PHYSICAL REVIEW A VOLUME 50, NUMBER 2 AUGUST 1994

Sub-Doppler cooling of three-level Λ atoms in space-shifted standing light waves

D. Kosachiov
State Technical University, 195251, St. Petersburg, Russia

Yu. Rozhdestvensky
S. I. Vavilov State Optical Institute, 199034, St. Petersburg, Russia

M. Olsen, L. Plimak, and D. F. Walls
Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand
(Received 21 March 1994)

We present an investigation of an alternative mechanism for sub-Doppler cooling of atoms, based on coherent population transfer in three-level Λ systems. The mechanism considered is that of a Λ atom interacting with two standing light waves with a mutual spatial phase shift $\varphi \neq 0$. The spatial dependence of the level populations of the Λ atom for different values of φ is presented. For $\varphi \neq 0$, this clearly demonstrates coherent population transfer in an atom with transverse motion along the space-shifted nodes and antinodes of the two standing waves. We show that this allows translational temperatures well below the Doppler limit $T_D = \hbar\gamma/k_B$ to be achieved.

PACS number(s): 32.80.Pj

I. INTRODUCTION

At present, mechanisms for the translational cooling of neutral atoms allow temperatures below the Doppler limit $T_D = \hbar\gamma/k_B$, determined by the natural linewidth 2γ [1], to be achieved. These mechanisms are based on coherent phenomena in multilevel atoms, resulting, in principle, in deep cooling of the atoms. For example, coherent population trapping [2] allows temperatures below T_D, as well as below the temperature $T_R = R/k_B$ [1], determined by the recoil energy $R = \hbar^2k^2/2M$, to be obtained (M is the atomic mass and $\hbar k$ is the photon momentum).

Another coherent effect existing in three-level Λ schemes, namely, coherent population transfer, is also widely investigated at present ([3] see also references therein). The essence of this effect is in the population transfer between the two lower levels of a Λ system interacting with two time-shifted light pulses. The absence of population in the upper level is characteristic of this effect. The population exchange between lower levels can be described in a similar manner to the well known quasicrossing of molecular terms [4]. The low upper level population, with substantial population transfer between lower levels, seems promising for the cooling of atoms, since this should result in high light pressure forces with low velocity diffusion. In general, the light pressure force is defined by the interaction of the induced dipole moment of the atom with the light wave. In turn, the existence of the induced dipole moment is connected with population transfer in the system. At the same time, at low field intensities diffusion of the atoms in velocity space is mainly connected with momentum fluctuations due to the randomness of the direction of spontaneous emission and is therefore proportional to the population of the upper level. Since the coherent population transfer implies that this population is nearly zero, the whole situation looks favorable for deep translational cooling of the atoms.

In practice, the coherent population transfer can be realized by means of placing the Λ atom into the field of two standing waves of nearly equal frequencies, shifted in space. For a drifting atom, the spatial separation of the antinodes of these waves transforms into a time shift between two series of the light pulses. These considerations result in the alternative method for the sub-Doppler cooling of Λ atoms presented in this paper.

II. STATEMENT OF THE PROBLEM

We consider three-level atoms in the Λ configuration (Fig. 1) normally incident on two standing wavelight fields. The cooling of the atomic motion is in the direction along the standing waves (z direction). The two standing light waves have a spatial phase shift φ with respect to each other,

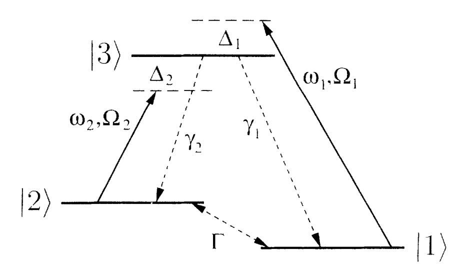

FIG. 1. Three-level Λ atom in the field of the two standing waves.

