COHERENT X-RAY RADIATION OF A RELATIVISTIC ELECTRON IN A BILAYER AMORPHOUS LAYER – SINGLE CRYSTAL TARGET

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A theory of coherent X-ray radiation is proposed, which is generated when a relativistic electron is transversing a hybrid target consisting of two parallel plates: amorphous and crystalline. Within a two-wave approximation of the dynamic diffraction theory, expressions describing the spectral-angular density of coherent radiation from this target are obtained.

Keywords: dynamic diffraction, parametric X-ray radiation, diffraction transition radiation, relativistic electron.

INTRODUCTION

Emission of a relativistic electron in crystalline or amorphous media has been commonly dealt with separately. In this work, within the framework of a two-wave approximation of the dynamic diffraction theory we for the first time address coherent X-ray radiation of a relativistic electron transversing a complex target consisting of amorphous and crystalline components. At the first and second boundaries in the structure of the target, transition radiation (TR) is generated [1, 2], which later gives rise to diffraction transition radiation (DTR) produced on a system of parallel atomic planes of the single-crystal plate along the Bragg direction [3–6]. When a relativistic electron transverses the single-crystal layer of the target, its Coulomb field is scattered on a system of parallel atomic planes of the crystal, producing parametric X-ray radiation (PXR) [7–9], with the PXR photons moving together with the DTR photons along the Bragg scattering direction. The process of coherent X-ray scattering of relativistic electrons in a crystal, including PXR and DTR, was developed in [10–14] within the framework of a two-wave approximation of the dynamic theory of X-ray diffraction. It is noteworthy that in [10, 11] coherent X-ray radiation is treated in a particular case of symmetrical reflection, where the reflecting system of atomic planes of a crystal is parallel with respect to the surface of the target in the case of the Bragg scattering geometry and normal to it in the case of the Laue scattering geometry. In [12–14], a dynamic theory of coherent X-ray radiation of relativistic electrons in a crystal was developed for a general case of asymmetrical reflection of the electron's field with respect to the target surface, where the system of parallel reflecting layers of the target could be located at an arbitrary angle to the surface.

In this work, for the first time we investigate coherent radiation of a relativistic electron transversing a composite medium consisting of the amorphous and crystalline plates. In the Laue scattering geometry, we consider the dynamic diffraction of PXR and DTR photons in a crystalline plate under conditions of an asymmetric reflection of the field with respect to the target surface. In this case, the emitted photons leave the target through its boundary. Within the framework of a two-wave approximation of the dynamic diffraction theory, we obtained a set of expressions describing the spectral-angular distribution of PXR and DTR in this structure. We also consider a possibility of

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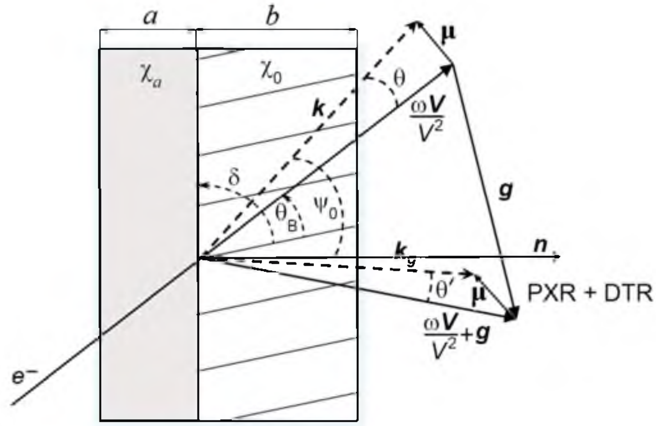

Fig. 1. Geometry of the process of radiation of a relativistic electron in a composite structure.

increasing the intensity of DTR of a relativistic electron without increasing its energy for the case where an amorphous plate is located in front of the crystalline plate.

1. RADIATION AMPLITUDE

Let us look at the process of coherent X-ray radiation of a relativistic electron transversing a bilayer composite structure at velocity V in the Laue scattering geometry. Assume a target composed of the amorphous and crystalline plates (Fig. 1), whose thicknesses are a and b, respectively. Let us denote the dielectric susceptibility of the amorphous medium as χ_a and those of the crystalline medium χ_0 and $\chi_{\boldsymbol{g}}$.

