New Supersymmetric and Exactly Solvable Model of Correlated Electrons

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A new lattice model is presented for correlated electrons on the unrestricted 4^L-dimensional electronic Hilbert space $\otimes_{n=1}^L \mathbf{C}^4$ (where L is the lattice length). It is a supersymmetric generalization of the Hubbard model, but differs from the extended Hubbard model proposed by Essler, Korepin, and Schoutens. The supersymmetry algebra of the new model is superalgebra gl(2|1). The model contains one symmetry-preserving free real parameter which is the Hubbard interaction parameter U, and has its origin here in the one-parameter family of inequivalent typical 4-dimensional irreducible representations of gl(2|1). On a one-dimensional lattice, the model is exactly solvable by the Bethe ansatz.

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The Hubbard model and the t-J model, both models of correlated electrons on a lattice and exactly solvable in one dimension, have been extensively studied due to their promising role in theoretical condensed-matter physics and possibly in high-T_c superconductivity [1]. The t-J model is a lattice model on the restricted 3^L-dimensional electronic Hilbert space $\otimes_{n=1}^L \mathbf{C}^3$ (throughout the Letter, L is the lattice length), where the occurrence of two electrons on the same lattice site is forbidden. With the special choice of parameters $t = 1$ and $J = 2$, the t-J model becomes supersymmetric with the symmetry algebra being the superalgebra gl(2|1) [2,3]. In [4,5], Essler, Korepin, and Schoutens (EKS) proposed a model, the so-called extended Hubbard model, of correlated electrons on the unrestricted 4^L-dimensional electronic Hilbert space $\otimes_{n=1}^L \mathbf{C}^4$. This EKS model, which allows doubly occupied sites and combines and extends some of the interesting features of the Hubbard model and the t-J model, is exactly solvable in one dimension and has gl(2|2) supersymmetry.

In this Letter, we propose another direction of generalization of the Hubbard model. Specifically, we propose a new model on the same unrestricted 4^L-dimensional electronic Hilbert space $\otimes_{n=1}^L \mathbf{C}^4$, but with quite different interaction terms from the ones in the EKS model. Our model has gl(2|1) supersymmetry and contains one symmetry-preserving free real parameter which is exactly the Hubbard interaction parameter U; this real parameter U has its origin here in the one-parameter family of inequivalent typical 4-dimensional irreducible representations (irreps) of gl(2|1). The model can naturally be regarded as a modified Hubbard model with additional nearest-neighbor interactions and is again exactly solvable on a one-dimensional lattice. The exact solvability of our model in one dimension comes from the fact that as an abstract dynamical model it is derived from a gl(2|1)-invariant rational R matrix which satisfies the (graded) quantum Yang-Baxter equation (QYBE).

It seems that only a gl(2|1)-symmetric lattice model on the unrestricted 4^L-dimensional electronic Hilbert space could be a natural candidate for the lattice analog of $N = 2$ superconformal field theory, of which the gl(2|1) = osp(2|2) algebra defines the underlying symmetry, and which is a critically fixed point of the $N = 2$ supersymmetric Landau-Ginzburg model [6]. This gives another motivation for our model.

Let us begin by introducing some notation as in [4]. Electrons on a lattice are described by canonical Fermi operators $c_{i,\sigma}$ and $c_{i,\sigma}^\dagger$ satisfying the anticommutation relations given by $\{c_{i,\sigma}^\dagger, c_{j,\tau}\} = \delta_{ij}\delta_{\sigma\tau}$, where $i, j = 1, 2, \ldots, L$ and $\sigma, \tau = \uparrow, \downarrow$. The operator $c_{i,\sigma}$ annihilates an electron of spin σ at site i, which implies that the Fock vacuum $|0\rangle$ satisfies $c_{i,\sigma}|0\rangle = 0$. At a given lattice site i there are four possible electronic states:

$$|0\rangle, \quad |\uparrow\rangle_i = c_{i,\uparrow}^\dagger|0\rangle, \quad |\downarrow\rangle_i = c_{i,\downarrow}^\dagger|0\rangle, \quad |\uparrow\downarrow\rangle_i = c_{i,\downarrow}^\dagger c_{i,\uparrow}^\dagger|0\rangle. \tag{1}$$

