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Using distant globular clusters as a test for gravitational theories

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ABSTRACT

We propose to determine the stellar velocity dispersions of globular clusters in the outer halo of the Milky Way in order to decide whether the dynamics of the Universe on large scales is governed by dark matter or modified Newtonian dynamics (MOND). We show that, for a number of Galactic globular clusters, both the internal and the external accelerations are significantly below the critical acceleration parameter a_0 of MOND. This leads to velocity dispersions in the case of MOND which exceed their Newtonian counterparts by up to a factor of 3, providing a stringent test for MOND. Alternatively, in cases where high velocity dispersions are found, these would provide the first evidence that globular clusters are dark matter dominated.

Key words: globular clusters: general – dark matter.

1 INTRODUCTION

Dark matter is now generally believed to be the dominating mass component of the Universe, starting with the discovery of Zwicky (1933) that the speed of galaxies in the Coma cluster is too large to keep them gravitationally bound unless they are much heavier than one would estimate on the basis of visible matter alone. Although the currently favoured Λ cold dark matter (ΛCDM) model has proven to be remarkably successful on large scales (Spergel et al. 2003), high-resolution *N*-body simulations are still in contradiction with observations on subgalactic scales where they predict orders of magnitude more substructure than what is seen (Moore et al. 1999; Klypin et al. 1999) and also a wrong spatial distribution of the subhaloes (Kroupa, Theis & Boily 2005). Additional arguments regarding supporting and contradictory observations of structure formation in CDM models are presented by Grebel, Gallagher & Harbeck (2003) and Grebel & Gallagher (2004). The above discrepancies might be resolved if more realistic simulations that can better treat the dynamics of the interstellar gas and feedback processes become available, or they could show that our current understanding of cosmology and large-scale structure formation is still missing important ingredients.

An alternative to the dark matter hypothesis could be modified Newtonian dynamics (MOND), which was proposed by Milgrom (1983a,b) and Bekenstein & Milgrom (1984) as a way to explain the rotation curves of galaxies without the need to assume large amounts of otherwise unseen dark mass in the outer parts of galaxies. According to MOND, Newtonian dynamics breaks down in the limit of very weak accelerations, and the acceleration $\boldsymbol{a}$ experienced by a particle is given by the following (heuristic) equation:

$$\mu\left(\frac{|\boldsymbol{a}|}{a_0}\right)\boldsymbol{a} = \boldsymbol{g}_{\mathrm{N}}, \qquad (1)$$

where

$$\mu(x) = \begin{cases} x & \text{if} \quad x \ll 1 \\ 1 & \text{if} \quad x \gg 1. \end{cases} \qquad (2)$$

Here $\boldsymbol{g}_{\mathrm{N}}$ is the standard Newtonian acceleration and a_0 a constant, observationally determined to be $a_0 = 1.2 \times 10^{-8}$ cm s^{-2} (Begeman, Broeils & Sanders 1991; Sanders & McGaugh 2002). The above formulation of MOND conserves angular momentum and energy only for spherical mass configurations, but a more general formulation exists which obeys these conservation laws for all cases (Bekenstein & Milgrom 1984).

Predictions from MOND have been shown to be in good agreement with the observed rotation curves of galaxies (Begeman et al. 1991; Sanders 1996; Sanders & Verheijen 1998) and can also explain the velocities of galaxies in groups with reasonable mass-to-light ratio values for individual galaxies (Milgrom 2002). In addition, Milgrom (1995) showed that the velocity dispersion of seven dwarf galaxies which was available at the time was compatible with the predictions from MOND, a conclusion also found by Lokas (2002) with updated data for the Fornax and Draco dwarf galaxies. These successes are remarkable given the fact that, unlike the dark matter hypothesis, MOND has only one free parameter (a_0) that can be adjusted to explain observations. In addition, Bekenstein (2004) has recently developed a relativistic formulation of MOND, putting the theory on a more solid theoretical basis.