The Fourier transform of the electric field

$$\boldsymbol{E}(\boldsymbol{k},\omega)=\int dt\, d^3\boldsymbol{r}\, \boldsymbol{E}(\boldsymbol{r},t)\exp(i\omega t-i\boldsymbol{k}\boldsymbol{r}) \tag{1}$$

will be found from the Maxwell equation

$$(k^2-\omega^2(1+\chi_0))\boldsymbol{E}(\boldsymbol{k},\omega)-\boldsymbol{k}(\boldsymbol{k}\boldsymbol{E}(\boldsymbol{k},\omega))-\omega^2{\sum_{\boldsymbol{g}}}'\chi_{-\boldsymbol{g}}\boldsymbol{E}(\boldsymbol{k}+\boldsymbol{g},\omega)=4\pi i\omega\boldsymbol{J}(\boldsymbol{k},\omega), \tag{2}$$

where $\boldsymbol{J}(\boldsymbol{k},\omega)=2\pi e\boldsymbol{V}\delta(\omega-\boldsymbol{k}\boldsymbol{V})$ is the electron current density.

Since the electromagnetic field induced is nearly transverse within the X-ray frequency range, so the incident electromagnetic wave $\boldsymbol{E}(\boldsymbol{k},\omega)$ and the one diffracted in the crystal $\boldsymbol{E}(\boldsymbol{k}+\boldsymbol{g},\omega)$ are controlled by two amplitudes with different values of transverse polarization

$$\begin{aligned}\boldsymbol{E}(k,\omega)&=E_0^{(1)}(\boldsymbol{k},\omega)\boldsymbol{e}_0^{(1)}+E_0^{(2)}(\boldsymbol{k},\omega)\boldsymbol{e}_0^{(2)},\\ \boldsymbol{E}(\boldsymbol{k}+\boldsymbol{g},\omega)&=E_{\boldsymbol{g}}^{(1)}(\boldsymbol{k},\omega)\boldsymbol{e}_1^{(1)}+E_{\boldsymbol{g}}^{(2)}(\boldsymbol{k},\omega)\boldsymbol{e}_1^{(2)},\end{aligned} \tag{3}$$

where vectors $\boldsymbol{e}_0^{(1)}$ and $\boldsymbol{e}_0^{(2)}$ are perpendicular to vector $\boldsymbol{k}$, while vectors $\boldsymbol{e}_1^{(1)}$ and $\boldsymbol{e}_1^{(2)}$ are perpendicular to vector $\boldsymbol{k}_{\boldsymbol{g}}=\boldsymbol{k}+\boldsymbol{g}$. Vectors $\boldsymbol{e}_0^{(2)}$, $\boldsymbol{e}_1^{(2)}$ lie within the plane of vectors $\boldsymbol{k}$ and $\boldsymbol{k}_{\boldsymbol{g}}$ (π-polarization), and vectors $\boldsymbol{e}_0^{(1)}$ and $\boldsymbol{e}_1^{(1)}$

are perpendicular to it (σ-polarization), and $\boldsymbol{g}$ is the reciprocal lattice vector determining the system of reflecting atomic planes in the crystal plate. Within the framework of a two-wave approximation of the dynamic diffraction theory, equation (2) is reduced to a well-known system of equations [15]

$$\begin{cases} (\omega^2(1+\chi_0)-k^2)E_0^{(s)}+\omega^2\chi_{-\boldsymbol{g}}C^{(s)}E_{\boldsymbol{g}}^{(s)}=8\pi^2 ie\omega\theta VP^{(s)}\delta(\omega-\boldsymbol{kV}), \\ \omega^2\chi_{\boldsymbol{g}}C^{(s)}E_0^{(s)}+(\omega^2(1+\chi_0)-k_{\boldsymbol{g}}^2)E_{\boldsymbol{g}}^{(s)}=0, \end{cases} \tag{4}$$

where $\chi_{\boldsymbol{g}}$, $\chi_{-\boldsymbol{g}}$ are the coefficients of a Fourier expansion of the dielectric susceptibility of the crystal over the reciprocal lattice vectors $\boldsymbol{g}$

$$\chi(\omega,\boldsymbol{r})=\sum_{\boldsymbol{g}}\chi_{\boldsymbol{g}}(\omega)\exp(i\boldsymbol{gr})=\sum_{\boldsymbol{g}}\left(\chi'_{\boldsymbol{g}}(\omega)+i\chi''_{\boldsymbol{g}}(\omega)\right)\exp(i\boldsymbol{gr}). \tag{5}$$

