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WHICH GLOBULAR CLUSTERS CONTAIN INTERMEDIATE-MASS BLACK HOLES?

HOLGER BAUMGARDT,¹ JUNICHIRO MAKINO,² AND PIET HUT³ Received 2004 September 9; accepted 2004 October 20

ABSTRACT

It has been assumed that intermediate-mass black holes (IMBHs) in globular clusters can only reside in the most centrally concentrated clusters, with a so-called core-collapsed density profile. While this would be a natural guess, it is in fact wrong. We have followed the evolution of star clusters containing IMBHs with masses between $125 \le M_{\rm BH} \le 1000 M_{\odot}$ through detailed *N*-body simulations, and we find that a cluster with an IMBH, in projection, appears to have a relatively large "core" with surface brightness only slightly rising toward the center. This makes it highly unlikely that any of the "core-collapsed" clusters will harbor an IMBH. On the contrary, the places to look for an IMBH are those clusters that can be fitted well by medium-concentration King models. The velocity dispersion of the visible stars in a globular cluster with an IMBH is nearly constant well inside the apparent core radius. For a cluster of mass M_C containing an IMBH of mass $M_{\rm BH}$, the influence of the IMBH becomes significant only at a fraction $2.5M_{\rm BH}/M_C$ of the half-mass radius, deep within the core, where it will affect only a small number of stars. In conclusion, observational detection of an IMBH may be possible, but will be challenging.

Subject headings: black hole physics — globular clusters: general — methods: *n*-body simulations — stellar dynamics

Online material: color figure

1. INTRODUCTION

Over the last few years, four lines of evidence have accumulated pointing to the possible presence of a $\sim 10^3 M_{\odot}$ black hole in some globular clusters. The first hint follows from an extrapolation of the $M_{\rm BH}-M_{\rm bulge}$ relation found for supermassive black holes in galactic nuclei (Magorrian et al. 1998), which leads to a prediction of a typical central black hole mass of $\sim 10^3 M_{\odot}$ for globular clusters (Kormendy & Richstone 1995; van der Marel 2001). The empirical $M_{\rm BH}-M_C$ relation also comes naturally from rapid mass segregation and the Spitzer instability applied to a standard initial mass function (IMF) in young, dense star clusters (Gürkan et al. 2004).

The second hint is related to the discovery of a new class of ultraluminous, compact X-ray sources (ULXs). Their high luminosities and strong variability suggest that they are intermediatemass black holes (IMBHs), rather than binaries containing a normal stellar-mass black hole, and they may occur preferentially in young star clusters (Zezas et al. 2002).

The third hint stems from an analysis of the central velocity dispersions of specific globular clusters. Gerssen et al. (2002, 2003) and Gebhardt et al. (2002) have published evidence for black holes in M15 and G1 with masses of the order of 10^3 and $10^4 M_{\odot}$, respectively (since M31's G1 is 1 order of magnitude more massive than typical globular clusters in our Galaxy, both values fall on the $M_{\rm BH}-M_{\rm bulge}$ relation).

The fourth hint is based on detailed *N*-body simulations by Portegies Zwart et al. (2004) of the evolution of a young (~10 Myr) star cluster in M82, the position of which coincides with an ULX with luminosity $L > 10^{40}$ ergs s⁻¹. They found that runaway merging of massive stars could have led to the formation of an IMBH of ~10³ M_{\odot} . Since globular clusters in their youth may have resembled this type of star cluster, it

Sternwarte, Universität Bonn, Auf dem Hügel 71, 53121 Bonn, Germany.
 Department of Astronomy, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku,

Tokyo 113-0033, Japan. ³ Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540. is altogether likely that at least some globular clusters harbor IMBHs.

None of these four hints in itself carries enough weight to be convincing. Given our lack of understanding of the formation process of globular clusters, there is no strong reason to expect the $M_{\rm BH}-M_{\rm bulge}$ relation to carry over to globular clusters. ULXs may just be unusual forms of X-ray binaries containing a massive but still stellar-mass black hole. The velocity dispersion profiles of M15 and G1 can be reproduced by simulations without central black holes (Baumgardt et al. 2003a, 2003b). Still, the fact that the four arguments are so different in character does suggest that we have to take the possible existence of IMBHs very seriously.

The question arises: which globular clusters contain IMBHs? The intuitive answer would be: clusters with a steep central luminosity profile, both because a higher density might suggest an easier formation of a large black hole and because such a black hole could be expected to draw more stars inward.

