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FAULT DIAGNOSIS IN NONLINEAR SYSTEMS USING INTERCONNECTED SLIDING MODE OBSERVERS

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Abstract: This paper presents a new technique for fault diagnosis and estimation of unknown inputs in a class of nonlinear systems. The novelty of the approach is governed by the use of two interconnected sliding mode observers. The first of the two observers is used for fault diagnosis and the second is used for the reconstruction of unknown inputs. The two observers exchange their respective reconstructed signals online and in real time. Conditions for the convergence are derived. The design is such that the state trajectories do not leave the sliding manifold even in presence of unknown inputs and faults. This allows for faults and unknown inputs to be reconstructed based on information retrieved from the equivalent output error injection signals.

Keywords: Nonlinear observer, fault diagnosis, sliding mode.

1. INTRODUCTION

With the increasing demand for automation as a means of achieving optimal performance, high quality product, and higher efficiency, engineering processes have become more complex to manage and operate. The increased complexity, however, brings issues such as process reliability, safety and integrity to the fore of practical consideration. As these issues are related to the conditions under which industrial plants operate, it is of paramount importance that these deviations (due to faults) from normal operating conditions are detected and identified in real time. This in turn will prompt the necessary actions to prevent or mitigate the consequences of major disruptions to operation of industrial plants and physical harm to operators.

Model based fault detection has been the focus of much research over the past three decades; since the landmark results of (Beard 1971). The usual strategy is to generate residuals which reflect the difference between the actual and estimated values of outputs. In the no-fault case, the residual is equal to zero. In the event of a fault the residual signal acquires a nonzero value. If the system is affected by disturbances, the effect of faults has to be differentiated from that of disturbances in order to prevent false alarms (robust fault detection) (Frank and Ding 1997). Significant developments have been made in this area in linear systems and the existence conditions for unknown input observer based fault detection filters have been established (Massoumnia, Verghese et al. 1989).

However, in case of nonlinear systems advances have been less rapid. With the exception of (Seliger and Frank 1991; Koenig and Mammar 2001), little work has so far been reported in the field of fault detection in nonlinear systems with unknown inputs. Since most engineering systems possess some degree of nonlinearity, the area of designing fault detection filter for nonlinear systems with unknown inputs warrants further investigation.

Recently, sliding mode observer theory has emerged highly efficient in accounting for the effect of disturbances in nonlinear systems. Here the dynamics of the system are altered by high speed switching. As well as insensitivity to external disturbances, other main features of sliding mode theory are high accuracy and finite time convergence, which make it one of key tools in robust state estimation. To this end, some interesting results on nonlinear sliding mode observers for robust state estimation have been published (Xiong and Saif 2001; Koshkouei and Zinober 2004). In (Sreedhar, Fernandez et al. 1993), a sliding mode observer is considered to perform residual based fault detection but the paper assumes availability of the full state. This restriction of (Sreedhar, Fernandez et al. 1993) is overcome in (Koshkouei and Zinober 2004), where a disturbance decoupled subsystem is used to estimate unavailable states using a sliding mode observer. However, the technique is not extended to robust fault detection.

The focus of above approaches is mainly on state estimation and/or fault detection by residual generation. However, they are ineffective in detecting, isolating and directly reconstructing faults (or fault *identification* (Chen and Patton 1999)) and unknown inputs, simultaneously. The need for reconstruction of unknown inputs not only facilitates fault detection and identification (as shown in this paper) but also plays an important role in the enhancement of system robustness properties.

The technique presented in this paper involves the use a network of two interconnected sliding mode observers. The two observers simultaneously reconstruct faults and unknown inputs, respectively, and exchange their corresponding estimates online. As a result, a robust fault detection scheme is obtained for nonlinear uncertain systems.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Following class of nonlinear systems is frequently considered in the literature (see for example (Persis and Isidori 2001), (Hammouri, Kinnaert et al. 1999)) for robust fault studies:

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i + D(x)d + E(x)\mu \qquad (1)$$

$$y_j = h_j(x), \quad j = 1,\dots,p \qquad (2)$$

where, $x \in \Re^n$, $d \in \Re^{q_D}$ and $\mu \in \Re^{q_E}$ denote the states, unknown inputs and faults, respectively. $u = \begin{bmatrix} u_1 & \dots & u_m \end{bmatrix}^T$ and $y = \begin{bmatrix} y_1 & \dots & y_p \end{bmatrix}^T$ represent the control input and output measurements, respectively. Vectors d and μ are unknown but bounded, that is, $\|d\| \le \alpha_d$ and $\|\mu\| \le \alpha_\mu$. These bounds can easily be ascertained from history of the plant. In the ensuing analysis we assume that $p \ge m \ge \max(q_E, q_D)$.

The approach taken in this paper is based on block diagonalisation, through input-output linearization, of the system of equations (1)-(2). This can be done by the application of a nonlinear transformation, of the form $z = \phi(x)$, based on the r*elative degree* $[r_1,\dots,r_m]$ of the system (Isidori 1996). As a result an *equivalent* linear form is obtained, which forms the basis for the design of the two interconnected sliding mode observers.

We assume that the system (1)-(2) has a vector *relative degree* $[r_1,\dots,r_m]$ and, for simplicity,

$$\textstyle\sum_{i=1}^{m} r_i = r = n \qquad (3)$$

Remark 1: For the case where $r < n$ there always exist $n - r$ functions, $\phi_j(x)$, such that $L_g\phi_j(x) = 0$; $j = r+1,\dots,n$, provided that the distribution spanned by the vector fields $g_1(x),\dots,g_m(x)$ is involutive (Isidori 1996). This implies that the approach presented in this paper can well be extended to the case when $r < n$ provided that the zero dynamics associated with the remaining $n - r$ states are asymptotically stable.