Part of the trouble in deciding whether MOND or dark matter is the better candidate in explaining the velocities of stars on galactic scales stems from the fact that cosmological structure formation is still not sufficiently understood, so, within certain limits, the mass

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and size distribution of dark matter can be adjusted to fit the observational data. It is therefore highly desirable to test MOND for objects in which no dark matter is thought to exist. Ideal candidates for such objects are globular clusters, which still form today as a result of collisions between gas clouds during major mergers of galaxies (Whitmore & Schweizer 1995), and which are believed to have formed in the same way in the early Universe (Ashman & Zepf 1992).

Testing gravity for low accelerations by using nearby globular clusters has already been tried by Scarpa, Marconi & Gilmozzi (2003). However, nearby clusters experience an acceleration from the Milky Way that is larger than the critical MOND constant a_0, and should therefore be governed by Newtonian dynamics if MOND is correct.

In the present paper we investigate the effects of MOND for a number of low-mass globular clusters in the outer halo of the Milky Way. It will be shown that, if MOND is true, these clusters would have mass-to-light ratios far larger than their Newtonian values, allowing an independent test of the predictions of MOND. The paper is organized as follows. In section 2 we calculate expected velocity dispersions and mass-to-light ratios for a number of globular clusters for the Newtonian case and in the MOND regime. Section 3 discusses how the validity of MOND could be constrained if the velocity dispersions of these clusters were known, and in Section 4 we draw our conclusions.

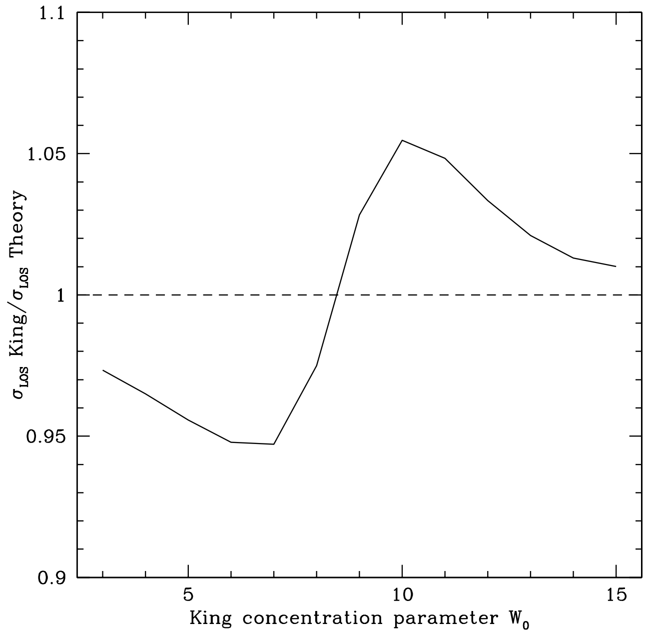

Figure 1. Ratio of the line-of-sight velocity dispersion of stars in different King models with the prediction from equation (4) (solid line). For all models, the velocity dispersion agrees with the prediction to within 5 per cent.

2 EVALUATING THE EFFECT OF MOND FOR GLOBULAR CLUSTERS

2.1 Line-of-sight velocity dispersions for different cases

In the case of Newtonian gravity, the virial theorem connects the mass M_C, radius r and average velocity dispersion σ of a cluster through the following equation (Binney & Tremaine 1986, equation 4-80a):

$$\sigma^2 = \frac{GM_C}{r_v}, \qquad (3)$$

where r_v is the virial radius, which in many stellar systems can be approximated by the three-dimensional half-mass radius r_h as $r_v \approx 2.5\, r_h$ if the clusters are stationary systems and sufficiently unperturbed that the assumption of virial equilibrium is valid. A similar relation exists between the three-dimensional half-mass radius and the easier to observe two-dimensional, projected half-mass radius r_{hp}: $r_{hp} = \gamma r_h$ with $\gamma \approx 0.74$.

If we assume an isotropic velocity distribution, the line-of-sight velocity dispersion is related to the three-dimensional one through $\sigma_{LOS} = \sigma/\sqrt{3}$, and we obtain the following equation for the observed velocity dispersion of a star cluster in the case of Newtonian gravity:

$$\sigma_{LOS,N} = 0.335\sqrt{\frac{GM_C}{r_{hp}}}. \qquad (4)$$

In order to test the accuracy of equation (4), we created N-body representations of King (1966) models with dimensionless central concentrations in the range $3 \leqslant W_0 \leqslant 15$ and checked our theoretical estimate against the N-body data. King models with concentrations in this range are usually used to represent density profiles of globular clusters. Fig. 1 compares the average velocity dispersion of stars in these models with the prediction of equation (4). For all models, the agreement is within 5 per cent, which is accurate enough for our purpose of predicting velocity dispersions in real clusters.

