ФИЗИКА

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COHERENT X-RAY RADIATION ALONG THE VELOCITY OF RELATIVISTIC ELECTRON CROSSING A PERIODIC LAYERED MEDIUM

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Abstract. In the present work, the dynamic theory of coherent X-radiation generated by a relativistic electron crossing the multilayered medium in the direction of the electron velocity vector is developed in Bragg scattering geometry for general case of asymmetric reflection. The comparison of the analogous radiation generated by a relativistic electron in the single crystal medium is made. The developed theory predicts the existence condition for the radiation and describes its spectral and angular characteristics.

Keywords: Relativistic electron; periodic layered target; coherent X-radiation dynamic theory

1. Introduction

Traditionally, the relativistic particle radiation in a periodic lamellar structure was considered as resonance transition radiation (RTR) [1, 2]. Significant contribution to the study of X-ray transition radiation was made by a group of physicists from Japan [3-5].

In the work [4] for the first time the periodic target consisting of plates of several hundred nanometers thick was used and the energy of the radiated photons was of 2-4 keV for the fundamental harmonic. The authors of [4] assert that the radiation intensity obtained in this experiment exceeds the synchrotron radiation intensity achievable on the modern electron accelerators. A theoretical description of RTR in the above-mentioned media was presented in the work [6]. Subsequently in the work [7] the RTR of the relativistic electron in a layered medium was considered together with parametric X-radiation (PXR). In the work [8] analogous with the coherent X-radiation in a single crystal medium [9-12], the radiation in a multilayered periodic structure was considered in the dynamic approximation as the scattering of the pseudo photons of the relativistic electron coulomb field in the amorphous layers of the structure. In [8] the coherent radiation in a periodic multilayered structure was considered for the first time as the result of the contribution of two radiation mechanisms, namely PXR and diffracted transition radiation (DTR).

The theory of the radiation of the relativistic electron in layered periodic media [8] describes properly the experimental data presented in the same work [13]. The data were obtained in the experiment, where the structure was used with the layers of about 1 nanometer thick, and the photons were generated with energy of about 15 keV. A detailed comparison of the theory [8] and the experiment [13] are presented in the work [14].

It's necessary to note that in all the cited works the radiation process in a layered medium was considered in Bragg geometry only for the case of symmetric reflection, where the angle between target surface and reflecting planes/ (layers) is equal to zero. Later, in our works [15-16] the dynamic theory of coherent X-radiation by the relativistic electron, crossing a layered medium in Laue geometry for the general case of asymmetric reflection of the relativistic electron coulomb field in respect to the entrance surface of the target was built. It was clarified in these works that the radiation yield in a periodic layered target significantly exceeds the yield in the crystal and the additional opportunity of the yield increase by means of the choice of optimal reflection asymmetry was shown. It was revealed in these works that the radiation yield in the periodic layered target significantly exceeds the yield in the crystalline one, and the additional opportunity of the yield increase by the choice of the optimal reflection asymmetry was proved.

The theory of parametric X-radiation (PXR) of a relativistic charged particle in a single crystal medium forecasts the radiation not only in the Bragg direction, but also in the direction along the particle velocity vector

(FPXR) [17-19]. FPXR is a result of the dynamic diffraction of pseudo photons of the particle coulomb field on the atomic planes in the crystal. The attempts of the experimental study of FPXR are known [20-24], but the first report about FPXR detection in the crystal target was recently made in the work [24]. The detailed theoretical description of the dynamic effect of FPXR and accompanying background of transition radiation (TR) in a crystal in the case of symmetric reflection was provided in the works [25-27].

The general case of asymmetric reflection was presented in FPXR theory built in Laue scattering geometry [28] and in a theory of transition radiation (TR) built in Bragg geometry [29].

In the present work the dynamic theory of the coherent radiation along the velocity of the relativistic electron crossing the periodic layered medium in Bragg scattering geometry was built for general case of asymmetric reflection, when the reflecting layers in the target are situated under a free angle relative to the target surface (symmetric reflection is a special case of the reflection). The expressions for spectral-angular characteristics of FPXR and TR in a periodic layered medium are derived on the basis of two wave approximation of the dynamic diffraction theory.

2. Radiation amplitude

Let us assume that a relativistic electron with the velocity $\mathbf{V}$ passes through the multilayered structure, which consists of periodically situated amorphous layers of the thicknesses a and b ($T = a + b$ – the structure period). The substance of the layers a and b have the dielectric susceptibility χ_a and χ_b correspondently (fig.1).