$$E(z,t) = 2E_1 \cos\omega_1 t \cos(kz) + 2E_2 \cos\omega_2 t \cos(kz+\varphi), \quad (1)$$

where E_m and ω_m are the amplitude and frequency of the wave resonant with the atomic transition $|3\rangle \rightarrow |m\rangle$ ($m = 1,2$). The two wave vectors are assumed to be approximately equal, $k_1 \approx k_2 \approx k$. In the rotating wave approximation, the dynamics of the atomic density matrix in the field (1) are described by the following equations [5]:

$$i\frac{d}{dt}\rho_{11} = i\gamma\rho_{33} - \Omega_1 \cos(kz)(\rho_{31} - \rho_{13}), \quad (2a)$$

$$i\frac{d}{dt}\rho_{22} = i\gamma\rho_{33} - \Omega_2 \cos(kz+\varphi)(\rho_{32} - \rho_{23}), \quad (2b)$$

$$i\frac{d}{dt}\rho_{33} = -2i\gamma\rho_{33} + \Omega_1 \cos(kz)(\rho_{31} - \rho_{13}) + \Omega_2 \cos(kz+\varphi)(\rho_{32} - \rho_{23}), \quad (2c)$$

$$i\frac{d}{dt}\rho_{13} = (-i\gamma + \Delta_1)\rho_{13} - \Omega_1 \cos(kz)(\rho_{33} - \rho_{11}) + \Omega_2 \cos(kz+\varphi)\rho_{12}, \quad (2d)$$

$$i\frac{d}{dt}\rho_{23} = (-i\gamma + \Delta_2)\rho_{23} + \Omega_1 \cos(kz)\rho_{21} - \Omega_2 \cos(kz+\varphi)(\rho_{33} - \rho_{22}), \quad (2e)$$

$$i\frac{d}{dt}\rho_{12} = (-i\Gamma + \Delta_1 - \Delta_2)\rho_{12} - \Omega_1 \cos(kz)\rho_{32} + \Omega_2 \cos(kz+\varphi)\rho_{13}. \quad (2f)$$

Here $\rho_{ji} = \rho_{ij}^*$, and $\frac{d}{dt} = \frac{\partial}{\partial t} + v_z\frac{\partial}{\partial z}$, and $\Delta_m = \omega_m - \omega_{m3}$ are the detunings of the standing waves from the atomic transitions (cf. Fig. 1); $\Omega_m = dE_m/2\hbar$ are the Rabi frequencies; γ represents the spontaneous decay rates $|3\rangle \rightarrow |m\rangle$ ($m = 1,2$), assumed equal for both levels; and Γ is the decay rate of the atomic coherence ρ_{12} ($\Gamma \ll \gamma$). When obtaining (2), the anisotropy of the spontaneous decays was neglected.

The light pressure force acting on a Λ atom in the field (1) can be found as in [1,5]

$$F_z = -\hbar k[\Omega_1(\rho_{13} + \rho_{31})\sin(kz) + \Omega_2(\rho_{23} + \rho_{32})\sin(kz+\varphi)], \quad (3)$$

where ρ_{13} and ρ_{23} are the nondiagonal elements of the Λ atom's density matrix found from (2), for $t \gg \gamma^{-1}$. The latter determines the time scale on which the atomic motion can be considered classically. Formally, it means that when solving system (2) time derivatives on the left can be neglected.

In the system thus obtained, we expand the ρ-matrix elements in a spatial Fourier series

$$\rho_{ij} = \sum_{n=-\infty}^{+\infty} \rho_{ij}^{(n)} \exp inkz, \quad (4)$$

which accounts directly for the spatial structure of the light field. This leads to an infinite system of recurrent algebraic equations for $\rho_{ij}^{(n)}$ (recalling that the time derivatives are neglected). A solution to this system will be sought by following the continuous fraction method [6]. The nondiagonal matrix elements ρ_{13} and ρ_{23} may be expressed in terms of ρ_{11}, ρ_{22}, and ρ_{12}, this resulting in the system being rewritten as a system of recurrent matrix equations,

$$A_{n+2}\vec{X}_{n+2} + B_n\vec{X}_n + C_{n-2}\vec{X}_{n-2} = \frac{3}{2}i\vec{\gamma}\delta_{n0}, \qquad n = 0, \pm 2, \pm 4, \ldots . \quad (5)$$

Here $\vec{X}_n = \left\{\rho_{11}^{(n)}, \rho_{22}^{(n)}, \rho_{12}^{(n)}, \rho_{21}^{(n)}\right\}$ and $\vec{\gamma} = \{\gamma, \gamma, 0, 0\}$. $A_n, B_n,$ and C_n are 4×4 matrices, the elements of which are functions such as $f(\varphi)/[i\gamma - nkv_z \pm \Delta_p]$ ($p = 1,2$), where $f(\varphi)$ are rational functions of $\cos\varphi$ and $\sin\varphi$.