Quantities $C^{(s)}$ and $P^{(s)}$ in the system of equations (4) have been defined as follows:

$$C^{(s)}=\boldsymbol{e}_0^{(s)}\boldsymbol{e}_1^{(s)},\ C^{(1)}=1,\ C^{(2)}=\cos 2\theta_{\mathrm{B}},$$

$$P^{(s)}=\boldsymbol{e}_0^{(s)}(\boldsymbol{\mu}/\mu),\ P^{(1)}=\sin\varphi,\ P^{(2)}=\cos\varphi. \tag{6}$$

Here $\boldsymbol{\mu}=\boldsymbol{k}-\omega\boldsymbol{V}/V^2$ is the component of the virtual photon momentum, perpendicular to the particle velocity $\boldsymbol{V}$, its absolute value being $\mu=\omega\theta/V$, where $\theta \ll 1$ is the angle between vectors $\boldsymbol{k}$ and $\boldsymbol{V}$, θ_{B} is the Bragg angle, and φ is the azimuthal radiation angle calculated from the plane formed by the velocity vectors $\boldsymbol{V}$ and the reciprocal lattice vector $\boldsymbol{g}$ in the crystal. The length of vector $\boldsymbol{g}$ could be expressed via the Bragg angle and the Bragg frequency ω_{B} as follows: $g=2\omega_{\mathrm{B}}\sin\theta_{\mathrm{B}}/V$. The angle between vector $\dfrac{\omega\boldsymbol{V}}{V^2}$ and the wave vector of the incident wave $\boldsymbol{k}$ is denoted as θ, and the angle between vector $\dfrac{\omega\boldsymbol{V}}{V^2}+\boldsymbol{g}$ and the wave vector of the diffracted wave $\boldsymbol{k}_{\boldsymbol{g}}$ is denoted as θ'. The system of equations (4) for parameter $s=1$ describes σ-polarized fields, and for $s=2$ – π-polarized ones.

Let us solve the dispersion equation following from the system of equations (4)

$$(\omega^2(1+\chi_0)-k^2)(\omega^2(1+\chi_0)-k_{\boldsymbol{g}}^2)-\omega^4\chi_{-\boldsymbol{g}}\chi_{\boldsymbol{g}}C^{(s)2}=0, \tag{7}$$

using standard procedures of the dynamic theory of diffraction of X-ray waves in a crystal [16]. We need the lengths of the wave vectors $\boldsymbol{k}$ and $\boldsymbol{k}_{\boldsymbol{g}}$ of photons in the crystal given by

$$k=\omega\left(1+\frac{\chi_0}{2}\right)+\lambda_0,\ k_{\boldsymbol{g}}=\omega\left(1+\frac{\chi_0}{2}\right)+\lambda_{\boldsymbol{g}}. \tag{8}$$

The dynamic contributions λ_0 and $\lambda_{\boldsymbol{g}}$ for X-ray waves are related as [16]

$$\lambda_{\boldsymbol{g}}=\frac{\omega\beta}{2}+\lambda_0\frac{\gamma_{\boldsymbol{g}}}{\gamma_0}. \tag{9}$$

Since these contributions are small ($|\lambda_0| << \omega$, $|\lambda_g| << \omega$), when substituting (8) into (7), we can neglect the terms quadratic with respect to λ_0 and λ_g. This yields us two solutions for each of the incident and diffracted waves

$$\lambda_g^{(1,2)} = \frac{\omega}{4}\left(\beta \pm \sqrt{\beta^2 + 4\chi_g\chi_{-g}C^{(s)^2}\frac{\gamma_g}{\gamma_0}}\right),\ \lambda_0^{(1,2)} = \omega\frac{\gamma_0}{4\gamma_g}\left(-\beta \pm \sqrt{\beta^2 + 4\chi_g\chi_{-g}C^{(s)^2}\frac{\gamma_g}{\gamma_0}}\right), \tag{10}$$

where $\beta = \alpha - \chi_0\left(1 - \frac{\gamma_g}{\gamma_0}\right)$, $\alpha = \frac{1}{\omega^2}(k_g^2 - k^2)$, $\gamma_0 = \cos\psi_0$, $\gamma_g = \cos\psi_g$, ψ_0 is the angle between the incident wave vector $\boldsymbol{k}$ and that normal to the surface of the plate $\boldsymbol{n}$, and ψ_g is the angle between the wave vector $\boldsymbol{k}_g$ and vector $\boldsymbol{n}$ (see Fig. 1). Since the dynamic contributions are small, we can show that $\theta \approx \theta'$ (see Fig. 1), hence in what follows we shall denote angle θ' as θ.

