

Testing quantised inertia on emdrives with dielectrics

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Abstract

Truncated cone-shaped cavities with microwaves resonating within them (emdrives) move slightly towards their narrow ends, in contradiction to standard physics. This effect has been predicted by a model called quantised inertia (MiHsC) which assumes that the inertia of the microwaves is caused by Unruh radiation, more of which is allowed at the wide end. Therefore, photons going towards the wide end gain inertia, and to conserve momentum the cavity must move towards its narrow end, as observed. A previous analysis with quantised inertia predicted a controversial photon acceleration, which is shown here to be unnecessary. The previous analysis also mispredicted the thrust in those emdrives with dielectrics. It is shown here that having a dielectric at one end of the cavity is equivalent to widening the cavity at that end, and when dielectrics are considered then quantised inertia predicts these results as well as the others, except for Shawyer's first test where the thrust is predicted to be the right size but in the wrong direction. As a further test, quantised inertia predicts that an emdrive's thrust can be enhanced by using a dielectric at the wide end.

1 Introduction

Shawyer [1] demonstrated that when microwaves resonate within a truncated-cone shaped cavity a small, unexplained thrust and acceleration occurs towards the narrow end. There is no explanation for this behaviour in standard physics because it violates the conservation of momentum, and Shawyer's own attempt to explain it using special relativity is not convincing, as this theory also should obey the conservation of momentum [2].

Nethertheless, this anomaly has also been seen by [3], tentatively by [4] and more solidly by a NASA team [5,6], most recently in a vacuum, proving that

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the effect is not due to moving air. Their results are shown in Table 1 (rows 4-7 and 9-11).

One way to explain the emdrive involves a modification of inertial mass. McCulloch [7,8] has proposed a new model for inertia that assumes that when an object accelerates, say, to the right, an information horizon forms to its left and the object perceives Unruh radiation which is also suppressed by the horizon to the left. Therefore a gradient in the Unruh radiation appears that pushes the object back against its initial acceleration, predicting standard inertia [8,9]. Furthermore, this model predicts that some of the Unruh radiation will also be suppressed, this time isotropically, by the distant cosmic horizon which will make this mechanism less efficient for very low accelerations for which Unruh waves are of cosmic scale, reducing inertia in a new way [7]. Quantised inertia modifies the standard inertial mass (m) to a modified one (m_i) as follows:

$$m_i = m \left(1 - \frac{2c^2}{|a|\Theta} \right) = m \left(1 - \frac{\lambda_U}{4\Theta} \right) \quad (1)$$

where c is the speed of light, Θ is twice the Hubble distance, ' $|a|$ ' is the magnitude of the relative acceleration of the object relative to surrounding matter and λ_U is the peak wavelength of the Unruh radiation it sees ($\lambda_U \sim 8c^2/a$). Eq. 1 predicts that for terrestrial accelerations (eg: $9.8m/s^2$) the second term in the bracket is tiny and standard inertia is recovered, but in low acceleration environments, for example at the edges of galaxies (when a is small and λ_U is large) the second term in the bracket becomes significant and the inertial mass decreases in a new way. In this way, quantised inertia can explain galaxy rotation without the need for dark matter [10,11] and cosmic acceleration without the need for dark energy [7,12].

The difficulty of demonstrating this model on Earth is the huge size of Θ in Eq. 1 which makes the effect negligible unless the acceleration is tiny, as in deep space. One way to make the effect more obvious is to reduce the distance to the horizon Θ , and this is what the emdrive may be doing since the photons within it are accelerating so fast that the Unruh waves they see will be short enough to interact, at least electromagnetically, with the metal cavity walls. McCulloch [13,14] showed that assuming that the inertial mass of the photons is determined by quantised inertia and the width of the cavity, and assuming the conservation of momentum, a new force is predicted of size

$$F = \frac{-6PQL}{c} \left(\frac{1}{(L + 4w_b)} - \frac{1}{(L + 4w_s)} \right) \quad (2)$$

where P is the power input, Q is the quality factor of the cavity, L is the axial length, c is the speed of light and w_s and w_b are the widths of the small and big ends respectively. This formula predicts that the photons input into the cavity continually gain mass when going to the wide end and lose it going the other way (so their collective centre of mass is continually being shifted rightwards)

so to conserve momentum the cavity itself must move the other way (see Fig. 1, upper schematic). Eq. 2 predicted the observed emdrive thrusts quite well, except for the tests of Shawyer1 and the various NASA tests. Interestingly, these tests all used dielectrics. In this paper the previous derivation is improved and the effects of dielectrics are considered. The predictions are shown, in all but one case, to be much better.

