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Paul Stockman, Stephen Oxburgh, Tomáš Tyc, Johannes Courtial, "What do forbidden light-ray fields look like?," Proc. SPIE 9193, Novel Optical Systems Design and Optimization XVII, 919304 (12 September 2014); doi: 10.1117/12.2061409

Event: SPIE Optical Engineering + Applications, 2014, San Diego, California, United States

What do forbidden light-ray fields look like?

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ABSTRACT

Ray-optically, optical components change a light-ray field on a surface immediately in front of the component into a different light-ray field on a surface behind the component. In the ray-optics limit of wave optics, the incident and outgoing light-ray directions are given by the gradient of the phase of the incident and outgoing light field, respectively. But as the curl of any gradient is zero, the curl of the light-ray field also has to be zero. The above statement about zero curl is true in the absence of discontinuities in the wave field. But exactly such discontinuities are easily introduced into light, for example by passing it through a glass plate with discontinuous thickness. This is our justification for giving up on the global continuity of the wave front, thereby compromising the quality of the field (which now suffers from diffraction effects due to the discontinuities) but also allowing light-ray fields that appear to be (but are not actually) possessing non-zero curl and thereby significantly extending the possibilities of optical design. Here we discuss how the value of the curl can be seen in a light-ray field. As curl is related to spatial derivatives, the curl of a light-ray field can be determined from the way in which light-ray direction changes when the observer moves. We demonstrate experimental results obtained with light-ray fields with zero and apparently non-zero curl.

Keywords: generalised refraction, micro-optics, light fields that appear forbidden

1. INTRODUCTION

In recent years a new class of optical interfaces called METATOYs has emerged.[1] METATOYs are microstructured windows that change the direction of transmitted light rays without significantly offsetting the light rays. If the light-ray offset is neglected, METATOYs appear to perform refraction according to generalised laws of refraction. These generalised laws of refraction can appear to be wave-optically forbidden, in the sense that the outgoing light-ray field can appear to be such that there cannot exist a corresponding wave front.[1]

Examples of METATOYs include arrays of micro-telescopes[4,5] and arrays of Dove prisms.[6,7] Such windows have been approximated experimentally with arrays of confocal lenticular arrays (arrays of cylindrical lenses).[8] Combinations of Dove-prism arrays form more complex METATOYs that rotate the incident light-ray direction around the window normal[9] or any other direction.[10] Fig.1 shows a few of the effects associated with such components. Light-ray rotation by an angle α around the window normal can be elegantly described as refraction at the interface between media with a refractive-index ratio of the form $\exp(\mathrm{i}\alpha)$,[11] which in turn has inspired descriptions of the effect of such windows on ray bundles as imaging between complex object and image distances.[2]

The apparent wave-optical prohibition of light-ray fields produced by METATOYs suggests that METATOYs should offer new possibilities for optical engineering. Here we show how it is possible to spot whether or not a light-ray field is (apparently) prohibited.

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Novel Optical Systems Design and Optimization XVII, edited by G. Groot Gregory, Arthur J. Davis,
Proc. of SPIE Vol. 9193, 919304 · © 2014 SPIE · CCC code: 0277-786X/14/$18
doi: 10.1117/12.2061409