For ease of solving the problem, let us present the length of the wave vector of free photons in an amorphous medium $k_a = \omega\sqrt{1+\chi_a}$ as

$$k_a = \omega\left(1 + \frac{\chi_0}{2}\right) + \frac{\gamma_0}{\gamma_g}\left(\lambda_g' - \frac{\omega\beta}{2}\right), \tag{11}$$

where

$$\lambda_g' = \lambda_g^* - \frac{\gamma_g}{\gamma_0}\omega\left(\frac{\gamma^{-2} + \theta^2 - \chi_a}{2}\right),\ \lambda_g^* = \frac{\omega\beta}{2} + \frac{\gamma_g}{\gamma_0}\lambda_0^*,\ \lambda_0^* = \omega\left(\frac{\gamma^{-2} + \theta^2 - \chi_0}{2}\right), \tag{12}$$

and give the free photon emitted in the Bragg direction by

$$k_0 = \omega\left(1 + \frac{\chi_0}{2}\right) + \lambda_g'', \tag{13}$$

where $\lambda_g'' = -\omega\frac{\chi_0}{2}$.

Using the above-introduced notations and the system of equations (4), write the expressions for the fields. In vacuum, in front of the target the field consists of pseudo-photons of the Coulomb field of a relativistic electron incident on the target

$$E_0^{(s)\mathrm{vac\,I}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\gamma_g}{\gamma_0}\left(-\chi_0 - \frac{2}{\omega}\frac{\gamma_0}{\gamma_g}\lambda_g + \beta\frac{\gamma_0}{\gamma_g}\right)^{-1}\delta\left(\lambda_g - \lambda_g^*\right). \tag{14}$$

In an amorphous medium, the field consists of the Coulomb field of an electron and that of the emitted free photons $E_a^{(s)}$

$$E_0^{(s)\mathrm{sr}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\gamma_g}{\gamma_0}\left(-\chi_0 + \chi_a - \frac{2}{\omega}\frac{\gamma_0}{\gamma_g}\lambda_g + \beta\frac{\gamma_0}{\gamma_g}\right)^{-1}\delta\left(\lambda_g - \lambda_g^*\right) + E_a^{(s)}\delta\left(\lambda_g - \lambda_g'\right). \tag{15}$$

In a crystal medium, for the incident and diffracted waves the field consists of the Coulomb field of a relativistic electron and two fields of free X-ray waves propagating in the crystal

$$E_0^{(s)\mathrm{cr}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{-\omega^2\beta - 2\omega\frac{\gamma_{\boldsymbol{g}}}{\gamma_0}\lambda_0}{4\frac{\gamma_{\boldsymbol{g}}}{\gamma_0}\left(\lambda_0 - \lambda_0^{(1)}\right)\left(\lambda_0 - \lambda_0^{(2)}\right)} \delta\left(\lambda_0 - \lambda_0^*\right) + E_0^{(s)^{(1)}}\delta(\lambda_0 - \lambda_0^{(1)}) + E_0^{(s)^{(2)}}\delta(\lambda_0 - \lambda_0^{(2)}) , \quad (16a)$$

$$E_{\boldsymbol{g}}^{(s)\mathrm{cr}} = -\frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{\omega^2\chi_{\boldsymbol{g}}C^{(s)}}{4\frac{\gamma_0^2}{\gamma_{\boldsymbol{g}}^2}\left(\lambda_{\boldsymbol{g}} - \lambda_{\boldsymbol{g}}^{(1)}\right)\left(\lambda_{\boldsymbol{g}} - \lambda_{\boldsymbol{g}}^{(2)}\right)} \delta\left(\lambda_{\boldsymbol{g}} - \lambda_{\boldsymbol{g}}^*\right) + E_{\boldsymbol{g}}^{(s)^{(1)}}\delta(\lambda_{\boldsymbol{g}} - \lambda_{\boldsymbol{g}}^{(1)}) + E_{\boldsymbol{g}}^{(s)^{(2)}}\delta(\lambda_{\boldsymbol{g}} - \lambda_{\boldsymbol{g}}^{(2)}) . \quad (16b)$$

The incident and diffracted fields in the crystal are related by the expression $E_0^{(s)\mathrm{cr}} = (2\omega\lambda_{\boldsymbol{g}} / \omega^2\chi_g C^{(s)})E_{\boldsymbol{g}}^{(s)\mathrm{cr}}$ following from the second equation of the system of equations (4).