A solution to (5) can be found as an infinite converging matrix chain fraction [6],

$$\vec{X}_0 = \cfrac{Q(0)}{\mathbb{1} + \cfrac{Q(2)}{\mathbb{1} + Q(4)\cdots}}, \quad (6)$$

where $Q(n)$ are combinations of the direct and inverse matrices A_n, B_n, and C_n. By making use of (6), the force (3) can also be found as an infinite chain fraction. Since we will be interested later in atomic distributions with a spatial distribution of $\Delta z \gg \lambda$, only the spatially homogeneous part of the light pressure force will be considered [5],

$$F_z^0 = \frac{1}{\lambda}\int_0^{\lambda} F_z dz. \quad (7)$$

III. RESULTS AND DISCUSSION

Let us now consider the behavior of the light pressure force F_z^0 for small atomic velocities. In Fig. 2, the dependence of this force on the atomic velocity for different values of the phase shift φ [cf. (1)] between the standing waves is presented. It can be seen that for $\varphi \neq 0$ this dependence is dispersive near zero velocity and the friction coefficient at $v_z = 0$ is greatly increased. For $\varphi = 0$ there is no dispersive behavior. Note that, for values of the detunings and Rabi frequencies given in Fig. 2(a), for $\varphi = 0$ "heating" of the atoms takes place whereas for $\varphi \neq 0$ we have "cooling." This means that in an experiment we can switch from the collimation (narrowing of the velocity distribution) to decollimation of the atomic beam by simply changing the spatial phase shift φ while leaving all other conditions (i.e., the Rabi frequencies and detunings) unchanged.

Figure 3 demonstrates how the velocity dependence of the light force changes with increasing detunings of the light waves for a given value of φ. It can be seen that the amplitude of the force diminishes and the velocity domain where the dispersive dependence takes place gets narrower, but the derivative of the force at $v_z = 0$ remains unchanged for a wide range of detuning values. It should be emphasized that the dispersive character of the dependence of the light force near zero velocity is not associated with the saturation of the transitions in the

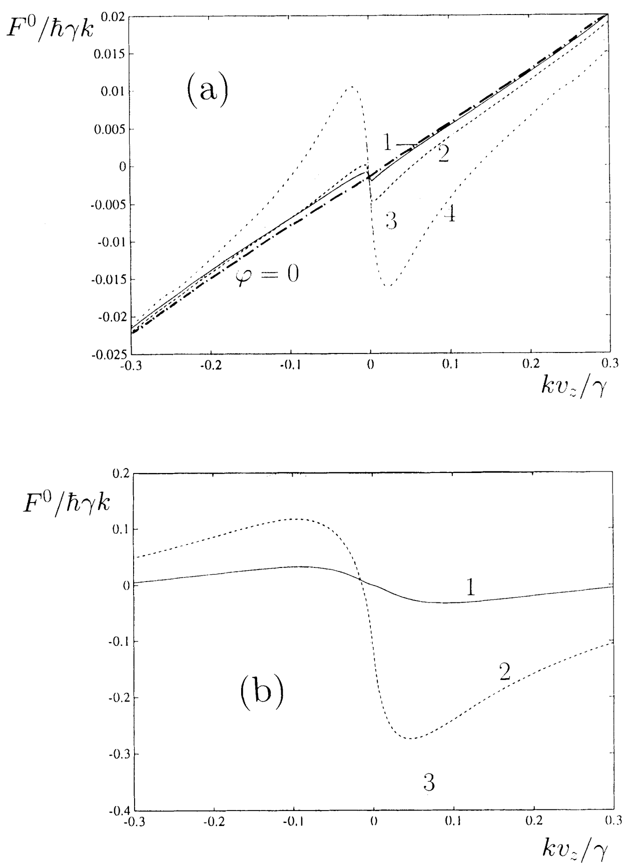

FIG. 2. Dependence of the light pressure force F_z^0 on the velocity v_z of the Λ atom for different values of the phase shift φ. (a) $\Delta_1 = \gamma, \Delta_2 = 2\gamma, \Omega_1 = \Omega_2 = 0.5\gamma$, and $\Gamma = 0.01\gamma$. 1, $\varphi = \frac{\pi}{24}$; 2, $\varphi = \frac{\pi}{12}$; 3, $\varphi = \frac{\pi}{6}$; and 4, $\varphi = \frac{\pi}{4}$. (b) $\Delta_1 = 4\gamma, \Delta_2 = 8\gamma, \Omega_1 = \Omega_2 = 4\gamma$, and $\Gamma = 0.01\gamma$. 1, $\varphi = 0$; 2, $\varphi = \frac{\pi}{6}$; and 3, $\varphi = \frac{\pi}{4}$.

