. 4. Mutation operation.

The selection operator is to ensure the best individuals can survive in the next generation. And it is designed based on the fitness function, defined as $f(n) = Z$, where the Z is defined by formula (3). The more the fitness is, the more likely the best individual can survive. We adopt the Roulette method to determine whether the each individual can survive in the next generation or not.

For convergence, three different criteria have been used in the literature (Shafahi et al., 2010). These are (1) the best solution does not change after a given number of iterations; (2) iterations reach the maximum number; and (3) the algorithm running exceeds the permitted time. In this article, we adopt the second criterion as the convergence criterion.

4.2 Active-set algorithm

In our model, according to the solution of the upper level, the lower level model in this article is a minimization problem. Active-set algorithm is a two-phase iterative method that provides an estimate of the active set at the solution and reduces the complexity of the search. Recently, Zhang et al. (2009) and He et al. (2013) showed that the active-set algorithm was very efficient and consistently produced good results with different initial solutions demonstrating the potential of applying the active-set algorithm to large-scale networks. The active set algorithm has been developed in Matlab Optimization Tool Package. In this article, we implement this algorithm within the Matlab R2012.

4.3 Solution algorithm of LTTO

The framework developed to solve the LTTO problem using GA is described as follows:

Step 0. Initialization. Set the initialized values of parameters λ, $\psi_{sll'}$, ξ, ζ, M, T_{min}, T_{max}, t_{ls}^{min}, t_{ls}^{max}, $t_{l,0}^{min}$, $t_{l,0}^{max}$, the maximum generations G, population size N and block running time t_{lb}^{R}. Select the feasible departure time $t_{l,0}^{D}$. Set the iteration counter $g = 0$.

Step 1. Solving the lower level problem. For a fixed $t_{l,0}^{D}(g)$, solve the lower level problem with active-set algorithm offered by the Optimization Tool in Matlab, and obtain the optimized dwell time $t_{ls}^{Dw}(g)$.

Step 2. Solving the upper level problem. According to the solution $t_{ls}^{Dw}(g)$ from the lower level, implement GA to get the optimized departure time $t_{l,0}^{D}(g)$ of the upper level.

Step 2.1. Adopt the mutation operation of the optimized solution and create a new chromosome.

Step 2.2. Check the feasibility of the new chromosome. If infeasible, delete and return to step 2.1.

Step 2.3. Repeat the above two steps until N chromosomes are constructed.

Step 2.4. Adopt the crossover operator to the N chromosomes.

Step 2.5. Calculate the objective values of total chromosomes and adopt the selection operator.

Step 3. Verify termination criterion. If the iteration g exceeds the maximum generation G, take $t_{l,0}^{D}(g)$ and $t_{ls}^{Dw}(g)$ as an optimal solution to the LTTO; otherwise, increment $g := g+1$, and go to step 1.

5 Numerical example

In this section, we present the application of LTTO model and the solution algorithm to an example URN. Figure 5 illustrates the example network, which contains 6 directional lines and 5 transfer stations. Table 2 presents the initial timetable in terms of the last train's arrival and departure times at each of the transfer stations. The times have been converted into absolute values starting from zero. From which, the running time in each block between two stations can be calculated. For example, for Line 1, the last train departs from S2 at time 10.5 and arrives at S3 at time 30.5, then the result of running time between S2 and S3 is 20 minutes. In this experiment, we assume that the dwell time is given in the range of [0.5, 3] minute. For simplicity, passengers' transfer walking time and train headways are set as 3 minutes and 5 minutes, respectively. If not specifically defined, the subsidy function is set as exponential function $f_l(t) = \exp(t)$ and $t = t_{l,0}^{D} + \sum_{s \in S(l)} t_{ls}^{Dw} + \sum_{b \in B(l)} t_{lb}^{R}$, the social benefit parameter α is set as 1, and the parameter φ_l is also set as 1 for simplicity in rest of the paper.

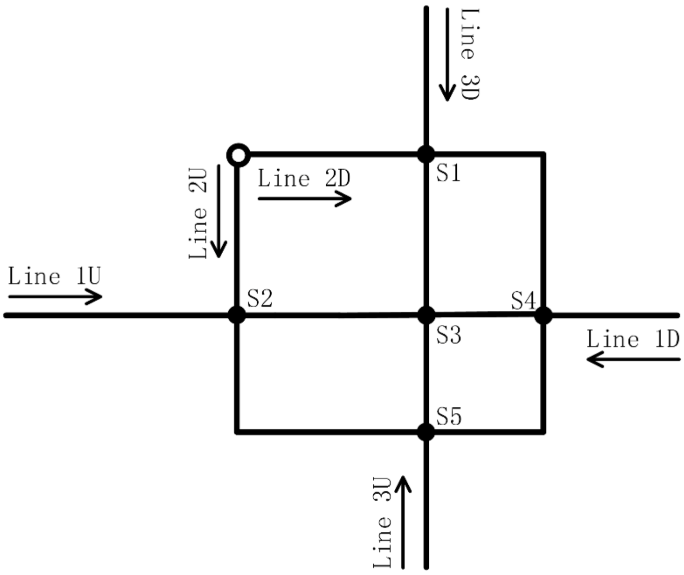

Fig. 5. Illustration of the sample URN

Table 2
The initial arrival and departure time (min) of the last train at stations

Line		1		2		3	
Direction		Up	Down	Up	Down	Up	Down
S1	Arr.	--	--	51.5	10	41	10
	Dep.	--	--	52	10.5	41.5	10.5
S2	Arr.	10	41	10	51.5	--	--
	Dep.	10.5	41.5	10.5	52	--	--
S3	Arr.	30.5	20.5	--	--	20.5	30.5
	Dep.	31	21	--	--	21	31
S4	Arr.	41	10	41	20.5	--	--
	Dep.	41.5	10.5	41.5	21	--	--
S5	Arr.	--	--	30.5	31	10	41
	Dep.	--	--	31	31.5	10.5	41.5