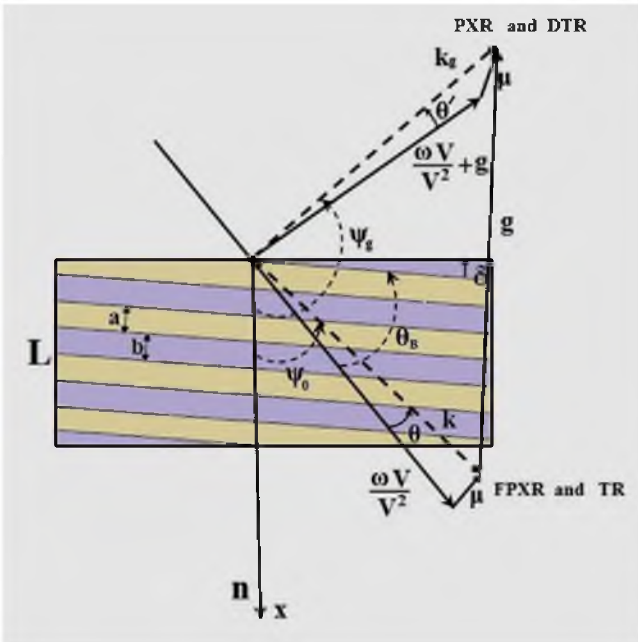

Fig. 1. The radiation process geometry and table of notations for the using magnitudes. θ and θ' are the radiation angles, θ_B is the Bragg angle (the angle between the electron velocity $\mathbf{V}$ and the layers of the target), δ is the angle between the target surface and the layers of the target, $\mathbf{k}$ and $\mathbf{k}_g$ are the wave vectors of the incident and diffracted photons

For studying the electromagnetic radiation accompanying this process we will use the two-wave approximation of the dynamic diffraction theory. Let us consider the Fourier image of the electromagnetic field

$$\mathbf{E}(\mathbf{k},\omega) = \int dt\, d^3\mathbf{r}\, \mathbf{E}(\mathbf{r},t)\exp(i\omega t - i\mathbf{kr}) . \tag{1}$$

As the relativistic particle coulomb field could be represented practically as transverse, the incident $\mathbf{E}_0(\mathbf{k},\omega)$ and the diffracted $\mathbf{E}_\mathbf{g}(\mathbf{k},\omega)$ electromagnetic waves are determined by two amplitudes with different values of transverse polarization:

$$\begin{aligned} \mathbf{E}_0(\mathbf{k},\omega) &= E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)}, \\ \mathbf{E}_\mathbf{g}(\mathbf{k},\omega) &= E_\mathbf{g}^{(1)}(\mathbf{k},\omega)\mathbf{e}_1^{(1)} + E_\mathbf{g}^{(2)}(\mathbf{k},\omega)\mathbf{e}_1^{(2)}, \end{aligned} \tag{2}$$

where the vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to the vector $\mathbf{k}_\mathbf{g} = \mathbf{k} + \mathbf{g}$. The vectors $\mathbf{e}_0^{(2)}$, $\mathbf{e}_1^{(2)}$ lie in the plane of vectors $\mathbf{k}$ and $\mathbf{k}_\mathbf{g}$ (π-polarization) and the vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_1^{(1)}$ are normal to it (σ-polarization). The vector $\mathbf{g}$ is analogous to the reciprocal lattice vector in the crystal, and it is perpendicular to the layers and its length is $g = \frac{2\pi}{T}n$, $n = 0, \pm1, \pm2, \ldots$

The system of equations for the Fourier image of the electromagnetic field in two-wave approximation of the dynamic diffraction theory is as follows:

$$\begin{cases}(\omega^2(1+\chi_0) - k^2)E_0^{(s)} + \omega^2\chi_{-\mathbf{g}}C^{(s,\tau)}E_\mathbf{g}^{(s)} = 8\pi^2 ie\omega\theta VP^{(s)}\delta(\omega - \mathbf{kV}),\\ \omega^2\chi_\mathbf{g}C^{(s,\tau)}E_0^{(s)} + (\omega^2(1+\chi_0) - k_\mathbf{g}^2)E_\mathbf{g}^{(s)} = 0,\end{cases} \tag{3}$$

where $\chi_\mathbf{g}$, $\chi_{-\mathbf{g}}$ are the coefficients of Fourier expansion of dielectric susceptibility in the reciprocal lattice vector $\mathbf{g}$:

$$\chi(\omega, \mathbf{r}) = \sum_\mathbf{g} \chi_\mathbf{g}(\omega)\exp(i\mathbf{gr}) = \sum_\mathbf{g} \left(\chi'_\mathbf{g}(\omega) + i\chi''_\mathbf{g}(\omega)\right)\exp(i\mathbf{gr}). \tag{4}$$

The quantities $C^{(s,\tau)}$ and $P^{(s)}$ are defined in (3) as follows:

$$C^{(s,\tau)} = \mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)} = (-1)^\tau C^{(s)},\ C^{(1)} = 1,\ C^{(2)} = \left|\cos 2\theta_B\right|,$$
$$P^{(s)} = \mathbf{e}_0^{(s)}(\boldsymbol{\mu}/\mu),\ P^{(1)} = \sin\varphi,\ P^{(2)} = \cos\varphi, \tag{5}$$