The field emitted in vacuum behind the target along the Bragg direction will be given by the following:

$$E_{\boldsymbol{g}}^{(s)\mathrm{vac\,II}} = E_{\boldsymbol{g}}^{(s)\mathrm{Rad}}\delta\left(\lambda_{\boldsymbol{g}} - \lambda_{\boldsymbol{g}}''\right) . \quad (17)$$

To determine the amplitude of the radiation field $E_{\boldsymbol{g}}^{(s)\mathrm{Rad}}$, let us use the boundary conditions on three boundaries of the composite target under study

$$\int E_0^{(s)\mathrm{vac}\,I}\, d\lambda_{\boldsymbol{g}} = \int E_0^{(s)\mathrm{sr}} d\lambda_{\boldsymbol{g}} , \ \int E_0^{(s)\mathrm{sr}} e^{i\frac{\lambda_{\boldsymbol{g}}}{\gamma_g}a}\, d\lambda_{\boldsymbol{g}} = \int E_0^{(s)\mathrm{cr}} e^{i\frac{\lambda_{\boldsymbol{g}}}{\gamma_g}a}\, d\lambda_{\boldsymbol{g}} ,$$

$$\int E_{\boldsymbol{g}}^{(s)\mathrm{cr}} e^{i\frac{\lambda_{\boldsymbol{g}}}{\gamma_g}a}\, d\lambda_{\boldsymbol{g}} = 0 , \ \int E_{\boldsymbol{g}}^{(s)\mathrm{cr}} e^{i\frac{\lambda_{\boldsymbol{g}}}{\gamma_g}(a+b)}\, d\lambda_{\boldsymbol{g}} = \int E_{\boldsymbol{g}}^{(s)\mathrm{vac\,II}} e^{i\frac{\lambda_{\boldsymbol{g}}}{\gamma_g}(a+b)}\, d\lambda_{\boldsymbol{g}} . \quad (18)$$

Since in this work the investigation of the process of radiation of a relativistic electron in a composite medium is limited to the rectilinear motion of the electron, so two mechanisms contribute to the total radiation yield, specifically, diffracted transition radiation and parametric X-ray radiation. From the total amplitude of coherent radiation, let us single out the amplitudes corresponding to these radiation mechanisms and write them as two summands

$$E_{\boldsymbol{g}}^{(s)\mathrm{Rad}} = E_{\mathrm{PXR}}^{(s)} + E_{\mathrm{DTR}}^{(s)} , \quad (19a)$$

$$E_{\mathrm{PXR}}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} e^{i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{(a+b)}{\gamma_g}} \frac{\omega^2\chi_{\boldsymbol{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\boldsymbol{g}}}\left(\lambda_{\boldsymbol{g}}^{(1)} - \lambda_{\boldsymbol{g}}^{(2)}\right)}$$

$$\times\left[\left(\frac{1}{\chi_0 - \theta^2 - \gamma^{-2}} + \frac{\omega}{2\frac{\gamma_0}{\gamma_{\boldsymbol{g}}}\left(\lambda_{\boldsymbol{g}}^* - \lambda_{\boldsymbol{g}}^{(1)}\right)}\right)\left(e^{i\frac{\lambda_{\boldsymbol{g}}^{(1)} - \lambda_{\boldsymbol{g}}^*}{\gamma_g}b} - 1\right) - \left(\frac{1}{\chi_0 - \theta^2 - \gamma^{-2}} + \frac{\omega}{2\frac{\gamma_0}{\gamma_{\boldsymbol{g}}}\left(\lambda_{\boldsymbol{g}}^* - \lambda_{\boldsymbol{g}}^{(2)}\right)}\right)\left(e^{i\frac{\lambda_{\boldsymbol{g}}^{(2)} - \lambda_{\boldsymbol{g}}^*}{\gamma_g}b} - 1\right)\right], \quad (19b)$$

