Extraction of timelike entanglement from the quantum vacuum

S. Jay Olson* and Timothy C. Ralph
Department of Physics, University of Queensland, St Lucia, Queensland 4072, Australia
(Received 8 September 2011; published 4 January 2012)

An intriguing property of the massless quantum vacuum state is that it contains entanglement between both spacelike *and* timelike separated regions of space-time. The implications of timelike entanglement and its connection to standard entanglement, however, are unexplored. Here we show that timelike entanglement can be extracted from the massless Minkowski vacuum and converted into standard entanglement "at a given time" between two inertial, two-state detectors at the same spatial location: one coupled to the field in the past and the other coupled to the field in the future. The procedure used here demonstrates a time correlation as a requirement for extraction; e.g., if the past detector was active at a quarter to 12:00, then the future detector must wait to become active at precisely a quarter past 12:00 in order to achieve entanglement.

DOI: 10.1103/PhysRevA.85.012306 PACS number(s): 03.67.Bg, 03.65.Ud

I. INTRODUCTION

Quantum theory introduces stronger than classical correlations in states that have the property of nonseparability, i.e., entangled states. Although entanglement seems like an exotic phenomenon, it is quite ubiquitous; even the zero-particle state of the quantum vacuum can be seen to be a highly entangled state between spacelike separated regions of space-time, a fact that leads to a number of important quantum field theoretical effects such as Gibbons-Hawking radiation in de Sitter space-time [1] and Unruh-Davies radiation for accelerated observers in Minkowski space-time [2,3].

Recently, it has been shown that for massless fields in the Minkowski vacuum state, in addition to the usual entanglement between spacelike separated regions or "entanglement at a given time," field entanglement exists between timelike separated regions of space-time, or "entanglement between different times" [4]. This timelike entanglement gives rise to a thermal effect whose temperature is the same as the Unruh-Davies temperature, multiplied by a factor of the speed of light. Nevertheless, the conceptual meaning and implications of quantum nonseparability across time has not been explored and is not immediately clear: what is the relationship to ordinary, constant-time entanglement?

Here we demonstrate a basic property of timelike entanglement in the vacuum, that it can be extracted and transformed into spacelike entanglement. More precisely, we show that two particle detectors, initially in the separable state $|00\rangle$ at time $t=-\infty$, one that operates in the past region P (see Fig. 1) and another that waits to operate in the timelike separated future region F, may become entangled (nonseparable) with one another at time $t=\infty$. The timelike field entanglement in the Minkowski vacuum is thus transformed into bipartite entanglement between detectors at a constant time.

We also find that in our extraction procedure, timelike entanglement manifests itself in a novel type of time correlation. The detectors we describe are constructed such that they select out a preferred moment in time, which we take to be $t=0$. If the past detector is active in the vicinity of time t_p, then we find that the future detector must be active in the vicinity of time $t_f=-t_p$ (i.e., the detectors must be active symmetrically across $t=0$) in order for the entanglement extraction procedure to be successful. Choices of interaction times that are not sufficiently symmetrical across $t=0$ will kill the extraction of entanglement. Finally, we discuss some implications of timelike entanglement extraction for quantum technology, which we exemplify with a thought experiment describing "quantum teleportation in time."

*stephanjayolson@gmail.com

II. TIMELIKE ENTANGLEMENT: DEFINITION

To establish our terminology, consider a general case of two subsets of the space-time manifold M, denoted by R_1 and R_2; the states of the field restricted to each region are described by Hilbert spaces $\mathcal{H}_{R_1}$ and $\mathcal{H}_{R_2}$, respectively. If the field operators commute between the two regions, then $\mathcal{H}_{R_1}$ and $\mathcal{H}_{R_2}$ may be regarded as describing independent systems. Then the state of the field ρ restricted to $R_1 \cup R_2$ is called *entangled* if it is not separable, that is, if it cannot be expressed as $\rho=\sum_i p_i \rho_1^i \otimes \rho_2^i$, with the ρ_1^i density operators on $\mathcal{H}_{R_1}$ and the ρ_2^i density operators on $\mathcal{H}_{R_2}$, with $p_i \geqslant 0$.

We now introduce a distinction. If all points in R_1 are spacelike separated with respect to all points in R_2, then we say that the field in R_1 is *spacelike entangled* with respect to the field in R_2. This is the usual conception of entanglement, which carries over intuitively from Schrödinger-picture quantum mechanics, where entanglement is a property of a state described at a single time. If, on the other hand, all the points in R_1 are timelike separated with respect to all points in R_2, then we refer to the nonseparability of the state as *timelike entanglement*.

It is useful to contrast timelike entanglement with the following two observations on finite-dimensional systems, which are similar enough to potentially cause confusion.

1. Consecutive (i.e., timelike separated) measurements *on the same system* are known to exhibit stronger-than-classical correlations, violating the Leggett-Garg inequalities [5]: This is entirely distinct from the timelike entanglement we discuss here, which refers to the nonseparability of the state of a multicomponent system, as guaranteed by the commutation of the field operators between R_1 and R_2.

