МАТЕМАТИЧЕСКАЯ ФИЗИКА, МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ

MSC 78A40

COHERENT X-RADIATION GENERATED BY RELATIVISTIC ELECTRON ALONG ITS VELOCITY IN PERIODICALLY LAYERED MEDIUM

S.V. Blazhevich, Yu.P. Gladkih, A.V. Noskov, O.N. Satler

Belgorod State National Research University,
Pobedy St., 85, Belgorod, 308015, Russia, e-mail: blazh@bsu.edu.ru

Abstract. Dynamic theory of coherent X-radiation generated by relativistic electron crossing multilayered medium is developed. It is found the radiation along the electron velocity vector in Bragg's scattering geometry for general asymmetric case. The comparison of analogous radiation generated by relativistic electron in the single crystal medium is done. The developed theory predicts some conditions under which this coherent radiation exists and also it describes its spectral and angular characteristics.

Keywords: relativistic electron, periodically layered target, coherent X-radiation, dynamic theory.

1. Introduction. Traditionally, the relativistic particle radiation in a periodic lamellar structure was studied as resonance transition radiation (RTR) [1, 2]. Significant contribution to investigation of the X-ray transition radiation was made Japanese physicists [3-5]. In the work [4] the periodic target consisting of plates with the thick of several hundred nanometers was used for the X-ray generation for the first time. Radiated photons had the energy 2-4 keV at the fundamental harmonic. Authors of this work asserted that the radiation intensity obtained in the experiment exceeded the synchrotron radiation intensity achievable on the modern electron accelerators. Theoretical description of RTR in the above-mentioned media was presented in the work [6]. Subsequently, in the work [7] the RTR of relativistic electron in layered medium was studied together with the parametric X-radiation (PXR). In the work [8] the radiation in multilayered periodic structure which is analogous to the coherent X-radiation in a single crystal medium [9-12] was considered. It was done in the dynamic approximation as the scattering of pseudo photons of the relativistic electron coulomb field in the structure of amorphous layers. In [8] the coherent radiation in periodic multilayered structure was considered for the first time as the result of contributions of two radiation mechanisms, namely, PXR and diffracted transition radiation (DTR).

The theory of the relativistic electron radiation in layered periodic media describing some experimental data was presented in the work [13]. Data were obtained in the experiment where the used structure consisted of layers with the thick of one nanometer, and photons were generated with energy of 15 keV. A detailed comparison of the theory [8] and the experiment [9] are presented in the work [14].

It is necessary to note that in all cited works the radiation process in layered medium was considered in Bragg's geometry only for the symmetric reflection case where the angle between target surface and reflecting planes (layers) is equal to zero. Later, in our works [15-16], dynamic theory of the coherent X-radiation of relativistic electron crossing layered medium in Laue's geometry was built for the general case with electron asymmetric reflection in respect to the entrance surface of target. It was clarified in these works that the radiation yield in periodic layered target significantly exceeds the yield in the crystal. It was shown that there exists the additional opportunity of the yield increase by means of the choice of optimal reflection asymmetry. It was revealed in these works that the radiation yield in the periodic layered target significantly exceeds the yield in the crystalline one, and the additional opportunity of the yield increase by the choice of optimal reflection asymmetry was proved. The theory of the parametric X-radiation (PXR) of relativistic charged particle in single crystal medium forecasts the radiation not only in the Bragg direction but also in the direction along the particle velocity (FPXR) [17-19]. FPXR is the result of dynamic diffraction of pseudo photons of the particle coulomb field on atomic planes in the crystal. Some attempts of experimental study of FPXR are known [20-24] but first report about FPXR detection in the crystal target was recently made in the work [24]. The detailed theoretical description of the dynamic effect of FPXR and accompanying background of transition radiation (TR) in crystal in symmetric reflection case was provided in works [25-27].

The general case of asymmetric reflection was studied in the FPXR theory in Laue scattering geometry [28] and it was studied in the theory of transition radiation (TR) in Bragg geometry [29]. In present work the dynamic theory of the coherent radiation along the velocity of relativistic electron crossing periodic layered medium in Bragg scattering geometry was built for general case of asymmetric reflection when reflecting layers in the target are situated under a free angle relative to the target surface (symmetric reflection is the special case of reflection). The expressions of spectral-angular characteristics of FPXR and TR in a periodic layered medium are derived on the basis of two wave approximation of the dynamic diffraction theory.

2. Radiation amplitude. Let the relativistic electron with the velocity V passes through the multilayered structure which consists of periodically situated amorphous layers having thicknesses a and b ($T = a + b$ is the structure period). The substance of the layers a and b have the dielectric susceptibility χ_a and χ_b correspondently (fig.1). For studying the electromagnetic radiation accompanying this process we use the two-wave approximation of the dynamic diffraction theory. Let us consider the Fourier image of the electromagnetic field

$$\mathbf{E}(\mathbf{k},\omega) = \int \mathbf{E}(\mathbf{r},t)\exp(i\omega t - i\mathbf{k}\mathbf{r})dtd^3r\,. \tag{1}$$

Since the relativistic particle coulomb field could be represented practically transverse, the incident $\mathbf{E}_0(\mathbf{k},\omega)$ and the diffracted $\mathbf{E}_{\mathbf{g}}(\mathbf{k},\omega)$ electromagnetic waves are determined by two amplitudes with different values of transverse polarization:

$$\mathbf{E}_0(\mathbf{k},\omega) = E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)},$$

$$\mathbf{E}_{\mathbf{g}}(\mathbf{k},\omega) = E_{\mathbf{g}}^{(1)}(\mathbf{k},\omega)\mathbf{e}_1^{(1)} + E_{\mathbf{g}}^{(2)}(\mathbf{k},\omega)\mathbf{e}_1^{(2)} \tag{2}$$

where the vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to vector $\mathbf{k_g} = \mathbf{k} + \mathbf{g}$. Vectors $\mathbf{e}_0^{(2)}$,$\mathbf{e}_1^{(2)}$ lie in the plane of vectors $\mathbf{k}$ and $\mathbf{k_g}$ (π-polarization), and vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_1^{(1)}$ are normal to it (σ-polarization). The vector $\mathbf{g}$ is analogous to the reciprocal lattice vector in the crystal, and it is perpendicular to layers. Its length is $\mathbf{g} = \frac{2\pi}{T}n, n = 0, \pm1, \pm2, \cdots$.