where $\boldsymbol{\mu} = \mathbf{k} - \omega\mathbf{V}/V^2$ is a component of the momentum of the virtual photon, which is perpendicular to the particle velocity $\mathbf{V}$ ($\mu = \omega\theta/V$, where θ <<1 is the angle between vectors $\mathbf{k}$ and $\mathbf{V}$), θ_B is the Bragg angle, φ is azimuthal radiation angle, counted from the plane formed by the velocity vector $\mathbf{V}$ and $\mathbf{g}$ vector perpendicular to the reflecting layers. The magnitude of the vector $\mathbf{g}$ can be expressed by the Bragg angle θ_B and the Bragg frequency ω_B: $g = 2\omega_B \sin\theta_B / V$. The angle between vector $\frac{\omega\mathbf{V}}{V^2}$ and the incident wave vector $\mathbf{k}$ is notated as θ, the angle between vector $\frac{\omega\mathbf{V}}{V^2} + \mathbf{g}$ and the wave vector of the diffracted wave $\mathbf{k}_\mathbf{g}$ is notated as θ'. The system of equations (3) with $s = 1$ and $\tau = 2$ describes the π-polarized fields. In this case, $\tau = 2$ if $2\theta_B < \frac{\pi}{2}$, otherwise $\tau = 1$.

The quantities χ_0 and $\chi_\mathbf{g}$ in this periodic structure are nominated as follows:

$$\chi_0(\omega) = \frac{a}{T}\chi_a + \frac{b}{T}\chi_b, \tag{6a}$$

$$\chi_\mathbf{g}(\omega) = \frac{\exp(-iga) - 1}{igT}(\chi_b - \chi_a). \tag{6b}$$

The following expressions will be obtained from (6) and will be used further:

$$\chi'_0 = \frac{a}{T}\chi'_a + \frac{b}{T}\chi'_b \tag{7a}$$

$$\chi''_0 = \frac{a}{T}\chi''_a + \frac{b}{T}\chi''_b, \tag{7b}$$

$$\mathrm{Re}\sqrt{\chi_\mathbf{g}\chi_{-\mathbf{g}}} = \frac{2\sin\left(\frac{ga}{2}\right)}{gT}(\chi'_b - \chi'_a), \tag{7c}$$

$$\operatorname{Im}\sqrt{\chi_g\chi_{-g}} = \frac{2\sin\left(\frac{ga}{2}\right)}{gT}\left(\chi_b'' - \chi_a''\right)$$

(7d)

Solving the following dispersion equation (8) which is obtained from system (3) by means of standard methods of the dynamical theory of X-ray scattering of waves in a crystal

$$(\omega^2(1+\chi_0)-k^2)(\omega^2(1+\chi_0)-k_g^2)-\omega^4\chi_{-g}\chi_g C^{(s)^2}=0, \tag{8}$$

we will obtain projections of the wave vectors $\mathbf{k}$ and $\boldsymbol{k}_g$.

Let us search projections of the wave vectors $\mathbf{k}$ and $\boldsymbol{k}_g$ in the following form:

$$k_x = \omega\cos\psi_0 + \frac{\omega\chi_0}{2\cos\psi_0} + \frac{\lambda_0}{\cos\psi_0}, \tag{9a}$$

$$k_{gx} = \omega\cos\psi_g + \frac{\omega\chi_0}{2\cos\psi_g} + \frac{\lambda_g}{\cos\psi_g}. \tag{9b}$$

We will use the known expression connected to the dynamic additions λ_0 and λ_g:

$$\lambda_g = \frac{\omega\beta}{2} + \lambda_0\frac{\gamma_g}{\gamma_0}, \tag{10}$$

where $\beta = \alpha - \chi_0\left(1-\frac{\gamma_g}{\gamma_0}\right)$, $\alpha = \frac{1}{\omega^2}(k_g^2 - k^2)$, $\gamma_0 = \cos\psi_0$, $\gamma_g = \cos\psi_g$, ψ_0 is the angle between the incident wave vector $\mathbf{k}$ and the normal to the plate (target) surface $\boldsymbol{n}$, ψ_g - the angle between the wave vector $\mathbf{k}_g$ and vector $\mathbf{n}$ (see fig.1).

Let us find the wave vectors k and k_g

$$k = \omega\sqrt{1+\chi_0}+\lambda_0,\ k_g = \omega\sqrt{1+\chi_0}+\lambda_g. \tag{11}$$

Taking into account that $k_{\parallel} \approx \omega\sin\psi_0$, $k_{g\parallel} \approx \omega\sin\psi_g$, we will obtain:

$$\lambda_g^{(1,2)} = \frac{\omega}{4}\left(\beta \pm \sqrt{\beta^2 + 4\chi_g\chi_{-g}C^{(s)^2}\frac{\gamma_g}{\gamma_0}}\right), \tag{12a}$$

$$\lambda_0^{(1,2)} = \omega\frac{\gamma_0}{4\gamma_g}\left(-\beta \pm \sqrt{\beta^2 + 4\chi_g\chi_{-g}C^{(s)^2}\frac{\gamma_g}{\gamma_0}}\right), \tag{12b}$$

Since the dynamic additions are small ($|\lambda_0| << \omega$, $|\lambda_g| << \omega$), it can be shown that $\theta \approx \theta'$ (see fig.1) and we will further designate these both angles as θ.