By $n_{i,\sigma} = c_{i,\sigma}^\dagger c_{i,\sigma}$ we denote the number operator for electrons with spin σ on site i, and we write $n_i = n_{i,\uparrow} + n_{i,\downarrow}$. The spin operators S, $S^\dagger$, S^z (in the following, the global operator $\mathcal{O}$ will be always expressed in terms of the local one $\mathcal{O}_i$ as $\mathcal{O} = \sum_{i=1}^L \mathcal{O}_i$ in one dimension),

$$S_i = c_{i,\uparrow}^\dagger c_{i,\downarrow}, \quad S_i^\dagger = c_{i,\downarrow}^\dagger c_{i,\uparrow}, \quad S_i^z = \tfrac{1}{2}(n_{i,\downarrow} - n_{i,\uparrow}), \tag{2}$$

form an sl(2) algebra and they commute with the Hamiltonians that we consider below.

In what follows, we only consider periodic lattice of length L. The well-known Hubbard model Hamiltonian takes the following form:

$$\begin{aligned} H^{\text{Hubbard}}(U) = &- \sum_{\langle i,j\rangle} \sum_{\sigma=\uparrow,\downarrow} (c_{i,\sigma}^\dagger c_{j,\sigma} + c_{j,\sigma}^\dagger c_{i,\sigma}) \\ &+ U \sum_{\langle i,j\rangle} \Big[\Big(n_{i,\uparrow} - \tfrac{1}{2}\Big)\Big(n_{i,\downarrow} - \tfrac{1}{2}\Big) \\ &+ \Big(n_{j,\uparrow} - \tfrac{1}{2}\Big)\Big(n_{j,\downarrow} - \tfrac{1}{2}\Big)\Big], \end{aligned} \tag{3}$$

where $\langle i, j\rangle$ denote nearest-neighbor links on the lattice. It contains the hopping term for electrons and on-site interaction term for electron pairs (coupling U).

In [4], Essler, Korepin, and Schoutens proposed a supersymmetric generalization of the Hubbard model. The

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supersymmetry algebra in their model is gl(2|2). We present here another supersymmetric generalization of the Hubbard model. The Hamiltonian for our new model on a general d-dimensional lattice reads

$$\begin{aligned} H^Q(U) \equiv \sum_{\langle i,j\rangle} H^Q_{i,j}(U) &= H^{\text{Hubbard}}(U) + \frac{U}{2}\sum_{\langle i,j\rangle}\sum_{\sigma=\uparrow,\downarrow}(c^\dagger_{i,\sigma}c^\dagger_{i,-\sigma}c_{j,-\sigma}c_{j,\sigma} + \text{H.c.}) \\ &+ \left(1 + \frac{U}{|U|}\sqrt{U+1}\right)\sum_{\langle i,j\rangle}\sum_{\sigma=\uparrow,\downarrow}(c^\dagger_{i,\sigma}c_{j,\sigma} + c^\dagger_{j,\sigma}c_{i,\sigma})(n_{i,-\sigma} + n_{j,-\sigma}) \\ &- \left(1 + \frac{U}{|U|}\sqrt{U+1}\right)^2\sum_{\langle i,j\rangle}\sum_{\sigma=\uparrow,\downarrow}(c^\dagger_{i,\sigma}c_{j,\sigma} + c^\dagger_{j,\sigma}c_{i,\sigma})n_{i,-\sigma}n_{j,-\sigma} \\ &+ \frac{U+2}{2}\sum_{\langle i,j\rangle}(n_i + n_j). \end{aligned} \tag{4}$$

As will be seen, the supersymmetry algebra underlying this model is gl(2|1). Remarkably, the model still contains the parameter U as a free parameter without breaking the supersymmetry. Also, this model is exactly solvable on the one-dimensional periodic lattice, as seen below. Throughout this Letter, we will restrict U to the range $U > -1$.