2. Bell measurements on an entangled pair of particles should exhibit the same nonclassical correlations regardless

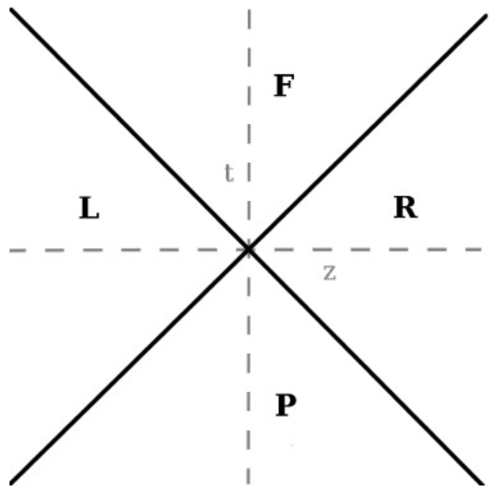

FIG. 1. Space-time divided into quadrants contained by the future and past light cones (F and P) and the right and left Rindler wedges (R and L). The massless Minkowski vacuum can be expanded in terms of modes restricted to R and L or in terms of modes restricted to F and P.

of how one separates the measurements in time (whether they are timelike or spacelike separated): In this finite-dimensional example, translation forward or backward in time just corresponds to the evolution of each isolated system. In the field theoretical context, however, the systems we study are identified by their position in space-time, such that a time translation of R_1 to a new subset $R_{1'}$ in the general case corresponds to a *different system* with independent degrees of freedom, not a simple unitary evolution of $\mathcal{H}_{R_1}$.

In what follows, we study the timelike entanglement between the regions F and P of Minkowski space-time (see Fig. 1), first by demonstrating the timelike nonseparability of the vacuum state explicitly for two-dimensional (2D) Minkowski space and then by studying an entanglement extraction procedure for full four-dimensional (4D) Minkowski space.

III. EXAMPLES: TIMELIKE ENTANGLEMENT IN THE 2D MINKOWSKI VACUUM AND SINGLE-DETECTOR RESPONSE IN THE 4D MINKOWSKI VACUUM

A. 2D vacuum state

The form of the Minkowski vacuum restricted to F-P reveals timelike entanglement explicitly for the simplified case of a $(1+1)$-dimensional space-time [4]. In $1+1$ dimensions, a free massless scalar field has the property that the left- and right-moving sectors of the field may be quantized independently (i.e., $\hat{\phi}(x,t) = \hat{\phi}(V) + \hat{\phi}(U)$, with $[\hat{\phi}(V),\hat{\phi}(U)] = 0$ for all $U = t - z$ and $V = t + z$). Thus, we now examine the left-moving sector of a free massless scalar field, and we use the approximation of a discrete set of modes labeled by ω_i.

For entanglement to be defined, we require the field in the future to be quantized as an independent system from the field in the past. For a scalar field $\hat{\phi}(x)$, this is satisfied when $[\hat{\phi}(x_F),\hat{\phi}(x_P)] = 0$, where x_F and x_P are any timelike separated points. This condition is satisfied for massless fields, and in the limit of an arbitrarily small but nonzero mass the commutator is bounded by a maximum value of $m^2/8\pi$ for any timelike separated points [6,7], and thus a sufficiently small mass also allows the approximation of independent systems between F and P.

It is well known that in 2D space-time, the Minkowski vacuum restricted to the right (R) and left (L) Rindler wedges (see Fig. 1) can be expressed as an entangled state of the Rindler modes in the following way, which can be understood as the basis for the Unruh effect [8]:

$$|0_M\rangle = \prod_i C_i \sum_{n_i=0}^{\infty} \frac{e^{-\pi n_i \omega_i/a}}{n_i!} \left(\hat{a}^{R\dagger}_{\omega_i} \hat{a}^{L\dagger}_{\omega_i}\right)^{n_i} |0_R\rangle, \tag{1}$$

where $|0_R\rangle$ is the Rindler vacuum and $\hat{a}^{R\dagger}_{\omega_i}$ is the creation operator for a right Rindler particle, corresponding to the solution $g^R_\omega(\tau+\epsilon) = (4\pi\omega)^{-1/2} e^{-i\omega(\tau+\epsilon)}$ in the coordinate system $t = a^{-1}e^{a\epsilon}\sinh(a\tau), z = a^{-1}e^{a\epsilon}\cosh(a\tau)$, with t and z being the usual Minkowski coordinates (and analogously for $\hat{a}^{L\dagger}_{\omega_i}$).

The Minkowski vacuum restricted to F-P takes a symmetrical form when expressed in terms of the "conformal modes" g^F_ω and g^P_ω [4]:

$$|0_M\rangle = \prod_i C_i \sum_{n_i=0}^{\infty} \frac{e^{-\pi n_i \omega_i/a}}{n_i!} \left(\hat{a}^{F\dagger}_{\omega_i} \hat{a}^{P\dagger}_{\omega_i}\right)^{n_i} |0_T\rangle. \tag{2}$$

Here F is coordinatized by $t = a^{-1}e^{a\eta}\cosh(a\zeta)$ and $z = a^{-1}e^{a\eta}\sinh(a\zeta)$, with P coordinatized by $t = -a^{-1}e^{a\bar{\eta}}\cosh(a\bar{\zeta})$ and $z = -a^{-1}e^{a\bar{\eta}}\sinh(a\bar{\zeta})$. The creation operators correspond to the solutions $g^F_\omega(\eta+\zeta) = (4\pi\omega)^{-1/2}e^{-i\omega(\eta+\zeta)}$ and $g^P_\omega(\bar{\eta}+\bar{\zeta}) = (4\pi\omega)^{-1/2}e^{-i\omega(\bar{\eta}+\bar{\zeta})}$, with $|0_T\rangle$ being their vacuum.