It is convenient to represent the solution of the system (3) for the incident field of the periodic structure in the following form:

$$E_0^{(s)medium} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{-\omega^2\beta - 2\omega\frac{\gamma_g}{\gamma_0}\lambda_0}{4\frac{\gamma_g}{\gamma_0}\left(\lambda_0-\lambda_0^{(1)}\right)\left(\lambda_0-\lambda_0^{(2)}\right)}\delta(\lambda_0-\lambda_0^*) + $$
$$+E_0^{(s)^{(1)}}\delta(\lambda_0-\lambda_0^{(1)}) + E_0^{(s)^{(2)}}\delta(\lambda_0-\lambda_0^{(2)}) \tag{13}$$

where $\lambda_0^* = \omega\left(\frac{\gamma^{-2}+\theta^2-\chi_0}{2}\right)$, $\gamma = 1/\sqrt{1-V^2}$ is Lorentz factor of a particle, $E_0^{(s)^{(1)}}$ and $E_0^{(s)^{(2)}}$ are free incident fields in the concerned media.

For the field in a vacuum before the periodic structure, the solution of system (3) can be represented in form:

$$E_0^{(s)vac\,I} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{-\chi_0 - \frac{2}{\omega}\lambda_0}\delta\left(\lambda_0 - \lambda_0^*\right). \tag{14}$$

The expression for the field in the vacuum behind the target can be written as:

$$E_0^{(s)vac\,II} = \frac{8\pi^2 ieV\Theta P^{(s)}}{\omega}\frac{1}{-\chi_0 - \frac{2\lambda_0}{\omega}}\delta(\lambda_0 - \lambda_0^*) + E_0^{(s)Rad}\delta\left(\lambda_0 + \frac{\omega\chi_0}{2}\right), \tag{15}$$

where $E_0^{(s)Rad}$ is the amplitude of the coherent radiation field along the velocity of the electron.

From the second equation of the system (3) the expression relating the incident and diffracted field in the crystal will follow:

$$E_0^{(s)medium} = \frac{2\omega\lambda_{\boldsymbol{g}}}{\omega^2\chi_{\boldsymbol{g}}C^{(s,\tau)}}E_{\boldsymbol{g}}^{(s)medium}. \tag{16}$$

Using the usual boundary conditions on the input and the exit surface of target:

$$\int E_0^{(s)vac\,I}d\lambda_{\boldsymbol{g}} = \int E_0^{(s)medium}d\lambda_{\boldsymbol{g}}\,,\ \int E_{\boldsymbol{g}}^{(s)medium}d\lambda_{\boldsymbol{g}} = \int E_{\boldsymbol{g}}^{(s)\,\mathrm{vac}}d\lambda_{\boldsymbol{g}}\,,$$

$$\int E_{\boldsymbol{g}}^{(s)medium}\exp\left(i\frac{\lambda_{\boldsymbol{g}}}{\gamma_{\boldsymbol{g}}}L\right)d\lambda_{\boldsymbol{g}} = 0\,, \tag{17}$$

we will obtain the expression for the amplitude of the radiation field.

$$E_0^{(s)Rad} = \frac{8\pi^2 ieV\Theta P^{(s)}}{\omega}\frac{1}{\lambda_{\boldsymbol{g}}^{(1)}\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L\right) - \lambda_{\boldsymbol{g}}^{(2)}\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L\right)}\times$$

$$\times\left[\lambda_{\boldsymbol{g}}^{(2)}\left(\frac{\omega}{-\omega\chi_0 - 2\lambda_0^*} + \frac{\omega}{2(\lambda_0^* - \lambda_0^{(2)})}\right)\left(1-\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L\right)\right)\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L\right) -\right.$$

$$\left.-\lambda_{\boldsymbol{g}}^{(1)}\left(\frac{\omega}{-\omega\chi_0 - 2\lambda_0^*} + \frac{\omega}{2(\lambda_0^* - \lambda_0^{(1)})}\right)\left(1-\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L\right)\right)\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L\right)\right]\exp\left(i\frac{\lambda_0^* + \frac{\omega\chi_0}{2}}{\gamma_0}L\right) \tag{18}$$

Before the analysis of spectral-angular characteristics of the radiation, it is necessary to note that three mechanisms of the radiation make contributions to the total radiation yield: bremsstrahlung, transition radiation (TR) and parametric radiation in forward direction (FPXR). The amplitude $E_0^{(s)Rad}$ contains the contributions of the radiations analogous to FPXR and TR in the crystal.