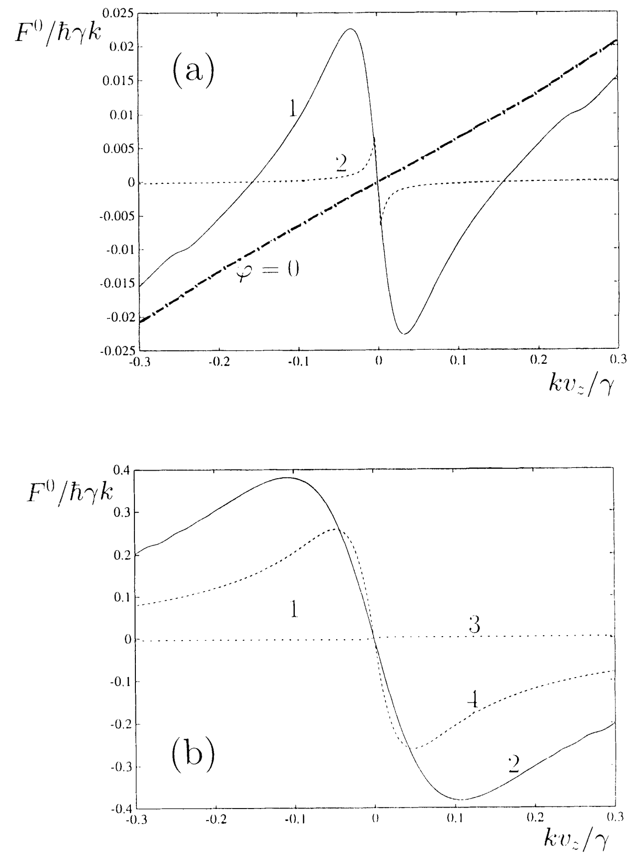

FIG. 3. Dependence of the light pressure force F_z^0 on the velocity v_z of the Λ atom for different values of the detunings. (a) $\varphi = \frac{\pi}{2}, \Omega_1 = \Omega_2 = 0.5\gamma$, and $\Gamma = 0.01\gamma$. 1, $\Delta_1 = \gamma$, and $\Delta_2 = 2\gamma$; 2, $\Delta_1 = 5\gamma$, and $\Delta_2 = 10\gamma$; the dashed curve corresponds to $\varphi = 0, \Delta_1 = \gamma$, and $\Delta_2 = 2\gamma$. (b) $\Delta_1 = 8\gamma, \Delta_2 = 16\gamma, \Omega_1 = \Omega_2 = 4\gamma$, and $\Gamma = 0.01\gamma$, 1, $\varphi = 0$; 2, $\varphi = \frac{\pi}{2}$. $\Delta_1 = 8\gamma, \Delta_2 = 16\gamma, \Omega_1 = \Omega_2 = 4\gamma$, and $\Gamma = 0.01\gamma$; 3, $\varphi = 0$; and 4, $\varphi = \frac{\pi}{2}$.

Λ atom, since curves Figs. 2(a) and 3(a) were obtained under low saturation conditions,

$$\left(\frac{\Delta_m^2}{\gamma^2} + 1\right)^{\frac{1}{2}} \gg \frac{2\Omega_m^2}{\gamma^2}, \quad m = 1, 2. \qquad (8)$$