This symmetry reflects the fact that the conformal modes g^F_ω are in fact the same solutions as g^R_ω, continued from R into F.

Also mirroring the case of spacelike entanglement, the state of the field in F (or P) alone is a thermal state of the conformal modes:

$$\hat{\rho}_F = \prod_i \left[C_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i \omega_i/a} \left|n_i^F\right\rangle\!\left\langle n_i^F\right| \right]. \tag{3}$$

By symmetry, these conclusions hold also for the right-moving sector of the field. Combining both sectors of the field (labeling left moving by $\leftarrow$ and right moving by $\rightarrow$) such that $\mathcal{H}_L = \mathcal{H}_L^{\leftarrow} \otimes \mathcal{H}_L^{\rightarrow}$ and $\mathcal{H}_R = \mathcal{H}_R^{\leftarrow} \otimes \mathcal{H}_R^{\rightarrow}$, it can readily be seen that the Hilbert spaces describing the field in F and P are built from field modes extended from R and L such that $\mathcal{H}_F = \mathcal{H}_R^{\leftarrow} \otimes \mathcal{H}_L^{\rightarrow}$ and $\mathcal{H}_P = \mathcal{H}_L^{\leftarrow} \otimes \mathcal{H}_R^{\rightarrow}$. This is just the statement that left-moving modes in R propagate to F and right-moving modes in R propagated from P and so on.

B. 4D single-detector response

For a particle detector to couple to the Rindler modes g^R_ω in R, its trajectory should be an integral curve of the timelike Killing vector ∂_τ, i.e., an accelerated trajectory. This ensures that the proper time in which the detector evolves coincides with the time dependence of the Rindler modes, and the detector thus becomes sensitive to Rindler particles.

The situation in F is somewhat different since the conformal modes g^F_ω we have described above evolve in η, which does not define a timelike Killing vector in F. As a consequence, a conventional detector moving along an integral curve of ∂_η (an inertial trajectory) does not define a detector for the g^F_ω. To couple to the g^F_ω, we must introduce a different type of detector, which "evolves naturally" in η while on an inertial trajectory. Such a detector can be seen to register a thermal response, consistent with the thermal distribution of the conformal modes in F, as we now review.

The condition for the detector to evolve in η while on the (3 + 1)D inertial trajectory $x = y = z = 0$, $t = a^{-1}e^{a\eta}$ can be expressed in terms of its free Schrödinger equation, written in terms of η. Specifically, we require

$$i\frac{\partial}{\partial\eta}\Psi = H_0\Psi, \tag{4}$$

where the eigenvalues of H_0 are taken to have a constant gap E. To see what sort of detector this represents in the laboratory, we transform this equation to Minkowski time t, which reads

$$i\frac{\partial}{\partial t}\Psi = \frac{H_0}{at}\Psi. \tag{5}$$

In other words, the essential feature of this detector is that its energy gap between "ground" and "excited" states must be scaled as $1/at$ in Minkowski time.

The interaction term H_I may be taken to be the standard Unruh-DeWitt term, which for a two-state detector takes the form $H_I = \alpha\hat{\phi}(x(t))[|0\rangle\langle 1| + |1\rangle\langle 0|]$ [9]. We thus consider the full Hamiltonian:

$$i\frac{\partial}{\partial\eta}\Psi = (H_0 + e^{a\eta}H_I)\Psi. \tag{6}$$

The interaction term acquires the exponential factor due to the change of variables to conformal time.

From here, one finds the detector response function to be

$$F(E) = \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta' e^{-iE(\eta-\eta')} e^{a(\eta+\eta')} D^+(\eta,\eta'), \tag{7}$$

where $D^+(\eta,\eta') = \langle 0_M|\phi(\eta)\phi(\eta')|0_M\rangle$.

The limits of integration correspond to a detector that becomes active at $t = 0$. Another symmetry between the F-P case and the R-L case now becomes important: the two-point function along the inertial trajectory $x = y = z = 0$, $t = a^{-1}e^{a\eta}$ can be calculated to take the form

$$D^+(\eta,\eta') = \frac{a^2 e^{-a(\eta+\eta')}}{4\sinh^2\left[\frac{a}{2}(\eta-\eta') - i\epsilon\right]}, \tag{8}$$

while the two-point function along a uniformly accelerated trajectory $t = a^{-1}\sinh(a\tau)$, $x = y = 0$, $z = a^{-1}\cosh(a\tau)$ takes the form

$$D^+(\tau,\tau') = \frac{a^2}{4\sinh^2\left[\frac{a}{2}(\tau-\tau') - i\epsilon\right]}. \tag{9}$$

This symmetry leads to a formally identical response function integral in the two cases and thus to the same thermal signature when the integral is evaluated by standard techniques [8,10]. In the case of an accelerated trajectory, the acceleration plays the role of temperature, giving $T_U = \hbar a/2\pi ck$, with 1 K corresponding to an acceleration on the order of 10^{20} m/s^2. In the inertial case, it is the magnitude of the scaling of the detector energy gap a that plays the role of temperature, giving $T = \hbar a/2\pi k$, with 1 K corresponding to a scaling on the order of 100 GHz. Our main result in the following section concerns two such scaled detectors, one active in F and one in P.