The system of equations for the Fourier image of electromagnetic field in two-wave approximation of the dynamic diffraction theory is as follows [30]:

$$\begin{cases} (\omega^2(1+\chi_0) - k^2)E_0^{(s)} + \omega^2\chi_{-\mathbf{g}}C^{(s,\tau)}E_{\mathbf{g}}^{(s)} = 8\pi^2 ie\omega\theta V P^s\delta(\omega - \mathbf{kV}), \\ \omega^2\chi_{\mathbf{g}}C^{(s,\tau)}E_0^{(s)} + (\omega^2(1+\chi_0) - k_{\mathbf{g}}^2)E_{\mathbf{g}}^{(s)} = 0 \end{cases} \tag{3}$$

where $\chi_{\mathbf{g}}, \chi_{-\mathbf{g}}$ are coefficients of Fourier expansion of dielectric susceptibility on the reciprocal lattice vector $\mathbf{g}$:

$$\chi(\omega, \mathbf{r}) = \sum_{\mathbf{g}} \chi_{\mathbf{g}}(\omega) exp(i\mathbf{gr}) = \sum_{\mathbf{g}} (\chi'_{\mathbf{g}}(\omega) + i\chi''_{\mathbf{g}}(\omega) exp(i\mathbf{gr})\,. \tag{4}$$

The quantities $C^{(s,\tau)}$ and $P^{(s)}$ in (3) are defined as follows:

$$C^{(s,\tau)} = \mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)} = (-1)^\tau C^{(s)}, \quad C^{(1)} = 1, \quad C^{(2)} = |\cos 2\theta_B|\,,$$

$$P^{(s)} = \mathbf{e}_0^{(s)}(\mu/\mu), \quad P^{(1)} = \sin\varphi, \quad P^{(2)} = \cos\varphi \tag{5}$$

where $\mu = \mathbf{k} - \omega\mathbf{V}/\mathbf{V^2}$ is the momentum component of the virtual photon which is perpendicular to the particle velocity $\mathbf{V}$ ($\mu = \omega\theta/V$, $\theta \ll 1$ is the angle between vectors $\mathbf{k}$ and V), θ_B is the Bragg angle, φ is the azimuthal radiation angle counted from the plane formed by the velocity vector $\mathbf{V}$ and $\mathbf{g}$ vector being perpendicular to reflecting layers. The magnitude of the vector $\mathbf{g}$ can be expressed by the Bragg angle θ_B and the Bragg frequency ω_B: $g = 2\omega_B \sin\theta_B/V$. The angle between vector $\frac{\omega\mathbf{V}}{V^2}$ and the incident wave vector $\mathbf{k}$ is notated as θ_B, the angle between vector $\frac{\omega\mathbf{V}}{V^2} + \mathbf{g}$ and the wave vector of the diffracted wave $\mathbf{k_g}$ is notated as θ'. The system of equations (3) with $s = 1$ and $\tau = 2$ describes the π-polarized fields. In this case $\tau = 2$, if $2\theta_B < \frac{\pi}{2}$, otherwise $\tau = 1$. The quantities χ_0 and $\chi_{\mathbf{g}}$ in this periodic structure are nominated as follows:

$$\chi_0(\omega) = \frac{a}{T}\chi_a + \frac{b}{T}\chi_b\,, \tag{6a}$$

$$\chi_{\mathbf{g}}(\omega) = \frac{exp(-iga) - 1}{igT}(\chi_b - \chi_a)\,. \tag{6b}$$

The following expressions will be obtained from (6) and will be used further:

$$\chi'_0 = \frac{a}{T}\chi'_a + \frac{b}{T}\chi'_b\,, \tag{7a}$$

$$\chi''_0 = \frac{a}{T}\chi''_a + \frac{b}{T}\chi''_b\,, \tag{7b}$$

$$\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}} = \frac{2\sin(\frac{ga}{2})}{gT}(\chi'_b - \chi'_a)\,, \tag{7c}$$

$$\mathrm{Im}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}} = \frac{2\sin(\frac{ga}{2})}{gT}(\chi_b'' - \chi_a'') \,. \tag{7d}$$

Solving the following dispersion equation (8) which is obtained from the system (3) by means of standard methods of the dynamical theory of X-ray ways scattering in crystal [31]

$$(\omega^2(1+\chi_0) - k^2)(\omega^2(1+\chi_0) - k_{\mathbf{g}}^2) - \omega^4\chi_{-\mathbf{g}}\chi_{\mathbf{g}}{C^{(s)}}^2 = 0 \tag{8}$$

we obtain projections of the wave vectors k and $k_{\mathbf{g}}$.