The main message of this paper is that both of these arguments are wrong. Since the most plausible formation scenario of an IMBH is runaway merging in the first 10^7 yr after the formation of a cluster, dynamical relaxation makes a comparison with current conditions irrelevant, and thus invalidates the first intuitive argument. More importantly, dynamical *N*-body simulations reported in this paper clearly show that the second intuitive argument is false as well. We find that IMBHs, whenever they are formed, quickly puff up the core to a size far larger than that of the so-called core collapsed clusters. In fact, we show that globular clusters with IMBHs have the appearance of normal King model clusters except in the central regions. We discuss the observational implications in § 4, after describing our simulation methods in § 2 and our numerical results in § 3.

2. MODELING METHOD

The reason why clusters with unresolved cores have been the primary candidate for harboring an IMBH is that there

$M_{ m BH} \ (M_{\odot})$	Ν	W_0	$R_{h, \text{ init}}$ (pc)	$M_{c,{ m fin}}\ (M_{\odot})$	$R_{h, \text{fin}}$ (pc)	$R_{h, \text{ pro}}$ (pc)	R_C (pc)	$R_C/R_{h, \rm pro}$	$\log T_{r,h}$ (yr)
125	65536	5	2.03	21451.3	10.12	4.14	0.55	0.13	9.54
125	65536	9	2.03	21749.6	12.34	6.20	0.65	0.11	9.67
125	131072	7	4.91	45534.8	12.31	5.98	0.81	0.14	9.82
250	131072	7	4.91	45311.2	12.60	6.46	0.71	0.11	9.84
500	131072	7	4.91	44771.1	13.70	7.76	0.64	0.08	9.89
1000	131072	7	4.91	45300.4	14.07	7.96	0.58	0.08	9.91

 TABLE 1

 Results of the N-Body Runs

should be a density cusp with $\rho \propto r^{-7/4}$ around the black hole. The formation of such a cusp was first predicted by Bahcall & Wolf (1976) and later confirmed by numerical simulations (Cohn & Kulsrud 1978; Marchant & Shapiro 1980; Baumgardt et al. 2004a; Preto et al. 2004). The projected density profile therefore should have a cusp with slope -3/4.

A cusp in density, however, does not necessarily imply the existence of a cusp in luminosity, since there is no guarantee that M/L is constant. Quite the contrary, numerical simulations of core collapse have demonstrated that in post-core-collapse clusters, M/L shows a sharp rise toward the center: neutron stars and heavy white dwarfs dominate the central regions as a result of mass segregation (Baumgardt et al. 2003a). A similar rise in M/L must exist in the density cusp around an IMBH.

In previous studies, Baumgardt et al. (2004a, 2004b) have followed the evolution of star clusters with central black holes with masses of 1%-10% of the cluster mass. They found a distinct density cusp around the central black hole but no clear luminosity cusp, since the central cusp is dominated by remnant stars. The projected luminosity profile was effectively flat at the center, and the evolved clusters looked just like normal King clusters. Unfortunately, the black hole mass used in Baumgardt et al. (2004b) was too large to allow a direct comparison with observations of globular clusters. In this paper, we report the results of new simulations, starting with a more realistic central black hole with a mass of 0.1%-1% of the total cluster mass.

The setup of our runs is similar to that of the runs made by Baumgardt et al. (2004a, 2004b), and we refer the reader to these papers for a detailed description. We simulated the evolution of star clusters using the collisional N-body program NBODY4 (Aarseth 1999) on the GRAPE-6 computers at Tokyo University. Our simulations include two-body relaxation, stellar evolution, and the tidal disruption of stars by the central black hole. Initially, no binaries were present in our models. Each cluster started with a spectrum of stellar masses between 0.1 and $30 M_{\odot}$, distributed according to a Kroupa (2001) mass function, and massive central black holes initially at rest at the cluster center. The initial density profile for most models was given by a King $W_0 = 7$ model with half-mass radius $R_h = 4.8$ pc. Stellar evolution was modeled according to the fitting formulae of Hurley et al. (2000), assuming a retention fraction of neutron stars of 15%. Simulations were run for T = 12 Gyr.

If the $M_{\rm BH}-M_{\rm bulge}$ relation found by Magorrian et al. (1998) for galactic nuclei holds for globular clusters as well, the mass expected for the central black hole in an average globular cluster of mass $M = 1.5 \times 10^5 \ M_{\odot}$ would be around $1000 \ M_{\odot}$. Black hole masses of $10^3 - 3 \times 10^3 \ M_{\odot}$ were also found as the end result of runaway merging of massive stars in the dense star cluster MGG-11 by Portegies Zwart et al. (2004), although their values are likely to be upper limits, since stellar mass loss of the runaway star was not included.