The Hamiltonian (4) is obviously invariant under spin reflection $c_{i,\uparrow} \leftrightarrow c_{i,\downarrow}$. It can be viewed as an extended Hubbard model with additional nearest-neighbor interaction terms in a different fashion from the one proposed in [4]. The physical nature of the additional terms is the following. The second term is nothing but a pair-hopping term. The third and fourth terms are so-called bond-charge interaction terms. And the last term is just a chemical potential. Clearly one can add to the above Hamiltonian an arbitrary chemical potential (coefficient μ) term $\mu\sum_i n_i$ and an external magnetic field (coefficient h) term $h\sum_i(n_{i,\downarrow} - n_{i,\uparrow})$, which commute with $H^Q(U)$ but break its gl(2|1) supersymmetry.

An interesting feature of our model is the discontinuity at $U = 0$. When $U \to 0^+$, the Hamiltonian (4) contains a hopping term plus a bond-charge interaction term (up to a chemical potential). But as $U \to 0^-$, only a hopping term (and a chemical potential) survives.

Our local Hamiltonian $H^Q_{i,j}(U)$ does not act as graded permutation of the electron states (1) at sites i and j, in contrast to the Hamiltonian in [4]. Nevertheless, it is supersymmetric. There are four supersymmetry generators for $H^Q(U)$: $Q_\uparrow, Q^\dagger_\uparrow, Q_\downarrow$, and $Q^\dagger_\downarrow$ with the corresponding local operators given by

$$\begin{aligned} Q_{i,\uparrow} &= -\sqrt{\alpha}\, n_{i,\downarrow}c_{i,\uparrow} + \sqrt{\alpha+1}\,(1 - n_{i,\downarrow})c_{i,\uparrow}, \\ Q_{i,\downarrow} &= -\sqrt{\alpha}\, n_{i,\uparrow}c_{i,\downarrow} + \sqrt{\alpha+1}\,(1 - n_{i,\uparrow})c_{i,\downarrow}, \end{aligned} \tag{5}$$

where $0 \le \arg\sqrt{Z} < \pi, Z = \alpha$ or $\alpha + 1$, and $\alpha \ge 0$ or $\alpha < -1$ is the inverse of U, i.e., $\alpha = 1/U$. These generators, together with $S, S^\dagger, S^z$, and two others ($E^2_2 + E^3_3$ and E^3_3, defined below), form the superalgebra gl(2|1). To prove this, we denote the generators of gl(2|1) by E^β_γ, $\beta,\gamma = 1,2,3$ with grading $[1] = [2] = 0$, $[3] = 1$. In a typical 4-dimensional representation of gl(2|1), the highest weight itself of the representation depends on the free parameter α, thus giving rise to a one-parameter family of inequivalent irreps [7]. Choose a basis $|4\rangle = (0,0,0,1)$, $|3\rangle = (0,0,1,0)$, $|2\rangle = (0,1,0,0)$, $|1\rangle = (1,0,0,0)$, with $|1\rangle$, $|4\rangle$ even (bosonic) and $|2\rangle$, $|3\rangle$ odd (fermionic). Then in this typical 4-dimensional representation, E^β_γ are 4×4 supermatrices of the form

$$\begin{aligned} &E^1_2 = |2\rangle\langle 3|, \quad E^2_1 = |3\rangle\langle 2|, \quad E^1_1 = -|3\rangle\langle 3| - |4\rangle\langle 4|, \quad E^2_2 = -|2\rangle\langle 2| - |4\rangle\langle 4|, \\ &E^2_3 = \sqrt{\alpha}\,|1\rangle\langle 2| + \sqrt{\alpha+1}\,|3\rangle\langle 4|, \quad E^3_2 = \sqrt{\alpha}\,|2\rangle\langle 1| + \sqrt{\alpha+1}\,|4\rangle\langle 3|, \\ &E^1_3 = -\sqrt{\alpha}\,|1\rangle\langle 3| + \sqrt{\alpha+1}\,|2\rangle\langle 4|, \quad E^3_1 = -\sqrt{\alpha}\,|3\rangle\langle 1| + \sqrt{\alpha+1}\,|4\rangle\langle 2|, \\ &E^3_3 = \alpha|1\rangle\langle 1| + (\alpha+1)(|2\rangle\langle 2| + |3\rangle\langle 3|) + (\alpha+2)|4\rangle\langle 4|. \end{aligned} \tag{6}$$