5.1 Transfer feasibility analysis

Here, we analyze the transfer feasibility with the initial last-train timetable. To illustrate the analysis, we take transfer station S3 as an example. At station $\mathrm{S3}$, there are four crossing lines and eight transfer directions, i.e., $\text{Line 1U} \leftrightarrow \text{Line 3U}$, $\text{Line 1U} \leftrightarrow \text{Line 3D}$, $\text{Line 1D} \leftrightarrow \text{Line 3U}$, and $\text{Line 1D} \leftrightarrow \text{Line 3D}$. To accomplish the transfer, the departure time of connecting line should be after the arrival plus transfer time of the feeding line at station $\mathrm{S3}$ as shown in Eq. (11). As can be seen, eight directions have the same expressions with a feeding line l and a connecting line l'. Thus, we pick directions $l \leftrightarrow l'$ as the general illustration and have Lemma 1.

$$\begin{cases} t^{D}_{3U,s3} - t^{A}_{1U,s3} - t^{Tra}_{s3\text{-}1U\text{-}3U} \geq 0 \\ t^{D}_{1U,s3} - t^{A}_{3U,s3} - t^{Tra}_{s3\text{-}3U\text{-}1U} \geq 0 \\ t^{D}_{3D,s3} - t^{A}_{1U,s3} - t^{Tra}_{s3\text{-}1U\text{-}3D} \geq 0 \\ t^{D}_{1U,s3} - t^{A}_{3D,s3} - t^{Tra}_{s3\text{-}3D\text{-}1U} \geq 0 \\ t^{D}_{3U,s3} - t^{A}_{1D,s3} - t^{Tra}_{s3\text{-}1D\text{-}3U} \geq 0 \\ t^{D}_{1D,s3} - t^{A}_{3U,s3} - t^{Tra}_{s3\text{-}3U\text{-}1D} \geq 0 \\ t^{D}_{3D,s3} - t^{A}_{1D,s3} - t^{Tra}_{s3\text{-}1D\text{-}3D} \geq 0 \\ t^{D}_{1D,s3} - t^{A}_{3D,s3} - t^{Tra}_{s3\text{-}3D\text{-}1D} \geq 0 \end{cases} \Leftrightarrow \begin{cases} t^{A}_{3U,s3} + t^{Dw}_{3U\text{-}s3} \geq t^{A}_{1U,s3} + t^{Tra}_{s3\text{-}1U\text{-}3U} \\ t^{A}_{1U,s3} + t^{Dw}_{1U\text{-}s3} \geq t^{A}_{3U,s3} + t^{Tra}_{s3\text{-}3U\text{-}1U} \\ t^{A}_{3D,s3} + t^{Dw}_{3D\text{-}s3} \geq t^{A}_{1U,s3} + t^{Tra}_{s3\text{-}1U\text{-}3D} \\ t^{A}_{1U,s3} + t^{Dw}_{3D\text{-}s3} \geq t^{A}_{3D,s3} + t^{Tra}_{s3\text{-}3D\text{-}1U} \\ t^{A}_{3U,s3} + t^{Dw}_{3U\text{-}s3} \geq t^{A}_{1D,s3} + t^{Tra}_{s3\text{-}1D\text{-}3U} \\ t^{A}_{1D,s3} + t^{Dw}_{1D\text{-}s3} \geq t^{A}_{3U,s3} + t^{Tra}_{s3\text{-}3U\text{-}1D} \\ t^{A}_{3D,s3} + t^{Dw}_{3D\text{-}s3} \geq t^{A}_{1D,s3} + t^{Tra}_{s3\text{-}1D\text{-}3D} \\ t^{A}_{1D,s3} + t^{Dw}_{1D\text{-}s3} \geq t^{A}_{3D,s3} + t^{Tra}_{s3\text{-}3D\text{-}1D} \end{cases} \Leftrightarrow \begin{cases} t^{Dw}_{3U\text{-}s3} \geq t^{A}_{1U,s3} - t^{A}_{3U,s3} + t^{Tra}_{s3\text{-}1U\text{-}3U} \\ t^{Dw}_{1U\text{-}s3} \geq t^{A}_{3U,s3} - t^{A}_{1U,s3} + t^{Tra}_{s3\text{-}3U\text{-}1U} \\ t^{Dw}_{3D\text{-}s3} \geq t^{A}_{1U,s3} - t^{A}_{3D,s3} + t^{Tra}_{s3\text{-}1U\text{-}3D} \\ t^{Dw}_{3D\text{-}s3} \geq t^{A}_{3D,s3} - t^{A}_{1U,s3} + t^{Tra}_{s3\text{-}3D\text{-}1U} \\ t^{Dw}_{3U\text{-}s3} \geq t^{A}_{1D,s3} - t^{A}_{3U,s3} + t^{Tra}_{s3\text{-}1D\text{-}3U} \\ t^{Dw}_{1D\text{-}s3} \geq t^{A}_{3U,s3} - t^{A}_{1D,s3} + t^{Tra}_{s3\text{-}3U\text{-}1D} \\ t^{Dw}_{3D\text{-}s3} \geq t^{A}_{1D,s3} - t^{A}_{3D,s3} + t^{Tra}_{s3\text{-}1D\text{-}3D} \\ t^{Dw}_{1D\text{-}s3} \geq t^{A}_{3D,s3} - t^{A}_{1D,s3} + t^{Tra}_{s3\text{-}3D\text{-}1D} \end{cases} \quad (11)$$

Lemma 1. Half of the eight transfer directions cannot be coordinated by last trains. Only the passengers who ride feeder trains earlier than the connecting trains can transfer successfully.