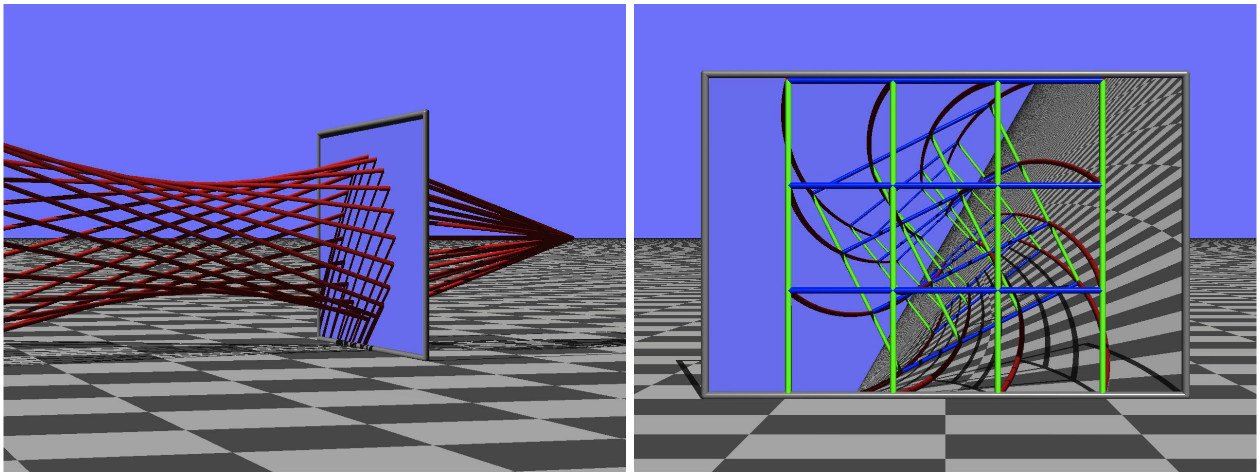

Figure 1. The effect of a ray-rotation sheet that rotates rays by 140° around the local surface normal. Left: A cone of rays is rotated into a hyperboloid as a result of ray-rotation. Such a hyperboloid is interpreted in Ref.[2] as a cone whose apex lies at a complex position. Right: A lattice of coloured cylinders as viewed throughout a 140° ray-rotation sheet. The images were created with our custom raytracer Dr TIM.[3]

2. WAVE-OPTICALLY ILLEGAL LIGHT-RAY FIELDS

When viewing optics from a geometrical perspective it can be shown that any light-ray field which can be described wave-optically must have zero curl, i.e.

$$\nabla \times \mathbf{d} = 0, \tag{1}$$

where $\mathbf{d}$ is the normalised local light-ray direction. This condition holds in the ray-optics limit,[12] i.e. where every wave locally looks like a plane wave. This is because for plane waves, $\mathbf{d}$ is proportional to the phase gradient (recall the curl of a gradient is zero $\nabla \times \nabla f = 0$), and in the ray-optics limit the length of the phase gradient is always $2\pi/\lambda$. Beyond the ray-optics limit, the length of the phase gradient can change from point to point,[13] however, after suitable normalisation, an analogous expression to the one above must still hold. If a generalised law of refraction is to be described wave-optically, the outgoing light-ray field it produces, $\mathbf{d}'$, must therefore satisfy Eq.(1). Those laws of refraction for which Eq.(1) is always satisfied are called zero-curl preserving.[14]

We concentrate on measuring the z component of the curl of a light-ray field. (The other components can be measured similarly.) The expression for the z component of the curl of a light-ray field $\mathbf{d}$,

$$\frac{\partial d_y}{\partial x} - \frac{\partial d_x}{\partial y} = \delta, \tag{2}$$

involves partial derivatives with respect to x and y, which means that it is related to the change in light-ray direction with position. For a wave-optically legal light-ray field, the rate of change of d_y with respect to x and of d_x with respect to y should be equal for the z component of the curl of $\mathbf{d}$ to be zero. For wave-optically forbidden fields, on the other hand, these rates should be different. We call this difference δ (see Eqn (2)).

3. PARALLAX FOR WAVE-OPTICALLY ALLOWED AND APPARENTLY FORBIDDEN LIGHT-RAY FIELDS

When viewing an object at a distance, it is known that as an observer moves horizontally or vertically, the object under scrutiny appears to change position relative to its background. This effect is known as parallax. In general, the further the observer is from an object, the smaller the change in light-ray direction the observer sees. As a clear example, Figure 2 shows parallax for a small green sphere sitting in front of a frame as the virtual camera moves in the x or y directions.