For $\alpha > 0$,

$$(E^\beta_\gamma)^\dagger = E^\gamma_\beta, \tag{7}$$

and we call the representation unitary of type I. For $\alpha < -1$, we have

$$(E^\beta_\gamma)^\dagger = (-1)^{[\beta]+[\gamma]}E^\gamma_\beta, \tag{8}$$

and we refer to the representation as unitary of type II. In this Letter, we are interested in these unitary representations. For a description and classification of the two types of unitary representations, see [8].

Further choosing

$$|4\rangle \equiv |0\rangle, \quad |3\rangle \equiv |\uparrow\rangle, \quad |2\rangle \equiv |\downarrow\rangle, \quad |1\rangle \equiv |\uparrow\downarrow\rangle, \tag{9}$$

one can easily establish that

$$S_i^\dagger = (E_2^1)_i, \quad S_i = (E_1^2)_i, \quad S_i^z = (E_1^1)_i - (E_2^2)_i,$$
$$Q_{i,\downarrow}^\dagger = (E_2^3)_i^\dagger, \quad Q_{i,\downarrow} = (E_2^3)_i, \quad Q_{i,\uparrow}^\dagger = (E_1^3)_i^\dagger, \quad Q_{i,\uparrow} = (E_1^3)_i. \tag{10}$$

The verification that the Hamiltonian $H^Q(U)$ commutes with all nine generators of gl(2|1) is a straightforward calculation.

The model is exactly solvable in one dimension by the Bethe ansatz. To show this, we first show that the local Hamiltonian $H_{i,i+1}^Q(U)$ on the one-dimensional lattice is actually derived from a gl(2|1)-invariant rational R matrix which satisfies the (graded) QYBE. To this end, let $U_q[\mathrm{gl}(2|1)]$ be the well-known quantum (or q) deformation of gl(2|1) and V be the $U_q[\mathrm{gl}(2|1)]$ module with highest weight $(0,0|\alpha)$, which affords the q-deformed version of the one-parameter family of the inequivalent typical 4-dimensional irreps [9,10]. Without loss of generality, we assume q to be real. We also assume q to be generic, i.e., it is not a root of unity. For $\alpha > 0$ or $\alpha < -1$, the module V is unitary of type I and II, respectively, and thus the tensor product $V \otimes V$ is completely reducible. We write $V \otimes V = V_1 \bigoplus V_2 \bigoplus V_3$, where V_1, V_2, and V_3 are $U_q[\mathrm{gl}(2|1)]$ modules with highest weights $(0,0|2\alpha)$, $(0,-1|2\alpha+1)$, and $(-1,-1|2\alpha+2)$, respectively [9], and let $\check{P}_k, k = 1,2,3$, be the projection operator from $V \otimes V$ onto V_k. The trigonometric R matrix $\check{R}(x) \in \mathrm{End}(V \otimes V)$, which satisfies the (graded) QYBE,

$$[I \otimes \check{R}(x)][\check{R}(xy) \otimes I][I \otimes \check{R}(y)] = [\check{R}(y) \otimes I][I \otimes \check{R}(xy)] \times [\check{R}(x) \otimes I], \tag{11}$$

was given in [9–12] in the form

$$\check{R}(x) = \frac{x - q^{2\alpha}}{1 - xq^{2\alpha}}\check{P}_1 + \check{P}_2 + \frac{1 - xq^{2\alpha+2}}{x - q^{2\alpha+2}}\check{P}_3. \tag{12}$$