C. Estimate of experimental parameters

Finally, we make an estimate of the experimental parameters required to observe single-detector thermalization due to the timelike Unruh effect described above. A "conformal observer" (one with constant conformal frequency gap) will see a constant temperature of approximately $T \approx 10^{-11}a$. The probability of excitation per inverse bandwidth at a temperature T and frequency ω_c is $\bar{n} \approx (e^{\omega_c\hbar/(k_B T)} - 1)^{-1}$. Therefore, if the conformal observer's two-level system has a resonant conformal frequency of ω_c and is coupled to the vacuum field for time $\Delta\eta \approx 1/\omega_c$, with $\bar{n} \gtrsim 0.1$, we expect an observable excitation rate.

We now need to translate these conformal parameters into laboratory frame parameters (i.e., Minkowski time intervals and frequencies). In particular, the time interval of observation in the laboratory frame corresponding to $\Delta\eta$ in conformal time is given by

$$\Delta t = \frac{\omega_c}{\omega_{mi}a}\left(e^{\frac{a}{\omega_c}} - 1\right), \tag{10}$$

where ω_{mi} is the initial frequency of the two-level systems in the laboratory frame. Also of interest is the ratio of the initial, ω_{mi}, and final, ω_{mf}, frequencies in the laboratory frame. This is given by

$$\frac{\omega_{mf}}{\omega_{mi}} = e^{-\frac{a}{\omega_c}}. \tag{11}$$

Scaling energy levels by more than a factor 3 would seem experimentally too demanding, so we will require $\omega_{mf}/\omega_{mi} \gtrsim 0.33$. This then gives us the constraint $a/\omega_c \lesssim 1.1$. Taking $a/\omega_c = 1$, we find $\bar{n} \approx 0.9$, clearly indicating an observable excitation rate.

The final consideration is the choice of appropriate initial and final Minkowski frequencies for the scaled detector. The constraint here is that we look for frequencies for which we do not expect thermal excitation in the laboratory frame due to the ambient temperature. For example, if we choose to work at room temperature, then we require the final frequency to be at or above the near infrared, $\omega_{mf} \gtrsim 5 \times 10^{13}$ s^{-1}. This leads to an observation time of $\Delta t \approx 20$ fs, during which the frequency must be scaled by a factor of 3, as noted above. At liquid-helium temperatures we can reduce our final frequency to the high microwave region, $\omega_{mf} \gtrsim 5 \times 10^{11}$ s^{-1}, giving $\Delta t \approx 2$ ps. Within a dilution refrigerator it is, in principle, possible to work at low microwave frequencies, $\omega_{mf} \gtrsim 5 \times 10^9$ s^{-1}, giving $\Delta t \approx 0.2$ ns.

We conclude that probably the best near-term prospect for observing thermalization due to timelike entanglement is in low-temperature microwave waveguides, possibly using a superconducting "artificial atom" as the two-level system [11].

IV. EXTRACTION OF TIMELIKE ENTANGLEMENT

Here we present our main results, in three components. First, we describe the extraction procedure in some detail and show that extraction of timelike entanglement from the vacuum is possible, in a manner analogous to the extraction of spacelike entanglement. Second, we note a basic symmetry property of our procedure, namely, that identical entanglement can, in principle, be extracted between regions of arbitrarily great timelike separation but that larger timelike separations require an exponentially longer interaction with the field for the detectors to achieve the same entanglement. Third, we show a fundamental time correlation in the extraction procedure.

A. Entanglement extraction procedure

We now consider two two-state, energy-scaled Unruh-DeWitt detectors, one of which is active in F, with the other active in P. Due to the field commutator, which satisfies $[\hat{\phi}(x_F),\hat{\phi}(x_P)] = 0$, the detectors interact entirely with independent systems.

In F, the detector moves along the inertial trajectory parametrized by conformal time η as $x = y = z = 0$, $t = a^{-1}e^{a\eta}$, while in the past, P, the trajectory is parametrized by $x = y = z = 0$, $t = -a^{-1}e^{a\bar{\eta}}$. The detectors are sensitive to the frequency E with respect to the conformal time in their respective quadrants. This requires the following energy scaling in terms of the Minkowski time t:

$$H_F = \frac{H_0}{at} + H_I, \tag{12}$$

$$H_P = \frac{-H_0}{at} + H_I, \tag{13}$$

where $H_0|0\rangle = E_0|0\rangle$ and $H_0|1\rangle = E_1|1\rangle$, with $E_1 - E_0 = E$, which is taken to be positive. The minus sign appearing in H_P cancels the negative value of t in the past, so that the interpretation of "ground state" $|0\rangle$ and "excited state" $|1\rangle$ is standard for both detectors.