Proof. For all lines $l \in L$, $l' \in L$, stations $s \in S(l) \cap S(l')$ and $t^{Tra}_{sll'} = t^{Tra}_{sl'l}$ in the network, passengers transfer from line l to line l' and from line l' to line l. Firstly, we assume passengers of both directions can transfer successfully. Therefore, equation (12) should be satisfied.

$$\begin{cases} t^{A}_{l',s} + t^{Dw}_{l's} \geq t^{A}_{l,s} + t^{Tra}_{sll'} \\ t^{A}_{l,s} + t^{Dw}_{ls} \geq t^{A}_{l',s} + t^{Tra}_{sl'l} \end{cases} \Leftrightarrow \begin{cases} t^{Dw}_{l's} \geq t^{A}_{l,s} - t^{A}_{l',s} + t^{Tra}_{sll'} \\ t^{Dw}_{ls} \geq t^{A}_{l',s} - t^{A}_{l,s} + t^{Tra}_{sl'l} \end{cases} \quad (12)$$

It should be noted that t^{Dw}_{ls} is generally less than $t^{Tra}_{sll'}$ in the real operations (e.g., in the Beijing subway, $t^{Dw}_{ls} \in [0.5,1]$ min, $t^{Tra}_{sll'} \in [2,7]$ min).

Case 1: $t^{A}_{l,s} = t^{A}_{l',s}$

Result 1: $t^{Dw}_{l's} \geq t^{Tra}_{sll'}$, $t^{Dw}_{ls} \geq t^{Tra}_{sl'l}$: passengers fail to transfer of both directions.

Case 2: $t^{A}_{l,s} > t^{A}_{l',s}$

Result 2: $t^{Dw}_{l's} > t^{Tra}_{sll'}$, $t^{Dw}_{ls} < t^{Tra}_{sl'l}$: passengers fail to transfer of direction $l \to l'$.

Case 3: $t^{A}_{l,s} < t^{A}_{l',s}$

Result 3: $t^{Dw}_{l's} < t^{Tra}_{sll'}$, $t^{Dw}_{ls} > t^{Tra}_{sl'l}$: passengers fail to transfer of direction $l' \to l$.

Therefore, according to Eq. (12), if two trains do not arrive at the same time, only one side of passengers can transfer successfully. This completes the proof for Lemma 1.

5.2 Sensitive analysis of subsidy parameter λ

Here experiments with different values of the subsidy parameter λ are conducted. Firstly, we consider λ=1. The results are shown in Table 3. As can be seen, 105 passengers can finish their transfers successfully. Then, the effects of different λ values on the passengers' transfers are studied. As the value of λ, the government subsidy increases continuously, but the total number of successful transfers do not increase continuously. Up to the value λ=10, the number of successful transfers remain the same as with no subsidy case. Only when $\lambda = 25$, is the maximum transfer number 110 obtained. Further increasing the value of λ will not increase the number of successful transfers in this test network. This suggests that there is an optimal subsidy to transfer the largest number of passengers. The results also show that with the sufficiently high subsidy, the maximum number of successful transfers can be met.

Table 3
Analysis of subsidy parameter.

λ	Departure time of each line	Dwell time at each station	Transfer passengers	Subsidy
1	8.09, 5.29, 1.81, 3.90, 5.14, 2.57	.5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5	105	12.74
5	9.25, 7.13, 6.17, 6.65, 8.19, 3.88	.5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5	105	14.87
10	9.87, 2.58, 4.42, 8.27, 9.82, 1.36	.5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5, .5	105	14.88
25	8.84, 1.18, 7.45, 7.11, 9.70, 7.20	3, 3, 3, 3, 3, 3, 2.9, 2.9, 2.9, 2.9, 2.9, 2.9, 2.9, 2.9, 3, 3, 3, 3, 3, 3	110	17.12
45	9.47, 7.51, 4.03, 9.30, 6.02, 9.68	3, 3, 3, 3, 3, 3, 2.7, 2.7, 2.7, 2.7, 2.5, 2.5, 2.5, 2.5, 3, 3, 3, 3, 3, 3	110	17.12
55	9.03, 5.10, 7.98, 6.73, 6.16, 9.98	3, 3, 3, 3, 3, 3, 2.6, 2.6, 2.6, 2.6, 2.8, 2.8, 2.8, 2.8, 3, 3, 3, 3, 3, 3	110	17.12
65	8.26, 9.42, 6.85, 9.76, 4.70, 8.79	3, 3, 3, 3, 3, 3, 2.8, 2.8, 2.8, 2.8, 2.8, 2.8, 2.8, 2.8, 3, 3, 3, 3, 3, 3	110	17.12
…	…	…	…	…

5.3 Optimization results

Following from the above, we get an optimal last train timetable as shown in Table 4, at $\lambda = 25$. This timetable is compared with the original Table 2 on two aspects: the number of transferred passengers and the transfer waiting time. On one hand, it is clear that the lager the number of successfully transferred passengers is, the better the service is. The binary variable $x_{sll'}$ in this aspect is an important indicator which shows the line-to-line accessibility. A passenger can reach his/her scheduled destination as long as he/she transfers smoothly at each node. On the other hand, passenger transfer waiting time indicates the efficiency of a timetable. Mohring et al. (1987) found that passengers perceived their waiting time to be almost twice as long as what it actually is. This illusion is intensified when passengers wait for the last trains. Thus, a well-designed timetable, with good coordination between feeder trains and connecting trains so that passengers can enjoy fast transfers, is much desired by passengers.