Note, however, that q and α are both free parameters which do not enter the (graded) QYBE. Setting $x = q^\theta$ and taking the $q = 1$ limit, one gets the corresponding rational R matrix [which also satisfies the (graded) QYBE]

$$\check{R}^r(\theta) = -\frac{\theta - 2\alpha}{\theta + 2\alpha}\check{P}_1^{(0)} + \check{P}_2^{(0)} - \frac{\theta + 2\alpha + 2}{\theta - 2\alpha - 2}\check{P}_3^{(0)}, \tag{13}$$

where $\check{P}_k^{(0)}$, $k = 1,2,3$, are classical ($q = 1$) versions of $\check{P}_k$, i.e., projection operator from $V^{(0)} \otimes V^{(0)}$ onto $V_k^{(0)}$, with $V^{(0)}$ and $V_k^{(0)}$ being the $q = 1$ versions of V and V_k, respectively. Note that $V^{(0)}$ and $V_k^{(0)}$ are actually gl(2|1) modules, and $V^{(0)}$ affords the representation (6). The projectors $\check{P}_k^{(0)}$ can easily be evaluated:

$$\begin{aligned}
\check{P}_1^{(0)} &= |\Psi_1^1\rangle\langle\Psi_1^1| + |\Psi_2^1\rangle\langle\Psi_2^1| + |\Psi_3^1\rangle\langle\Psi_3^1| + |\Psi_4^1\rangle\langle\Psi_4^1|,\\
\check{P}_3^{(0)} &= |\Psi_1^3\rangle\langle\Psi_1^3| + |\Psi_2^3\rangle\langle\Psi_2^3| + |\Psi_3^3\rangle\langle\Psi_3^3| + |\Psi_4^3\rangle\langle\Psi_4^3|,\\
\check{P}_2^{(0)} &= I - \check{P}_1^{(0)} - \check{P}_3^{(0)},
\end{aligned} \tag{14}$$

where $|\Psi_k^1\rangle$ and $|\Psi_k^3\rangle$, $k = 1,2,3,4$, form the symmetry adapted bases for the spaces $V_1^{(0)}$ and $V_3^{(0)}$, respectively. Note that $\check{R}^r(0) \equiv I$. By means of the matrix representation (6), one can show

$$\begin{aligned}
|\Psi_1^1\rangle &= |1\rangle \otimes |1\rangle, \quad |\Psi_2^1\rangle = \frac{1}{\sqrt{2}}(|2\rangle \otimes |1\rangle + |1\rangle \otimes |2\rangle),\\
|\Psi_3^1\rangle &= \frac{1}{\sqrt{2}}(|3\rangle \otimes |1\rangle + |1\rangle \otimes |3\rangle),\\
|\Psi_4^1\rangle &= \frac{1}{\sqrt{2(2\alpha+1)}}[\sqrt{\alpha+1}(|4\rangle \otimes |1\rangle + |1\rangle \otimes |4\rangle) + \sqrt{\alpha}(|2\rangle \otimes |3\rangle - |3\rangle \otimes |2\rangle)],\\
|\Psi_1^3\rangle &= \frac{1}{\sqrt{2(2\alpha+1)}}[\sqrt{\alpha}(|4\rangle \otimes |1\rangle + |1\rangle \otimes |4\rangle) + \sqrt{\alpha+1}(-|2\rangle \otimes |3\rangle + |3\rangle \otimes |2\rangle)],\\
|\Psi_2^3\rangle &= \frac{1}{\sqrt{2}}(|2\rangle \otimes |4\rangle + |4\rangle \otimes |2\rangle),\\
|\Psi_3^3\rangle &= \frac{1}{\sqrt{2}}(|3\rangle \otimes |4\rangle + |4\rangle \otimes |3\rangle), \quad |\Psi_4^3\rangle = |4\rangle \otimes |4\rangle),
\end{aligned} \tag{15}$$