Furthermore, for the system of equations (1)-(2) to assume a special canonical *observable* form, the following two matching conditions must be satisfied (Kwatny and Blankenship 2000):

$$A1: \; E(\phi^{-1}(z)) \in \Omega^{\perp} \qquad (4)$$

$$A2: \; D(\phi^{-1}(z)) \in \Omega^{\perp} \qquad (5)$$

where

$$\Omega = span\left\{dh_i(\phi^{-1}(z)), dL_f h_i(\phi^{-1}(z)),\dots, dL_f^{r_i-2}h_i(\phi^{-1}(z))\right\}.$$

It can be easily demonstrated that, provided *A1-A2* hold, the original system (1)-(2), upon input-output linearization results in the following new coordinates ((Khalil 2002; Sharma and Aldeen 2007)): $\forall\; i = 1,\dots,m$

$$\begin{aligned} \dot{z}_j^{(i)} &= z_{j+1}^{(i)}, \qquad j = 1,\dots,r_i - 1 \\ \dot{z}_{r_i}^{(i)} &= L_f^{r_i}h_i(\phi^{-1}(z)) + L_g L_f^{r_i-1}h_i(\phi^{-1}(z))u \\ &\quad + \rho_{r_i}^{(i)}(z)d + \psi_{r_i}^{(i)}(z)\mu \end{aligned} \qquad (6)$$

$$y = Cz \qquad (7)$$

where,

$\rho_{r_i}^{(i)}(z) = L_D L_f^{r_i-1}h_i(\phi^{-1}(z))$ and $\psi_{r_i}^{(i)}(z) = L_E L_f^{r_i-1}h_i(\phi^{-1}(z))$.

This canonical structure facilitates the design of fault and unknown input reconstruction filters as detailed next.

3. ESTIMATION OF FAULTS AND UNKNOWN INPUTS

This section introduces the main results of the paper in terms of a *Theorem*, which states the structures of two interconnected sliding mode based filters. One filter is used to reconstruct faults and the other is used to reconstruct unknown inputs. The two filters operate in parallel and provide updates to each other after each iteration.

Fault reconstruction filter: Based on (6)-(7), we introduce the following fault reconstruction filter as the first block of the interconnected filter system

$$\begin{aligned} \dot{w}_1^{(i)} &= w_2^{(i)} + \gamma_{w_1}^{(i)} sign\left(z_1^{(i)} - w_1^{(i)}\right) \\ \dot{w}_2^{(i)} &= w_3^{(i)} + \gamma_{w_2}^{(i)} sign\left(\tilde{z}_2^{(i)} - w_2^{(i)}\right) \\ &\vdots \\ \dot{w}_{r_i-1}^{(i)} &= w_{r_i}^{(i)} + \gamma_{w_{r_i-1}}^{(i)} sign\left(\tilde{z}_{r_i-1}^{(i)} - w_{r_i-1}^{(i)}\right) \\ \dot{w}_{r_i}^{(i)} &= L_f^{r_i}h_i(\phi^{-1}(w)) + L_g L_f^{r_i-1}h_i(\phi^{-1}(w))u \\ &\quad + \rho_{r_i}^{(i)}(w)\hat{d} + \gamma_{w_{r_i}}^{(i)} sign\left(\tilde{z}_{r_i}^{(i)} - w_{r_i}^{(i)}\right) \end{aligned} \qquad (8)$$

where $\gamma_{w_{(\cdot)}}^{(i)} > 0$, are constant gains whose choice governs the convergence of estimated states $w^{(i)}$ to true states $z^{(i)}$, as shown in the proof of the *theorem* below. They are defined as $\forall j = 3,\dots r_i$

$$\gamma_{w_j}^{(i)} = \begin{cases} \bar{k}_{w_j}^{(i)} > 0, \text{ if } \left(z_1^{(i)} - w_1^{(i)}\right) = \dots = \left(\tilde{z}_{j-1}^{(i)} - w_{j-1}^{(i)}\right) = 0 \\ k_{w_j}^{(i)} > 0, \qquad \text{otherwise} \end{cases} \qquad (9)$$

and

$$\begin{aligned}
&\tilde{z}_2^{(i)} = w_2^{(i)} + \gamma_{w_1}^{(i)} sign\left(z_1^{(i)} - w_1^{(i)}\right)\\
&\tilde{z}_3^{(i)} = w_3^{(i)} + \gamma_{w_2}^{(i)} sign\left(\tilde{z}_2^{(i)} - w_2^{(i)}\right)\\
&\vdots\\
&\tilde{z}_{r_i}^{(i)} = w_{r_i}^{(i)} + \gamma_{w_{r_i-1}}^{(i)} sign\left(\tilde{z}_{r_i-1}^{(i)} - w_{r_i-1}^{(i)}\right)
\end{aligned} \tag{10}$$

Also in (8), vector $\hat{d}$ represents the estimate of unknown disturbances obtained from the unknown input reconstruction filter described in the sequel.