Table 4
The optimized last train timetable

Line		1		2		3	
Direction		Up	Down	Up	Down	Up	Down
S1	Arr.	--	--	67.01	18.36	56.00	20.00
	Dep.	--	--	69.91	21.26	59.00	23.00
S2	Arr.	20.00	56.00	18.31	67.06	--	--
	Dep.	23.00	59.00	21.21	69.96	--	--
S3	Arr.	43.00	33.00	--	--	43.00	43.00
	Dep.	46.00	36.00	--	--	46.00	46.00
S4	Arr.	56.00	20.00	54.11	31.26	--	--
	Dep.	59.00	23.00	57.01	34.16	--	--
S5	Arr.	--	--	41.21	44.16	20.00	56.00
	Dep.	--	--	44.11	47.06	23.00	59.00

Table 5 presents the comparison results. As can be seen, the original timetable has 5 coordination directions and 65 passengers finishing transfer for last trains. The new timetable, however, has improved the number of coordination directions to 8, and with 110 successful passenger transfers. In order to show the advantages of our model, the passengers transfer waiting time of the two timetables are computed. Generally, passengers will get on the coming connecting train if they arrive early enough. Otherwise, they may fail the transfer. Therefore, two cases are analyzed and shown in Figure 6: (1) the last feeder train arrives earlier than the last connecting train, and (2) the last feeder train arrives later than the last connecting train. To capture the exact value of the waiting time $t_{sll'}^{w}$, Eqs. (13) and (14) are defined as follows.

Table 5
Comparisons of coordination for the original and optimized timetables

Station	Direction	Transfers	Original timetable		Optimized timetable	
			$x_{sll'}$	Waiting (min)	$x_{sll'}$	Waiting (min)
S1	L2 Down to L3 Up	5	1	3.5	1	2.64
	L2 Up to L3 Up	10	0	--	0	--
S2	L2 Up to L1 Down	15	1	3.5	1	2.69
	L2 Down to L1 Down	10	0	--	0	--
S3	L1 Up to L3 Up	20	0	--	0	--
	L1 Up to L3 Down	5	0	--	1	0
	L3 Down to L1 Up	25	0	--	1	1.0
S4	L2 Down to L1 Up	10	1	3	1	4.73
	L2 Up to L1 Up	15	0	--	1	1.85
S5	L2 Up to L3 Down	15	1	3	1	4.77
	L2 Down to L3 Down	20	1	4.5	1	3.46
	Total		5/65	5/17.5	8/110	8/21.14

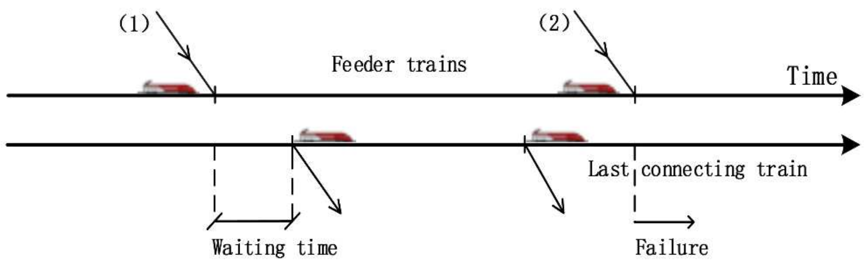

Fig. 6. Passenger transfer waiting time.

$$t_{sll'}^{w}=\begin{cases} t_{l',s}^{D}-t_{l,s}^{A}-t_{sll'}^{Tra}-h\cdot t_{l'}^{H},\ x_{sll'}\geq 0\ ; \\ M,\ x_{sll'}<0. \end{cases} \tag{13}$$

$$h=\begin{cases} \left[{(t_{l',s}^{D}-t_{l,s}^{A}-t_{sll'}^{Tra})}\big/{t_{l'}^{H}} \right],\ x_{sll'}\geq 0\ ; \\ 0,\ x_{sll'}<0. \end{cases} \tag{14}$$

Here, M is an arbitrary large number and $[\ ]$ indicates the integer portion of the argument. Thus, h captures the number of trains running on the connecting line l' before the advent of the last train.

For the original timetable, the total transfer waiting time for 5 coordination directions and 65 passengers is about 235 minutes. Therefore, the average transfer waiting time is about 3.61 minutes. However, in the new improved timetable, the average transfer waiting time for 8 coordination directions and 110 passengers is about 294.5 minutes. The average transfer waiting time is reduced to 2.67 min which has improved by 24.6%. The detailed comparisons can be found in Table 5 and Figure 7 (for simplicity, just one in ten is shown in Figure 7).

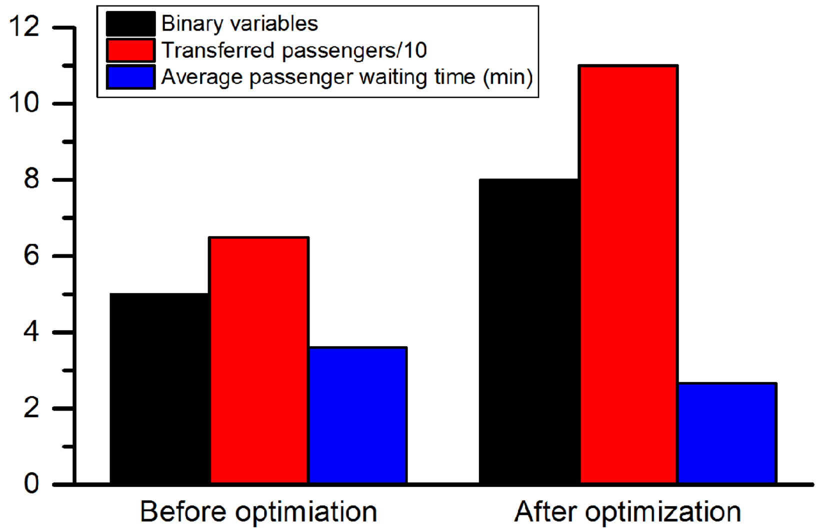

Fig. 7. Comparisons of timetable indicators.