$$E_{\text{DTR}}^{(s)}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}e^{i\left(\frac{\omega\chi_0}{2}+\lambda_{\boldsymbol{g}}^*\right)\frac{(a+b)}{\gamma_g}}\frac{\omega^2\chi_{\boldsymbol{g}}C^{(s)}\left(e^{i\frac{\lambda_{\boldsymbol{g}}^{(1)}-\lambda_{\boldsymbol{g}}^*}{\gamma_g}b}-e^{i\frac{\lambda_{\boldsymbol{g}}^{(2)}-\lambda_{\boldsymbol{g}}^*}{\gamma_g}b}\right)}{2\omega\frac{\gamma_0}{\gamma_{\boldsymbol{g}}}\left(\lambda_{\boldsymbol{g}}^{(1)}-\lambda_{\boldsymbol{g}}^{(2)}\right)}$$

$$\left[\left(\frac{1}{\theta^2+\gamma^{-2}-\chi_a}-\frac{1}{\theta^2+\gamma^{-2}}\right)e^{-i\frac{\omega a}{2\gamma_0}\left(\theta^2+\gamma^{-2}-\chi_a\right)}+\left(\frac{1}{\chi_a-\theta^2-\gamma^{-2}}-\frac{1}{\chi_0-\theta^2-\gamma^{-2}}\right)\right]. \tag{19c}$$

Expression (19b) represents the amplitude of the PXR field of a relativistic electron in a composite medium. which is generated when the electron transverses the plate located behind the amorphous one. Expression (19c) describes the amplitude of the diffracted transition radiation in the composite structure under study. The summands in the square brackets correspond to the transition radiation components emitted from the first and the second boundaries. respectively. which later on are diffracted in the crystal plate along the Bragg direction.

2. SPECTRAL-ANGULAR DENSITY OF RADIATION

Substituting (19b) and (19c) into the expression for the spectral-angular density of X-ray radiation

$$\omega\frac{d^2N}{d\omega d\Omega}=\omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E_{\boldsymbol{g}}^{(s)\text{Rad}}\right|^2, \tag{20}$$

we obtain an expression describing the spectral-angular densities of PXR and DTR of a relativistic electron in the composite bilayer structure under study

$$\omega\frac{d^2N_{\text{PXR}}^{(s)}}{d\omega d\Omega}=\frac{e^2}{4\pi^2}P^{(s)^2}\frac{\theta^2}{(\theta^2+\gamma^{-2}-\chi_0')^2}R_{\text{PXR}}^{(s)}, \tag{21a}$$

$$R_{\text{PXR}}^{(s)}=\left(1-\xi/\sqrt{\xi^2+\varepsilon}\right)^2\left[1+\exp\left(-2B^{(s)}\rho^{(s)}\Delta^{(1)}\right)-\right.$$

$$\left.-2\exp\left(-B^{(s)}\rho^{(s)}\Delta^{(1)}\right)\cos\left(B^{(s)}\left(\sigma^{(s)}+\left(\xi-\sqrt{\xi^2+\varepsilon}\right)/\varepsilon\right)\right)\right]\left(\left(\sigma^{(s)}+\left(\xi-\sqrt{\xi^2+\varepsilon}\right)/\varepsilon\right)^2+\rho^{(s)2}\Delta^{(1)2}\right)^{-1}, \tag{21b}$$

$$\omega\frac{d^2N_{\text{DTR}}^{(s)}}{d\omega d\Omega}=\frac{e^2}{4\pi^2}P^{(s)^2}G(\theta,\omega)R_{\text{DTR}}^{(s)}, \tag{22a}$$

$$G(\theta,\omega)=\theta^2\left(\frac{1}{\theta^2+\gamma^{-2}}-\frac{1}{\theta^2+\gamma^{-2}-\chi_a'}\right)^2\exp\left(-\frac{\omega\chi_a''}{\gamma_0}a\right)+\theta^2\left(\frac{1}{\theta^2+\gamma^{-2}-\chi_a'}-\frac{1}{\theta^2+\gamma^{-2}-\chi_0'}\right)^2$$

$$+2\theta^2\left(\frac{1}{\theta^2+\gamma^{-2}}-\frac{1}{\theta^2+\gamma^{-2}-\chi'_a}\right)\left(\frac{1}{\theta^2+\gamma^{-2}-\chi'_a}-\frac{1}{\theta^2+\gamma^{-2}-\chi'_0}\right)$$

$$\times\cos\left(\frac{\omega a}{2\gamma_0}(\theta^2+\gamma^{-2}-\chi'_a)\right)\exp\left(-\frac{\omega\chi''_a}{2\gamma_0}a\right), \quad (22b)$$