This dispersive characteristic of the light pressure force velocity dependence is associated with the spatial structure of the light field. Indeed, Fig. 4 demonstrates how the force F_z^0 calculated for the phase shift $\varphi = \frac{\pi}{2}$ depends on the number of terms taken into account in the converging continuous fraction (6). Curve 1 in Fig. 4 corresponds to only $Q(0)$ being taken into account in the chain fraction (6). This corresponds to the following approximation for the matrix elements [cf. (4)]:

$$\rho_{m3} = \rho_{m3}^{(+1)} \exp ikz + \rho_{m3}^{(-1)} \exp(-ikz), \quad m = 1, 2$$

$$\rho_{ll} = \rho_{ll}^{(0)}, \quad l = 1, 2, 3,$$

$$\rho_{12} = \rho_{12}^{(0)}.$$

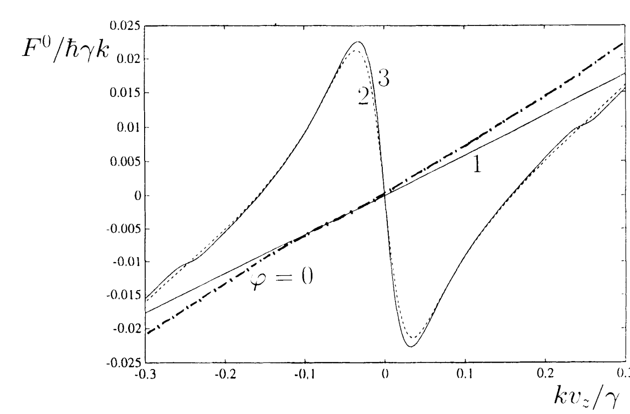

FIG. 4. Dependence of the light pressure force F_z^0 on the velocity v_z of the Λ atom for different orders taken into account in the chain fraction (6). $\varphi = \frac{\pi}{2}, \Delta_1 = \gamma, \Delta_2 = 2\gamma, \Omega_1 = \Omega_2 = 0.5\gamma$, and $\Gamma = 0.01\gamma$. 1, $n = 0$; 2, $n = 2$; and 3, $n = 4$. The dashed curve corresponds to $\varphi = 0$ for the same values of parameters.

Curve 1 exhibits no dispersive behavior and nearly coincides with the force dependence on the velocity for $\varphi = 0$ (Fig. 4). At the same time, curve 2 in Fig. 4 corresponds to $Q(0) \neq 0$ and $Q(2) \neq 0$ in the chain fraction (6), which, in turn, corresponds to the following approximation for the matrix elements:

$$\rho_{m3} = \rho_{m3}^{(+1)} \exp ikz + \rho_{m3}^{(-1)} \exp(-ikz) + \rho_{m3}^{(+3)} \exp 3ikz + \rho_{m3}^{(-3)} \exp(-3ikz), \quad m = 1,2$$

$$\rho_{ll} = \rho_{ll}^{(0)} + \rho_{ll}^{(+2)} \exp 2ikz + \rho_{ll}^{(-2)} \exp(-2ikz), \qquad l = 1,2,3$$

$$\rho_{12} = \rho_{12}^{(0)} + \rho_{12}^{(+2)} \exp 2ikz + \rho_{12}^{(-2)} \exp(-2ikz).$$

As soon as the spatial dependence of the populations of the lower levels of the Λ atom is included, the desired effect appears: curve 2 exhibits the pronounced dispersive shape and a considerably increased friction coefficient. Further quantitative improvements [$Q(4) \neq 0$ in the chain fraction (6)] do not bring qualitative changes (curve 3 in Fig. 4). This also demonstrates the quick convergence of the chain fraction and the consistency of computations using this representation.

Clearly the effect of interest is connected with the spatial dependence of the population of the two lower levels of the Λ atom. Consider first this dependence for different values of φ, assuming that $v_z = 0$. In this case, the expressions for the populations can be found as stationary analytical solutions to the system (2),