Unknown input reconstruction filter: Similarly, based on equations (6)-(7), following unknown input reconstruction filter is proposed as the second block of the interconnected filter system

$$\begin{aligned}
&\dot{\eta}_1^{(i)} = \eta_2^{(i)} + \gamma_{\eta_1}^{(i)} sign\left(z_1^{(i)} - \eta_1^{(i)}\right)\\
&\dot{\eta}_2^{(i)} = \eta_3^{(i)} + \gamma_{\eta_2}^{(i)} sign\left(\overline{z}_2^{(i)} - \eta_2^{(i)}\right)\\
&\vdots\\
&\dot{\eta}_{r_i-1}^{(i)} = \eta_{r_i}^{(i)} + \gamma_{\eta_{r_i-1}}^{(i)} sign\left(\overline{z}_{r_i-1}^{(i)} - \eta_{r_i-1}^{(i)}\right)\\
&\dot{\eta}_{r_i}^{(i)} = L_f^{r_i} h_i(\phi^{-1}(\eta)) + L_g L_f^{r_i-1} h_i(\phi^{-1}(\eta))u\\
&\qquad + \psi_{r_i}^{(i)}(\eta)\hat{\mu} + \gamma_{\eta_{r_i}}^{(i)} sign\left(\overline{z}_{r_i}^{(i)} - \eta_{r_i}^{(i)}\right)
\end{aligned} \tag{11}$$

where $\gamma_{\eta_{(\cdot)}}^{(i)} > 0$, defined below, are yet to be ascertained, $\forall j = 3,\ldots r_i$

$$\gamma_{\eta_j}^{(i)} = \begin{cases} \overline{k}_{\eta_j}^{(i)} > 0, \text{ if } \left(z_1^{(i)} - \eta_1^{(i)}\right) = \ldots = \left(\overline{z}_{j-1}^{(i)} - \eta_{j-1}^{(i)}\right) = 0 \\ k_{\eta_j}^{(i)} > 0, \quad \text{otherwise} \end{cases} \tag{12}$$

and

$$\begin{aligned}
&\overline{z}_2^{(i)} = \eta_2^{(i)} + \gamma_{\eta_1}^{(i)} sign\left(z_1^{(i)} - \eta_1^{(i)}\right)\\
&\overline{z}_3^{(i)} = \eta_3^{(i)} + \gamma_{\eta_2}^{(i)} sign\left(\overline{z}_2^{(i)} - \eta_2^{(i)}\right)\\
&\vdots\\
&\overline{z}_{r_i}^{(i)} = \eta_{r_i}^{(i)} + \gamma_{\eta_{r_i-1}}^{(i)} sign\left(\overline{z}_{r_i-1}^{(i)} - \eta_{r_i-1}^{(i)}\right)
\end{aligned} \tag{13}$$

Further, $\hat{\mu}$ in (11) denotes the reconstructed fault signal obtained from the fault reconstruction filter defined by (8). The expressions for both estimates $\hat{d}$ and $\hat{\mu}$ will be derived later on in this section.

Remark 2: The structure of two filters in light of definitions (10) and (13) ensures that the sliding manifolds (which represent zero estimation errors in the case at hand) are reached state by state (sequentially). That is, asymptotic convergence of the j^{th} state can take place only when all the previous states, $1,\ldots, j-1$, have converged to their true values. As a result, high gain dynamics are obtained with no or much reduced peaking phenomenon (Khalil 2002).

Remark 3: Constant filter gains, $\gamma_{(\cdot)}^{(i)} > 0$, are chosen as per (9) and (12). Initially, first set of gains, $k_{(\cdot)}^{(i)}$, is chosen to ensure estimation error dynamics remain uniformly *bounded*. Then, (as shown in the proof of the theorem below) the gains are switched, sequentially, to their new values, $\overline{k}_{(\cdot)}^{(i)}$, to guarantee the convergence of estimation error to zero, state by state.

In order to reconstruct faults and unknown inputs, the two filters (defined by (8) and (11)) run simultaneously in parallel. Each filter injects its reconstructed signal into the other online. In the following, we summarize the main results of this paper in terms of a *Theorem.* Then the conditions required for the stability and asymptotic convergence of error dynamics of each filter are established in the proof of the *Theorem*. In this respect, we assume that the following (Lipschitz) condition holds.

A3. Let $(z,\hat{z}) \in \Omega \times \Omega$ where the set $\Omega \subseteq \Re^n$. Then, there exists a constant $l_p > 0$ such that

$$\left\| L_f^{r_i} h_i(z) - L_f^{r_i} h_i(\hat{z}) + \left(L_g L_f^{r_i-1} h_i(z)) - L_g L_f^{r_i-1} h_i(\hat{z})\right)u \right\| \le l_p \left\| z - \hat{z} \right\|$$

Theorem: *If conditions A1-A3 hold and the nonlinear system, (1)-(2), has a well defined relative degree n, then the interconnection of (8) with (11) can act as fault and unknown input reconstruction filters, respectively, to simultaneously reconstruct fault signals* μ *and unknown inputs* d. *The estimates of fault signal and unknown input are given by* $\hat{\mu} \to \Omega_\mu^+ \left[\overline{k}_{w_{r_i}}^{(i)} sign\left(e_{w_{r_i}}^{(i)}\right)\right]_{eq,\,\forall i}$ *and* $\hat{d} \to \Omega_d^+ \left[\overline{k}_{\eta_{r_i}}^{(i)} sign\left(e_{\eta_{r_i}}^{(i)}\right)\right]_{eq,\,\forall i}$, *respectively, where*

$$\Omega_\xi = \begin{bmatrix} L_\xi L_f^{r_1-1} h_1(\phi^{-1}(w)) \\ \vdots \\ L_\xi L_f^{r_i-1} h_i(\phi^{-1}(w)) \\ \vdots \\ L_\xi L_f^{r_m-1} h_i(\phi^{-1}(w)) \end{bmatrix}_{m\times q_\xi} ; \left[\overline{k}_{\theta_{r_i}}^{(i)} sign\left(e_{\theta_{r_i}}^{(i)}\right)\right]_{eq,\,\forall i} = \begin{bmatrix} \left[\gamma_{\theta_{r_1}}^{(1)} sign\left(e_{\theta_{r_1}}^{(1)}\right)\right]_{eq} \\ \vdots \\ \left[\overline{k}_{\theta_{r_i}}^{(i)} sign\left(e_{\theta_{r_i}}^{(i)}\right)\right]_{eq} \\ \vdots \\ \left[\overline{k}_{\theta_{r_m}}^{(m)} sign\left(e_{\theta_{r_m}}^{(i)}\right)\right]_{eq} \end{bmatrix}_m ;$$

$$\xi = E, D;\; \tilde{\xi} = \begin{cases} \mu & \text{if } \xi = E \\ d & \text{if } \xi = D \end{cases} ;\theta = w, \eta$$

and Ω_μ^+ *and* Ω_d^+ *symbolize the pseudo inverse of* Ω_μ *and* Ω_d, *respectively.*