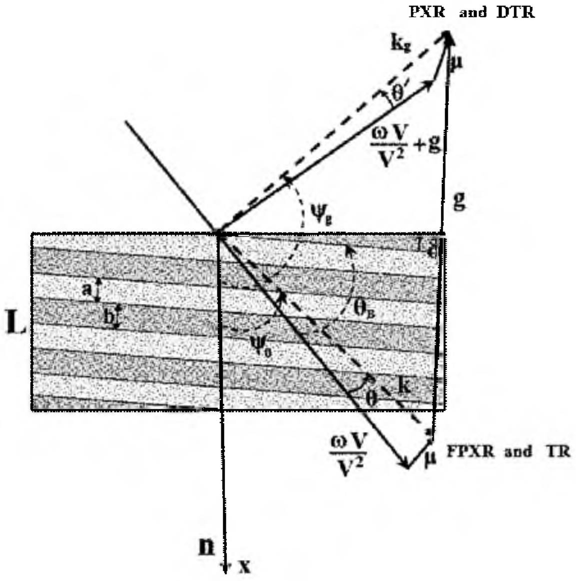

Fig. 1. Geometry of radiation process and notations of using radiation angles θ and θ'. θ_B is the Bragg angle (the angle between the electron velocity $\mathbf{V}$ and layers of the target), δ is the angle between the target surface and layers of target, $\mathbf{k}$ and $\mathbf{k_g}$ are wave vectors of the incident and diffracted photons.

Projections of the wave vectors k and $k_{\mathbf{g}}$ are presented in the following form:

$$k_x = \omega\cos\psi_0 + \frac{\omega\chi_0}{2\cos\psi_0} + \frac{\lambda_0}{\cos\psi_0} \,, \tag{9a}$$

$$k_{\mathbf{g}x} = \omega\cos\psi_{\mathbf{g}} + \frac{\omega\chi_0}{2\cos\psi_{\mathbf{g}}} + \frac{\lambda_{\mathbf{g}}}{\cos\psi_{\mathbf{g}}} \,. \tag{9b}$$

We use the known expression connected with the dynamic additions λ_0 and $\lambda_{\mathbf{g}}$ [31]:

$$\lambda_{\mathbf{g}} = \frac{\omega\beta}{2} + \lambda_0\frac{\gamma_{\mathbf{g}}}{\gamma_0} \tag{10}$$

where $\beta = \alpha - \chi_0(1 - \frac{\gamma_{\mathbf{g}}}{\gamma_0})$, $\alpha = \frac{1}{\omega^2}(k_{\mathbf{g}}^2 - k^2)$, $\gamma_0 = \cos\psi_0$, $\gamma_{\mathbf{g}} = \cos\psi_{\mathbf{g}} and \psi_0$ is the angle between the incident wave vector $\mathbf{k}$ and the normal to the plate (target) surface $\boldsymbol{n}$, $\psi_{\mathbf{g}}$ is the angle between the wave vector $\mathbf{k_g}$ and the vector $\mathbf{n}$ (see fig. 1).

Let us find the wave vectors $\mathbf{k}$ and $\mathbf{k_g}$

$$k = \omega\sqrt{1+\chi_0} + \lambda_0, k_{\mathbf{g}} = \omega\sqrt{1+\chi_0} + \lambda_{\mathbf{g}}\,. \tag{11}$$

Taking into account that $k_{\|} \approx \omega \sin\psi_0$, $k_{\mathbf{g}\|} \approx \omega \sin\psi_{\mathbf{g}}$ we obtain:

$$\lambda_{\mathbf{g}}^{(1,2)} = \frac{\omega}{4}\Big(\beta \pm \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\Big)\,, \tag{12a}$$

$$\lambda_0^{(1,2)} = \omega\frac{\gamma_0}{4\gamma_{\mathbf{g}}}\Big(-\beta \pm \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\Big)\,. \tag{12b}$$

Since the dynamic additions are small, ($|\lambda_0| \ll \omega$, $|\lambda_{\mathbf{g}}| \ll \omega$), it can be shown that $\theta \approx \theta'$ (see fig. 1), and we will further designate both these angles as θ.

It is convenient to represent the solution of the system (3) for the incident field of the periodic structure in the following form:

$$E_{0,med}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \cdot \frac{-\omega^2\beta - 2\omega\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0}{4\frac{\gamma_{\mathbf{g}}}{\gamma_0}(\lambda_0 - \lambda_0^{(1)})(\lambda_0 - \lambda_0^{(2)})}\delta(\lambda_0 - \lambda_0^*) +$$

$$+E_0^{(s)(1)}\delta(\lambda_0 - \lambda_0^{(1)}) + E_0^{(s)(2)}\delta(\lambda_0 - \lambda_0^{(2)}) \tag{13}$$

where $\lambda_0^* = \omega(\frac{\gamma^{-2}+\theta^2-\chi_0}{2})$, $\gamma = 1/\sqrt{1-V^2}$ is the Lorentz factor of particle, $E_0^{(s)(1)}$ and $E_0^{(s)(2)}$ are free incident fields in the media.

For the field in vacuum before the periodic structure, the solution of system (3) can be represented in form:

$$E_{0,vacI}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \cdot \frac{1}{-\chi_0 - 2\lambda_0/\omega}\,\delta(\lambda_0 - \lambda_0^*)\,. \tag{14}$$

The expression for the field in the vacuum behind the target can be written as:

$$E_{0,vacII}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \cdot \frac{1}{-\chi_0 - 2\lambda_0/\omega}\,\delta(\lambda_0 - \lambda_0^*) + E_{0,rad}^{(s)}\delta(\lambda_0 + \omega\chi_0/2) \tag{15}$$

where $E_{0,rad}^{(s)}$ is the amplitude of the coherent radiation field along the velocity of the electron.