$$\rho_{11} = \frac{Y_2(z)}{Y_1(z) + Y_2(z)},$$

$$\rho_{22} = \frac{Y_1(z)}{Y_1(z) + Y_2(z)},$$

$$\rho_{33} = \frac{2Y_1(z)Y_2(z)}{Y_1(z) + Y_2(z)}, \tag{9}$$

where $Y_1(z) = \Omega_1^2 \cos^2(kz)/(\Delta_1^2 + \gamma^2)$ and $Y_2(z) = \Omega_2^2 \cos^2(kz + \varphi)/(\Delta_2^2 + \gamma^2)$ and it is assumed that $\Delta_1, \Delta_2, \gamma \gg \Omega_1, \Omega_2$. One can see from (9) that there exist two physical situations. The first one occurs for $\varphi = 0$ when the populations of the lower levels of the Λ atom, ρ_{11} and ρ_{22}, are defined by the detunings and exhibit no spatial dependence. Conversely, if $\varphi \neq 0$ the strong spatial dependence of the populations emerges. In the node of the first standing wave [$\cos(kz) = 0$] only the second wave has any effect, so that $\rho_{11} = 1$ and $\rho_{22} = 0$. Similarly, if $\cos(kz + \varphi) = 0$, one finds $\rho_{11} = 0$ and $\rho_{22} = 1$. Thus, for an atom moving slowly enough, adiabatic population exchange between the lower levels takes place (cf. [7]).

If the velocity of the atom is a little larger, the adiabatic condition is violated and the second effect emerges — coherent population transfer in the Λ system. Basically, this effect takes place in the Λ system excited by time-delayed light pulses [3]. This is exactly how the moving atom "sees" the antinodes of the two standing waves: the spatial phase shift φ transforms into a time shift of φ/kv_z. Therefore, the velocity is "small" if $kv_z \ll \varphi\Gamma$, when only the adiabatic population transfer exists. The maximum of the average force due to the coherent population transfer corresponds to $kv_z \sim \varphi\Gamma$, and this force dies out when $kv_z \gg \varphi\Gamma$, which is in obvious agreement with the numerical data.

Note that in the Λ system the "adiabatic" average force at zero velocity may be nonzero, but its value is small compared to that due to the coherent population transfer. The existence of this residual force results in the atomic beam's deflection without affecting its cooling or heating.

Let us now estimate the atomic temperature assuming that $\varphi = \frac{\pi}{2}$ and $kv_z \approx 0$. For the parameters $\Delta_1 = \gamma, \Delta_2 = 2\gamma$, and $\Omega_{1,2} = 0.5\gamma$, the dynamic friction coefficient β found from Fig. 3 is equal to $7\hbar^2k^2/M$. At the same time, the diffusion coefficient is determined mainly by the quantity ρ_{33} given by (9), and if estimated following [5] is found to be

$$D_{zz} \approx \hbar^2 k^2 \gamma \rho_{33} \approx 0.06 \hbar^2 k^2 \gamma / M.$$

Then, the temperature of the cooled atoms can be found from the Einstein formula [5] to be

$$T_\Lambda = \frac{D_{zz}}{\beta} = 10^{-2} \frac{\hbar\gamma}{k_B} \ll T_D,$$

so that the spatially shifted standing light waves allow atomic temperatures well below the Doppler limit to be obtained. The velocity interval where the dispersive behaviour of the light force takes place is of the order of $0.1\gamma/k \approx 1\ \mathrm{ms}^{-1}$. The evolution of the velocity distribution of the Λ atoms in the standing waves is shown in Fig. 5. When drawing the curves in Fig. 5, the Liouville equation was solved with the force found by the method

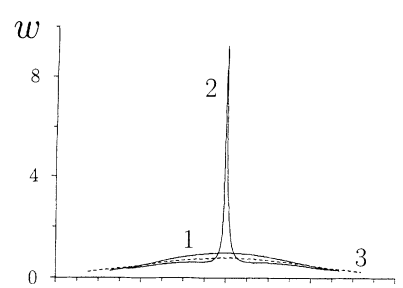

FIG. 5. Time evolution of the velocity distribution of the Λ atoms interacting with the field of the standing waves. 1, initial velocity distribution with $\Delta v_z = 0.5\gamma/k$, which corresponds to $T = T_D$ for ^{23}Na; 2, velocity distribution at $t = 5 \times 10^{-6}$ s for $\varphi = \frac{\pi}{2}, \Omega_1 = \Omega_2 = 0.5\gamma, \Delta_1 = \gamma, \Delta_2 = 2\gamma$, and $\Gamma = 0.01\gamma$; 3, velocity distribution at $t = 10^{-5}$ s for $\varphi = 0, \Omega_1 = \Omega_2 = 0.5\gamma, \Delta_1 = -\gamma, \Delta_2 = -2\gamma$, and $\Gamma = 0.01\gamma$.