At $t = -\infty$, we take the state of the detectors to be $|00\rangle$, and we wish to determine the state at $t = \infty$; specifically, we wish to know whether the state of the detectors is entangled. Since the energy gap of the P detector diverges as it approaches $t = 0$, we make the following assumption: After the P detector has interacted with the field in P but before $t = 0$, we assume that the detector energy scaling is adiabatically turned off (rather than allowed to "blow up" at $t = 0$), so that the state of the P detector is effectively "frozen" after its interaction with the field. Similarly, the energy scaling of the F detector is adiabatically turned on prior to its interaction with the field.

To determine entanglement, we follow an approach analogous to that of Reznik, Retzker, and Silman [9], who studied vacuum entanglement extraction from the spacelike separated Rindler wedges using two, two-state detectors (alternately interpreted as "entanglement generation" by exchange of virtual particles [12]). Specifically, we look for a positive value of the negativity of the two-detector state at $t = \infty$, which is the necessary and sufficient condition for the nonseparability of the (2×2)-dimensional system formed by our detectors [13,14].

To express the state at $t = \infty$, we use perturbation theory in the conformal time as in the single-detector case, but we now include the "window functions" $\chi_F(\eta)$ and $\chi_P(\bar{\eta})$, defined in F and P, respectively, describing the interval over which the detectors are switched on and off.

To second order in perturbation theory, the state at $t = \infty$ takes the following form:

$$\begin{aligned}
|\Psi\rangle = {} & |0_M\rangle|00\rangle - \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta'\, \chi_F(\eta)\chi_F(\eta')e^{a(\eta+\eta')}e^{-iE(\eta+\eta')}\hat{\phi}(\eta)\hat{\phi}(\eta')|0_M\rangle|00\rangle \\
& - \int_{-\infty}^{\infty} d\bar{\eta} \int_{-\infty}^{\infty} d\bar{\eta}'\, \chi_P(\bar{\eta})\chi_P(\bar{\eta}')e^{a(\bar{\eta}+\bar{\eta}')}e^{iE(\bar{\eta}+\bar{\eta}')}\hat{\phi}(\bar{\eta})\hat{\phi}(\bar{\eta}')|0_M\rangle|00\rangle \\
& - i\int_{-\infty}^{\infty} d\eta\, \chi_F(\eta)e^{a\eta}e^{-iE\eta}\hat{\phi}(\eta)|0_M\rangle|01\rangle - i\int_{-\infty}^{\infty} d\bar{\eta}\, \chi_P(\bar{\eta})e^{a\bar{\eta}}e^{iE\bar{\eta}}\hat{\phi}(\bar{\eta})|0_M\rangle|10\rangle \\
& - \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\bar{\eta}\, \chi_F(\eta)\chi_P(\bar{\eta})e^{a(\eta+\bar{\eta})}e^{-iE(\eta-\bar{\eta})}\hat{\phi}(\eta)\hat{\phi}(\bar{\eta})|0_M\rangle|11\rangle.
\end{aligned} \tag{14}$$

To simplify notation, we define the following (unnormalized) field states:

$$\begin{aligned}
C|0_M\rangle = {} & \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta'\, \chi_F(\eta)\chi_F(\eta')e^{a(\eta+\eta')}e^{-iE(\eta+\eta')}\hat{\phi}(\eta)\hat{\phi}(\eta')|0_M\rangle \\
& + \int_{-\infty}^{\infty} d\bar{\eta} \int_{-\infty}^{\infty} d\bar{\eta}'\, \chi_P(\bar{\eta})\chi_P(\bar{\eta}')e^{a(\bar{\eta}+\bar{\eta}')}e^{iE(\bar{\eta}+\bar{\eta}')}\hat{\phi}(\bar{\eta})\hat{\phi}(\bar{\eta}')|0_M\rangle,
\end{aligned} \tag{15}$$

$$|A_F\rangle = \int_{-\infty}^{\infty} d\eta\, \chi_F(\eta)e^{a\eta}e^{-iE\eta}\hat{\phi}(\eta)|0_M\rangle, \tag{16}$$

$$|A_P\rangle = \int_{-\infty}^{\infty} d\bar{\eta}\, \chi_P(\bar{\eta})e^{a\bar{\eta}}e^{iE\bar{\eta}}\hat{\phi}(\bar{\eta})|0_M\rangle, \tag{17}$$

$$|X\rangle = \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\bar{\eta}\, \chi_F(\eta)\chi_P(\bar{\eta})e^{a(\eta+\bar{\eta})}e^{-iE(\eta-\bar{\eta})}\hat{\phi}(\eta)\hat{\phi}(\bar{\eta})|0_M\rangle. \tag{18}$$

The state at $t = \infty$ can then be written in the following simplified form:

$$|\Psi\rangle = (1 - C)|0_M\rangle|00\rangle - i|A_F\rangle|01\rangle - i|A_P\rangle|10\rangle - |X\rangle|11\rangle. \tag{19}$$

We now trace over the field degrees of freedom to obtain the corresponding two-detector density matrix in the basis $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$:

$$\rho = \frac{1}{N}\begin{pmatrix} \langle 0_M|(1-C)^2|0_M\rangle & 0 & 0 & -\langle X|(1-C)|0_M\rangle \\ 0 & \langle A_F|A_F\rangle & -\langle A_P|A_F\rangle & 0 \\ 0 & -\langle A_F|A_P\rangle & \langle A_P|A_P\rangle & 0 \\ -\langle 0_M|(1-C^*)|X\rangle & 0 & 0 & \langle X|X\rangle \end{pmatrix}, \tag{20}$$

with N being the overall normalization factor.

