Quantised inertia from relativity and the uncertainty principle.

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Abstract

It is shown here that if we assume that what is conserved in nature is not simply mass-energy, but rather mass-energy plus the energy uncertainty of the uncertainty principle, and if we also assume that position uncertainty is reduced by the formation of relativistic horizons, then the resulting increase of energy uncertainty is close to that needed for a new model for inertial mass (MiHsC, quantised inertia) which has been shown to predict galaxy rotation without dark matter and cosmic acceleration without dark energy. The same principle can also be used to model the inverse square law of gravity, and predicts the mass of the electron.

1 Introduction

Although special relativity and quantum mechanics have been partially merged in quantum field theories, some aspects, and general relativity and quantum mechanics are still incompatible. For example, relativity is based on a smooth spacetime and demands locality, whereas quantum mechanics is modelled using discrete particles and quantum experiments seem to demand non-locality [1-5].

In some instances it has been possible to combine general relativity and quantum mechanics, at least partially, for example [6] proposed that the event horizons caused by the strong gravity within black holes would seperate pairs of particles produced by the quantum vacuum, leaving one to fall into the black hole and one to escape, giving rise to a new kind of radiation called Hawking radiation that originates from a combination of relativity (curved space) and quantum mechanics on a large scale. There is now some evidence that at least analogues of this process occur [7].

[8], [9] and [10] showed that when an object accelerates, say, to the left, an information horizon, very like an event horizon, forms to its right since information

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which is limited to the speed of light by relativity cannot now get to the object from behind that horizon. They showed that this horizon can seperate paired virtual particles in a similar way to a black hole event horizon, leading to the production of acceleration-dependent Unruh radiation. This conclusion is now generally accepted, but see [11] for remaining controversies. It is possible that Unruh radiation has already been observed [12].

An early inspiring attempt to implicate quantum mechanics and the zero point field in inertial mass was made by [13]. However, they required an arbitrary cutoff to make their scheme work. Also, [14] questioned whether Unruh radiation might account for inertial-MoND (Modified Newtonian Dynamics), but concluded that Unruh radiation was unlikely to be the cause of inertia because it was isotropic.

A new model for inertia was proposed by [15, 16]. It is called Modified inertia by a Hubble-scale Casimir effect, MiHsC or quantised inertia. This model assumes that the inertia of an object is due to the Unruh radiation it sees when it accelerates. The relativistic Rindler horizon that appears in the opposite direction to its acceleration damps the Unruh radiation on that side of the object producing an anisotropic radiation pressure that looks like inertial mass [16]. So inertia arises in this model from the interplay of relativity (horizons) and quantum mechanics (Unruh waves). Also, when accelerations are extremely low the Unruh waves become very long and are also damped, this time equally in all directions, by the Hubble horizon (Hubble-scale Casimir effect) [15]. This leads to a new loss of inertia as accelerations become tiny. So MiHsC modifies the standard inertial mass (m) to a modified one (m_i) as follows:

$$m_i = m\left(1 - \frac{2c^2}{|a|\Theta}\right) \tag{1}$$

where c is the speed of light, Θ is the diameter of the observable universe and '$|a|$' is the magnitude of the acceleration of the object relative to surrounding matter. Eq. 1 predicts that for terrestrial accelerations (eg: $9.8m/s^2$) the second term in the bracket is tiny and standard inertia is recovered, but in low acceleration environments, for example at the edges of galaxies (when a is tiny) the second term in the bracket becomes larger and the inertial mass decreases in a new way so that quantised inertia (MiHsC) can explain galaxy rotation without the need for dark matter [17] and cosmic acceleration without the need for dark energy [15,18]. There are also anomalies seen in Solar system probes [19] that can be explained by this model [15,20]. Quantised inertia does not significantly affect the predictions of general relativity for high accelerations and only becomes significant for very low accelerations or upon a change in acceleration.

Similarly, applying quantum mechanics on a large scale [21] derived Newtonian gravity from the uncertainty principle. The main aim of this paper is to extend [21] and show that both gravity and quantised inertia can be derived by allowing large-scale dynamics or horizons to determine the position uncertainty in the

Heisenberg uncertainty principle, and allowing the resulting energy uncertainty to become real.

2 Gravity from Uncertainty

Imagine there are two Planck masses orbiting each other. With Planck masses, we are still, just, in the quantum realm, Heisenberg's uncertainty principle applies to their mutual position uncertainty (Δx) given by the distance between them, and momentum (Δp), and the total uncertainty is twice that for a single particle

$$\Delta p \Delta x \sim \hbar \tag{2}$$

Now $E = pc$ so

$$\Delta \bar{E} \Delta \bar{x} \sim \hbar c \tag{3}$$

If a bigger mass M has N Planck masses in it, and another big mass m has n of them, then we can add up all the possible interactions (all the various uncertainties: $\hbar c$) between the various Planck masses

$$\Delta \bar{E} \Delta \bar{x} = \sum_{i=1}^{N} \sum_{j=1}^{n} (\hbar c)_{ij} \tag{4}$$