From the second equation of the system (3) the expression relating the incident and diffracted field in the crystal will follow:

$$E_{0,med}^{(s)} = \frac{2\omega\lambda_{\mathbf{g}}}{\omega^2\chi_g C^{(s,\tau)}}E_{\mathbf{g},med}^{(s)}. \tag{16}$$

Using the usual boundary conditions on the input surface and the exit one of target:

$$\int E_{0,vacI}^{(s)}d\lambda_{\mathbf{g}} = \int E_{0,med}^{(s)}d\lambda_{\mathbf{g}}\,, \qquad \int E_{\mathbf{g},med}^{(s)}d\lambda_{\mathbf{g}} = \int E_{\mathbf{g},vac}^{(s)}d\lambda_{\mathbf{g}}\,,$$

$$\int E^{(s)}_{\mathbf{g},med} \exp(i\lambda_{\mathbf{g}} L/\gamma_{\mathbf{g}}) d\lambda_{\mathbf{g}} = 0\,, \tag{17}$$

we obtain the expression for the amplitude of the radiation field

$$E^{(s)}_{0,rad} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \cdot \frac{1}{\lambda^{(1)}_{\mathbf{g}} \exp\left(i\frac{\lambda^{(2)}_0 - \lambda^*_0}{\gamma_0}L\right) - \lambda^{(2)}_{\mathbf{g}} \exp\left(i\frac{\lambda^{(1)}_0 - \lambda^*_0}{\gamma_0}L\right)} \times$$

$$\left[\lambda^{(2)}_{\mathbf{g}}\left(\frac{\omega}{-\omega\chi_0 - 2\lambda^*_0)} + \frac{\omega}{2(\lambda^*_0 - \lambda^{(2)}_0)}\right)\left(1 - \exp\left(i\frac{\lambda^{(2)}_0 - \lambda^*_0}{\gamma_0}L\right)\right)\exp\left(i\frac{\lambda^{(2)}_0 - \lambda^*_0}{\gamma_0}L\right) - \right.$$

$$\left. -\lambda^{(1)}_{\mathbf{g}}\left(\frac{\omega}{-\omega\chi_0 - 2\lambda^*_0)} + \frac{\omega}{2(\lambda^*_0 - \lambda^{(1)}_0)}\right)\left(1 - \exp\left(i\frac{\lambda^{(1)}_0 - \lambda^*_0}{\gamma_0}L\right)\right)\exp\left(i\frac{\lambda^{(2)}_0 - \lambda^*_0}{\gamma_0}L\right)\right] \times$$

$$\times \exp\left(i\frac{\lambda^*_0 + \frac{\omega\chi_0}{2}}{\gamma_0}L\right)\,. \tag{18}$$

Before the analysis of spectral-angular characteristics of the radiation, it is necessary to note that three radiation mechanisms give contributions to the total radiation yield: bremsstrahlung, transition radiation (TR) and parametric radiation in forward direction (FPXR). The amplitude $E^{(s)}_{0,rad}$ contains contributions of radiations analogous to FPXR and TR in the crystal.

Let us represent the expression for the radiation field (8) in the following form:

$$E^{(s)Rad}_0 = E^{(s)}_{FPXR} + E^{(s)}_{TR}\,, \tag{19a}$$

$$E^{(s)}_{FPXR} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \cdot \frac{\omega}{2\lambda^*} \cdot \frac{e^{i(\lambda^*_0 + \omega\chi_0/2)L/\gamma_0}}{\lambda^{(1)}_{\mathbf{g}} \exp\left(i\frac{\lambda^{(2)}_0 - \lambda^*_0}{\gamma_0}L\right) - \lambda^{(2)}_{\mathbf{g}} \exp\left(i\frac{\lambda^{(1)}_0 - \lambda^*_0}{\gamma_0}L\right)} \times$$

$$\times\left[\frac{\lambda^{(2)}_{\mathbf{g}}\lambda^{(2)}_0}{\lambda^*_0 - \lambda^{(2)}_0}\left(1 - \exp\left(i\frac{\lambda^{(2)}_0 - \lambda^*_0}{\gamma_0}L\right)\right)\exp\left(i\frac{\lambda^{(1)}_0 - \lambda^*_0}{\gamma_0}L\right) - \right.$$

$$\left. - \frac{\lambda^{(1)}_{\mathbf{g}}\lambda^{(1)}_0}{\lambda^*_0 - \lambda^{(1)}_0}\left(1 - \exp\left(i\frac{\lambda^{(1)}_0 - \lambda^*_0}{\gamma_0}L\right)\right)\exp\left(i\frac{\lambda^{(2)}_0 - \lambda^*_0}{\gamma_0}L\right)\right]\,, \tag{19b}$$

$$E^{(s)}_{TR} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} e^{i(\lambda^*_0 + \omega\chi_0/2)L/\gamma_0}\left(\frac{\omega}{\omega\chi_0 + 2\lambda^*_0} - \frac{\omega}{2\lambda^*_0}\right) \times$$

$$\times\left(1 - \frac{\frac{\gamma_{\mathbf{g}}}{\gamma_0}(\lambda^{(1)}_0 - \lambda^{(2)}_0)e^{i(\lambda^{(2)}_0 + \lambda^{(1)}_0 - 2\lambda^*_0)L/\gamma_0}}{\lambda^{(1)}_{\mathbf{g}} \exp\left(i\frac{\lambda^{(2)}_0 - \lambda^*_0}{\gamma_0}L\right) - \lambda^{(2)}_{\mathbf{g}} \exp\left(i\frac{\lambda^{(1)}_0 - \lambda^*_0}{\gamma_0}L\right)}\right)\,. \tag{19c}$$