Since it is not yet possible to perform a full N-body simulation of a massive globular cluster over a Hubble time, we have to scale down our simulations. Scaling down can be achieved by simulating either a smaller N cluster while keeping the mass of the central black hole unchanged, as done by Baumgardt et al. (2004b), or by scaling down the black hole mass and the cluster mass simultaneously while keeping the ratio of both constant. The first method has the advantage that the ratio of black hole to stellar mass is the same as in a real cluster, allowing a study of black hole wandering and relaxation processes in the cusp around the central black hole, while the second method is most suitable to compare the final velocity and density profile of a star cluster with observations. In the present paper we decided to employ both strategies and made several runs of star clusters containing N = 65,536 (64K) and N = 131,072 (128K) cluster stars and central black holes with masses of $M_{\rm BH} = 125, 250,$ 500, and 1000 M_{\odot} , respectively. For a N = 128K star cluster the final cluster mass is around $M_C = 45,000 M_{\odot}$, so a cluster with a 250 M_{\odot} IMBH would follow the Magorrian et al. (1998) relation.

The results of our runs are listed in Table 1, which gives, respectively, the mass of the black hole, the initial number of cluster stars, the initial depth of the central potential, the initial half-mass radius, the final cluster mass, the final half-mass radius, the projected final half-light radius, the final core radius, the ratio of the last two quantities, and the logarithm of the final half-mass relaxation time (Spitzer 1987):

$$T_{r,h} = 0.138 \frac{\sqrt{M_C} R_h^{3/2}}{\langle m \rangle \sqrt{G} \ln \Lambda},$$
(1)

where $\langle m \rangle$ is the mean mass of all stars in the cluster, and $\ln \Lambda$ is the Coulomb logarithm and is of order 10. The core radius was determined as the radius where the surface density has dropped to half its central value.

3. RESULTS

3.1. Density Profile

We first discuss the density profile of a cluster with an IMBH after it has evolved for a Hubble time. Portegies Zwart & McMillan (2002) and Rasio et al. (2004) have shown that a globular cluster has to start with a short enough central relaxation time to form an IMBH by runaway merging of main-sequence stars. Specific examples have been provided by Portegies Zwart et al. (2004), who found that the star cluster MGG-11 in the starburst galaxy M82, which has a mass of $3.5 \times 10^5 M_{\odot}$ and an initial half-mass radius of $R_h = 1.2$ pc, could have formed an



FIG. 1.—Two-dimensional density profile of bright stars for two N = 64K clusters starting with half-mass radii of $R_h = 2.0$ pc but different values for the initial central potential W_0 . The initial relaxation time was short enough that both clusters expanded by a factor $R_h \sim 5R_{h,init}$ and evolved toward the same density profile after a Hubble time.

IMBH while in the slightly larger cluster MGG-9 with a halfmass radius of $R_h = 2.6$ pc; the time for spiral-in of heavy mass stars was already too long and no runaway merging occurred. In order to study the dynamical evolution of clusters concentrated enough to form IMBHs, we first simulated two N = 64K clusters starting with a three-dimensional half-mass radius of $R_h = 2.0$ pc. These clusters have half-mass relaxation times equal to those of MGG-11. Both clusters start with black holes of $125 M_{\odot}$, in agreement with the Magorrian relation. Since the initial density profile could in principle influence the final density profile and the dynamical evolution of the cluster, we simulated two clusters starting from a King $W_0 = 5$ and a higher concentration $W_0 = 9$ model, respectively.

Figure 1 depicts the projected density profile of bright stars in both clusters at the start of the simulations and after a Hubble time. We defined bright stars to be all giants and all mainsequence stars with masses larger than 90% of the mass of turnoff stars. In order to improve statistics, we overlaid 10 snapshots spaced by 50 Myr and centered at 12 Gyr. Although initially quite different, the density profiles have become virtually indistinguishable after a Hubble time. The reason is that both clusters have expanded strongly: the final half-mass radii are about 5-6 times larger than the initial ones, and the expansion has erased the initial profile.

As shown in the right panel, this density profile can be fitted rather well by a King $W_0 = 7$ model in the range $0.1 \le R/R_h \le$ 5. Outside this range our models show extended halos, which will be truncated by a background tidal field in most realistic cases. Inside $R/R_h = 0.1$, there is some indication that the clusters have developed a weak cusp.

In order to improve statistics for the inner parts, it was necessary to add more particles to the simulation. We have simulated a set of N = 128K clusters, containing a range of IMBH masses between $125 \le M_{\rm BH} \le 1000 M_{\odot}$. The starting density profile was chosen to be a King $W_0 = 7$ model, close to the equilibrium profile found above. These calculations are quite challenging: the total amount of computing time used for the runs reported in this letter is more than half of a teraflops year, or well over 10^{19} floating point operations. It was only through the use of the GRAPE-6 system in the Astronomy Department of Tokyo University that we were able to perform these simulations.