Let us represent the expression for the radiation field (8) in the following form:

$$E_0^{(s)Rad} = E_{FPXR}^{(s)} + E_{TR}^{(s)}\,, \tag{19a}$$

$$E_{FPXR}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega}{2\lambda^*}\frac{e^{i\frac{\lambda_0^*+\frac{\omega\chi_0}{2}}{\gamma_0}L}}{\lambda_g^{(1)}\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L\right)-\lambda_g^{(2)}\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L\right)}\times$$
$$\times\left[\frac{\lambda_g^{(2)}\lambda_0^{(2)}}{\lambda_0^*-\lambda_0^{(2)}}\left(1-\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L\right)\right)\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L\right)-\right.$$
$$\left.-\frac{\lambda_g^{(1)}\lambda_0^{(1)}}{\lambda_0^*-\lambda_0^{(1)}}\left(1-\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L\right)\right)\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L\right)\right], \tag{19b}$$

$$E_{TR}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}e^{i\frac{\lambda_0^*+\frac{\omega\chi_0}{2}}{\gamma_0}}\left(\frac{\omega}{\omega\chi_0+2\lambda_0^*}-\frac{\omega}{2\lambda_0^*}\right)\times$$
$$\times\left(1-\frac{\frac{\gamma_g}{\gamma_0}(\lambda_0^{(1)}-\lambda_0^{(2)})e^{i\frac{\lambda_0^{(2)}+\lambda_0^{(1)}-2\lambda_0^*}{\gamma_0}L}}{\lambda_g^{(1)}\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L\right)-\lambda_g^{(2)}\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L\right)}\right), \tag{19c}$$

The summands in the square brackets of the expression (19b) represent two branches of the dispersion equation solution corresponding to the two X-ray waves excited in the periodical medium.

For further analysis of the radiation, we will represent the dynamic additions (10) and (11) in the following view:

$$\lambda_0^{(1,2)} = \frac{\omega\left|\chi_g' C^{(s)}\right|}{2\varepsilon}\left(\xi^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2}\mp\right.$$
$$\left.\mp\sqrt{\xi^{(s)2}-\varepsilon-i\rho^{(s)}((1+\varepsilon)\xi^{(s)}-2\kappa^{(s)}\varepsilon)-\rho^{(s)2}\left(\frac{(1+\varepsilon)^2}{4}-\kappa^{(s)2}\varepsilon\right)}\right), \tag{20a}$$

$$\lambda_g^{(1,2)} = \frac{\omega\left|\chi_g' C^{(s)}\right|}{2}\left(\xi^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2}\pm\right.$$
$$\left.\pm\sqrt{\xi^{(s)2}-\varepsilon-i\rho^{(s)}((1+\varepsilon)\xi^{(s)}-2\kappa^{(s)}\varepsilon)-\rho^{(s)2}\left(\frac{(1+\varepsilon)^2}{4}-\kappa^{(s)2}\varepsilon\right)}\right), \tag{20b}$$

where

$$\xi^{(s)}(\omega)=\eta^{(s)}(\omega)+\frac{1+\varepsilon}{2\nu^{(s)}},$$

$$\eta^{(s)}(\omega)=\frac{\alpha}{2\left|\mathrm{Re}\sqrt{\chi_g\chi_{-g}}\right|C^{(s)}}\equiv\frac{\sin^2\theta_B}{V^2C^{(s)}}\frac{gT}{\left|\chi_b'-\chi_a'\right|\left|\sin\left(\frac{ga}{2}\right)\right|}\left(1-\frac{\omega(1-\theta\cos\varphi\cot\theta_B)}{\omega_B}\right),$$

$$\nu^{(s)}=\frac{C^{(s)}\,\mathrm{Re}\sqrt{\chi_g\chi_{-g}}}{\chi_0'}\equiv\frac{2C^{(s)}\left|\sin\left(\frac{ga}{2}\right)\right|}{g}\left|\frac{\chi_b'-\chi_a'}{a\chi_a'+b\chi_b'}\right|,$$

$$\rho^{(s)} = \frac{\chi_0''}{\left|\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}\right|C^{(s)}} \equiv \frac{a\chi_a'' + b\chi_b''}{\left|\chi_b' - \chi_a'\right|C^{(s)}} \frac{g}{2\left|\sin\left(\frac{ga}{2}\right)\right|},$$

$$\kappa^{(s)} = \frac{\chi_{\mathbf{g}}'' C^{(s)}}{\chi_0''} \equiv \frac{2C^{(s)}\left|\sin\left(\frac{ga}{2}\right)\right|}{g}\left|\frac{\chi_b'' - \chi_a''}{a\chi_a'' + b\chi_b''}\right|, \quad \varepsilon = \frac{\gamma_{\mathbf{g}}}{\gamma_0}. \tag{21}$$

An important parameter in expression (20) is the parameter ε, which we will rewrite as

$$\varepsilon = \frac{\sin(\theta_B - \delta)}{\sin(\theta_B + \delta)}. \tag{22}$$

Parameter ε defines the degree of the reflection asymmetry of the field relative to the target entrance surface. Here θ_B is the angle between the electron velocity and the reflected layers, δ is the angle between the target entrance surface and the reflecting layers. For the fixed value of θ_B the parameter ε defines the entrance surface orientation relative to the reflecting layers (fig.2). When the incident angle of the electron on the target $(\theta_B + \delta)$ decreases the parameter δ becomes negative and then its absolute value increases, (in the extreme case $\delta \to -\theta_B$), which leads to the increase of ε. On the contrary, when the incident angle increases, the parameter ε decreases. In the case of symmetric reflection when $\delta = 0$, the asymmetry parameter $\varepsilon = 1$.