$$R_{\mathrm{DTR}}^{(s)}=\frac{\varepsilon^2}{\xi(\omega)^2+\varepsilon}\Big[\exp\left(-2B^{(s)}\rho^{(s)}\Delta^{(1)}\right)+\exp\left(-2B^{(s)}\rho^{(s)}\Delta^{(2)}\right)$$

$$-2\cdot\exp\left(-B^{(s)}\rho^{(s)}\frac{1+\varepsilon}{\varepsilon}\right)\cos\left(\frac{2B^{(s)}\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\Big], \quad (22c)$$

where

$$\Delta^{(2)}=\frac{\varepsilon+1}{2\varepsilon}+\frac{1-\varepsilon}{2\varepsilon}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)2}+\varepsilon}}+\frac{\kappa^{(s)}}{\sqrt{\xi^{(s)2}+\varepsilon}},\quad \Delta^{(1)}=\frac{\varepsilon+1}{2\varepsilon}-\frac{1-\varepsilon}{2\varepsilon}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)2}+\varepsilon}}-\frac{\kappa^{(s)}}{\sqrt{\xi^{(s)2}+\varepsilon}},$$

$$\sigma^{(s)}=\frac{1}{\nu^{(s)}}\left(\frac{\theta^2}{|\chi'_0|}+\frac{1}{\gamma^2|\chi'_0|}+1\right),\quad \nu^{(s)}=\frac{\chi'_{\boldsymbol g}C^{(s)}}{\chi'_0},\quad B^{(s)}=\frac{\omega|\chi'_{\boldsymbol g}|C^{(s)}}{2}\frac{b}{\gamma_0},\quad \kappa^{(s)}=\frac{\chi''_{\boldsymbol g}C^{(s)}}{\chi''_0},$$

$$\xi^{(s)}(\omega)=\frac{2\sin^2\theta_{\mathrm B}}{V^2|\chi'_{\boldsymbol g}|C^{(s)}}\left(1-\frac{\omega(1-\theta\cos\varphi\cot\theta_{\mathrm B})}{\omega_{\mathrm B}}\right)+\frac{1-\varepsilon}{2\nu^{(s)}},\quad \varepsilon=\frac{\sin(\delta+\theta_{\mathrm B})}{\sin(\delta-\theta_{\mathrm B})},\quad \rho^{(s)}=\frac{\chi''_0}{|\chi'_{\boldsymbol g}|C^{(s)}}. \quad (23)$$

In the resulting expressions (21) and (22), functions $R_{\mathrm{PXR}}^{(s)}$ and $R_{\mathrm{DTR}}^{(s)}$ represent the PXR and DTR spectra that within the dynamic diffraction theory describe propagation of free and bound X-ray photons through a crystal plate, respectively. Function $G(\theta,\omega)$ describes the angular dependence of the diffraction transition radiation and consists of three summands. The first summand corresponds to the transition radiation generated by a relativistic electron transversing the first boundary, which further propagates in the amorphous medium and is diffracted in the crystal plate in the Bragg direction. The second summand corresponds to the transition radiation generated on the second boundary (between the amorphous medium and the crystal) and to the radiation diffracted in the crystal plate along the Bragg direction. The third summand describes the interference of these two DTR waves.

Expressions (21) and (22), derived within the dynamic diffraction theory, which describe the spectral-angular distribution of PXR and DTR of a relativistic electron in a composite medium consisting of the amorphous and crystalline plates, represent the principal result of this work. These expressions take into consideration the asymmetry of reflection of the field relative to the surface of the target (parameter ε) in a crystalline plate. They allow the effects of dynamic diffraction to be revealed and the radiation yield to be optimized as a function of the target parameters.

The expressions describing the angular density of PXR and DTR, following from (21) and (22), are given by

$$\frac{dN_{\mathrm{PXR}}^{(s)}}{d\Omega}=\frac{e^2}{4\pi^2}P^{(s)2}\frac{\theta^2}{(\theta^2+\gamma^{-2}-\chi'_0)^2}\int_{-\infty}^{+\infty}R_{\mathrm{PXR}}^{(s)}(\omega)\frac{d\omega}{\omega}, \quad (24a)$$

$$\frac{dN_{\mathrm{DTR}}^{(s)}}{d\Omega}=\frac{e^2}{4\pi^2}P^{(s)2}\int_{-\infty}^{+\infty}G(\theta,\omega)R_{\mathrm{DTR}}^{(s)}(\omega)\frac{d\omega}{\omega}. \quad (24b)$$