Figure 2 depicts the projected density distribution of bright stars after the cluster evolution was simulated for 12 Gyr. Between $0.1 < R/R_h < 3$, the final profiles can be fitted by King $W_0 = 7$ profiles and by almost flat power-law profiles $\rho \sim r^{\alpha}$ inside $R/R_h = 0.1$. The measured slopes α lie between -0.1



Fig. 2.—Projected density distribution of bright stars after T = 12 Gyr for four clusters containing black holes between $125 \le M_{\rm BH} \le 1000 \ M_{\odot}$ and N = 128K stars. All clusters can be fitted by profiles in which the density is equal to a King $W_0 = 7$ model outside $R = 0.1R_h$, followed by a density increase in the inner parts. The profiles in the inner parts are nearly the same for all models. The inset shows the average profile of all N-body runs. Between $0.01 < R/R_h < 0.1$, it can be fitted by a power law with slope $\alpha = -0.25$. This is significantly flatter than the value found for galactic core-collapsed clusters, $\alpha = -0.8$. [See the electronic edition of the Journal for a color version of this figure.]



Fig. 3.—Velocity dispersion profiles for four cluster simulations that started with N = 128K stars and M = 125, 250, 500, and 1000 M_{\odot} black holes. Filled circles with error bars are the velocity dispersion of visible stars in the *N*-body runs. The influence of the central black hole grows with increasing mass. Also shown are estimates for observational error bars for a cluster with 5×10^5 stars in which the brightest 5% of all stars can be observed all the way into the center.

and -0.3 for the different models, with no clear trend with the mass of the central black hole. The mean profile of all models has a slope of $\alpha = -0.25$ (see inset). A look at the projected profiles of other stars shows that the heavy-mass stars, i.e., the heaviest white dwarfs and neutron stars, follow significantly steeper slopes near -0.5, reflecting their strong mass segregation. The overall density profile in the center, however, is quite close to the density profile of the bright stars, since the mass of the bright stars is close to the average mass of the stars in the core.

According to Noyola & Gebhardt (2004), slopes of the central surface brightness profiles of galactic globular clusters span a range of values between those for constant density core models and those for models with steep cusps up to $\alpha = -0.8$. The latter value would correspond to the luminosity profile expected for a cluster in core collapse (Baumgardt et al. 2003a). The above results show that core-collapse density profiles are too steep for clusters that contain IMBHs, but that several clusters

ters in the list from Noyola & Gebhardt (2004) have slopes compatible with the assumption that they contain IMBHs. We will come back to this point in § 4.

3.2. Velocity Dispersion Profile

We next discuss the chances of detecting an IMBH through observations of the radial velocity or proper motion profiles of a star cluster. The filled circles in Figure 3 show the projected velocity dispersion profile for four N = 128K star clusters. The lines show the predicted profiles calculated from the Jeans equation under the assumption that the velocity dispersion is isotropic, using as input the potential from the black hole and the cluster stars. These predictions form a very good fit to the *N*-body data, including the region where the influence of the black hole begins to dominate that of the stars, an effect that becomes stronger with increasing black hole mass.

Equations (2) and (3) of Baumgardt et al. (2004a) predict a linear relation between the radius R where the stellar

Name	Central Slope	$\log M_C$ (M_{\odot})	$R_{h, \text{ pro}}$ (pc)	R_C (pc)	$R_C/R_{h, \rm pro}$	С	$\log T_{r,h}$ (yr)					
NGC 5286	-0.20 ± 0.02	5.67	2.44	0.18	0.08	1.46	9.72					
NGC 5694	-0.21 ± 0.10	5.35	3.28	0.34	0.10	1.84	9.76					
NGC 5824 ^a	-0.38 ± 0.08	5.15	3.35	0.20	0.06	2.45	9.67					
NGC 6093	-0.13 ± 0.04	5.51	1.89	0.24	0.13	1.95	9.48					
NGC 6266	-0.15 ± 0.04	5.90	1.92	0.20	0.08	1.70	9.68					
NGC 6388	-0.14 ± 0.03	5.99	1.53	0.20	0.10	1.70	9.58					
NGC 6397 ^a	-0.29 ± 0.03	4.87	1.94	0.03	0.02	2.50	9.17					
NGC 6541 ^a	-0.36 ± 0.07	5.56	2.42	0.13	0.05	2.00	9.67					
NGC 6715 ^a	-0.16 ± 0.07	6.23	3.58	1.30	0.34	1.84	10.26					