5.4 Analysis of initial solutions and convergence test

The model is implemented in Matlab 2012, using a 2×2.5 MHz CPU and 2GB of RAM. To analyze the effects of initial solutions on the timetable, we give different initial solutions and the optimized results in Table 6. It is clear that, for all cases, the number of transferred passengers is the same. Therefore, it proves that our algorithm is efficient. Besides, the convergence can be seen in Figure 8.

Table 6

Iterative results of different initial solutions

Initial solutions	CPU time for First best solution (sec)	Optimal departure time	Transferred passengers	CPU time (sec)
100000	25.63	9.9931,9.9944,8.2713,8.7202,9.9996,9.9981	110	3060
010000	22.53	9.9989,9.9973,8.2338,9.5366,9.9896,9.9999	110	4062
001000	23.00	9.9967,9.9992,8.7528,9.8988,9.9995,9.9980	110	3026
000100	22.39	9.9893,9.9919,9.5958,8.6230,9.9981,9.9978	110	2700
000010	23.13	3.3728,9.9692,7.2960,7.7032,9.9260,1.8331	110	7369
000001	25.24	9.9962,9.9958,9.1048,9.5434,9.9976,9.9979	110	2870

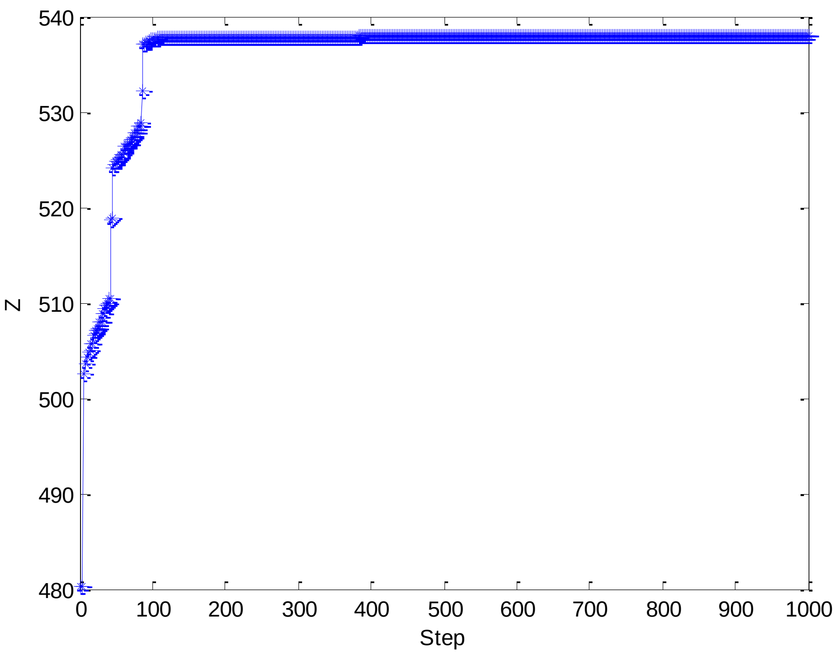

Fig. 8. Illustration of the convergent.

5.5 Comparisons of subsidy functions

In practical operations, the government adopts the subsidy policy to encourage the companies to extend the service time. We examine now some particular subsidy functions in this section, e.g., exponential function, linear function and quadratic function. Let us first assume that, during the last train operation, one additional minute's operation often results in a rapidly increasing subsidy, leading to a significant exponential correlation between extended operation time and the subsidy. Besides, we can also analyze the effects of different subsidy functions on the optimal results. To represent the relationships between the subsidy and the operation time, we propose the following government subsidy function,

$$f_l(t)=\begin{cases}\exp(\theta t)\\ \theta t\\ \theta t^2\end{cases},t=t_{l,0}^{D}+\sum_{s\in S(l)}t_{ls}^{Dw}+\sum_{b\in B(l)}t_{lb}^{R} \tag{15}$$

where θ is the parameter related to the relationship between time and cost. As illustrated in Table 7, the exponential type has the most transferred passengers, the most connected directions, and the least financial subsidy. On the contrary, the linear type obtains the fewest transferred passengers, the fewest connected directions, and the most financial subsidy. At the same time, the logarithmic type becomes the most "expensive" one, which costs 12.479 unit of subsidy and returns 90 transferred passengers. As can be seen, the quadratic type costs 0.139 unit of subsidy to ensure one passenger and 2.08 unit of subsidy to connect one transfer direction. Compared with that, the exponential type is the "cheapest", which only costs 0.038 unit of subsidy and 0.566 unit of subsidy, respectively. In the following Case Study section, we choose the exponential function for the detailed analysis.

Table 7

Effects of subsidy functions on optimal results

Function types	Passengers	Directions	Subsidy	Subsidy/ passengers	Subsidy/ directions
Exponential function	105	7	3.9638	0.038	0.566
Linear function	65	5	4.325	0.067	0.865
Quadratic function	90	6	12.479	0.139	2.08

6 Case study

The following case study considers the Beijing subway network including 7 two-directional lines and 17 transfer stations, as shown in Figure 9. The headway of all lines is set to be 10 minutes.