Summands in the square brackets of expression (19b) represent two branches of the dispersion equation solution corresponding to two X-ray waves excited in the periodical medium. For further analysis of the radiation we represent dynamic additions (10) and (11) in the following form:

$$\lambda_{\mathbf{g}}^{(1,2)} = \frac{\omega|\chi_0' C^{(s)}|}{2\varepsilon}\Bigg(\xi^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2} \mp$$

$$\mp\sqrt{\xi^{(s)2} - \varepsilon - i\rho^{(s)}\left((1+\varepsilon)\xi^{(s)} - 2\kappa^{(s)}\varepsilon\right) - \rho^{(s)2}\left((1+\varepsilon)^2/4 - \kappa^{(s)2}\varepsilon\right)}\Bigg)\,, \quad (20a)$$

$$\lambda_{\mathbf{g}}^{(1,2)} = \frac{\omega|\chi_0' C^{(s)}|}{2}\Bigg(\xi^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2} \pm$$

$$\pm\sqrt{\xi^{(s)2} - \varepsilon - i\rho^{(s)}\left((1+\varepsilon)\xi^{(s)} - 2\kappa^{(s)}\varepsilon\right) - \rho^{(s)2}\left((1+\varepsilon)^2/4 - \kappa^{(s)2}\varepsilon\right)}\Bigg) \quad (20b)$$

where $\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + (1+\varepsilon)/2\nu^{(s)}$,

$$\eta^{(s)}(\omega) = \frac{\alpha}{2\left|\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}\right| C^{(s)}} \equiv \frac{\sin^2\theta_B}{V^2 C^{(s)}} \cdot \frac{gT}{|\chi_b' - \chi_a'|\left|\sin\left(\frac{ga}{2}\right)\right|}\left(1 - \frac{\omega(1-\theta\cos\varphi\,\mathrm{ctg}\theta_b)}{\omega_B}\right),$$

$$\nu^{(s)} = \frac{C^{(s)}\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}}{\chi_0'} \equiv \frac{2C^{(s)}\left|\sin(\frac{ga}{2})\right|}{g}\left|\frac{\chi_b' - \chi_a'}{a\chi_a' - b\chi_b'}\right|,$$

$$\rho^{(s)} = \frac{\chi_0''}{\left|\mathrm{Re}\sqrt{\chi_{\mathbf{g}}\chi_{-\mathbf{g}}}\right| C^{(s)}} \equiv \frac{a\chi_a'' - b\chi_b''}{\chi_b' - \chi_a' C^{(s)}}\,\frac{g}{2\left|\sin\left(\frac{ga}{2}\right)\right|},$$

$$\kappa^{(s)} = \frac{\chi_{\mathbf{g}}'' C^{(s)}}{\chi_0''} \equiv \frac{2C^{(s)}\left|\sin\left(\frac{ga}{2}\right)\right|}{g}\left|\frac{\chi_b'' - \chi_a''}{a\chi_a'' - b\chi_b''}\right|, \qquad \varepsilon = \frac{\gamma_{\mathbf{g}}}{\gamma_0}. \quad (21)$$

The important parameter in expression (20) is the following, which we rewrite in the form

$$\varepsilon = \frac{\sin(\theta_B - \delta)}{\sin(\theta_B + \delta)}. \quad (22)$$

The parameter ε defines the degree of field reflection asymmetry relative to the target entrance surface. Here θ_B is the angle between the electron velocity and reflected layers, δ is the angle between the target entrance surface and reflecting layers. For the fixed value θ_B the parameter ε defines the entrance surface orientation relative to reflecting layers (fig. 2). When the incident angle of the electron on the target $(\theta_B + \delta)$ decreases, the parameter δ becomes negative and then its absolute value increases, (in the extreme case $\delta \to -\theta_B$). It leads to the increase of ε. On the contrary, when the incident angle increases, the parameter ε decreases. In the case of symmetric reflection when $\delta = 0$, the asymmetry parameter $\varepsilon = 1$.

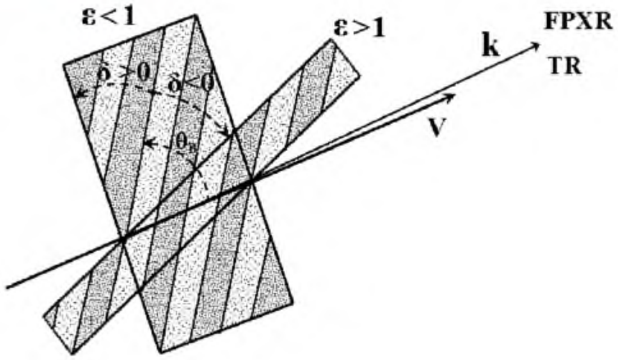

Fig. 2. Asymmetric ($\varepsilon > 1$, $\varepsilon < 1$) reflections of radiation from the periodic layered structure.

3. Spectral-angular density of the radiation. Let us consider σ-polarized waves (s=1). Substituting (19b) and (19c) into the well-known expression [30] for the spectral angular density of X-ray radiation

$$\omega\frac{d^2N}{d\omega d\Omega} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E^{(s)}_{\mathbf{g},rad}\right|^2, \tag{26}$$

we find the expressions which describes the spectral angular density of FPXR and TR mechanisms

$$\omega\frac{d^2N_{FPXR}}{d\omega d\Omega} = \frac{e^2}{\pi^2}\cdot\frac{\theta_\perp^2}{(\theta_\perp^2+\gamma^{-2}+|a\chi'_a+b\chi'_b|/T)}R_{FPXR}\ , \tag{27a}$$

$$R_{FPRX} = \frac{1}{\left|\left(\xi - K - i\frac{\rho(1+\varepsilon)}{2}\right)\exp\left(-iB\frac{K}{\varepsilon}\right) - \left(\xi + K - i\frac{\rho(1+\varepsilon)}{2}\right)\exp\left(iB\frac{K}{\varepsilon}\right)\right|^2}\times$$

$$\times\left|\frac{\left(1-\exp\left(-iB\left(\sigma-\frac{\xi-K}{\varepsilon}+i\frac{\rho(1-\varepsilon)}{2\varepsilon}\right)\right)\right)}{\sigma-\frac{\xi-K}{\varepsilon}+i\frac{\rho(1-\varepsilon)}{2\varepsilon}}\exp\left(iB\frac{K}{\varepsilon}\right)-\right.$$