2 Method

We first calculate the collective mass of the microwave photons in the cavity (m_L) from the energy input by the magnetron, E:

$$m_L = \frac{E}{c^2} \quad (3)$$

The energy (E) is equal to the power input (P) times the time that the photons last before dissipating (t):

$$m_L = \frac{Pt}{c^2} \quad (4)$$

The time for a photon to dissipate is the Q factor (number of bounces from end to end) times the time taken to go from end to end, so

$$m_L = \frac{P}{c^2} \times Q \times \frac{L}{c/\bar{n}} \quad (5)$$

where \bar{n} is the average refractive index inside the cavity. So

$$m_L = \frac{PQL\bar{n}}{c^3} \quad (6)$$

Now we consider the conservation of momentum for the cavity of constant mass m_c and speed v_c and the light inside it of mass m_L (as above) and speed c which is

$$\frac{d}{dt}(m_c v_c + m_L c) = 0 = m_c a_c + m_L \frac{dc}{dt} + c \frac{dm_L}{dt} \quad (7)$$

The first term on the right hand side is the force on the cavity, to be found. The second term, the acceleration of light, is zero. Note that unlike McCulloch (2015, 2016) this derivation assumes that there is no change in light speed in the cavity, just a change in photon mass. So

$$F_c = -c \frac{dm_L}{dt} = -c \frac{dm_L}{dx} \frac{dx}{dt} = -c^2 \frac{dm_L}{dx} \quad (8)$$

In quantised inertia (see Eq. 1) the mass of the photons inside a cosmic horizon of radius S would be $m = m_L(1 - \lambda_U/4S)$ where m_L is the unmodified mass (Eq. 6). In this case, as in the previous paper, it is assumed that the cavity walls act like a horizon for the Unruh waves that are assumed to cause the photons' inertia.

The previous derivation is now generalised by assuming there is a dielectric in the cavity, so that $m = m_L(1 - \lambda_U/4nS)$ where n is the refractive index of the dielectric, since n reduces the light speed and reduces the wavelength (since the frequency is constant) so that more waves fit within the horizon or cavity. Therefore changing the partial derivative in Eq. 8 to a finite difference, and noting that a dielectric can change the refractive indices at the ends to n_s and n_b (for the small and big ends) we get

$$F_c = -c^2 m_L \left(\frac{\left(1 - \frac{\lambda_U}{4n_s S_s}\right) - \left(1 - \frac{\lambda_U}{4n_b S_b}\right)}{L} \right) \quad (9)$$

Since $\lambda_U \sim 8c^2/a$, where a is the acceleration of the photons as their speed changes direction from c to $-c$ as they bounce between the two ends of the cavity: $a = dv/dt = 2c/(L/c) = 2c^2/L$ (see [13,14]). So $\lambda_U = 4L$, leaving

$$F_c = m_L c^2 \left(\frac{1}{n_b S_b} - \frac{1}{n_s S_s} \right) \quad (10)$$

In the emdrive, the average cavity size measured from the central axis at each end plate (averaged from all directions) is approximately $(L + 4w)/6$ (see [14]) where w is the cavity width. So substituting for S_s and S_b

$$F_c = 6m_L c^2 \left(\frac{1}{n_b(L + 4w_b)} - \frac{1}{n_s(L + 4w_s)} \right) \quad (11)$$