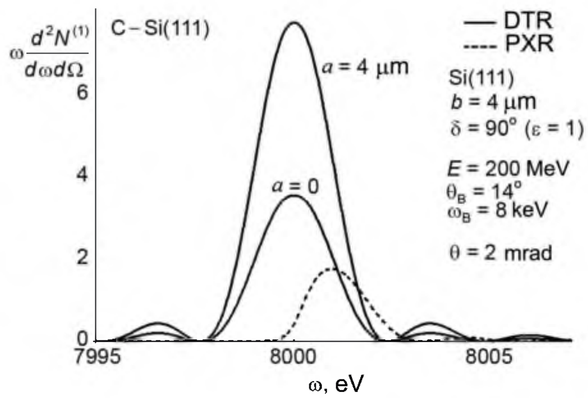

Fig. 2

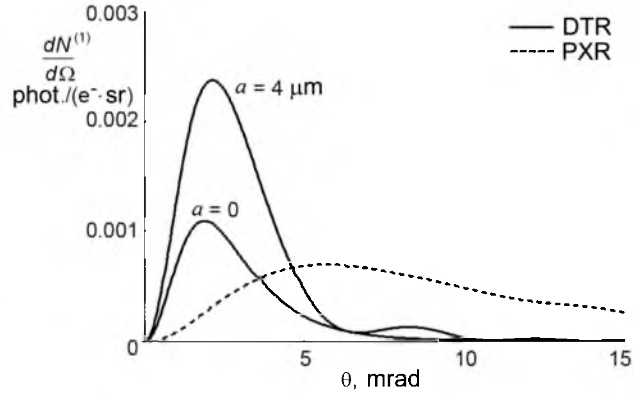

Fig. 3

Fig. 2. PXR spectrum and DTR spectra for the thickness of the amorphous carbon plate $a = 4$ μm and without any plate ($a = 0$).

Fig. 3. Angular density of PXR and angular densities of DTR for the parameters listed in Fig. 2.

For illustrative purposes, let us address the process of radiation of a relativistic electron transversing a composite bilayer structure consisting of amorphous carbon and crystalline silicon. Consider σ -polarized waves ($s = 1$). Note that the energy in the calculations was taken to be comparatively low $E = 200$ MeV, and the system of reflecting parallel atomic planes (111) of silicon was assumed to be perpendicular to the boundary of the target ($\delta = 90^\circ$), i.e., we deal with a conventional case of symmetrical reflection. Shown in Fig. 2 are the curves describing the PXR spectrum (dotted line) and DTR spectra for a fixed observation angle θ for two cases: in the first, the thickness of the amorphous target $a = 4$ μm and in the second there is no plate $(a = 0)$. It is evident that the presence of a carbon plate in front of the crystalline silicon plate considerably increases the spectral-angular density of DTR. It should also be noted that PXR is generated in the crystalline plate only and is independent of the presence of an amorphous plate. This circumstance could be used, e.g., to identify peaks in the PXR and DTR in real experiments.

Let us address the angular density of both types of radiation. The curves describing the PXR and DTR angular densities, which were built using formulas (24) for the same parameters as those in Fig. 2, are presented in Fig. 3. Figure 3 suggests that the angular density of PXR from the bilayer target under study can considerably increase compared to a single-layer crystalline target. It should be underlined that the principal contributions both into the spectral-angular (Fig. 2) and into the angular (Fig. 3) DTR density in the case under study come from the TR waves generated on the first boundary (vacuum-amorphous medium) and the interference terms of the TR waves from the first and the second boundaries. The contribution from the first wave and the interference term corresponds to the first and third summands in expression (22b). Note also that the curves in Figs. 2 and 3 have been built for the crystal plate thickness that is optimal for the DTR yield, in other words, for a thickness that would ensure maximum DTR yield. This result is appealing from the perspective of designing a monochromatic X-ray source relying on the radiation of relativistic electrons in periodic media.

SUMMARY

Within the framework of a two-wave approximation of the dynamic diffraction theory, we have developed a theory of coherent X-ray radiation of a relativistic electron transversing a composite medium consisting of amorphous and crystalline plates. Relying on this approximation, we have derived a set of expressions describing spectral-angular densities of parametric X-ray and diffraction transition radiation. We have demonstrated that the spectral-angular density of DTR from a composite target consisting of the amorphous and crystalline plates can significantly exceed that from a single crystalline plate of the same thickness.

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