TABLE 2 GC Candidates that Could Contain IMBHs

Note.—From the List of Noyola & Gebhardt (2004). ^a Unlikely to contain IMBHs, see text.

velocity dispersion is affected more by the IMBH than by the stars alone: $R/R_h = \gamma M_{\rm BH}/M_C$, where M_C is the cluster mass. The results in Figure 3 are compatible with this relation and give $\gamma \approx 2.5$. The error bars in Figure 3 show the statistical error for a star cluster containing 5×10^5 stars and in which the brightest 5% of all stars can be observed in the center. For the cluster with the lowest mass black hole, the black hole dominates only at radii $R < 0.005R_h$, corresponding to radii of R < 0.5 for a typical globular cluster. There are too few stars inside this radius, so the velocity error is too large to discern between the black-hole and the no-black-hole case. IMBHs with masses $M_{\rm BH}/M_C < 0.3\%$ can therefore not be detected by radial velocity measurements in star clusters. For the cluster with a $M_{\rm BH} = 250 \ M_{\odot}$ black hole, the detection might be possible at the 2 σ level, and higher mass black holes might be detected even under less favorable conditions. Nevertheless, even the largest simulated IMBH with 1000 M_{\odot} , which has a mass significantly above the Magorrian et al. relation, creates a central rise that is hardly significant if only the brightest cluster stars can be observed. Observational detection of an IMBH in a star cluster will therefore be a challenging task.

4. GALACTIC GLOBULAR CLUSTER CANDIDATES

In this section, we compare the projected density profile of bright stars in our simulations with the observed central surface brightness profiles of galactic globular clusters. Noyola & Gebhardt (2004) have determined surface brightness profiles for 37 globular clusters from previously published *Hubble Space Telescope (HST)* WFPC2 images. They found that the slopes of central surface brightness profiles follow a range of values, from 0 (i.e., flat cores) to -0.8. As was shown in § 3.1, the most promising candidate clusters for IMBHs have central surface brightness slopes of -0.25 and outer profiles that can be fitted by King models with $W_0 = 7$, corresponding to a concentration parameter c = 1.5. Slightly different values for W_0 and c might be possible if the tidal field plays an important role and removes stars in the halo.

Table 2 lists all clusters whose profiles are compatible with a central slope between -0.2 and -0.3 and incompatible with a flat core from the list of Noyola & Gebhardt (2004). We have also listed the central concentration c of the clusters, the projected half-light radii as given by Trager et al. (1995), and the core radii as determined by Noyola & Gebhardt (2004). Core and half-light radii were transformed into physical units with

the cluster distances from Harris (1996).⁴ The final column gives the half-mass relaxation times, calculated from the cluster masses and the half-light radii, assuming that the (three-dimensional) half-mass radius is twice as large as the (two-dimensional) half-light radius. This is approximately the case in our runs.

It can be seen that a total of 9 clusters out of 37 have central surface brightness slopes in agreement with our simulations. Among these, NGC 6397 is an unlikely candidate, since the central slope is rather steep, and its concentration c is more compatible with a core-collapsed cluster. The same could be true for NGC 5824 and NGC 6541. The half-mass relaxation time of NGC 6715 is rather long compared to what our clusters reach after a Hubble time. For the remaining five clusters, the central slopes, the ratio of the core to the half-light radius, and the relaxation times are in good agreement of what we would predict for a cluster with an IMBH. It would therefore be extremely interesting to obtain accurate radial velocity dispersions for these clusters in order to either detect IMBHs or place upper limits on their possible masses.

5. CONCLUSIONS

We have followed the evolution of star clusters containing central IMBHs with masses $125 \le M_{\rm BH} \le 1000 M_{\odot}$. All clusters show a final density profile corresponding to a King $W_0 = 7$ model outside the cluster core. Inside the core, the projected distribution of bright stars is almost flat, with only a weak rise toward the center in the form of a power law of slope $\alpha \sim -0.25$. We conclude that the luminosity profiles of several galactic globular clusters are in good agreement with the assumption that they contain IMBHs.

A definite detection of an IMBH in a globular cluster can only be made through observations of the velocity dispersion profile of stars deep within the core, since the radius where the influence of the black hole dominates over the cluster stars is given by $R/R_h = 2.5M_{\rm BH}/M_C$. This radius is 1 order of magnitude smaller than the core radius if the IMBH mass follows the Magorrian relation. About 25 stars would have to be observed inside this radius to detect the black hole at a 2 σ level.

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⁴ Catalog available at http://physun.physics.mcmaster.ca/Globular.html.

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