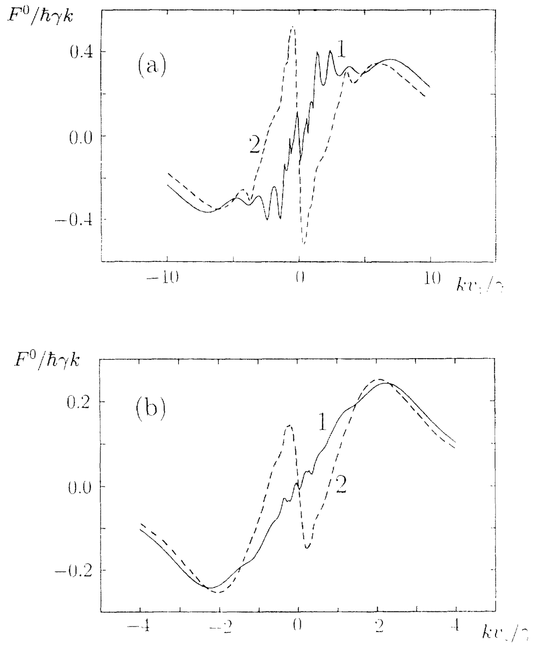

FIG. 6. Dependence of the light pressure force F_z^0 on the velocity v_z of the Λ atom for considerable saturation of the atomic transitions. 1, $\varphi = 0$; 2, $\varphi = \frac{\pi}{2}$. (a) $\Omega_1 = \Omega_2 = 1.5\gamma, \Delta_1 = \gamma, \Delta_2 = 2\gamma$, and $\Gamma = 0.01\gamma$. (b) $\Omega_1 = \Omega_2 = 5\gamma, \Delta_1 = 3\gamma, \Delta_2 = 6\gamma$, and $\Gamma = 0.01\gamma$.

described above. The narrowing of velocity distribution at $\varphi = \frac{\pi}{2}$ is clearly evident.

Finally, Fig. 6 demonstrates how the velocity dependence of the light force changes with increasing intensity of the light waves.

Note that all the considerations above retain their validity if the standing waves affecting the Λ atom fluctuate, but, at the same time, exhibit a high degree of mutual coherence. In practice, this can be easily realized if one of the waves is generated from the other using an optoacoustic modulator.

When this manuscript was already prepared for publication, the authors became aware of a recent paper [8]. Proceeding from a different standpoint, the authors of Ref. [8] proposed the same mechanism for the sub-Doppler laser cooling and presented its experimental realization. It may easily be seen that our results are in good qualitative agreement with the phenomena observed. More detailed discussion of the results [8] using our non-phenomenological quantum computational approach may be subject to further investigations.

In conclusion, two effects mainly determine the spatial dependence of the dynamics of the atomic level populations: adiabatic inhomogeneous optical pumping and coherent population transfer. Together these effects essentially change the population dynamics for $\varphi \neq 0$, which in turn results in substantial change in the character of the interaction between the atomic system and the field of the two standing waves as compared to the case of zero spatial shift $\varphi = 0$.

ACKNOWLEDGMENTS

The research was supported by the New Zealand Foundation for Research, Science and Technology; the University of Auckland Research Committee; and the New Zealand Lotteries Grants Board.

[1] J. Opt. Soc. Am. B **6** (11) (1989), special issue on laser cooling and trapping of atoms.

[2] E. Korsunsky, D. Kosachiov, B. Matisov, Yu. Rozhdestvensky, L. Windholz, and C. Neureiter, Phys. Rev. A **48**, 1419 (1993).

[3] V. Gaubatz, P. Rudecki, S. Schiemann, and K. Bergmann, J. Chem. Phys. **92**, 5363 (1990); B. W. Shore *et al.*, Phys. Rev. A **45**, 5297 (1992).

[4] A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1962).

[5] V. G. Minogin and V. S. Letochov, *Laser Light Pressure on Atoms* (Gordon & Breach, New York, 1987).

[6] R. Vilaseca, G. Orriols, L. Roso, R. Corbalan, and E. Arimondo, Appl. Phys. B **34**, 73 (1984).

[7] A. Sidorov, R. Grimm, and V. S. Letokhov, J. Phys. B **24**, 3733 (1991).

[8] R. Gupta, C. Xie, S. Padua, H. Batelaan, and H. Metcalf, Phys. Rev. Lett. **71**, 3087 (1993).