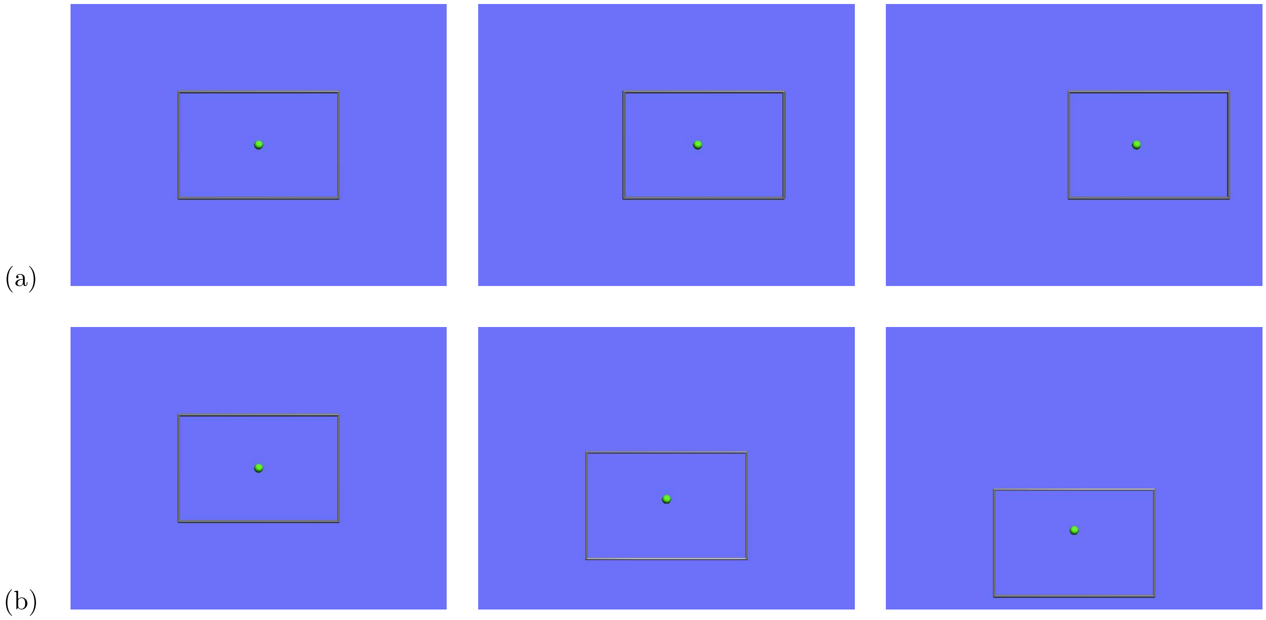

Figure 2. Parallax. A green sphere is seen in front of a fray frame. Subsequent frames show both objects as the observer moves to the left (a) or upwards (b). Then the relative position of the ball and the frame seem to change with respect to each other. These images, like all other ray-tracing simulations in this paper, were created with our custom raytracer Dr TIM.[3]

To see how light-ray rotation around the normal of a plane can turn a wave-optically allowed light-ray field into one that is apparently forbidden, consider the light-ray field from a point light source positioned at $(0, 0, -1)$, seen by an observer close to the origin. The un-normalised light-ray field from this source can then be written as

$$\mathbf{d} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}. \tag{3}$$

We consider this field close to the origin, where its length is 1, and so it is normalised (as required). The z component of the curl of this light-ray field is

$$(\nabla \times \mathbf{d})_z = \frac{\partial d_y}{\partial x} - \frac{\partial d_x}{\partial y} = 0. \tag{4}$$

After transmission through a 90° ray-rotating sheet in the $z = 0$ plane, the light-ray field is

$$\mathbf{d}' = \begin{pmatrix} -y \\ x \\ 1 \end{pmatrix}, \tag{5}$$

whose z component of the curl is

$$(\nabla \times \mathbf{d}')_z = \frac{\partial d'_y}{\partial x} - \frac{\partial d'_x}{\partial y} = 1 - (-1) = 2 \neq 0. \tag{6}$$

Fig. 3 shows a ray-tracing simulation of an object seen through a ray-rotating window for different observer positions. It shows that, for the light-ray field due to the object, the partial derivatives involved in Eqn (2) have opposite signs, and so add up to a non-zero z component of the curl.

We now validate this with experimental demonstrations using ray-rotation sheets which rotate rays by 90° around the local sheet normal. Fig. 4 shows an Rubik's cube seen through such a ray-rotating window. The