$$\left.-\frac{\left(1-\exp\left(-iB\left(\sigma-\frac{\xi+K}{\varepsilon}+i\frac{\rho(1-\varepsilon)}{2\varepsilon}\right)\right)\right)}{\sigma-\frac{\xi+K}{\varepsilon}+i\frac{\rho(1-\varepsilon)}{2\varepsilon}}\exp\left(-iB\frac{K}{\varepsilon}\right)\right|^2, \tag{27b}$$

$$\omega\frac{d^2N_{TR}}{d\omega d\Omega} = \frac{e^2}{\pi^2}\theta_\perp^2\left(\frac{1}{\theta_\perp^2+\gamma^{-2}} - \frac{1}{\theta_\perp^2+\gamma^{-2}+\frac{|a\chi'_a+b\chi'_b|}{T}}\right)^2 R_{TR}\,, \tag{28a}$$

$$R_{TR} = \left|1+\frac{2K\exp\left(-iB\left(\sigma-\frac{\xi}{\varepsilon}+\frac{i\rho(1-\varepsilon)}{2\varepsilon}\right)\right)}{\left(\xi-K-i\frac{\rho(1+\varepsilon)}{2}\right)\exp\left(-iB\frac{K}{\varepsilon}\right)-\left(\xi+K-i\frac{\rho(1+\varepsilon)}{2}\right)\exp\left(iB\frac{K}{\varepsilon}\right)}\right|^2, \tag{28b}$$

The following notations are put into formulas (27) and (28):

$$\sigma(\theta,\gamma) = \frac{gT}{2\left|\sin\left(\frac{ga}{2}\right)\right| |\chi_b' - \chi_a'|}\left(\theta_\perp^2 + \gamma^{-2} + \frac{|a\chi_a' + b\chi_b'|}{T}\right),$$

$$K = \sqrt{\xi^2 - \varepsilon - i\rho^{(s)}((1+\varepsilon)\xi^{(s)} - 2\kappa^{(s)}\varepsilon) - \rho^{(s)2}\left(\frac{(1+\varepsilon)^2}{4} - \kappa^{(s)2}\varepsilon\right)},$$

$$\xi(\omega) = \frac{gT\sin^2\theta_B}{\left|\sin\left(\frac{ga}{2}\right)\right| |\chi_b' - \chi_a'|}\left(1 - \frac{\omega}{\omega_B}\right) + \frac{1+\varepsilon}{2\nu^{(1)}}, \quad \rho = \frac{a\chi_a'' + b\chi_b''}{|\chi_b' - \chi_a'|}\frac{g}{2\left|\sin\left(\frac{ga}{2}\right)\right|},$$

$$\kappa = \frac{2\left|\sin\left(\frac{ga}{2}\right)\right|}{g}\left|\frac{\chi_b'' - \chi_a''}{a\chi_a'' + b\chi_B''}\right|, \quad B = \frac{2\omega_B\left|\sin\left(\frac{ga}{2}\right)\right| |\chi_b' - \chi_a'|}{gT\sin(\theta_B + \delta)}, \quad \theta_\perp = \theta\sin\phi, \tag{29}$$

In accordance with (27b), two waves that contribute to the FPXR may exist in periodic layered medium. The contribution of first or second waves could be significant if, respectively, the first or the second of the next equations has a solution:

$$\mathrm{Re}\left(\sigma - \frac{\xi - K}{\varepsilon} + i\frac{\rho(1-\varepsilon)}{2\varepsilon}\right) \approx \sigma - \frac{\xi - \sqrt{\xi^2 - \varepsilon}}{\varepsilon} = 0, \tag{30a}$$

$$\mathrm{Re}\left(\sigma - \frac{\xi + K}{\varepsilon} + i\frac{\rho(1-\varepsilon)}{2\varepsilon}\right) \approx \sigma - \frac{\xi + \sqrt{\xi^2 - \varepsilon}}{\varepsilon} = 0, \tag{30b}$$

Since $\sigma > 1$ one may show that the equation (30b) has a solution under the condition $\varepsilon > \sigma^{-2}$, and the equation (30a) is solvable at the condition that $\varepsilon < \sigma^{-2}$. Thus, under different values of asymmetry parameter, the first or the second of X-ray waves may contribute in the FPXR.

Let us consider the direction of energy transfer of two waves responsible for the formation of the FPXR. For this purpose, we consider the velocity group of radiation waves along the OX-axis neglecting absorption. Projections of wave vectors connected with waves along the OX-axis (9a) in periodic layered structure in the case of non-absorbing targets are as follows:

$$k_x^{(1,2)} = \omega\sin(\theta_B + \delta) + \frac{\omega\chi_0}{2\sin(\theta_B + \delta)} + \frac{\omega}{2\sin(\theta_B - \delta)}\frac{\left|\sin\left(\frac{ga}{2}\right)\right| |\chi_b' - \chi_a'| C^{(x)}}{gT}\times$$

$$\times\left(\xi(\omega) \mp \sqrt{\xi(\omega)^2 - \epsilon}\right), \tag{31}$$

The velocity group of these X-ray waves (31) have the form:

$$V_{gr} = \left(\frac{\partial k_x^{(1,2)}}{\partial\omega}\right)^{-1} \approx \left(\sin(\theta_B + \delta) - \frac{\sin^2\theta_B}{\sin(\theta_B - \delta)}\left(1 \mp \frac{\xi}{\sqrt{\xi(\omega)^2 - \varepsilon}}\right)\right)^{-1}, \tag{32}$$

It may be shown that the velocity group of waves which correspond to the first branch of the dispersion relation solution is positive $(\partial k_x^{(1)}/\partial\omega)^{-1} > 0$ and the energy of wave is transferred from the input surface to the output surface of the target. The velocity group of the second wave is always negative $(\partial k_x^{(2)}/\partial\omega)^{-1} < 0$. Consequently, the energy of wave transfers from the output surface to the input one of the target. This fact leads to the suppression of the second wave of FPRX in periodic layered medium in the case of crystal with considerable thickness when the transmitted energy is completely absorbed.