In the case of MOND, solutions exist only for several special cases since the function μx is defined only for limiting values of x. If the acceleration a that stars experience is much larger than a_0, $\mu \approx 1$ and the MOND solution is the same as in the Newtonian case. For this to be true it does not matter if the acceleration a is the internal acceleration a_{int} due to stars in a cluster or if it comes from an external gravitational field a_{ext}. External accelerations are, for example, important for all globular clusters that have galactocentric distances R_{GC} < few × 10 kpc or for the dynamics of the Solar system. In both cases the external acceleration due to the Milky Way is larger than a_0 and the dynamics can be described by Newtonian gravity, no matter how small the internal accelerations are. Hence MOND can be important only for star clusters in the outer halo of the Milky Way.

If both a_{int} and a_{ext} are much smaller than a_0, one is in the deep MOND regime with $\mu(x) \approx x$. If in this case the external acceleration is larger than the internal one $a_{ext} \gg a_{int}$, the system is again nearly Newtonian but with an effective gravitational constant G that is larger than the standard Newtonian one by a factor a_0/a_{ext} (Milgrom 1986). The line-of-sight velocity dispersion is therefore equal to

$$\sigma_{LOS,M1} = 0.335\sqrt{\frac{GM_C}{r_{hp}}}\sqrt{\frac{a_0}{a_{ext}}}. \qquad (5)$$

Finally, if $a_{ext} \ll a_{int}$ and both are smaller than a_0, the cluster is effectively isolated and the acceleration of the cluster stars is given by $g_M = \sqrt{a_0 g_N}$, where g_N is the acceleration in the Newtonian case. The line-of-sight velocity dispersion of a star cluster is then given by (Milgrom 1994)

$$\sigma_{LOS,M2} = 0.471(a_0 G M_C)^{1/4}, \qquad (6)$$

i.e. independent of the radius of the cluster.

Table 1. Distant globular clusters for which the predictions of MOND and Newtonian dynamics differ. For explanation of symbols see text.

Cluster Name	$\log M_C$ ($M_\odot$)	r_{hp} (pc)	R_T (pc)	R_{GC} (kpc)	a_{int} (10^{-9} cm s^{-2})	a_{ext} (10^{-9} cm s^{-2})	$\sigma_{LOS,N}$ (km s^{-1})	$\sigma_{LOS,M}$ (km s^{-1})	M/L MOND
AM 1	4.10	17.7	151.3	123.2	1.84	1.05	0.58	1.77	18.2
Eridanus	4.27	10.5	145.5	95.2	3.79	1.36	0.93	1.96	8.8
Pyxis	4.52	15.6	101.1	41.7	3.38	3.11	1.01	2.25	9.9
Pal 3	4.50	17.8	173.5	95.9	2.90	1.35	0.92	2.23	11.6
Pal 4	4.63	17.2	212.0	111.8	3.48	1.16	1.09	2.40	9.6
Pal 14	4.11	24.7	103.4	69.0	1.33	1.88	0.50	1.27	12.8
Pal 15	4.42	15.7	87.6	37.9	2.98	3.42	0.90	1.68	7.0
Arp 2	4.34	15.9	56.3	21.4	2.68	6.06	0.81	1.14	4.0

2.2 Application to globular clusters

Table 1 shows the predicted velocity dispersions of globular clusters for the Newtonian and the MOND cases. We have calculated the dispersions for all Galactic globular clusters in the list of Harris (1996), but present only those in Table 1 for which there is a noticeable difference between the two cases. Clusters in Table 1 are generally far away in the Galactic halo so that the external acceleration due to the Milky Way is small, and also have small masses and large half-mass radii so that their internal accelerations become small.