Proof of theorem:

Define the observation error of the *fault reconstruction filters* as

$$e_w^{(i)} = \left(z^{(i)} - w^{(i)}\right) \tag{14}$$

and that of *unknown input reconstruction filter* as

$$e_\eta^{(i)} = \left(z^{(i)} - \eta^{(i)}\right) \tag{15}$$

Then, due to (6) and (8) the error dynamics of the fault reconstruction filter are obtained as

$$\begin{aligned}
&\dot{e}_{w_1}^{(i)} = e_{w_2}^{(i)} - \gamma_{w_1}^{(i)} sign\left(z_1^{(i)} - w_1^{(i)}\right)\\
&\dot{e}_{w_2}^{(i)} = e_{w_3}^{(i)} - \gamma_{w_2}^{(i)} sign\left(\tilde{z}_2^{(i)} - w_2^{(i)}\right)\\
&\vdots\\
&\dot{e}_{w_{r_i}}^{(i)} = \Delta L_f^{r_i} h_i(w) + \Delta L_g L_f^{r_i-1} h_i(w)u + \delta_{r_i,w}^{(i)}\\
&\qquad + \psi_{r_i}^{(i)}(z)\mu - \gamma_{w_{r_i}}^{(i)} sign\left(\tilde{z}_{r_i}^{(i)} - w_{r_i}^{(i)}\right)
\end{aligned} \tag{16}$$

where,

$$\Delta L_f^{r_i} h_i(w) = \left(L_f^{r_i} h_i(\phi^{-1}(z)) - L_f^{r_i} h_i(\phi^{-1}(w))\right)$$

$$\Delta L_g L_f^{r_i-1} h_i(w) = \left(L_g L_f^{r_i-1} h_i(\phi^{-1}(z)) - L_g L_f^{r_i-1} h_i(\phi^{-1}(w))\right)$$

$$\delta_{r_i,w}^{(i)} = \left(L_D L_f^{r_i-1} h_i(\phi^{-1}(z))d - L_D L_f^{r_i-1} h_i(\phi^{-1}(w))\hat{d}\right).$$

Similarly, from (6) and (11) the error dynamics of the unknown input reconstruction filter are obtained as

$$\begin{aligned}
&\dot{e}_{\eta_1}^{(i)} = e_{\eta_2}^{(i)} - \gamma_{\eta_1}^{(i)} sign\left(z_1^{(i)} - \eta_1^{(i)}\right) \\
&\dot{e}_{\eta_2}^{(i)} = e_{\eta_3}^{(i)} - \gamma_{\eta_2}^{(i)} sign\left(\bar{z}_2^{(i)} - \eta_2^{(i)}\right) \\
&\vdots \\
&\dot{e}_{\eta_{r_i}}^{(i)} = \Delta L_f^{r_i} h_i(\eta) + \Delta L_g L_f^{r_i-1} h_i(\eta) u + \delta_{r_i,\eta}^{(i)} \\
&\qquad + \rho_{r_i}^{(i)}(z) d - \gamma_{\eta_{r_i}}^{(i)} sign\left(\bar{z}_{r_i}^{(i)} - \eta_{r_i}^{(i)}\right)
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
&\Delta L_f^{r_i} h_i(\eta) = \left(L_f^{r_i} h_i(\phi^{-1}(z)) - L_f^{r_i} h_i(\phi^{-1}(\eta))\right) \\
&\Delta L_g L_f^{r_i-1} h_i(\eta) = \left(L_g L_f^{r_i-1} h_i(\phi^{-1}(z)) - L_g L_f^{r_i-1} h_i(\phi^{-1}(\eta))\right) \\
&\delta_{r_i,\eta}^{(i)} = \left(L_E L_f^{r_i-1} h_i(\phi^{-1}(z))\mu - L_E L_f^{r_i-1} h_i(\phi^{-1}(\eta))\hat{\mu}\right)
\end{aligned}$$

In the following, Lyapunov stability theory is used to establish the reachability criteria of trajectories onto the *sliding manifolds* $e_{j_v}^{(i)} = 0$; $\forall\, j = w,\eta$ and $\forall\, v = 1,\ldots,r_i$ in proper sequence of r_i steps. While sequential convergence is being obtained, it is ensured in *step 1* that the error dynamics remain bounded.

Step 1. $0 \le t^{(i)} \le t_1^{(i)}$

Assume that $z_1^{(i)}(0) \ne w_1^{(i)}(0)$ & $z_1^{(i)}(0) \ne \eta_1^{(i)}(0)$. Then, due to (9) and (12), following fault and unknown input reconstruction filter error dynamics are resulted

$$\begin{aligned}
&\dot{e}_{\theta_1}^{(i)} = e_{\theta_2}^{(i)} - \gamma_{\theta_1}^{(i)} sign\left(e_{\theta_1}^{(i)}\right) \\
&\dot{e}_{\theta_2}^{(i)} = e_{\theta_3}^{(i)} - k_{\theta_2}^{(i)} sign\left(\tilde{z}_2^{(i)} - \theta_2^{(i)}\right) \\
&\vdots \\
&\dot{e}_{\theta_{r_i}}^{(i)} = \Delta L_f^{r_i} h_i(w) + \Delta L_g L_f^{r_i-1} h_i(w) u + \delta_{r_i,\theta}^{(i)} + \tilde{\theta}_{r_i}^{(i)} \\
&\qquad - k_{\theta_{r_i}}^{(i)} sign\left(\tilde{z}_{r_i}^{(i)} - \theta_{r_i}^{(i)}\right)
\end{aligned} \tag{18}$$