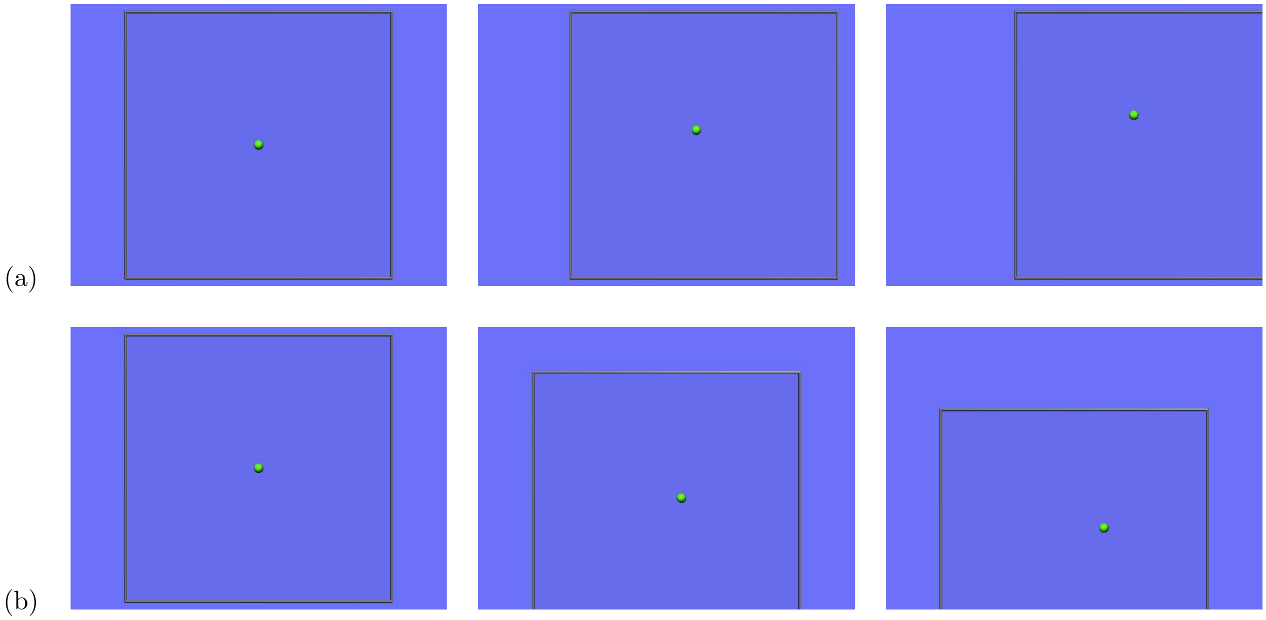

Figure 3. Parallax of an object (a green sphere) seen through a 90° ray-rotating window (framed). (a) As the virtual camera moves to the left, the apparent movement of the object has an upwards component. This demonstrates that $\partial d_y/\partial x > 0$. (b) As the virtual camera moves upwards, the apparent movement of the object has a rightwards component, showing that $\partial d_x/\partial y < 0$. The images were created with our custom raytracer Dr TIM.[3]

images produced are somewhat blurred. This is partly due to the low quality of the ray-rotation sheet, which is manufactured from pairs of ray-flipping sheets, each manufactured as described in Ref.,[8] but this is also a manifestation of the wave-optical illegality of the outgoing field. Because the outgoing field is not composed of one continuous phase front, each element in the ray-rotation sheet produces one individual phase front, and as such the image becomes pixellated, leading to diffraction effects. Analysis of the signs of the partial derivatives reveals that the z component of the light-ray-direction field is again non-zero. A more detailed quantitative analysis reveals that the z component of the curl of the light-ray direction is approximately

$$\delta = 0.0021 - (-0.0039) = 0.006 \pm 0.003. \tag{7}$$

4. CONCLUSION

We have described how light-ray fields which cannot be described wave-optically differ visually from conventional fields. Backed up by ray-tracing simulations, we have presented here the first experimental observation of such wave-optically forbidden light-ray fields. As such light-ray fields should offer new possibilities for optical design, this should provide a significant impetus for research into such light-ray fields.

ACKNOWLEDGMENTS

S.O. is funded by a studentship from the UK's Engineering and Physical Sciences Research Council (EPSRC).