Galactocentric distances R_{GC} and projected half-mass radii are taken from Harris (1996). We have assumed that for the clusters of Table 1 the half-light radius is equal to the half-mass radius, i.e. mass follows light. This is probably a good assumption since the half-mass relaxation times of most clusters in Table 1 are of the order of a Hubble time or even larger, so dynamical evolution is not likely to play an important role for these clusters. Cluster masses were calculated from the absolute luminosities by assuming a stellar mass-to-light ratio of $M/L = 2$. This value is in agreement with observed mass-to-light ratios of globular clusters (Mandushev, Spassova & Staneva 1991; Pryor & Meylan 1993). Mandushev et al. (1991), for example, fitted single-mass King models to a sample of 32 clusters with reliable central velocity dispersions and determined an average $M/L = 1.21$ for their sample, while Pryor & Meylan (1993) used multi-component King–Michie models for 56 clusters and estimated an average global mass-to-light ratio of $M/L = 2.3$. Since multi-component models can take the effect of mass segregation into account, the latter value is probably closer to the truth.

The tidal radii R_T in Table 1 were calculated from the cluster masses by assuming a flat rotation curve of the Milky Way with $V_G = 200$ km s^{-1} for all galactocentric distances. For most clusters they are a factor of 5 to 10 larger than the half-mass radii, which means that tidal effects play no significant role for the internal dynamics of the clusters in our sample. This is a significant improvement compared with the situation for dwarf galaxies which generally have larger r_h/R_T ratios, and for which there is an ongoing discussion on the extent to which the high mass-to-light ratios found are caused by tidal effects (Kroupa 1997; Klessen & Kroupa 1998; Odenkirchen et al. 2001; Fleck & Kuhn 2003; Wilkinson et al. 2004).

Internal accelerations were calculated at the half-mass radii of the clusters from $g_M = \sqrt{a_0 g_N}$, assuming that MOND is true, while the external accelerations were calculated from

$$a_{ext} = \frac{V_G^2}{R_G} \tag{7}$$

and hold in both the Newtonian and the MOND cases. The line-of-sight velocity dispersions for the MOND case were calculated from equations (5) and (6), depending on whether the internal or external acceleration is larger. While this procedure is not strictly valid within the framework of MOND, it is accurate enough to give an estimate of the effect that MOND would have. The mass-to-light ratio values in the last column, finally, are calculated from the $\sigma_{LOS,M}$ values assuming that an observer interprets them for the Newtonian case. They should be compared with the Newtonian input value of $M/L = 2$ which we assumed.

3 DISCUSSION

From Table 1, it is evident that if MOND is true, σ_{LOS} and the deduced mass-to-light ratio are increased by a significant amount over the Newtonian case. Three clusters would have M/L values in excess of 10, which is far larger than what is found in any Galactic globular cluster. In the analysis of Mandushev et al. (1991), for example, no cluster had a mass-to-light ratio larger than 3.0, and low-mass clusters with $\log M_C < 4.5$ generally had $M/L < 1.5$. Similarly, in the analysis of Pryor & Meylan (1993), low-mass clusters with $\log M_C < 5$ had an average $M/L = 1.8$ and none had M/L larger than 3.2. If a high mass-to-light ratio were found in any cluster, it could therefore be interpreted in only one of three ways.

(i) The initial mass function of this cluster was radically different from that of ordinary globular clusters and heavily weighted either towards high-mass stars which have by now turned into compact remnants, or towards very low-mass stars in order to create such a large mass-to-light ratio.

(ii) The cluster contains a significant amount of cold dark matter.

(iii) Newtonian dynamics has to be modified in the limit of low accelerations.

Possibility (i) can almost certainly be excluded since such high mass-to-light ratios have not been found for any stellar population to date. In addition, Stetson et al. (1999) and Sarajedini (1997) determined ages and metallicities for four of the clusters in our list (Eridanus, Pal 3, 4 and 14) which place them among other Galactic globular clusters, indicating that there is nothing unusual about these clusters.

Peebles (1984) and others after him have suggested that globular clusters formed in CDM mini-haloes in the early Universe, in which case they should contain significant amounts of dark matter. So possibility (ii) seems entirely possible and a detection of dark matter dominated globular clusters would be a direct confirmation of this idea. In this case halo globular clusters could help to bridge the gap between the predicted number of dark matter sub-clumps in the haloes of galaxies and the observed number of clumps in the form of dwarf galaxies (Côté et al. 2002). Since the formation history and dark matter content would basically be the same, halo globular clusters could in such a case be viewed as smaller-sized versions of dwarf galaxies.