where,

$$\theta = w,\eta;\ \tilde{\theta}_{r_i}^{(i)} = \begin{cases} \psi_{r_i}^{(i)}(z)\mu & \text{if } \theta = w \\ \rho_{r_i}^{(i)}(z) d & \text{if } \theta = \eta \end{cases};\ \tilde{z}_{(.)}^{(i)} = \begin{cases} \tilde{z}_{(.)}^{(i)} & \text{if } \theta = w \\ \bar{z}_{(.)}^{(i)} & \text{if } \theta = \eta \end{cases}.$$

Setting a composite Lyapunov function as

$$V_1^{(i)} = V_{w_1}^{(i)} + V_{\eta_1}^{(i)} = \frac{\left(e_{w_1}^{(i)}\right)^2}{2} + \frac{\left(e_{\eta_1}^{(i)}\right)^2}{2} \tag{19}$$

leads to

$$\dot{V}_1^{(i)} = e_{w_1}^{(i)} \dot{e}_{w_1}^{(i)} + e_{\eta_1}^{(i)} \dot{e}_{\eta_1}^{(i)} \tag{20}$$

From (20) it is clear that the *reachability condition* can be satisfied if $e_{w_1}^{(i)} \dot{e}_{w_1}^{(i)} + e_{\eta_1}^{(i)} \dot{e}_{\eta_1}^{(i)} < 0$ (Perruquetti and Barbot 2002). By writing

$$\sum_{\theta=w,\eta} \left(e_{\theta_1}^{(i)} \dot{e}_{\theta_1}^{(i)}\right) = \sum_{\theta=w,\eta} \left(e_{\theta_1}^{(i)} e_{\theta_2}^{(i)} - \gamma_{\theta_1}^{(i)} \left|e_{\theta_1}^{(i)}\right|\right) \le -\sum_{\theta=w,\eta} \left|e_{\theta_1}^{(i)}\right| \left(\gamma_{\theta_1}^{(i)} - \left|e_{\theta_2}^{(i)}\right|\right) \tag{21}$$

it is easy to conclude that if $\left|e_{w_2}^{(i)}\right|_{\max} < \gamma_{w_1}^{(i)}$ and $\left|e_{\eta_2}^{(i)}\right|_{\max} < \gamma_{\eta_1}^{(i)}$, then the *reachability condition* is satisfied and $e_{w_1}^{(i)} \to 0$ & $e_{\eta_1}^{(i)} \to 0$ in finite time.

It may be noted that in *step 1*, as a consequence of (10) and (13), $k_{w_j}^{(i)} sign\left(\tilde{z}_j^{(i)} - w_j^{(i)}\right) \to k_{w_j}^{(i)} sign\left(e_{w_1}^{(i)}\right)$ and $k_{\eta_j}^{(i)} sign\left(\bar{z}_j^{(i)} - \eta_j^{(i)}\right) \to k_{\eta_j}^{(i)} sign\left(e_{\eta_1}^{(i)}\right)$; $\forall j = 2,\ldots,r_i$ and hence the following result due to (Sanchis and Nijmeijer 1998) is applicable.

Assuming condition *A3* holds, if gains $\gamma_{\theta_1}^{(i)},\ k_{\theta_2}^{(i)},\ldots,k_{\theta_{r_i}}^{(i)}$; $\theta = w,\eta$ are chosen such that

$$\gamma_{\theta_1}^{(i)} > \sqrt{\frac{\lambda_{\max}\left(P_\theta^{(i)}\right)}{\lambda_{\min}\left(P_\theta^{(i)}\right)}} \left\|\tilde{e}_\theta^{(i)}(0)\right\| + \max\left(\alpha_d, \alpha_\mu\right) \tag{22}$$

and $\left(1 - 2l_p \lambda_{\max}\left(P_\theta^{(i)}\right)\sqrt{r_i - 1}\right) > 0$ (22a)

then, $e_{\theta_i}^{(i)} \to 0; \forall t > 0$ and $\tilde{e}_\theta^{(i)} = \left[e_{\theta_2}^{(i)} \ \ldots \ e_{\theta_{r_i}}^{(i)}\right]^T$ converges to the ball $\tilde{e}_\theta^{(i)} < \varsigma_\theta^{(i)*}$ in finite time. Here $P_\theta^{(i)}$ is a positive definite and symmetric matrix such that

$$P_\theta^{(i)} A_\theta^{(i)} + A_\theta^{(i)T} P_\theta^{(i)} = -I$$

with $A_\theta^{(i)}$ being Hurwitz matrix defined by

$$A_\theta^{(i)} = \begin{bmatrix} -k_{\theta_2}^{(i)} / \gamma_{\theta_1}^{(i)} & 1 & \ldots & 0 \\ -k_{\theta_3}^{(i)} / \gamma_{\theta_1}^{(i)} & 0 & \ldots & 0 \\ \vdots & \vdots & \ldots & 1 \\ -k_{\theta_{r_i}}^{(i)} / \gamma_{\theta_1}^{(i)} & 0 & \ldots & 0 \end{bmatrix} \text{ and } \varsigma_\theta^{(i)*} > \sqrt{\frac{\lambda_{\max}\left(P_\theta^{(i)}\right)}{\lambda_{\min}\left(P_\theta^{(i)}\right)}} \varsigma_\theta^{(i)};$$

$$\varsigma_\theta^{(i)} = \frac{2\lambda_{\max}\left(P_\theta^{(i)}\right)\left(\sqrt{r_i - 1}\left(1 + \max_{j \in [2,r_i]}\left\{\gamma_{\theta_1}^{(i)}, k_{\theta_j}^{(i)}\right\} / \gamma_{\theta_1}^{(i)}\right)\max\left(\alpha_d, \alpha_\mu\right)\right)}{1 - 2l_p \lambda_{\max}\left(P_\theta^{(i)}\right)\sqrt{r_i - 1}}$$

As a result, at the end of *step 1*, $e_{\theta_1}^{(i)} \to 0$ and remaining error trajectories will be confined to the ball ς^*.