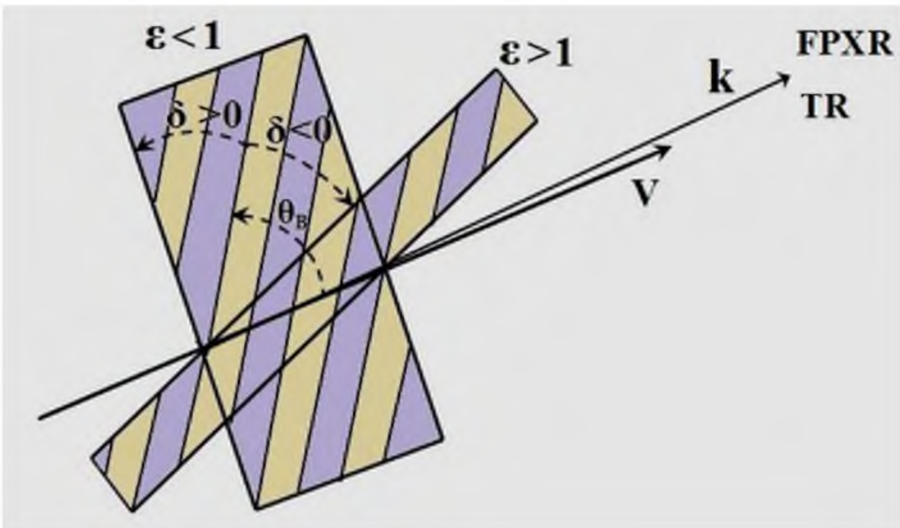

Fig. 2. Asymmetric ($\varepsilon > 1$, $\varepsilon < 1$) reflections of the radiation from the periodic layered structure

2. Spectral-angular density of the radiation

Let us consider the σ - polarized waves (s=1). Substituting (19b) and (19c) into the well-known expression for spectral angular density of X-ray radiation

$$\omega \frac{d^2 N}{d\omega d\Omega} = \omega^2 (2\pi)^{-6} \sum_{s=1}^{2} \left|E_0^{(s)Rad}\right|^2, \tag{23}$$

we will find the expressions which describe spectral angular density of FPXR and TR mechanisms.

$$\omega\frac{d^2N_{FPXR}}{d\omega d\Omega}=\frac{e^2}{\pi^2}\frac{\theta_\perp^2}{\left(\theta_\perp^2+\gamma^{-2}+\frac{\left|a\chi'_a+b\chi'_b\right|}{T}\right)^2}R_{FPXR}, \tag{24a}$$

$$R_{FPXR}=\frac{1}{\left|\left(\xi-K-i\frac{\rho(1+\varepsilon)}{2}\right)\exp\left(-iB\frac{K}{\varepsilon}\right)-\left(\xi+K-i\frac{\rho(1+\varepsilon)}{2}\right)\exp\left(iB\frac{K}{\varepsilon}\right)\right|^2}\times$$

$$\times\left|\frac{\left(1-\exp\left(-iB\left(\sigma-\frac{\xi-K}{\varepsilon}+i\frac{\rho(1-\varepsilon)}{2\varepsilon}\right)\right)\right)}{\sigma-\frac{\xi-K}{\varepsilon}+i\frac{\rho(1-\varepsilon)}{2\varepsilon}}\exp\left(iB\frac{K}{\varepsilon}\right)-\right.$$

$$\left.-\frac{\left(1-\exp\left(-iB\left(\sigma-\frac{\xi+K}{\varepsilon}+i\frac{\rho(1-\varepsilon)}{2\varepsilon}\right)\right)\right)}{\sigma-\frac{\xi+K}{\varepsilon}+i\frac{\rho(1-\varepsilon)}{2\varepsilon}}\exp\left(-iB\frac{K}{\varepsilon}\right)\right|^2, \tag{24b}$$

$$\omega\frac{d^2N_{TR}}{d\omega d\Omega}=\frac{e^2}{\pi^2}\theta_\perp^2\left(\frac{1}{\theta_\perp^2+\gamma^{-2}}-\frac{1}{\theta_\perp^2+\gamma^{-2}+\frac{\left|a\chi'_a+b\chi'_b\right|}{T}}\right)^2R_{TR}, \tag{25a}$$

$$R_{TR}=\left|1+\frac{2K\exp\left(-iB\left(\sigma-\frac{\xi}{\varepsilon}+\frac{i\rho(1-\varepsilon)}{2\varepsilon}\right)\right)}{\left(\xi-K-i\frac{\rho(1+\varepsilon)}{2}\right)\exp\left(-iB\frac{K}{\varepsilon}\right)-\left(\xi+K-i\frac{\rho(1+\varepsilon)}{2}\right)\exp\left(iB\frac{K}{\varepsilon}\right)}\right|^2. \tag{25b}$$