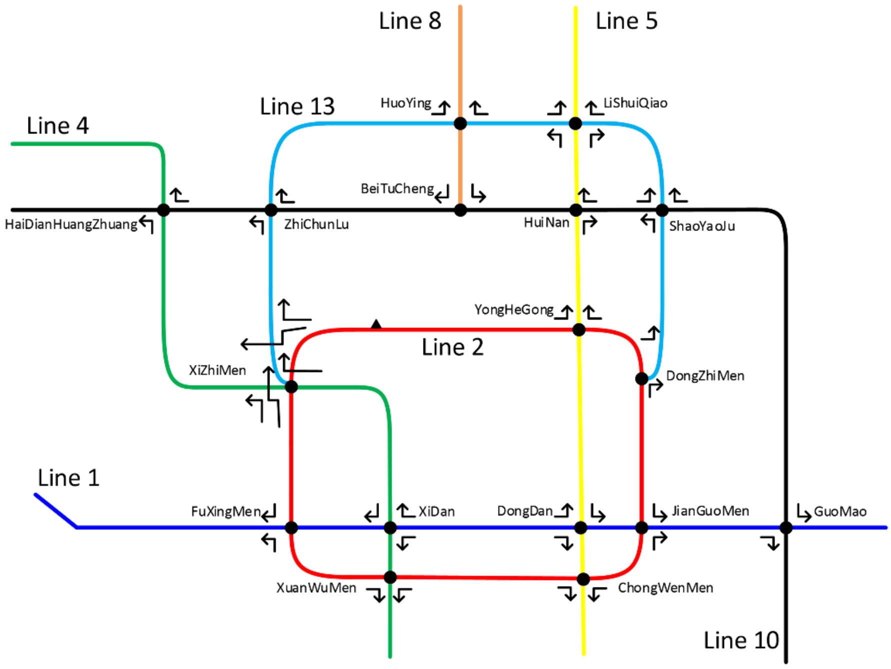

Fig. 9. Map of the Beijing subway 2012 (Kang et al., 2015).

6.1 Transfer directions

According to Lemma 1, only one direction can be coordinated in the Beijing subway. In such a case, it is especially important for passengers who transfer from the urban areas to the suburban habitations. A very important reason is that suburban transfer is more urgent than urban transfer in last trains, since passengers may have other alternatives in the urban area, e.g. bus, taxi, etc. But for suburban passengers, it will cost them more time and fare if they fail to transfer. Therefore, in the real operation of Beijing subway, key transfer directions are identified and shown by the arrows in Figure 9.

6.2 Optimization results

We analyze the possible transfer directions for the last trains using the original timetable. As shown in Table 8, we can see that nearly half of main concerned directions fail to coordinate under the original timetable. An optimized, new, timetable is derived based on our proposed model and is presented in Table 8. It can be seen from Table 8 that with the current timetable, there are 22 coordinated directions, whilst the optimized timetable improves the coordination directions to 28.

Table 8

The optimal last trains timetable of Beijing subway

Station	Transfer Direction	Passengers	Original timetable			Optimized timetable		
			$t^A_{l,s}$	$t^D_{l',s}$	$x_{sll'}$	$t^A_{l,s}$	$t^D_{l',s}$	$x_{sll'}$
FuXing Men	L2 U to L1 D	5	23:06	23:40	1	22:14:21	23:11:35	1
	L2 D to L1 D	10	23:19	23:40	1	22:54:39	23:11:35	1
XiDan	L4 D to L1 D	15	23:20	23:37	1	23:19:01	23:05:35	0
	L1 D to L4 U	10	23:37	23:25	0	23:02:35	22:59:27	0
	L1 D to L4 D	20	23:37	23:20	0	23:02:35	23:22:01	1
DongDan	L5 D to L1 U	5	23:23	23:35	1	23:14:45	22:51:19	0
	L1 U to L5 U	17	23:35	23:23	0	22:48:19	23:08:21	1
	L1 U to L5 D	10	23:35	23:23	0	22:48:19	23:17:45	1
JianGuo Men	L2 U to L1 U	15	23:22	23:38	1	22:31:51	22:57:19	1
	L2 D to L1 U	15	23:02	23:38	1	22:32:39	22:57:19	1
GuoMao	L10 D to L1 U	20	23:03	23:42	1	23:25:11	23:04:19	0
	L1 U to L10 D	9	23:42	23:03	0	23:01:19	23:28:11	1
XiZhiMen	L2 U to L4 U	11	22:59	23:35	1	22:06:51	23:12:27	1
	L2 D to L4 U	14	23:27	23:35	1	23:09:39	23:12:27	1
	L2 U to L13 D	9	22:59	23:45	1	22:06:51	22:37:15	1
	L2 D to L13 D	7	23:27	23:45	1	23:09:39	22:37:15	0
	L4 U to L13 D	12	23:35	23:45	1	23:09:27	22:37:15	0
YongHe Gong	L2 U to L5 U	17	22:33	23:33	0	22:43:51	23:21:21	1
	L2 D to L5 U	8	22:51	23:33	1	22:12:39	23:21:21	1
DongZhi Men	L2 U to L13 U	20	23:29	22:42	0	22:39:21	22:33:43	0
	L2 D to L13 U	15	22:55	22:42	0	22:19:39	22:33:43	1
ChongWenMen	L2 U to L5 D	19	23:16	23:25	1	22:25:21	23:22:45	1
	L2 D to L5 D	17	23:08	23:25	1	22:37:39	23:22:45	1
XuanWu	L2 U to L4 D	18	23:10	23:22	1	22:18:51	23:27:01	1