Step 2. $t_1^{(i)} \le t^{(i)} \le t_2^{(i)}$

Here the conditions that ensure both $e_{w_2}^{(i)}$ and $e_{\eta_2}^{(i)}$ converge to zero are established. After *step 1*, $e_{w_1}^{(i)} \to 0$ and $e_{\eta_1}^{(i)} \to 0$ and remaining errors are bounded. As a result and due to (9)-(10) and (12)-(13), the error dynamics of the interconnected filter system can be written as

$$\begin{aligned}
&0 = e_{\theta_2}^{(i)} - \left(\gamma_{\theta_1}^{(i)} sign\left(e_{\theta_1}^{(i)}\right)\right)_{eq} \\
&\dot{e}_{\theta_2}^{(i)} = e_{\theta_3}^{(i)} - \bar{k}_{\theta_2}^{(i)} sign\left(e_{\theta_2}^{(i)}\right) \\
&\dot{e}_{\theta_3}^{(i)} = e_{\theta_4}^{(i)} - k_{\theta_3}^{(i)} sign\left(\tilde{z}_3^{(i)} - \theta_3^{(i)}\right) \\
&\vdots \\
&\dot{e}_{\theta_{r_i}}^{(i)} = \Delta L_f^{r_i} h_i(\theta) + \Delta L_g L_f^{r_i-1} h_i(\theta) u + \delta_{r_i,\theta}^{(i)} + \tilde{\theta}_{r_i}^{(i)} \\
&\qquad - k_{\theta_{r_i}}^{(i)} sign\left(\tilde{z}_{r_i}^{(i)} - \theta_{r_i}^{(i)}\right)
\end{aligned} \tag{23}$$

where $\theta = w,\eta$ and the equivalent error signals, $\left(\gamma_{w_1}^{(i)} sign\left(z_1^{(i)} - w_1^{(i)}\right)\right)_{eq}$ and $\left(\gamma_{\eta_1}^{(i)} sign\left(z_1^{(i)} - \eta_1^{(i)}\right)\right)_{eq}$ are extracted by the use of a low pass filter. The equivalent error signals ensure that the trajectories remain confined to the sliding manifolds $e_{w_1}^{(i)} = e_{\eta_1}^{(i)} = 0$ while reachability onto the manifold $e_{w_2}^{(i)} = e_{\eta_2}^{(i)} = 0$ is being achieved. Furthermore, by setting a composite Lyapunov function as

$$V_2^{(i)} = V_{w_2}^{(i)} + V_{\eta_2}^{(i)} = \sum_{v=1}^{2}\left(\sum_{\theta=w,\eta}\left(e_{\theta_v}^{(i)}\right)^2 / 2\right) \tag{24}$$

we obtain

$$\dot{V}_2^{(i)} = \sum_{\theta=w,\eta} \left(e_{\theta_2}^{(i)} e_{\theta_3}^{(i)} - \bar{k}_{\theta_2}^{(i)} \left| e_{\theta_2}^{(i)} \right| \right) \le - \sum_{\theta=w,\eta} \left| e_{\theta_2}^{(i)} \right| \left(\bar{k}_{\theta_2}^{(i)} - \left| e_{\theta_3}^{(i)} \right| \right) \quad (25)$$

From this, it is straightforward to conclude that if $\left|e_{w_3}^{(i)}\right|_{\max} < \bar{k}_{w_2}^{(i)}$ and $\left|e_{\eta_3}^{(i)}\right|_{\max} < \bar{k}_{\eta_2}^{(i)}$ then $e_{w_2}^{(i)} \to 0$ and $e_{\eta_2}^{(i)} \to 0$ in finite time. Existence of such $\bar{k}_{\theta_2}^{(i)}$ is guaranteed due to the boundedness of $\left|e_{\theta_3}^{(i)}\right|$ ensured in *step 1*.

Steps 3 to r_{i-1} follow in the same way as *Step 2*.

Step r_i. $t_{r_i-1}^{(i)} \le t^{(i)} \le t_{r_i}^{(i)}$

It is clear from the above analysis that after time $t = t_{r_i-1}^{(i)}$, $e_{j_\nu}^{(i)}$ $(j = w,\eta;\ \nu = 1,\ldots,r_i-1)$ will converge to zero in proper sequence. Thus, the resulting error dynamics can be written as

$$\begin{aligned} 0 &= e_{\theta_2}^{(i)} - \left(\gamma_{\theta_1}^{(i)} sign\left(e_{\theta_1}^{(i)}\right)\right)_{eq} \\ 0 &= e_{\theta_3}^{(i)} - \left(\bar{k}_{\theta_2}^{(i)} sign\left(e_{\theta_2}^{(i)}\right)\right)_{eq} \\ &\vdots \\ \dot{e}_{\theta_{r_i}}^{(i)} &= \Delta L_f^{r_i} h_i(\theta) + \Delta L_g L_f^{r_i-1} h_i(\theta) u + \delta_{r_i,\theta}^{(i)} + \tilde{\theta}_{r_i}^{(i)} \\ &\quad - \bar{k}_{\theta_{r_i}}^{(i)} sign\left(e_{\theta_{r_i}}^{(i)}\right) \end{aligned} \quad (26)$$