The following notations are put into the formulas (27) and (28):

$$\sigma(\theta,\gamma)=\frac{gT}{2\cdot\left|\sin\left(\frac{ga}{2}\right)\right|\left|\chi'_b-\chi'_a\right|}\cdot\left(\theta_\perp^2+\gamma^{-2}+\frac{\left|a\chi'_a+b\chi'_b\right|}{T}\right),$$

$$K=\sqrt{\xi^2-\varepsilon-i\rho^{(s)}((1+\varepsilon)\xi^{(s)}-2\kappa^{(s)}\varepsilon)-\rho^{(s)2}\left(\frac{(1+\varepsilon)^2}{4}-\kappa^{(s)2}\varepsilon\right)}$$

$$\xi(\omega)=\frac{gT\sin^2\theta_B}{\left|\sin\left(\frac{ga}{2}\right)\right|\left|\chi'_b-\chi'_a\right|}\cdot\left(1-\frac{\omega}{\omega_B}\right)+\frac{1+\varepsilon}{2\nu^{(1)}},\ \rho=\frac{a\chi''_a+b\chi''_b}{\left|\chi'_b-\chi'_a\right|}\frac{g}{2\left|\sin\left(\frac{ga}{2}\right)\right|},$$

$$\kappa=\frac{2\left|\sin\left(\frac{ga}{2}\right)\right|}{g}\left|\frac{\chi''_b-\chi''_a}{a\chi''_a+b\chi''_b}\right|,\ B=\frac{2\omega_B\left|\sin\left(\frac{ga}{2}\right)\right|\left|\chi'_b-\chi'_a\right|}{gT\sin(\theta_B+\delta)}L,\ \theta_\perp=\theta\sin\varphi. \tag{26}$$

In accordance with (24b), two waves that contribute to the FPXR can exist in a periodic layered medium.

The contribution of the first or second wave could be significant if, respectively, the first or the second of the next equations has a solution:

$$\mathrm{Re}\left(\sigma - \frac{\xi - K}{\varepsilon} + i\frac{\rho(1-\varepsilon)}{2\varepsilon}\right) \approx \sigma - \frac{\xi - \sqrt{\xi^2 - \varepsilon}}{\varepsilon} = 0, \tag{27a}$$

$$\mathrm{Re}\left(\sigma - \frac{\xi + K}{\varepsilon} + i\frac{\rho(1-\varepsilon)}{2\varepsilon}\right) \approx \sigma - \frac{\xi + \sqrt{\xi^2 - \varepsilon}}{\varepsilon} = 0. \tag{27b}$$

Since $\sigma > 1$ it can be shown that equation (27b) has a solution under the condition $\varepsilon > \frac{1}{\sigma^2}$, and the equation (27a) is solvable on the condition that $\varepsilon < \frac{1}{\sigma^2}$. Thus, under different values of asymmetry parameter, the first or the second of x-ray waves can contribute in the FPXR.

Let us consider the direction of the energy transfer of the two waves responsible for the formation of the FPXR. For this purpose, we will consider the group velocities of the radiation waves along the OX axis, neglecting absorption. The projections of the wave vectors of the waves along the OX axis (9a) in the periodic layered structure in the case of non-absorbing targets are as follows:

$$k_x^{(1,2)} = \omega\sin(\theta_B + \delta) + \frac{\omega\chi_0}{2\sin(\theta_B + \delta)} + \frac{\omega}{2\sin(\theta_B - \delta)}\frac{\left|\sin\left(\frac{ga}{2}\right)\right|\left|\chi'_b - \chi'_a\right|C^{(s)}}{gT}\cdot\left(\xi(\omega) \mp \sqrt{\xi(\omega)^2 - \varepsilon}\right) \tag{28}$$

The group velocities of these X-ray waves (31) have the form:

$$V_{гр} = \left(\frac{\partial k_x^{(1,2)}}{\partial\omega}\right)^{-1} \approx \left(\sin(\theta_B + \delta) - \frac{\sin^2\theta_B}{\sin(\theta_B - \delta)}\left(1 \mp \frac{\xi(\omega)}{\sqrt{\xi(\omega)^2 - \varepsilon}}\right)\right)^{-1}. \tag{29}$$

It can be shown that the group velocity of the waves which correspond to the first branch of the dispersion relation solution is positive $\left(\partial k_x^{(1)} / \partial\omega\right)^{-1} > 0$ and the energy of the wave is transferred from the input surface to the output surface of the target. The group velocity of the second wave is always negative $\left(\partial k_x^{(2)} / \partial\omega\right)^{-1} < 0$, consequently, the energy of the wave transfers from the output to the input surface of the target. This fact leads to the suppression of the second wave of the FPRX in a periodic layered medium in the case of a crystal of a considerable thickness when the transmitted energy is completely absorbed.

Thus, for a sufficiently large thickness of the crystal, FPXR corresponding to the second branch of the dispersion relation solution is suppressed. However, on the conditions $\varepsilon < \frac{1}{\sigma^2}$, the FPXR which corresponds to the first of generated x-ray waves in periodic layered medium will be material.