Thus, for a sufficiently large thickness of the crystal, FPXR corresponding to the second branch of the dispersion relation solution is suppressed. However, at the condition $\varepsilon < \sigma^{-2}$, the FPXR which corresponds to the first of generated X-ray waves in periodic layered medium is material.

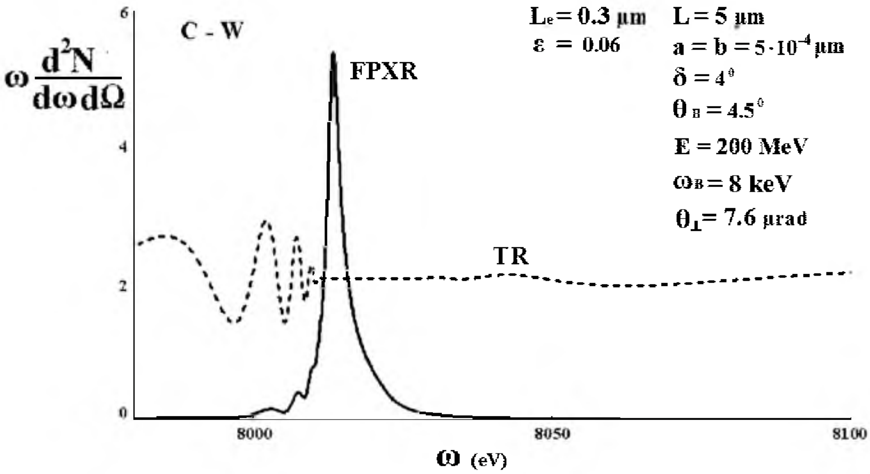

Fig. 3. Spectral-angular density of FPXR and TR of relativistic electron in periodic layered medium consisting of carbon and tungsten layers (the case $\varepsilon < 1$).

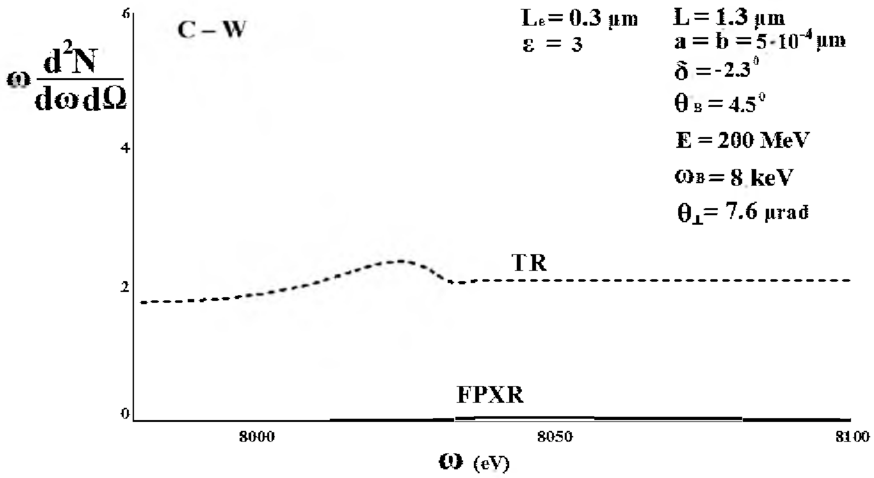

Fig. 4. Spectral-angular density of FPXR and TR of relativistic electron in periodic layered medium consisting of carbon and tungsten layers (the case $\varepsilon > 1$).

Let us demonstrate this claim by numerical calculations performed by formulas (27) and (28). In fig. 3 and fig. 4 curves are constructed describing the spectral and angular density FPXR and TR of relativistic electron at the energy E=200MeV which crosses the periodic layered structure C-W, that consists of layers of carbon and tungsten. Furthermore, curves in fig. 3 are constructed for the case where the asymmetry parameter $\varepsilon < \sigma^{-2}$ and the contribution comes from the first branch FPXR with positive velocity group of X-ray waves. In fig. 4 curves are plotted for the case $\varepsilon > \sigma^{-2}$ where the contribution of the first branch is absent, and the contribution of the second one is suppressed.

It is necessary to note that in fig. 3 and fig. 4 curves are constructed with the same path length of electrons $L_e = \frac{L}{\sin(\theta_B+\delta)} \approx 0.3\mu m$ and photons $L_e \approx L_{pk}$ in the target. In this case, the length of photon absorption in the structure $L_{abs} = \frac{T}{\omega|a\chi_a'' + b\chi_b''|} \approx 6.2\mu m$ is much more then L_{ph}.

In this case, the transition radiation consists of the radiation produced at the exit surface of target. It should be noted that the width of the peak FPXR in this case, as it follows from fig. 3, is equal 25eV which is much wider than in the crystalline medium (crystal 1-2 eV). This fact will ease the experimental research and the identification of FPXR in periodic layered structure.

4. Conclusion. In the work the dynamic theory of coherent X-rays along the velocity of relativistic particle in periodic layered structure in Bragg's scattering geometry is built up for the general case of asymmetric reflection of particle field relative to the entrance surface of target. On the basis of the two-wave approximation of dynamical theory of diffraction, expressions which describe spectral and angular characteristics of the radiation from two radiation mechanisms FPXR and TR are obtained.

Existence of the dynamic effect of FPXR in periodic layered structure is shown for the first time. It is also shown that the spectral-angular density of FPXR considerably depends on the asymmetry of electron field reflection relative to the surface of target under fixed path of the electron in it. It is shown that the spectral peak of relativistic electron parametric X-ray radiation in the forward direction is essentially larger than the peak of the emission spectrum of single crystal which may ease its experimental observation and investigation.