Using eq. 6 for the unmodified mass of the microwaves (m_L) we get

$$F_c = \frac{6PQL\bar{n}}{c} \left(\frac{1}{n_b(L + 4w_b)} - \frac{1}{n_s(L + 4w_s)} \right) \quad (12)$$

Since the dielectric occupies only about one tenth or less of the cavity's length then the average refractive index $\bar{n} \sim 1$, so

$$F_c = \frac{6PQL}{c} \left(\frac{1}{n_b(L + 4w_b)} - \frac{1}{n_s(L + 4w_s)} \right) \quad (13)$$

In this paper it is assumed that if one end has no dielectric then $n_{s,b} = 1$, and if it does then either $n_{s,b} = 1.46$ (the refractive index of the polyethylene used in the NASA tests) or $n_{s,b} = 6.16$ for Sawyer’s first test ([15]).

3 Results

Table 1 summarises the various experimental results from Sawyer [1] in rows 1 and 2, the Cannae drive in row 3 [3], the earlier NASA tests done in air by [5], in rows 4 to 7, the vacuum test by [4] and the 2016 NASA tests done in vacuum [6] in rows 9-11.

In Table 1, column 1 names the experiment. Column 2 shows the input power (in Watts). Column 3 shows the Q factor (dimensionless). Column 4 shows the axial length of the cavity. Column 5 shows the width of the big and small ends (metres). Column 6 states whether the dielectric was absent (-), or at the narrow end (narrow). Column 7 shows the thrust predicted by quantised inertia ignoring the dielectric (using Eq. 2), column 8 shows the prediction of quantised inertia considering the dielectric (Eq. 13) and column 9 shows the thrust observed in the experiments. For the NASA experiments in 2016 the observed thrusts can be summarised to be 1.2 mN/kW.

Expt	P	Q	L	w_{big}/w_{small}	Dielectric	F_{-d}	F_{+d}	F_{Obs}
	W		m	metres	location	mN	mN	mN
Shawyer1	850	5900	0.156	0.16/0.1275	narrow	3.8	-15.84	16
Shawyer2	1000	45,000	0.345	0.28/0.1289	-	149	148.85	80-214
Cannae	10.5	11×10^6	0.03	0.22/0.2	-	7.3	7.34	9
NASA 2014	16.9	7,320	0.2286	0.2794/0.1588	narrow	0.23	0.03	0.091
NASA 2014	16.7	18,100	“	“	narrow	0.57	0.07	0.05
NASA 2014	2.6	22,000	“	“	narrow	0.11	0.01	0.055
NASA 2014	50	6730	“	“	narrow	0.64	0.08	0.03
Tajmar1	700	20	0.1008	0.1062/0.075	-	0.02	0.02	0.02-0.11
NASA 2016a	40	7123	0.229	0.279/0.159	narrow	0.54	0.06	0.04
NASA 2016b	60	“	“	“	“	0.81	0.09	0.09
NASA 2016c	80	“	“	“	“	1.08	0.13	0.09

Table 1. A summary of the fully-documented and published emdrive experiments so far. Column 1 shows the experiment name, column 2 shows the input power, column 3 the Q factor, column 4 the cavity’s axial length, column 5 shows the cavity end widths. Columns 6 shows whether there was no dielectric (-) or if it was at the narrow end. Column 7 and 8 show the thrusts predicted by ignoring and considering the dielectric respectively, and column 9 shows the observed thrust. When considering the dielectric quantised inertia predicts the data well, except for the first Sawyer result, where the observed thrust was the same size but in the opposite direction.

A comparison of the results ignoring the dielectric with those observed shows

that the earlier formula (eq. 2) predicted the experiments without dielectrics but not those with dielectrics (Shawyer1 and the NASA tests). It tended to overpredict the NASA results by up to a factor of ten. This can also be seen in Figure 2 which shows a comparison of the thrust predicted by quantised inertia (along the x-axis) and that observed (on the y-axis). Accurate predictions should line up along the diagonal line, but the predictions using Eq. 2 (ignoring the dielectric), which are shown in Fig. 2 by the open squares, are well to the right of the diagonal line for all the NASA experiments, which had dielectrics at the narrow end of the cavity, and to the left of it for Shawyer1 which also had a dielectric at the narrow end.