Let us demonstrate this claim by numerical calculations performed by the formulas (24) and (25). In Fig. 3 and Fig. 4 the curves are constructed describing the spectral and angular density FPXR and TR of the relativistic electron of energy E=200MeV which crosses the periodic layered structure C-W, that consists of the layers of carbon and tungsten. Furthermore, the curves in Figure 3 are constructed for the case where the asymmetry parameter $\varepsilon < \frac{1}{\sigma^2}$ and contribution comes from the first branch FPXR with positive group velocity of the X-ray waves. In Figure 4 the curves are plotted for the case $\varepsilon > \frac{1}{\sigma^2}$ where the contribution of the first branch is absent, and the contribution of the second one is suppressed because of the negative group velocity of the correspondent waves.

It is necessary to note that in Figures 3 and Fig. 4 the curves are constructed with the same path length of the electrons $L_e = \dfrac{L}{\sin(\theta_B + \delta)} \approx 0.3\,\mu m$ and the photons $L_e \approx L_{ph}$ in the target. In this case, the length of the photon absorption in the structure $L_{abs} = \dfrac{T}{\omega\left|a\chi''_a + b\chi''_b\right|} \approx 6.2\,\mu m$ is much more then L_{ph}.

In this case, the transition radiation consists of the radiation produced at the exit surface of the target. It should be noted that the width of the peak FPXR in this case, as it follows from Figure 3, is about 25eV, which is much wider than in the crystalline medium (crystal 1-2 eV). This fact will ease the experimental research and the identification of FPXR in a periodic layered structure.

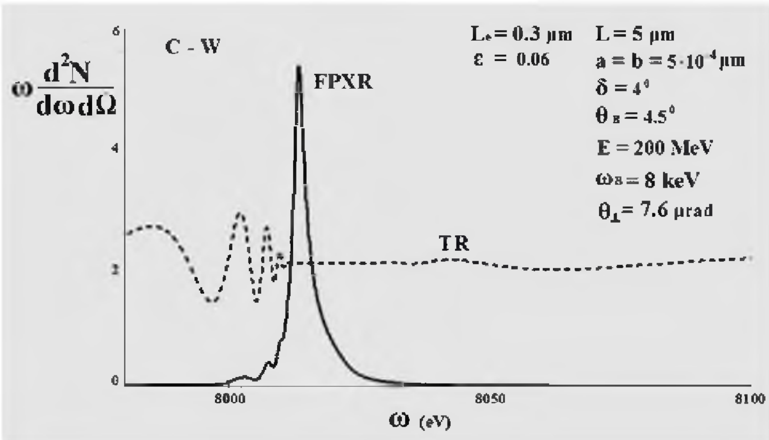

Fig. 3. Spectral-angular density of FPXR and TR of the relativistic electron in the periodic layered medium consisting of carbon and tungsten layers (the case of $\varepsilon < 1$)

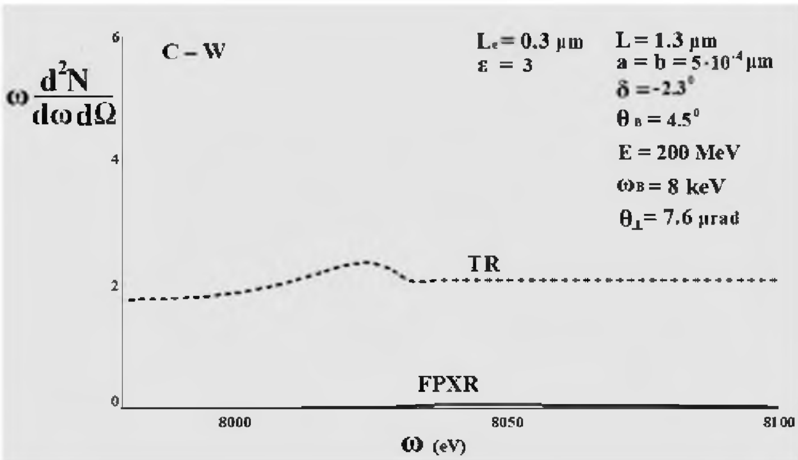

Fig 4. Spectral-angular density of FPXR and TR of the relativistic electron in the periodic layered medium consisting of carbon and tungsten layers (the case of $\varepsilon > 1$)

3. Conclusion

In the present work a dynamic theory of coherent X-rays along the velocity of the relativistic particle in a periodic layered structure in Bragg scattering geometry is built up for the general case of the asymmetric reflection of the particle field relative to the entrance surface of the target.

On the basis of the two-wave approximation of the dynamical theory of diffraction, the expressions which describe the spectral and angular characteristics of the radiation from the two radiation mechanisms FPXR and TR are obtained.

The very existence of the dynamic effect of the FPXR in a periodic layered structure is shown for the first time. It is also shown that the spectral-angular density of the FPXR considerably depends on the asymmetry of the electron field reflection relative to the surface of the target under fixed path of the electron in the target. It is shown that the spectral peak of the relativistic electron parametric X-ray radiation in the forward direction is many times larger than the peak of the emission spectrum of a single crystal, which may ease its experimental observation and investigation.

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