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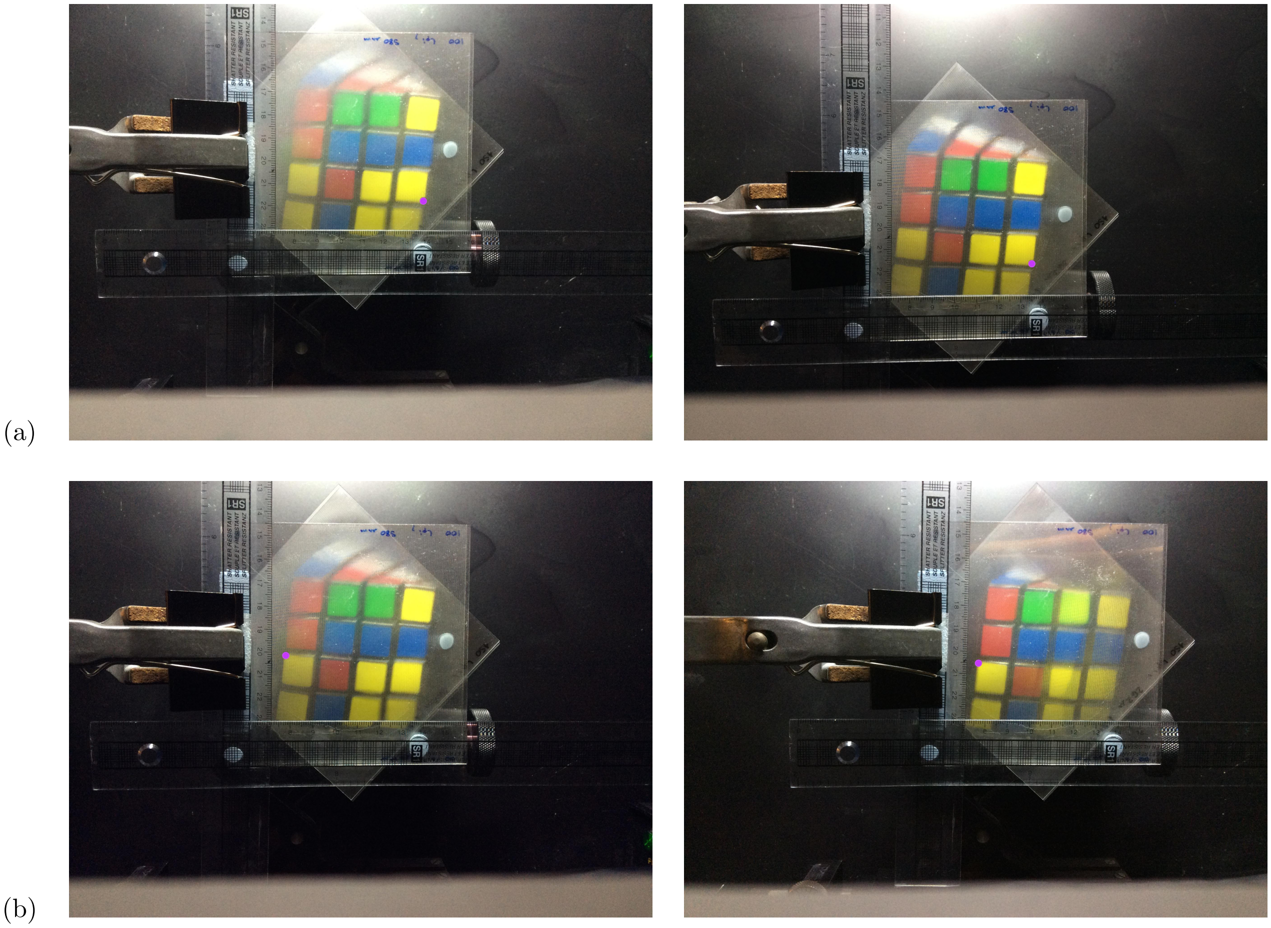

Figure 4. Experimental observation of parallax of standard point light sources and point light sources seen through a 90° ray-rotating window. Any feature of an object not seen through the window is an example of a standard point light source; any feature on the Rubik's cube seen through the window, for example the corner of the yellow square marked with a purple dot, gives rise to a wave-optically forbidden light-ray field. (a) The picture on the right was taken from a camera position 3 cm higher than that from which the left picture was taken. The purple dot appears (slightly) further left in the right image, which means the incident light-ray direction has a more positive x component. This implies that $\partial d_x/\partial y > 0$. (b) The right picture was taken with the camera moved to the left compared to the left picture; in it, the purple dot appears slightly lower. This means that $\partial d_y/\partial x < 0$. This provides confirmation of the effect of wave-optical illegality for a $\pi/2$ ray-rotating window.

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