To second order, the negativity $\mathcal{N}(\rho)$ is approximately

$$\mathcal{N}(\rho) \approx \frac{|\langle 0_M|X\rangle| - \sqrt{\langle A_F|A_F\rangle\langle A_P|A_P\rangle}}{N}. \tag{21}$$

The condition for nonseparability of the two-detector state is that $\mathcal{N}(\rho) > 0$. When the window functions χ_F and χ_P are symmetrical about $t = 0$ (as well as the scaling constant a and conformal frequency gap E, so that $\langle A_F|A_F\rangle = \langle A_P|A_P\rangle$), this nonseparability condition amounts to the following:

$$\left|\int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\bar\eta\ \chi_F(\eta)\chi_P(\bar\eta)e^{a(\eta+\bar\eta)}e^{-iE(\eta-\bar\eta)}\langle 0_M|\hat\phi(\eta)\hat\phi(\bar\eta)|0_M\rangle\right|$$
$$> \left|\int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta'\ \chi_F(\eta)\chi_F(\eta')e^{a(\eta+\eta')}e^{-iE(\eta-\eta')}\langle 0_M|\hat\phi(\eta)\hat\phi(\eta')|0_M\rangle\right| \tag{22}$$

We have used the notation that $\hat\phi(\eta) = \hat\phi(x(\eta))$ along the trajectory $t = a^{-1}e^{a\eta}$, $\vec{x} = 0$ in F, while in P we have that $t = -a^{-1}e^{a\bar\eta}$, $\vec{x} = 0$.

The quantity $\langle 0_M|\hat\phi(x)\hat\phi(x')|0_M\rangle$ takes the ordinary regularized form $-\frac{1}{4\pi^2}[(t - t' - i\epsilon)^2 - (\vec{x} - \vec{x}')^2]^{-1}$ [10], and thus for the given inertial trajectories, a coordinate transformation yields (for appropriately rescaled infinitesimal regulator ϵ)

$$\langle 0_M|\hat\phi(\eta)\hat\phi(\eta')|0_M\rangle = \frac{-a^2e^{-a(\eta+\eta')}}{16\pi^2\sinh^2\left(\frac{a(\eta-\eta')}{2} - i\epsilon\right)}, \tag{23}$$

$$\langle 0_M|\hat\phi(\eta)\hat\phi(\bar\eta)|0_M\rangle = \frac{-a^2e^{-a(\eta+\bar\eta)}}{16\pi^2\cosh^2\left(\frac{a(\eta-\bar\eta)}{2} - i\epsilon\right)}. \tag{24}$$

We now consider a specific paired set of window functions:

$$\chi_F(\eta) = e^{-\eta^2}, \tag{25}$$

$$\chi_P(\bar\eta) = e^{-\bar\eta^2}. \tag{26}$$

The entanglement condition $\mathcal{N} > 0$ thus reduces to the following:

$$\left|\int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\bar\eta\ e^{-\eta^2-\bar\eta^2}e^{-iE(\eta-\bar\eta)}\cosh^{-2}\left(\frac{a(\eta-\bar\eta)}{2}\right)\right| > \left|\int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta'\ e^{-\eta^2-\eta'^2}e^{-iE(\eta-\eta')}\sinh^{-2}\left(\frac{a(\eta-\eta')}{2} - i\epsilon\right)\right|. \tag{27}$$

We express this compactly as

$$I_X > I_A. \tag{28}$$

The first quantity, I_X, is not singular and may be numerically integrated. For $E = 1$ and $a = 2$, the value of I_X is approximately 1.561. The second quantity, I_A, is formally identical to the response function integral of an accelerated detector with a window function, where E would represent a fixed proper-energy gap and the integration variable η would represent the proper time (rather than the conformal time it represents here).

To make I_A convenient to compute, we first make use of the identity $\csc^2(\pi x) = \frac{1}{\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(x-k)^2}$, so that we can write

$$\begin{aligned} I_A &= \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta'\, e^{-\eta^2-\eta'^2} e^{-iE(\eta-\eta')} \sinh^{-2}\left(\frac{a(\eta-\eta')}{2} - i\epsilon\right) \\ &= \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta'\, e^{-\eta^2-\eta'^2} e^{-iE(\eta-\eta')} \left(\frac{4}{[a(\eta-\eta')-i\epsilon]^2} + \sum_{k=1}^{\infty}\frac{4}{[a(\eta-\eta')+i\pi k]^2} + \frac{4}{[a(\eta-\eta')-i\pi k]^2}\right) \\ &= I_{\text{inertial}} + \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\eta'\, e^{-\eta^2-\eta'^2} e^{-iE(\eta-\eta')} \left(\sum_{k=1}^{\infty}\frac{4}{[a(\eta-\eta')+i\pi k]^2} + \frac{4}{[a(\eta-\eta')-i\pi k]^2}\right). \end{aligned} \tag{29}$$