However, observations of interacting and starburst galaxies have shown that the formation of globular clusters is still an ongoing process, triggered mainly by major galaxy mergers (Whitmore & Schweizer 1995). Similarly, the relatively high metallicities and lower ages found by Stetson et al. (1999) and Sarajedini (1997) speak against a primordial formation scenario of these globular clusters and for an accretion scenario (Mackey & Gilmore 2004). It therefore appears unlikely that the clusters in Table 1 contain a significant amount of dark matter.

Another possible interpretation of high velocities would be that Newton's law of gravity has to be changed for low accelerations. Observations of additional halo clusters for which MOND does not predict a velocity enhancement would help to distinguish between the dark matter hypothesis and MOND. Also, checking the evolutionary state of the clusters that are in the MOND regime could help to decide between MOND and dark matter, since the relaxation time in the MOND regime is significantly smaller than in the Newtonian case (Ciotti & Binney 2004), meaning that the clusters would be dynamically more evolved if MOND were true.

A second possible outcome is that the observed velocity dispersion is in complete agreement with the Newtonian value and far lower than what MOND predicts. In such a case MOND would probably have to be discarded in its present form, since it would be impossible to reconcile the high rotational velocities and velocity dispersions seen in galaxies with the low velocities seen in the clusters of Table 1: assuming Newtonian gravity, Kochanek (1996) and Sakamoto, Chiba & Beers (2003) found from an analysis of the orbital motion of Galactic satellites that the total mass of the Milky Way within 50 kpc is respectively $(4.9 \pm 1.1) \times 10^{11}$ and $(5.5 \pm 0.2) \times 10^{11}\,M_{\odot}$, giving rise to a rotational velocity of $V_{\rm G} = 210$ km s^{-1}. On the other hand, the mass of the Milky Way in the disc and bulge in the form of visible matter is of the order of $5.5 \times 10^{10}\,M_{\odot}$ (Dehnen & Binney 1998), giving rise to a rotational velocity of $V_{\rm G} = 164\,(a_0/10^{-8}\,{\rm cm\,s^{-2}})^{1/4}$ km s^{-1} within the framework of MOND (equation 6). Values of a_0 below 5×10^{-9} cm s^{-2} seem therefore to be very hard to reconcile with the orbital velocities of Galactic satellites, even given the uncertainties in the above numbers. Similarly, the galaxies analysed by Begeman et al. (1991) give a mean $a_0 = (1.21 \pm 0.27) \times 10^{-8}$ cm s^{-2}, excluding values of a_0 below 6.7×10^{-9} cm s^{-2} at the 2σ level. On the other hand, even an a_0 as low as 5×10^{-9} cm s^{-2} would still give a velocity dispersion of 1.42 km s^{-1} for AM 1 and 1.80 km s^{-1} for Pal 3, which is twice the Newtonian value. Hence velocity dispersion measurements will have the power to falsify or support MOND.

4 CONCLUSIONS

We have calculated the velocity dispersions for a number of globular clusters in the halo of the Milky Way, and shown that in the case of MOND they would significantly exceed the corresponding Newtonian values, allowing a test of MOND and dark matter theories for a new class of objects and for length-scales one order of magnitude smaller than where they could be tested before. We have shown that interesting insights can be obtained about the formation of globular clusters and the role of dark matter almost independently of the actual results, so an observational effort to determine the velocity dispersions for clusters in our sample would be highly rewarding. Measurements of this kind are feasible with the current generation of 8- to 10-m-class telescopes.

Compared with dwarf galaxies, the studied globular clusters also have half-mass radii one order of magnitude smaller than their tidal radii, which means that they are effectively isolated. Their velocity dispersions are therefore much more straightforward to interpret, since tidal effects are not likely to play an important role. Also, unlike tests for MOND based on the velocity dispersion of stars in the haloes of nearby globular clusters, the interaction of the cluster stars with the Galactic tidal field and the proper separation of members and non-members are not likely to be problems for our clusters.

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