Again, choosing a composite Lyapunov function as

$$V_{r_i}^{(i)} = \sum_{\nu=1}^{r_i} \left(\sum_{\theta=w,\eta} \left(e_{\theta_\nu}^{(i)}\right)^2 / 2 \right)$$

results in

$$\begin{aligned} \dot{V}_{r_i}^{(i)} &= \sum_{\theta=w,\eta} \dot{e}_{\theta_{r_i}}^{(i)} e_{\theta_{r_i}}^{(i)} \\ &\le \sum_{\theta=w,\eta} \left|e_{\theta_{r_i}}^{(i)}\right| \left(\left| \Delta L_f^{r_i} h_i(\theta) + \Delta L_g L_f^{r_i-1} h_i(\theta) u + \delta_{r_i,\theta}^{(i)} + \tilde{\theta}_{r_i}^{(i)} - \bar{k}_{\theta_{r_i}}^{(i)} \right| \right) \end{aligned} \quad (27)$$

This implies that if gains $\bar{k}_{w_{r_i}}^{(i)}$ and $\bar{k}_{\eta_{r_i}}^{(i)}$ are chosen s.t.

$$\bar{k}_{w_{r_i}}^{(i)} > \left| \Delta L_f^{r_i} h_i(w) + \Delta L_g L_f^{r_i-1} h_i(w) u + \delta_{r_i,w}^{(i)} + \psi_{r_i}^{(i)}(z)\alpha_\mu \right| \quad (28)$$

$$\bar{k}_{\eta_{r_i}}^{(i)} > \left| \Delta L_f^{r_i} h_i(\eta) + \Delta L_g L_f^{r_i-1} h_i(\eta) u + \delta_{r_i,\eta}^{(i)} + \rho_{r_i}^{(i)}(z)\alpha_d \right| \quad (29)$$

then $e_{w_{r_i}}^{(i)}$ & $e_{\eta_{r_i}}^{(i)}$ will asymptotically converge to zero. The existence of such $\bar{k}_{\theta_{r_i}}^{(i)}$ follows from the Lipschitz property of nonlinearities.

Fault signal estimation:

As $\dot{e}_{w_{r_i}}^{(i)} \to 0$, rearrangement of (26) $(\theta = w)$ yields

$$\left[L_E L_f^{r_i-1} h_i(\phi^{-1}(w))\right]\mu \to \left[\bar{k}_{w_{r_i}}^{(i)} sign\left(e_{w_{r_i}}^{(i)}\right)\right]_{eq} \quad (30)$$

Hence, combination of equation (30) for all $i = 1,\ldots,m$ results in

$$\Omega_\mu \hat{\mu} \to \left[\bar{k}_{w_{r_i}}^{(i)} sign\left(e_{w_{r_i}}^{(i)}\right)\right]_{eq,\ \forall i} \quad (31)$$

where,

$$\Omega_\mu = \begin{bmatrix} L_E L_f^{r_1-1} h_1(\phi^{-1}(w)) \\ \vdots \\ L_E L_f^{r_i-1} h_i(\phi^{-1}(w)) \\ \vdots \\ L_E L_f^{r_m-1} h_i(\phi^{-1}(w)) \end{bmatrix}_{m\times q_E} ; \left[\bar{k}_{w_{r_i}}^{(i)} sign\left(e_{w_{r_i}}^{(i)}\right)\right]_{eq,\ \forall i} = \begin{bmatrix} \left[\gamma_{w_{r_1}}^{(1)} sign\left(e_{w_{r_1}}^{(1)}\right)\right]_{eq} \\ \vdots \\ \left[\bar{k}_{w_{r_i}}^{(i)} sign\left(e_{w_{r_i}}^{(i)}\right)\right]_{eq} \\ \vdots \\ \left[\bar{k}_{w_{r_m}}^{(m)} sign\left(e_{w_{r_m}}^{(i)}\right)\right]_{eq} \end{bmatrix}_m \quad (32)$$

From (31), we deduce that the estimate of the faults $\hat{\mu}$ is obtained as

$$\hat{\mu} \to \Omega_\mu^+ \left[\bar{k}_{w_{r_i}}^{(i)} sign\left(e_{w_{r_i}}^{(i)}\right)\right]_{eq,\ \forall i} \quad (33)$$

Unknown input estimation:

Similarly, as $\dot{e}_{\eta_{r_i}}^{(i)} \to 0$, rearrangement of (26) $(\theta = \eta)$ gives

$$\left[L_D L_f^{r_i-1} h_i(\phi^{-1}(\eta))\right]d \to \left[\bar{k}_{\eta_{r_i}}^{(i)} sign\left(e_{\eta_{r_i}}^{(i)}\right)\right]_{eq} \quad (34)$$

Hence, upon combination of equation (34) for all $i = 1,\ldots,m$, we obtain

$$\Omega_d \hat{d} \to \left[\bar{k}_{\eta_{r_i}}^{(i)} sign\left(e_{\eta_{r_i}}^{(i)}\right)\right]_{eq,\ \forall i} \quad (35)$$

where,

$$\Omega_d = \begin{bmatrix} L_D L_f^{r_1-1} h_1(\phi^{-1}(\eta)) \\ \vdots \\ L_D L_f^{r_i-1} h_i(\phi^{-1}(\eta)) \\ \vdots \\ L_D L_f^{r_m-1} h_m(\phi^{-1}(\eta)) \end{bmatrix}_{m\times q_D} ; \left[\bar{k}_{\eta_{r_i}}^{(i)} sign\left(e_{\eta_{r_i}}^{(i)}\right)\right]_{eq,\ \forall i} = \begin{bmatrix} \left[\gamma_{\eta_{r_1}}^{(1)} sign\left(e_{\eta_{r_1}}^{(1)}\right)\right]_{eq} \\ \vdots \\ \left[\bar{k}_{\eta_{r_i}}^{(i)} sign\left(e_{\eta_{r_i}}^{(i)}\right)\right]_{eq} \\ \vdots \\ \left[\bar{k}_{\eta_{r_m}}^{(m)} sign\left(e_{\eta_{r_m}}^{(i)}\right)\right]_{eq} \end{bmatrix}_m \quad (36)$$