The double summation on the right hand side is equal to the number of Planck masses in mass m (m/m_P) times the number in M (M/m_P), where m_P is the reduced Planck mass, so

$$\Delta \bar{E} = \frac{\hbar c m M}{m_P^2 \Delta \bar{x}} \tag{5}$$

Now let us imagine that the Planck masses within m and M are being buffeted from all sides by particles from the zero point field and moving at random. The net effect, forgetting horizons for a moment, will be zero. Sometimes random motion will increase the distance between the two objects, Δx, so their uncertainty in energy, ΔE, decreases, and sometimes it will decrease Δx, so the uncertainty in energy, ΔE, will increase. This latter event means that energy will suddenly be available that wasn't before, extracted from the decrease in position uncertainty, and if the objects continue to move together then more energy will be released in this way allowing the motion to continue. What if we assume that the sum of the kinetic energy and the energy uncertainty is conserved?

$$^{1}/_{2}m(\Delta v)^2 + \frac{\hbar c m M}{m_P^2 \Delta \bar{x}} = constant \tag{6}$$

Differentiating

$$m\Delta v \frac{d(\Delta v)}{dt} = \frac{\hbar c m M}{m_P^2 \Delta x^2}\frac{d(\Delta x)}{dt} \tag{7}$$

Since the right-most fraction can be written as Δv we get

$$m(\Delta a) = \frac{\hbar c m M}{m_P^2 \Delta x^2} \tag{8}$$

Now we assume that $m(\Delta a) = F$ (force) and that the uncertainty of the average position ($\triangle x$) is the orbital radius r

$$F \sim \frac{\hbar c}{m_P^2}\frac{mM}{r^2} \tag{9}$$

This looks like Newton's gravity law, and if we insert the value of the Planck mass, for which the value of G must be assumed, we get

$$F = \frac{GMm}{r^2} \tag{10}$$

The force required to drive the motion only becomes available for objects moving closer together since this reduces Δx and increases ΔE (the inevitability of attraction was not discussed in [21]). In this model, gravity is a process by which quantum mechanics applies at this large scale and converts position uncertainty to energy uncertainty, which shows up as an acceleration-dependent heat (Unruh radiation) and so it satisfies the second law of thermodynamics: increasing entropy. It has therefore been shown that Newton's gravity law can be produced if a summation is made for all interactions between masses equal to the Planck mass, but this requires an assumption of the value of G [21].

3 Quantised Inertia from Uncertainty

Again, using Heisenberg's momentum-position uncertainty principle we get

$$\Delta p \Delta x \sim \hbar \tag{11}$$

Since $E = pc$ we can write

$$\Delta E \Delta x \sim \hbar c \tag{12}$$

The energy uncertainty is then $\Delta E \sim \hbar c/\Delta x$. The new proposal here is that if the particle in question accelerates and a relativistic Rindler horizon forms then this destroys knowledge of all positions beyond the horizon and decreases the uncertainty in position Δx. From Eq. 12 we would then expect the uncertainty in energy to go up. Now, as above we assume that what is conserved in nature is not mass-energy, but rather mass-energy plus the energy uncertainty identified above, as follows

$$m_1 c^2 + \frac{\hbar c}{\Delta x_1} = m_2 c^2 + \frac{\hbar c}{\Delta x_2} \tag{13}$$

where the m_1 and m_2 are the initial and final inertial masses and Δx_1 and Δx_2 are the initial and final positional uncertainties. Note that the energy uncertainty terms are usually many orders of magnitude smaller than the mass-energy terms. Rewriting we get

$$m_2 - m_1 = dm = \frac{\hbar}{c}\left(\frac{1}{\Delta x_2} - \frac{1}{\Delta x_1}\right) \tag{14}$$

Now we can start to consider relativistic horizons. For an minimally-accelerated object (a zero acceleration cannot exist in MiHsC) the maximum uncertainty in position has to be due to the cosmic horizon, and equal to the radius of the cosmos, so $\Delta x_1 = \Theta/2$ so that

$$dm = \frac{\hbar}{c}\left(\frac{1}{\Delta x_2} - \frac{2}{\Theta}\right) \tag{15}$$

If an object then is subjected to an acceleration, a, then a Rindler horizon forms at a distance $d = c^2/a$ away. So the new uncertainty in position is smaller $\Delta x_2 = c^2/a$ so that

$$dm = \frac{\hbar}{c}\left(\frac{a}{c^2} - \frac{2}{\Theta}\right) \tag{16}$$

Rearranging we get

$$dm = \frac{\hbar a}{c^3}\left(1 - \frac{2c^2}{a\Theta}\right) \tag{17}$$

Now an acceleration 'a' is associated with Unruh radiation of wavelength λ where, using Unruh's expression for the Unruh temerature $T = \hbar a/2\pi ck$ and Wien's law $T = \beta hc/k\lambda$ where $\beta = 0.2$, it follows that that $a = 4\pi^2 c^2 \beta/\lambda$. Also $E = hc/\lambda$. Using these to replace the 'a' in the factor, we get

$$dm = \frac{\hbar}{c^3} \times \frac{4\pi^2 \beta c E}{2\pi\hbar} \times \left(1 - \frac{2c^2}{a\Theta}\right) \tag{18}$$