The integrated sum in the final line can be evaluated numerically, while the term I_{inertial} has been evaluated by Satz [15] with careful attention paid to regularization, in the context of a fixed-gap detector. For arbitrary window function, it takes the form

$$\begin{aligned} I_{\text{inertial}} &= \pi E \int_{-\infty}^{\infty} d\eta\, \chi(\eta)^2 - 2\int_{-\infty}^{\infty} d\eta\, \chi(\eta) \int_0^{\infty} ds\, \chi(\eta-s)\left(\frac{1}{s} - \frac{\cos(Es)}{s^2}\right) \\ &\quad - 2\int_0^{\infty}\frac{ds}{s^2}\int_{-\infty}^{\infty} d\eta\, \chi(\eta)\,[\chi(\eta)-\chi(\eta-s)], \end{aligned} \tag{30}$$

where a change of variables was made to $s = \eta' - \eta''$. Numerically evaluating this quantity and combining with the sum-integral term for our chosen window function $\chi_F(\eta) = e^{-\eta^2}$ yield a value of I_A that is approximately 1.273 (i.e., smaller than $I_X = 1.561$), yielding a positive value for the negativity, demonstrating the nonseparability of the detectors at $t = \infty$.

B. Time translation of the window functions

The value of the negativity calculated in the previous manner is invariant under a simultaneous translation of the Gaussian window functions in the conformal times η and $\bar{\eta}$ by an amount x to a new pair of window functions $\chi_{xF}(\eta) = e^{-(\eta-x)^2}$ and $\chi_{xP}(\bar{\eta}) = e^{-(\bar{\eta}-x)^2}$. However, different values for x result in different widths of the window functions when expressed in the Minkowski time t (see Fig. 2). As the window functions are shifted away from the $t = 0$ origin, the detectors must remain active for far longer in order to achieve the same degree of entanglement. This supports the intuition that most of the field entanglement is concentrated in the region close to the edges of the light cone.

Quantitatively, the price to be paid in terms of the total "volume" of interaction time (in terms of Minkowski time t) to achieve identical entanglement is exponential in the conformal time displacement x. That is, the ratio $\int_0^{\infty}\chi_{xF}(\eta)dt/\int_0^{\infty}\chi_F(\eta)dt = e^{ax}$. This is analogous to the exponential falloff of entanglement with distance in the spacelike separated case noted by Reznik, Retzker, and Silman [9].

C. Time correlation

The negativity does not remain constant if only one of the window functions is shifted, spoiling the symmetry about $t = 0$. In fact, the entanglement can be completely destroyed by shifting one of the window functions sufficiently far away from its symmetrical location in time. One can readily verify that the quantities $\langle A_F|A_F\rangle$ and $\langle A_P|A_P\rangle$ in Eq. (21) are each independently invariant under a translation $\chi_F(\eta) \to \chi_F(\eta - x)$ and $\chi_F(\eta) \to \chi_F(\eta - x)$, and thus the quantity I_A in Eq. (28) remains unchanged under the translation of a single window function. However, the quantity $|\langle 0_M|X\rangle|$ in Eq. (21) does not possess this symmetry if only one of the two window functions is shifted, and thus the resulting value of $I_X = |\int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\bar{\eta}\, \chi_F(\eta - x)\chi_P(\bar{\eta})e^{-iE(\eta-\bar{\eta})}\cosh^{-2}(\frac{a(\eta-\bar{\eta})}{2})|$ will determine whether or not entanglement has been extracted. We illustrate this by plotting the quantity $I_X - I_A$, which is proportional to the negativity, as a function of x. The detector state at $t = \infty$ is thus separable only when $I_X - I_A$ is positive. The window functions used to generate the plot are the following:

$$\chi_F = e^{-(\eta-x)^2}, \quad \chi_P = e^{-\bar{\eta}^2}. \tag{31}$$

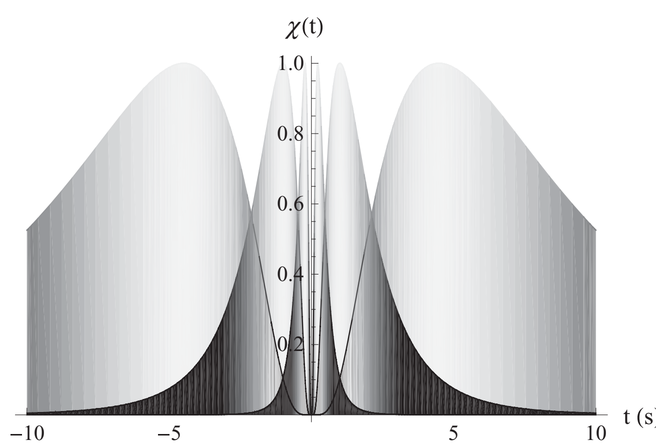

FIG. 2. Three paired sets of window functions, $\chi_{xF}(\eta) = e^{-(\eta-x)^2}$ and $\chi_{xP}(\bar{\eta}) = e^{-(\bar{\eta}-x)^2}$, symmetrically paired across $t = 0$ and plotted in Minkowski time t, which produce identical entanglement between the detectors (corresponding to the detector parameter $a = 1$ and window function parameters $x = -1.5$, $x = 0$, and $x = 1.5$). As the window functions are shifted away from $t = 0$ (corresponding to larger values of x), the detectors remain active for an exponentially longer duration to achieve identical entanglement.

