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Theorem about the Number and Structure of the Singular Points N-Dimensional Dynamical System of Population Dynamics Lotka-Volterra in Context of Informational Analysis and Modeling

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Abstract: By elementary methods of combinatorial mathematics and uniqueness of solutions systems of linear algebraic equations for non degenerate cases proved a theorem about the number and structure of the singular points of n-dimensional dynamical system of population a dynamics Lotka-Volterra model. Showed that the number of singular points for this system is equal to 2^n and their structure on a combination of zero and nonzero coordinates coincides with the binomial coefficients.

Key words: Lotka-Volterra's model · Population dynamics · Number of singular points · Binomial coefficients · Solution systems of linear algebraic equations

INTRODUCTION

A multidimensional model of population dynamics was proposed by Vito Volterra in [1, 2], but since it was Lotka who developed this type of parallel equation in biophysical and chemical kinetics [3], the equations of population dynamics bears the names of both scholars. This model attracted attention of various great scientists, e.g. G. Nicolis and Prigogine [4] R. May [5], etc. Considering this model the scholars mainly studied the stability of a non-trivial of the singular point. For example, B. Goh [6] in the study of models of mutualism shows that a necessary and sufficient condition for the local and global stability of a non-trivial singular point of the Lotka-Volterra model is the positivity of all the top (main) minors of the Jacobi matrix for this model. Later, Z. Lu and E. Takeuchi [7] proved a number of theorems on the global stability of Lotka-Volterra equations. In papers on economic dynamics [8, 9] it was observed that the n-dimensional system of equations of population dynamics of Lotka-Volterra has 2^n singular points, but still no evidence for that was provided. The possibility of using of such equations in the information analysis and modeling of the interactions of different types of R & D results is shown in [10]. The original n-dimensional Lotka-Volterra m odel, in our opinion, can be used in the simulation of competitive interactions n scientific fronts within the broad area of research in which a variety of variants for the suppression of some scientific fronts by the others, as well as their co-existence, will be observed. The following will state and prove a theorem on the number and structure of the singular points of the n-dimensional Lotka-Volterra model.

Main Part

Theorem: The number of singular points of the n-dimensional system of nonlinear ordinary differential equations of Lotka-Volterra with positive coefficients and non-degenerate cases of systems of linear algebraic equations which arise in the determination of the coordinates of singular points equals 2^n and their structure with respect to the zero and non-zero coordinates coincides with binomial coefficients.

Proof: Let us consider a system of Lotka-Volterra equations in the form

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$$\frac{dx_i}{dt} = x_i\left[a_i - \sum_{j=1}^{n}\gamma_{ij}x_j\right] \quad (1)$$

For convenience, the proof of the theorem the right sides of this system equations will be rewritten equal to zero, as follows:

$$\begin{cases} x_1(\alpha_1 - \gamma_{11}x_1 - \gamma_{12}x_{2-...-}\gamma_{1i}x_i - ... - \gamma_{1n}x_n) = 0 \\ x_2(\alpha_1 - \gamma_{21}x_1 - \gamma_{22}x_{2-...-}\gamma_{2i}x_i - ... - \gamma_{2n}x_n) = 0 \\ \\ x_i(\alpha_i - \gamma_{i1}x_1 - \gamma_{i2}x_{2-...-}\gamma_{ii}x_i - ... - \gamma_{in}x_n) = 0 \\ \\ x_n(\alpha_n - \gamma_{n1}x_1 - \gamma_{n2}x_{2-...-}\gamma_{ni}x_i - ... - \gamma_{nn}x_n) = 0 \end{cases} \quad (2)$$

We will consider the cases of non-degenerate solutions of systems of linear algebraic equations, each of them has only unique solution.

From (2), two singular points are immediately identified-the zero and non-trivial (non-zero), the latter is the solution of the n-dimensional system of linear algebraic equations in the parentheses of the original system (2). From the point of view of combinatorial mathematics, these singular points correspond to the following combinations:

$$C_n^n = \binom{n}{n} = 1 - n$$

of zeros from n variables;

$$C_n^0 = \binom{n}{0} = 1 - 0$$

of zeros from n variables.

In the first case we have only one zero singular point, in the second-only one non-zero singular point.

Further, the number of singular points with a combination of one zero coordinates of n variables is equal to C_n^1`, the number of singular points with a combination of two zero coordinates of n variables is equal to c_n^2, the number of singular points with a combination i zero coordinates of n variables is equal to c_n^i, the number of singular points with a combination of *(n-1)* zero coordinates of n variables equals to $c_n^{n-1} = n$.

Consequently, the total number of singular points is equal to

$$N = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + ... + \binom{n}{i} + ... + \binom{n}{n-1} + \binom{n}{n} =$$

$$C_n^0 + C_n^1 + C_n^2 + ... + C_n^i + C_n^n = (1+1)^n = 2^n,$$

$$C_n^i = \binom{n}{i} = \frac{n}{(n-i)i}, n \geq i$$

Thus the total number of the singular points is equal 2^n and their structure with respect to the zero and non-zero coordinates of repeats the consequent set of coefficients of Newton's binomial.

In this proof the following statement is meant. When we take all of the singular points with zero coordinates in the amount i, the rest of the system of linear algebraic equations of the (n-i)-order are the unique solutions (non-degenerate cases).

CONCLUSION

For an n-dimensional system of equations of population dynamics, introduced by V. Volterra and A.Lotka in the middle of 1920s, the theorem about the number and structure of the singular points of the classical system of equations wasn’t proven. In this paper, this theorem was proved by elementary methods of combinatorial mathematics and uniqueness of solutions of systems of linear algebraic equations for non-degenerate cases. From the view point of information analysis and modeling of information systems and processes, it should be noted that the dynamical system (1) basically can simulate the process of competitive interactions in scientific fronts within the broad field of scientific research. Then in such system we can observe 2^n variants of outcome of such interactions actions, where 2^n - 2 variants will be associated with the suppression of some scientific fronts by other more competitive ones.

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