Accordingly, by using (35), an estimate of d is given by

$$\hat{d} \to \Omega_d^+ \left[\bar{k}_{\eta_{r_i}}^{(i)} sign\left(e_{\eta_{r_i}}^{(i)}\right)\right]_{eq,\ \forall i} \quad (37)$$

4. APPLICATION EXAMPLE

Let us consider the nonlinear model of single link flexible joint robot system described by following equations (Raghavan and Hendrick 1994):

$$\begin{aligned} \dot{\theta}_m &= \omega_m \\ \dot{\omega}_m &= \frac{k}{J_m}(\theta_l - \theta_m) - \frac{B_R}{J_m}\omega_m + \frac{K_\tau}{J_m}u - \frac{K_\tau}{J_m}\mu + d_m \\ \dot{\theta}_l &= \omega_l \\ \dot{\omega}_l &= -\frac{k}{J_1}(\theta_l - \theta_m) - \frac{mgh}{J_1}\sin(\theta_l) + d_l \end{aligned}$$

where, (θ_m, ω_m) are, respectively, the position and angular velocity of the motor and (θ_l, ω_l) represent those of the link. The motor is excited by the excitation signal u. μ denotes a fault signal and $d^T = [d_m \quad d_l]^T$ stands for unknown input vector and/or any un-modelled dynamics. The variables (θ_m, θ_l) are assumed measurable. Symbols J_m, J_l, k and B_R represent the moment of inertia of the motor and link, spring constant and viscous friction, respectively. The values of these parameters used in the simulation are as in (Raghavan and Hendrick 1994). Aim of this study is to simulate and reconstruct a fault, μ, in the excitation signal of the motor, u, in the presence of an unknown inputs, (d_m, d_l).

For the simulation study, we assume the unknown inputs to be sinusoidal d_m (with $\alpha_{d_m} = 0.2$) and sawtooth d_l (with $\alpha_{d_l} = 0.2$) waves with 10 rad/s frequency. The upper bound on fault signal is assumed in order of $\alpha_\mu = 0.3$. Following sets of filter gains are chosen in *step 1* to satisfy (22)-(22a) and ensure boundedness: $\left(\gamma_{w_1}^{(1)}, k_{w_2}^{(1)}, k_{w_1}^{(2)}, k_{w_2}^{(2)}\right) \equiv (1,2,1,1.5)$, $\left(\gamma_{\eta_1}^{(1)}, k_{\eta_2}^{(1)}, k_{\eta_1}^{(2)}, k_{\eta_2}^{(2)}\right) \equiv (1,2,1,1.5)$. These gains are sequentially switched to the new values of $\left(\gamma_{w_1}^{(1)}, \bar{k}_{w_2}^{(1)}, \bar{k}_{w_1}^{(2)}, \bar{k}_{w_2}^{(2)}\right) \equiv (1,6,1,4)$, $\left(\gamma_{\eta_1}^{(1)}, \bar{k}_{\eta_2}^{(1)}, \bar{k}_{\eta_1}^{(2)}, \bar{k}_{\eta_2}^{(2)}\right) \equiv$ (1,6,1,4) to ensure convergence of errors to zero as per (28)-(29). The equivalent injection signals (as in (38)) are extracted by using a low pass filter with a cut off frequency of $300\ Hz$.

5. SIMULATION RESULTS

For the simulation study, we assume that system is initially ($t = 0$ s) at rest and fault free. At this time, the filters are switched on with different initial condition than that of the system. At time $t = 3$ s the motor's excitation is switched on resulting in a signal of $u = 0.4$ per unit to be applied. As a result, the angular positions of motor and the link change to new nonzero steady state values. Then at time $t = 6$ s an *instantaneous fault* occurs whereby 3/4th of the excitation signal is lost. Then, the fault is *gradually* cleared during the time interval of $t = 6$ s to $t = 10$ s. The estimated responses of unknown inputs d_m & d_l and fault signal μ are demonstrated in figures 1(a)-(c), respectively. It is clear from these figures that the proposed technique is able to asymptotically reconstruct the true values of fault and unknown inputs present in the nonlinear system. Figure 1(c) also reveals that the fault reconstruction filter is insensitive to any external disturbances or inputs, as demonstrated by the no response of the filter to the excitation signal at t = 3 seconds.

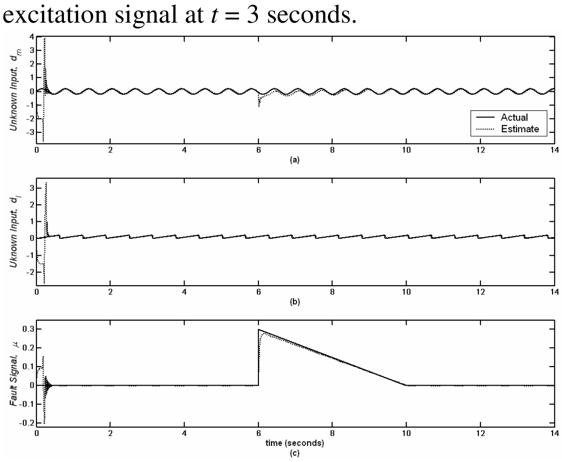

Fig.1. Estimation of (a) d_m (b) d_l (c) μ

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