which are easily seen to be orthonormal, so that

$$\begin{aligned}
\langle\Psi_k^1| &= (|\Psi_k^1\rangle)^\dagger, \quad \langle\Psi_k^3| = (|\Psi_k^3\rangle)^\dagger, \quad k = 1,2,3,4,\\
(|\beta\rangle \otimes |\gamma\rangle)^\dagger &= (-1)^{[|\beta\rangle][|\gamma\rangle]}(|\beta\rangle)^\dagger \otimes (|\gamma\rangle)^\dagger,\\
(|\beta\rangle)^\dagger &= \langle\beta|, \quad \forall\beta = 1,2,3,4.
\end{aligned} \tag{16}$$

Here $[|\beta\rangle]$ stands for the grading of the state $|\beta\rangle$, $[|\beta\rangle] = 0$ for even (bosonic) $|\beta\rangle$, and $[|\beta\rangle] = 1$ for odd (fermionic) $|\beta\rangle$. Readers should keep in mind that the multiplication rule for the tensor product is defined by

$$(a \otimes b)(c \otimes d) = (-1)^{[b][c]}(ac \otimes bd) \tag{17}$$

for any elements a, b, c, and d.

Using the rational R matrix (17) and denoting

$$\check{R}_{i,i+1}^r(\theta) = I \otimes \cdots I \otimes \underbrace{\check{R}^r(\theta)}_{i\,i+1} \otimes I \otimes \cdots \otimes I, \tag{18}$$

one may define [13] the local Hamiltonian

$$H_{i,i+1}^R(\alpha) = \frac{d}{d\theta}\check{R}_{i,i+1}^r(\theta)\Big|_{\theta=0} = -\frac{1}{\alpha}(\check{P}_1^{(0)})_{i,i+1} + \frac{1}{\alpha+1}(\check{P}_3^{(0)})_{i,i+1}. \tag{19}$$

By (14), (15), (16), and (9), and after tedious but straightforward manipulation, one gets, up to a constant,

$$H^Q_{i,i+1}(U) = -2(\alpha + 1)H^R_{i,i+1}(\alpha), \tag{20}$$

which implies that the local Hamiltonian $H^Q_{i,i+1}(U)$ is indeed derived from the gl(2|1)-invariant rational R matrix which satisfies the (graded) QYBE. [Note that the identity (20) also indicates that $H^Q(U)$ commutes with all the nine generators of gl(2|1), since the rational R matrix $\check{R}^r(\theta)$ is a gl(2|1) invariant.]

Now the exact solvability on the one-dimensional periodic lattice of our model is seen as the following four steps. *Step 1:* The Hamiltonian $H^Q(U)$ is self-adjoint and thus is diagonalizable. *Step 2:* Relation (20) immediately makes it clear [13] that on the one-dimensional periodic lattice the global Hamiltonian $H^Q(U)$ commutes with the transfer matrix $t(\theta)$ constructed from the rational R matrix (13) (see, e.g., [13] for the standard definition of the transfer matrix), for any value of the parameter θ. *Step 3:* Using the fact, established in the *rational* case, that $R^r(\theta)^\dagger = R^r(\theta)$, where $R^r(\theta) = P\check{R}^r(\theta)$ and P is the graded permutation operator of the electron states (1), one may show that the transfer matrix $t(\theta)$ is self-adjoint and consequently diagonalizable for any given parameter θ. (This result is in fact established for real θ but should also be valid for all complex θ by using analytical continuation arguments.) We remark here that the results in this step are actually quite general: They are valid for any (other) *rational* R matrices arising from *unitary* representations of any (other) quantum superalgebras. *Step 4:* It can easily be shown that $[t(\theta), t(\theta')] = 0, \forall\theta, \theta'$, and thus $t(\theta)$ is diagonalizable simultaneously for all θ. Summarizing the above four steps, one sees that the Hamiltonian $H^Q(U)$ satisfies the standard requirement for a model to be exactly solvable by the Bethe ansatz. This completes the proof for the exact solvability in one dimension of our model. The details of solution of the model is deferred to a separate publication.

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