References

1. Ter-Mikaelian M.L. High-Energy Electromagnetic Process in Condensed Media /New York: Wiley, 1972.
2. Piestrup M.A., Boyers D.G., Pincus C.I. et al. // Phys. Rev. A. – 1992. – 45. – P.1183.
3. Tanaka T et al. // Nucl. Instrum. Methods. – 1994. – B 93. – P.21.
4. Yamada K., Hosokawa T., Takenaka H. // Phys. Rev. A. – 1999. – 59. – P.3673.
5. Asano S. et al. // Phys. Rev. Lett. – 1993. – 70. – P.3247.
6. Kaplan A.E., Law C.T., Shkolnikov P.L. // Phys. Rev. E. – 1995. – 52. – P.6795.
7. Pardo B., Andre J.-M. // Phys. Rev. E. – 2002. – 65. – 036501.
8. Nasonov N.N., Kaplin V.V., Uglov S.R., Piestrup M.A., Gary C.K. // Phys. Rev. E. – 2003. – 68. – P.3604.
9. Garibian G.M., Yang C. // Exp. J. Theor. Phys. – 1971. – 61. – P.930.
10. Baryshevsky V.G., Feranchuk I.D. // Exp. J. Theor. Phys. – 1971. – 61. – P.944.
11. Baryshevsky V.G., Feranchuk I.D. // Physique J. (Paris). – 1983. – 44. – P.913.
12. Caticha A. // Phys.Rev. A. – 1989. – 40. – P.4322.

13. Kaplin V.V., Uglov S.R., Zabaev V.N., Piestrup M.A., Gary C.K., Nasonov N.N., Fuller M.K. // Appl. Phys. Lett. – 2000. – 76. – P.3647.
14. Nasonov N.N., Kaplin V.V., Uglov S.R., Zabaev V.N., Piestrup M.A., Gary C.K. // Nucl. Instrum. Methods B. – 2005. – 227. – P.41.
15. Blazhevich S.V., Kolosova I.V., Noskov A.V. // J. Exp. Theor. Phys. – 2012. – 114. – P.547.
16. Blazhevich S., Noskov A. // IL Nuovo Cimento. – 2011. – 34C. – P.231. (DOI 10.1393/ncc/i2011-10922-x).
17. Garibian G.M., Yang C.J. // Exp. Theor. Phys. – 1972. – 63. – P.1198.
18. Baryshevsky V.G., Feranchuk I.D. // Phys. Lett. A. – 1976. – 57. – P.183.
19. Baryshevsky V.G., Feranchuk I.D. // J. Physique. (Paris). – 1983. – 44. – P.913.
20. Yuan Luke C.L., Alley P.W., Bamberger A. et al // Nucl. Instr. and Meth. / In Phys. Res. A – 1985. – 234. – P.426.
21. Kalinin B.N., Naumenko G.A., Padalko D.V. et al // Nucl. Instr. and Meth. / In Phys. Res. B . – 2001. – 173. – P.253.
22. Kube G., Ay C., Backe H., Clawiter N. at al // in Abstracts V International Symposium “Radiation from Relativistic Electrons in Periodic Structures”, Lake Aya, Altai Mountains, Russia, 10-14 September, 2001.
23. Backe H., Clawiter N. at al // in Proc. Of the Intern. Symp. on Channeling–Bent Crystals–Radiation Processes, 2003, Frankfurt am Main, Germany, EP Systema Bt., Debrecen, P.41.
24. Aleinik A.N., Baldin A.N., Bogomazova E.A. et al // JETP Lett. – 2004. – 80. – P.393.
25. Kubankin A.S., Nasonov N.N., Sergienko V.I., Vnukov I.E. // Nucl. Instr. and Meth. / In Phys. Res. B. – 2003. – 201. – P.97.
26. Nasonov N., Noskov A. // Nucl. Instr. Meth. / In Phys. Res. B. – 201. – P.67.
27. Kubankin A., Nasonov N., Noskov A. // in Proc. 7 Int. Russian-Japanese Symposium “Interaction of fast charged particles with solids”, Nov. 24-30, 2002, Kyoto, Japan, P.217-225.
28. Blazhevich S., Noskov A. // J.Exp. Theor. Phys. – 2009. – 136. – P.1043.
29. Blazhevich S., Noskov A. // Nucl. Instr. Meth. / In Phys. Res. B. – 2006. – 252. – P.69.
30. Bazylev V., Zhevago N. Emission From Fast Particles Moving in a Medium and External Fields / Moscow: Nauka, 1987.
31. Pinsker Z. Dynamic Scattering of X-rays in Crystals / Berlin: Springer, 1984.

КОГЕРЕНТНОЕ РЕНТГЕНОВСКОЕ ИЗЛУЧЕНИЕ, ПОРОЖДЁННОЕ РЕЛЯТИВИСТСКИМ ЭЛЕКТРОНОМ ВДОЛЬ СКОРОСТИ ДВИЖЕНИЯ В ПЕРИОДИЧЕСКИ СЛОИСТОЙ СРЕДЕ

С.В. Блажевич, Ю.П. Гладких, А.В. Носков, О.Н. Сатлер

Белгородский государственный университет,
ул. Победы, 85, Белгород, 308015, Россия, e-mail: blazh@bsu.edu.ru

Аннотация. В работе рассматривается динамическая теория когерентного рентгеновского излучения, генерируемого релятивистским электроном, пересекающим многослойную среду. Излучение изучается в направлении вектора скорости электрона в геометрии рассеяния Брэгга для общего асимметричного случая. Сравниваются аналогичные излучения, генерируемые релятивистским электроном в монокристалле одной среды. Разработанная теория описывает условия, при которых когерентное излучение существует, и описывает его спектральные и угловые характеристики.

Ключевые слова: релятивистский электрон, периодически слоистая цель, когерентное рентгеновское излучение, динамическая теория.