So that

$$dm = \frac{4\pi^2 \beta}{2\pi} \times \frac{E}{c^2} \times \left(1 - \frac{2c^2}{a\Theta}\right) \tag{19}$$

Using $E = mc^2$ we get

$$dm = 2\pi\beta m\left(1 - \frac{2c^2}{a\Theta}\right) \tag{20}$$

This is the same as Eq. 1, except for the initial factor of $2\pi\beta \sim 1.26$ which could be due to the crudity of this model, which has treated the Rindler horizon as being a sphere around the object whereas it is a more complex shape. The important point is that Eqs. 1 and 20, by allowing quantum mechanics and relativity to interact in this way, can model the observed anomalous galactic rotation without dark matter [17] and the observed cosmic acceleration without dark energy [15,18].

4 Applications

4.1 Particle masses

An electron can be regarded as a photon that has become confined to a particular orbit and so Eq. 14 can be used to predict the mass-energy of the electron as follows

$$dm = \frac{\hbar}{c}\left(\frac{1}{\Delta x_2} - \frac{1}{\Delta x_1}\right) \tag{21}$$

Initially the photon is confined to the cosmic scale so $\Delta x_1 = \Theta/2$ and it is known that for it to form an electron it must have the Compton wavelength $\lambda_C = 2.426 \times 10^{-12} m$ so

$$dm = \frac{\hbar}{c}\left(\frac{1}{\lambda_C} - \frac{2}{\Theta}\right) \tag{22}$$

Neglecting the second term, which since $\Theta \sim 10^{26} m$ is about 38 orders of magnitude smaller than the first, we get

$$dm = \frac{\hbar}{c\lambda_c} = 9.1 \times 10^{-31} kg \tag{23}$$

This is very close to the mass of the electron measured in experiments. Similarly we can consider the protons and neutrons which are confined to the nucleus of radius $r_n = 1.75 \times 10^{-15} m$ (for hydrogen) so that

$$dm = \frac{\hbar}{c}\left(\frac{1}{r_n} - \frac{2}{\Theta}\right) = 1.3 \times 10^{-27} kg \tag{24}$$

This is close to the observed masses of the proton and neutron which are $1.67 \times 10^{-27} kg$. Equation 24 also predicts a small correction to the proton mass given by the second term in the bracket, which is about 41 orders of magnitude smaller than the first term in the bracket.

If we use the Planck length $1.616 \times 10^{-35} m$ instead this gives

$$dm = \frac{\hbar}{c}\left(\frac{1}{l_P} - \frac{2}{\Theta}\right) = 1.4 \times 10^{-7} kg \tag{25}$$

This is close to the Planck mass, which is $2.2176 \times 10^{-8} kg$. The agreement is very close if we use a scale of $2\pi l_P$

$$dm = \frac{\hbar}{c}\left(\frac{1}{2\pi l_P} - \frac{2}{\Theta}\right) = 2.2 \times 10^{-8} kg \tag{26}$$

Thus the assumption that what is conserved in nature is not mass-energy as previously assumed, but mass-energy plus the energy uncertainty and assuming the position uncertainty is determined by relativistic horizons, allows the calculation of some particle masses in this way as well as Newtonian gravity and quantised inertia (MiHsC).

5 Discussion

These derivations can be explained more intuitively as follows. For gravity: As the radius of an orbit decreases and so the uncertainty in position decreases, then the momentum ($dp = Fdx/c$) or force (F) on the orbiting body must increase, producing an inverse square law. In the above gravitational derivation, the correct value for the gravitational constant G can only be obtained when it is assumed that the gravitational interaction occurs between whole multiples of the Planck mass, but this last part of the derivation involves some circular reasoning since the Planck mass is defined using the value for G (this was not discussed in the precursor gravity paper, [21]). This paper also builds on [21] by showing how this formalism specifically implies attraction rather than repulsion (previously it could have been either).

For inertia: as an object accelerates, a relativistic Rindler horizon forms in the opposite direction. This curtails the object's observable space and reduces its uncertainty in position. The uncertainty principle then implies that the uncertainty in momentum (or energy) must increase, and the energy released agrees (within the uncertainty of the calculation) with the specific energy required for quantised inertia (MiHsC) which allows the prediction of galaxy rotation without dark matter and cosmic acceleration without dark energy.

6 Conclusion

The uncertainty principle of quantum mechanics states that if the uncertainty in position reduces, then the uncertainty in momentum increases. Relativity predicts that if an object accelerates, a Rindler horizon forms, curtailing its observable space.

If we combine these two principles, the formation of the Rindler horizon reduces position uncertainty, increasing energy uncertainty. It has already been shown, in a similar way, that if we accept this energy as being real, Newtonian gravity is the result, though a value for G has to be assumed.

It is shown here that using the same method, the model known as quantised inertia or MiHsC can also be derived, solving the problems of galaxy rotation and cosmic acceleration, and predicting the electron mass.

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