Men	L2 D to L4 D	8	23:14	23:22	1	22:47:39	23:27:01	1
HaiDian	L4 U to L10 U	12	23:47	23:53	1	23:24:27	23:36:58	1
HuangZhuang	L10 U to L4 U	20	23:53	23:47	0	23:33:58	23:27:27	0
BeiTuCheng	L8 D to L10 U	17	22:37	23:40	0	22:47:09	23:17:58	1
	L8 D to L10 D	14	22:37	22:40	0	22:47:09	22:55:11	1
HuiNan	L10 U to L5U	12	23:36	23:39	1	23:07:58	23:30:21	1
	L5 U to L10 D	6	23:39	22:44	0	23:27:21	23:02:11	0
LiShuiQiao	L13 D to L5 U	14	23:14	23:53	1	23:15:15	23:47:21	1
	L13 U to L5 U	19	23:01	23:53	1	22:55:43	23:47:21	1
	L5 U to L13 U	14	23:53	23:01	0	23:44:21	22:56:15	0
	L5 U to L13 D	15	23:53	23:14	0	23:44:21	23:18:15	0
HuoYing	L13 U to L8 U	16	23:06	23:11	1	23:01:15	23:12:29	1
	L13 D to L8 U	9	23:09	23:11	0	23:07:15	23:12:29	1
ZhiChunLu	L10 U to L13 D	13	23:48	23:50	0	23:26:58	22:45:15	0
	L13 D to L10 U	18	23:50	23:48	0	22:42:15	23:29:58	1
ShaoYaoJu	L10 U to L13 U	15	23:33	22:48	0	23:01:58	22:42:43	0
	L10 D to L13 U	8	22:47	22:48	0	23:05:11	22:42:43	0
	L13 U to L10 U	16	23:48	23:33	0	22:39:43	23:04:58	1

6.3 Absolute miss

To compare the frequencies of the "absolute miss" in our optimal timetable to that from the current timetable, we count the number of such occurrences at each interchange station. Table 9 presents the results from the two timetables. It is found that the number of absolute miss is reduced from 20 in the original timetable to 14 in our new timetable, 30% of improvement. The average waiting time for connections is improved by 169 seconds (per passenger), and the number of connected passengers is significantly increased from 201 to 384, about 91% improvement.

Table 9

Comparisons of original and optimized results

	Original	After optimization	Improvement
Absolute miss	20	14	6
Coordination	22	28	6
Transferred passengers	201	384	183
Total waiting time (sec)	70800	70330	470

Average waiting time (sec)	352	183	169

6.4 Transfer direction analysis of weighted parameter ψ_1

In practical operations, transfer direction is an important factor to consider for the last train scheduling. For example, the transfer directions to suburban lines are considered more important than those to the urban ones because passengers will spend much longer time and higher cost to arrive at their destinations if they miss the last train. This is a particular feature in Beijing city where a high percentage of commuters are distributed along the suburban lines. Therefore, we apply different weighted parameter $\psi_{sll'}$ to the different transfer directions on the timetable. As seen in Tables 10 and 11, the optimized timetable can improve the number of connected passengers to 386. Meanwhile, important transfer directions get a nice coordination. The passenger waiting time for the successful connections is 42709 sec, decreasing 27621 sec compared with no weighted model. Figure 10 shows the coordination for weighted transfer directions in which the red and the black arrows represent the missing direction and the coordination direction between two lines. As shown, each transfer direction is associated with the passenger waiting time. If the directions cannot be coordinated by last trains, red arrows without waiting times are depicted in Figure 10. This may indicate passengers transferring to avoid missing the last connecting trains.

Table 10

The optimized timetable for weighted directions

Station	Transfer Direction	$\psi_{sll'}$	$t^{A}_{l,s}$	$t^{D}_{l',s}$	$t^{Tra}_{sll'}$	$t^{w}_{sll'}$	$x_{sll'}$	$p_{sll'}$
FuXing Men	L2 U to L1 D	1.0	22:09:51	23:03:21	1’30”	2’00”	1	10
	L2 D to L1 D	1.0	23:01:36	23:03:21	1’30”	0’15”	1	25
XiDan	L4 D to L1 D	1.0	23:25:23	22:57:21	5’00”	∞	0	1
	L1 D to L4 U	1.5	22:54:21	23:07:39	5’00”	8’18”	1	2
	L1 D to L4 D	1.5	22:54:21	23:28:23	5’00”	9’01”	1	1
DongDan	L5 D to L1 U	1.0	23:08:06	22:57:13	3’00”	∞	0	5
	L1 U to L5 U	1.8	22:54:13	22:52:10	3’00”	∞	0	13

	L1 U to L5 D	1.8	22:54:13	23:11:06	3’00”	3’52”	1	9
JianGuoMen	L2 U to L1 U	1.0	22:27:21	23:03:13	1’30”	4’22”	1	12
	L2 D to L1 U	1.0	22:39:36	23:03:13	1’30”	2’07”	1	7
GuoMao	L10 D to L1 U	1.0	23:45:44	23:10:13	4’20”	∞	0	6
	L1 U to L10 D	1.5	23:07:13	23:48:44	4’20”	7’11”	1	2
XiZhiMen	L2 U to L4 U	1.2	22:02:21	23:20:39	2’00”	6’18”	1	10
	L2 D to L4 U	1.2	23:16:36	23:20:39	2’00”	2’03”	1	18
	L2 U to L13 D	1.9	22:02:21	22:29:12	6’00”	0’50”	1	4
	L2 D to L13 D	1.9	23:16:36	22:29:12	6’00”	∞	0	12
	L4 U to L13 D	1.6	23:17:39	22:29:12	7’00”	∞	0	18
YongHeGong	L2 U to L5 U	1.3	22:39:21	23:05:10	3’00”	2’49”	1	17
	L2 D to L5 U	1.3	22:19:36	23:05:10	3’00”	2’34”	1	27
DongZhiMen	L2 U to L13 U	1.7	22:26:36	22:49:20	4’00”	0’29”	1	14
	L2 D to L13 U	1.7	22:19:39	22:49:20	4’00”	8’44”	1	3
ChongWenMen	L2 U to L5 D	1.6	22:20:51	23:16:06	3’00”	2’14”	1	7
	L2 D to L5 D	1.6	22:44:36	23:16:06	3’00”	8’29”	1	2
XuanWuMen	L2 U to L4 D	1.2	22:14:21	23:33:23	4’00”	5’01”	1	3
	L2 D to L4 D	1.1	22:54:36	23:33:23	4’00”	4’47”	1	15
HaiDianHuangZhuang	L4 U to L10 U	1.4	23:32:39	23:31:20	4’00”	∞	0	14
	L10 U to L4 U	1.4	23:28:20	23:35:39	4’00”	3’19”	1	30
BeiTuCheng	L8 D to L10 U	1.1	23:06:36	23:12:20	4’00”	1’44”	1	3
	L8 D to L10 D	1.1	23:06:36	23:15:44	4’00”	5’08”	1	2
HuiNan	L10 U to L5U	1.3	23:02:20	23:14:10	1’30”	0’20”	1	43
	L5 U to L10 D	1.3	23:11:10	23:22:44	1’30”	0’04”	1	1
LiShuiQiao	L13 D to L5 U	1.1	23:07:12	23:31:10	3’30”	0’29”	1	1
	L13 U to L5 U	1.3	23:11:20	23:31:10	3’30”	6’20”	1	5
	L5 U to L13 U	1.9	23:28:10	23:14:20	3’30”	∞	0	20
	L5 U to L13 D	1.9	23:28:10	23:10:12	3’30”	∞	0	3
HuoYing	L13 U to L8 U	1.5	23:19:20	22:55:36	3’30”	∞	0	9
	L13 D to L8 U	1.5	22:59:12	22:55:36	3’30”	∞	0	5
ZhiChunLu	L10 U to L13 D	1.0	23:21:20	22:37:12	4’20”	∞	0	21
	L13 D to L10 U	0.9	22:34:12	23:24:20	4’20”	4’13”	1	1
ShaoYaoJu	L10 U to L13 U	1.2	22:56:20	22:58:20	3’50”	∞	0	25
	L10 D to L13 U	1.2	23:25:44	22:58:20	3’50”	∞	0	3
	L13 U to L10 U	0.8	22:55:20	22:59:20	3’50”	0’30”	1	2
	Total					42709”	28	386