The predictions of quantised inertia considering the dielectric (Eq. 13) agree far better with the NASA observations, but not the first Shawyer result where the prediction is now the right size but in the opposite direction. See the Table and the black diamonds on Figure 2. Quantised inertia is particularly close for the latest batch of NASA experiments which arguably had the greatest experimental controls. This is encouraging given that this model is rather approximate and does not yet take account of the resonance modes of the cavity, which should also affect the results. It should also be noted, in its favour, that quantised inertia has no adjustable parameters.

4 Discussion

Quantised inertia predicts that for a standard dielectric-less emdrive, see Figure 1, top panel, the inertial centre of mass of the highly-accelerated cloud of photons being input into the emdrive is continually being shifted towards the wide end (curved arrow) since more Unruh waves fit there, so the emdrive cavity itself has to move towards its narrow end (see the straight arrow) to conserve the momentum of the combined system. The additional effect considered in this paper is the insertion of the dielectric, see Fig. 1, bottom panel, which reduces the speed of the photons, and the wavelength of the Unruh radiation (frequency being constant) so that more Unruh waves fit in the end with the dielectric. This has the same effect as widening the cavity at that end, producing a thrust away from the end with the dielectric.

This is why a consideration of the dielectrics in most cases improved the fit of quantised inertia to the data. For example, in the case of the NASA tests, the dielectric was at the narrow end, thus it reduced their thrust. When the dielectric is considered then the thrust predicted by quantised inertia is reduced and agrees better with the NASA data (see Table 1, rows 4-7 and 9-11, and Fig. 2). The exception was the first experiment of Shawyer where the thrust is now predicted to be opposite to that observed.

More data is needed for comparison, and a more accurate modelling of the effects of quantised inertia is needed. This analysis for simplicity, assumed the microwaves only travelled along the axis and the three-dimensional resonance of the waves was only crudely modelled: a full 3-d model is needed.

This proposal predicts the observations quite well, but assumes that the inertial mass of photons is finite. In defence of this, it can be experimentally shown that they carry momentum. The inertial mass considered here is a collective one caused by their confinement in the cavity.

Both Eq. 13 and the simpler equation in [14], Eq. 2, predict that the thrust can be increased by increasing P and Q , and it should be possible to reverse the sign of the thrust by shortening the usual cavity length (L) or changing the frequency so that the Unruh waves fit better into the short end (w_s) than the wide end (w_b). This thrust reversal may have been seen in recent NASA experiments.

A new prediction is that the anomalous emdrive thrust could be enhanced by increasing the overall refractive index in the cavity (see Eq. 12), and also by inserting a dielectric at the wide end (see Eq. 13).

5 Conclusion

Nine tests in four independent labs have shown that when microwaves resonate within an asymmetric cavity an anomalous thrust is generated pushing the cavity towards its narrow end.

This thrust is predicted well, except for the case of Shawyer's first test, by a new model for inertia (quantised inertia) which assumes that the inertial mass of the photons is caused by Unruh radiation whose wavelengths have to fit exactly inside the cavity so that more are allowed at the wide end. This increases the photons' inertial mass as they travel towards the wide end, and to conserve momentum the cavity itself moves towards its narrow end.

It has been shown here that a change in the speed of the microwave photons, a criticism of a previous paper, is not needed and that the model predicts the data more successfully, except for Shawyer first experiment, when the effect of dielectrics is also considered.

Quantised inertia (Eq. 13) suggests that the thrust can be changed or reversed by altering the cavity's aspect ratio, by increasing P or Q , by using a dielectric uniformly within the cavity, or, more effectively, by adding a dielectric at the wide end.