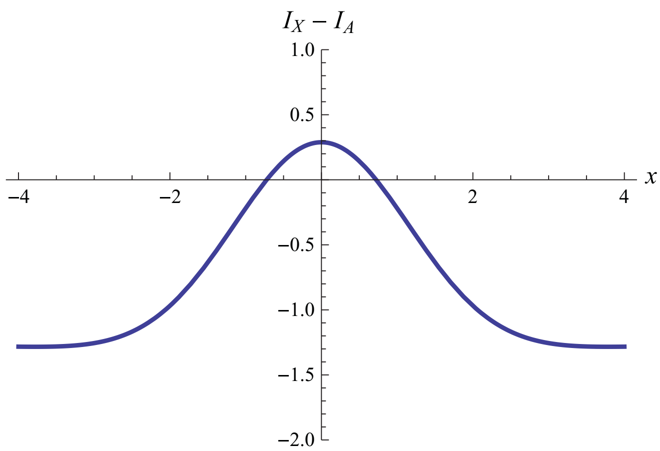

FIG. 3. (Color online) Numerical value of the quantity $I_X - I_A$ for choice of window functions $\chi_P = e^{-\bar{\eta}^2}$ and $\chi_F = e^{-(\eta - x)^2}$ for detector parameters $E = 1$ and $a = 2$. The detectors are entangled at $t = \infty$ for positive values of $I_X - I_A$ only. The position of the window functions must be sufficiently symmetrical about $t = 0$ (corresponding to a sufficiently small value of x) in order to extract entanglement from the vacuum between the timelike separated regions F and P.

This corresponds to a situation where the past detector has already interacted with the field at the time $\bar{\eta} = 0$, and we must now select the time at which the future detector will be active, with the quantity x signifying how far (in conformal time) it will be displaced from the point of symmetry. $I_X - I_A$ is plotted as a function of x in Fig. 3: positive values correspond to an entangled state of the detectors at $t = \infty$, while negative values correspond to a separable final state. Entanglement is maximized around the symmetrical point in time, while a sufficiently nonsymmetrical choice of x kills the extraction of timelike entanglement entirely.

Stated more dramatically, a detector that is switched on and off in the vicinity of a quarter to 12:00 can become entangled with a detector interacting with the field at the same spatial location in the future, but only if the later detector waits to be switched on and off at a quarter past 12:00.

V. DISCUSSION AND CONCLUSIONS

We have attempted to partially answer the question of how timelike entanglement in the Minkowski vacuum relates to the familiar notion of entanglement at a given time by showing that timelike entanglement may be extracted and converted into ordinary entanglement between two two-state detectors. We thus conclude that timelike entanglement may be regarded as a nonclassical resource in a manner analogous to the spacelike entanglement often studied in the Minkowski vacuum since a quantum information theoretic protocol may utilize conversion of timelike entanglement to spacelike entanglement as a step in the protocol. In addition, it may be possible to generate qualitatively new behavior in quantum information protocols simply by substituting timelike entanglement as the resource.

As a thought experiment to illustrate this possibility, we imagine a quantum teleportation protocol in which the entanglement resource is between a detector interacting in P and a detector interacting in F and all operations on the P detector and the qubit to be teleported take place before $t = 0$. Classical information alone is then sent into F, where the F detector must wait to interact with the field there at a particular time to form the other half of the entanglement resource. The classical information from P is then used to transform the F detector into the teleported qubit. Such a protocol might be called teleportation in time since there exists a period after $t = 0$ but before the future interaction time where it is not possible to recover the teleported qubit.

ACKNOWLEDGMENTS

We thank Nicolas Menicucci for stimulating discussions. In addition, we thank the Defence Science and Technology Organisation and the Australian Research Council for their support.

[1] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
[2] W. Unruh, Phys. Rev. D **14**, 870 (1976).
[3] P. Davies, J. Phys. A **8**, 609 (1975).
[4] S. J. Olson and T. C. Ralph, Phys. Rev. Lett. **106**, 110404 (2011).
[5] A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857 (1985).
[6] W. Greiner and J. Reinhardt, *Field Quantization* (Springer, Berlin, 1996).
[7] N. Bogolyubov and D. Shirkov, *Introduction to the Theory of Quantized Fields* (Wiley, New York, 1980).
[8] L. Crispino, A. Higuchi, and G. Matsas, Rev. Mod. Phys. **80**, 787 (2008).
[9] B. Reznik, A. Retzker, and J. Silman, Phys. Rev. A **71**, 042104 (2005).
[10] N. Birrell and P. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1984).
[11] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004).
[12] J. D. Franson, J. Mod. Opt. **55**, 2117 (2008).
[13] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996).
[14] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A **223**, 1 (1996).
[15] A. Satz, Classical Quantum Gravity **24**, 1719 (2007).