Table 11

Optimizing results for the weighted model

	No weighted	Weighted	Improvement
Total waiting time (sec)	70330	42709	27621
Transferred passengers	384	386	2
Average waiting time (sec)	183	110	73

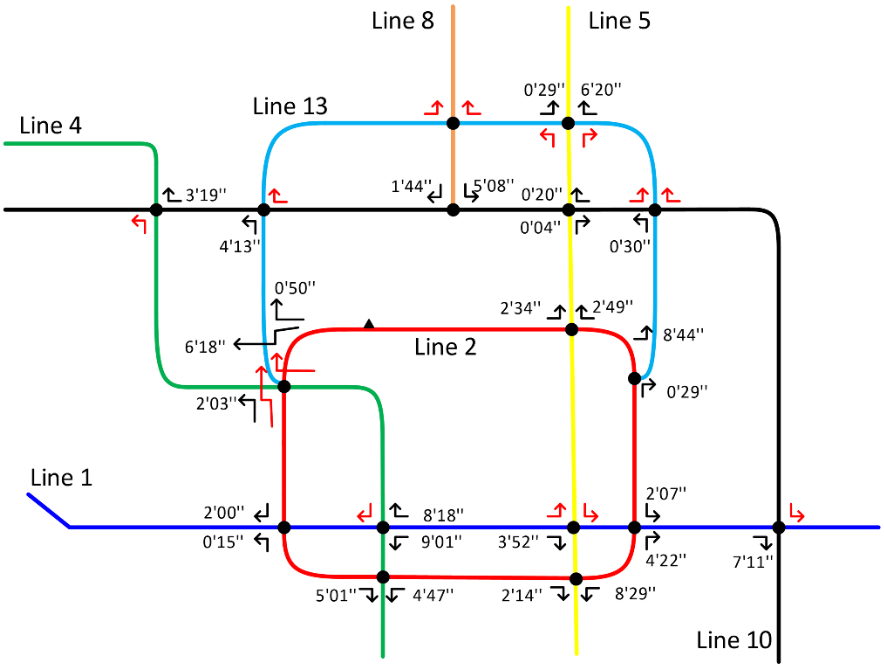

Fig. 10. Weighted transfer directions with transfer waiting time. In the figure, the red arrows represent the missing directions, while the black arrows show the coordination directions between two lines.

7 Conclusion

The problem of coordinating last train schedules is solved by a bi-level programming model, in which the upper level is to maximize the social service efficiency and the lower level is to minimize the difference between the operation cost and the subsidy. The government subsidy is modeled as a function of the arrival time of the last trains, and the effectiveness of the timetable is measured in terms of reductions in absolute misses and passenger wait time, and of improvements in connected passengers and transfer coordination. Furthermore, the model explicitly represents transfer directions, whereby a higher weight is put on transfer to suburban direction than to city center direction. This makes the model a weighted bi-level programming model. The upper and lower models are solved by GA and active-set approaches. The article fills a gap in the existing literature on train timetable design specifically for the last train services, and addresses explicitly the equilibrium between the government subsidy and train operating costs.

An application of the model to the Beijing subway indicates that the number of absolute misses is reduced from 20 with the current timetable to 14 with the optimal timetable, and the connected passengers are significantly increased from 201 to 384. In the weighted model, the important transfer directions produce a good coordination between transfer services, whereby the average passenger waiting time is reduced by 70 sec compared with that from non-weighted model.

Several extensions of the current work can be explored in future research. The subsidy function may be extended to include other factors, such as labor cost, energy consumption etc, which would require adding additional variables to the objective (and/or as constraints to) of the model, which will greatly increase the complexity of the problem. Another area is to model the imperfect running of the train services or the variability in train operations, adding for example random variables in the trains' arrival time, the departure time, the dwell time and/or the running time. Besides, more extensive validations of parameters, like social benefit parameter and operating cost parameter, will be conducted in our further study.

Geolocation information

Beijing, China

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Disclosure statement

No potential conflict of interest was reported by the authors.

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