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Figures

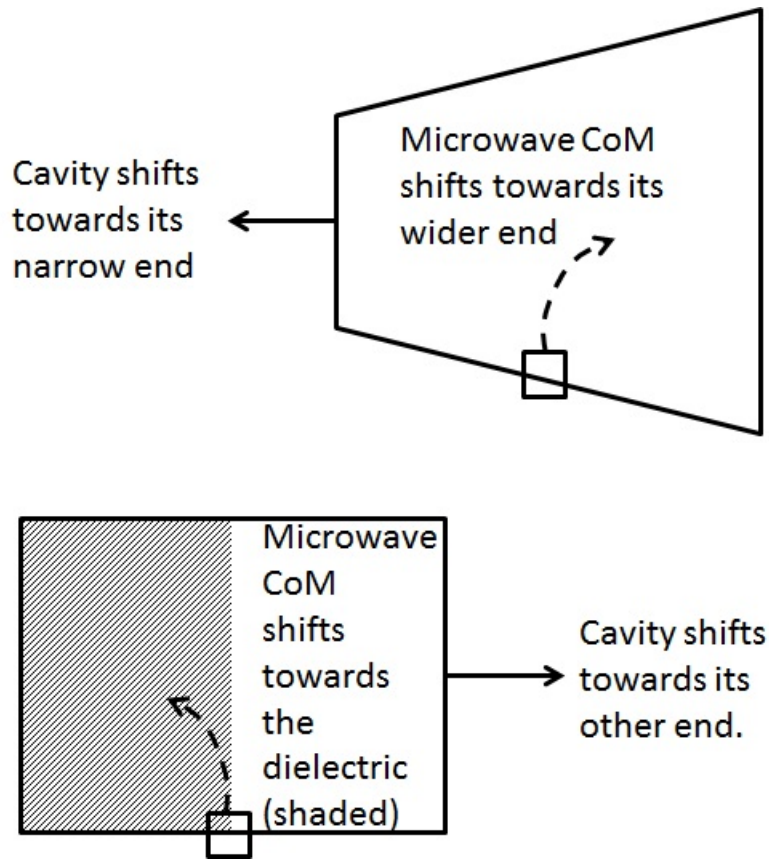


Figure 1. The emdrive cavity without a dielectric (top panel) showing that the centre of mass (CoM) of the input photon cloud of microwaves is continually being shifted towards the wide end. So the emdrive cavity, to conserve the momentum of the combined system, moves the other way, towards its narrow end. With a dielectric (bottom panel) the photons' centre of mass moves towards the dielectric. This paper considers an emdrive with both effects.

Observed vs Predicted EmDrive Thrust

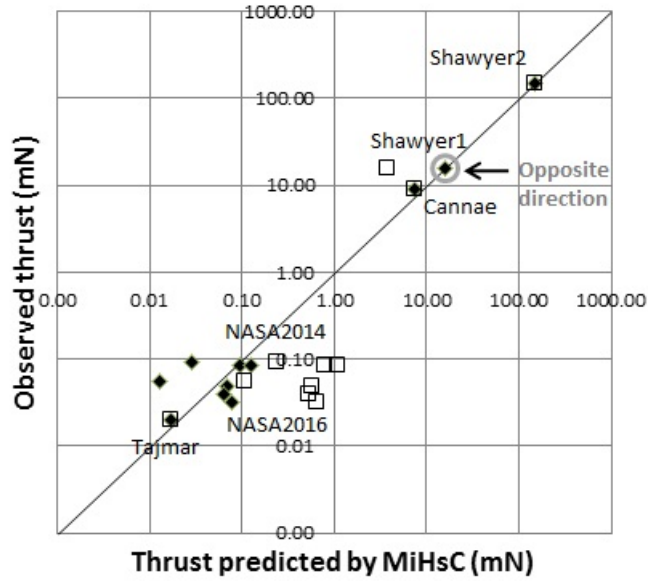


Figure 2. A comparison between the eleven observed emdrive thrusts (on the y axis) and the thrusts predicted by quantised inertia (on the x axis) without considering the effect of dielectrics (the empty squares) and considering the effect of the dielectrics (the black diamonds). When the dielectrics were considered the results were far better (closer to the diagonal line), except for Shawyer1 where the prediction was the right size but in the wrong direction.