Physics of Atomic Nuclei, Vol. 63, No. 11, 2000, pp. 2011–2014. Translated from Yadernaya Fizika, Vol. 63, No. 11, 2000, pp. 2105–2108.
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XXIX INTERNATIONAL CONFERENCE ON THE PHYSICS OF CHARGED-PARTICLE INTERACTION WITH CRYSTALS

Collimator-Size Effect on the Parametric-Radiation Spectrum

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Received February 11, 2000

Abstract—It is shown that, by varying the angular size of a collimator, the spectral distribution of parametric x-ray radiation can be split into two isolated peaks and that the center of the spectral distribution of this radiation is shifted in frequency in response to an increase in the collimator size. It is also predicted that an increase in the angular size of the collimator will lead to a substantial modification of the character of the orientation dependence of the radiation spectrum. © *2000 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

The motion of a fast charged particle in a crystal is accompanied by the scattering of its Coulomb field by atomic electrons of the crystal. Owing to the periodic arrangement of atoms in the crystal lattice, this scattering has a coherent character. Parametric x-ray radiation [1–4] generated under such conditions has a number of unique characteristics: it has a quasimonochromatic spectrum; it has a pencil-like character, propagating at a large angle with respect to the radiating-particle velocity; and the energy of the photons constituting this radiation can be smoothly changed.

Since the energy of a photon emitted in parametric x-ray radiation is tightly related to the angle of radiation observation, the properties of this radiation depend substantially on the collimator (or radiation-detector) sizes. The objective of this study is to analyze in detail the spectral and orientation properties of parametric x-ray radiation versus angular sizes of the collimator. We will show that these properties undergo qualitative changes with increasing collimator sizes.

2. GENERAL RELATIONS

Presently, the theory of parametric x-ray radiation has been firmly established. In a number of studies, the validity of its description on the basis of perturbation theory (kinematical approximation) was demonstrated theoretically and confirmed experimentally. Following [2] and using conventional methods, we find that the distribution of the intensity of parametric x-ray radiation with respect to spectral and angular variables can be represented as

$$\omega\frac{dN}{dtd\omega d^2\theta} = \frac{8\pi Z^2 e^6 n_0^2 |S(\mathbf{g})|^2 e^{-g^2u^2}}{m^2(1+g^2R^2)^2}$$
$$\times\frac{[\mathbf{v}(\mathbf{g}\cdot\mathbf{v})-\mathbf{g}(1-\sqrt{\varepsilon}(\mathbf{n}\cdot\mathbf{v}))/\varepsilon]^2-[(\mathbf{n}\cdot\mathbf{g})(\mathbf{g}\cdot\mathbf{v})-(\mathbf{n}\cdot\mathbf{g})(1-\sqrt{\varepsilon}(\mathbf{n}\cdot\mathbf{v}))/\varepsilon]^2}{[g^2(1-\sqrt{\varepsilon}(\mathbf{n}\cdot\mathbf{v}))+2\sqrt{\varepsilon}(\mathbf{n}\cdot\mathbf{g})(\mathbf{g}\cdot\mathbf{v})]^2}\frac{1}{1-\sqrt{\varepsilon}(\mathbf{n}\cdot\mathbf{v})}\delta\left(\omega-\frac{\mathbf{g}\cdot\mathbf{v}}{1-\sqrt{\varepsilon}(\mathbf{n}\cdot\mathbf{v})}\right), \quad (1)$$

where $\mathbf{g}$ is a reciprocal-lattice vector that specifies the reflecting crystallographic plane; $S(\mathbf{g})$ is a structural factor; $e^{-g^2u^2}$ is the Debye–Waller factor; n_0 is the density of crystal atoms; Z is the number of electrons in the atom; R is the screening radius; $\varepsilon = 1 - \omega_0^2/\omega^2$, ω_0 being plasmon frequency; $\mathbf{n}$ is a unit vector in the direction of emitted-photon momentum; and $\mathbf{v}$ is the velocity of the radiating particle. The disposition of the vectors $\mathbf{g}$, $\mathbf{v}$, and $\mathbf{e}$ lying in the same plane is shown in Fig. 1. The vector $\mathbf{n}$ is given by

$$\mathbf{n} = \mathbf{e}\left(1-\frac{1}{2}\theta^2\right)+\boldsymbol{\theta}, \quad \mathbf{e}\cdot\boldsymbol{\theta} = 0, \quad \boldsymbol{\theta} = \boldsymbol{\theta}_\perp+\boldsymbol{\theta}_\parallel, \quad (2)$$

where $\theta_\parallel$ and $\theta_\perp$ are, respectively the parallel and perpendicular components of the two-dimensional vector $\boldsymbol{\theta}$ appearing to be the angular variable in the distribution of the radiation. The orientation angle θ' is measured from the position of the exact Bragg resonance. A change in θ' corresponds to a rotation of the crystal as a discrete unit about the axis orthogonal to the plane of the figure. The angle between the vectors $\mathbf{v}$ and $\mathbf{e}$ remains unchanged upon this rotation.

In terms of the variables $\boldsymbol{\theta}$ and θ', formula (1) takes the conventional form in the kinematical theory of parametric x-ray radiation; that is,

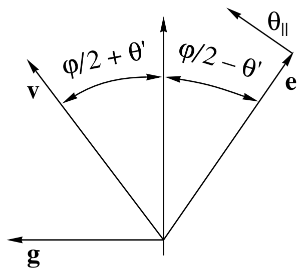

Fig. 1. Geometry of the parametric-radiation process (**v** is the radiating-particle velocity, while **g** is a reciprocal-lattice vector).

$$\omega \frac{dN}{dt d\omega d^2\theta} = \frac{e^2\omega_0^4|S(\mathbf{g})|^2 e^{-g^2u^2}}{\pi g^2(1+g^2R^2)^2} \times \frac{\theta_\perp^2 + (2\theta' + \theta_\parallel)^2\cos^2\varphi}{(\gamma^{*-2} + \theta_\perp^2 + (2\theta' + \theta_\parallel)^2)^2} \times \delta\left[\omega - \omega_b\left(1 + (\theta' + \theta_\parallel)\cot\frac{\varphi}{2}\right)\right], \quad (3)$$

where $\gamma^* = \gamma(1 + \gamma^2\omega_0^2/\omega^2)^{-1/2}$, $\gamma = (1 - v^2)^{-1/2} \gg 1$, and $\omega_b = g/2\sin(\varphi/2)$ is the Bragg frequency in whose neighborhood the spectrum of the radiation is concentrated.

Expression (3) (or expressions similar to it) is widely used to describe experimental results. In doing this, it is implied that the angular size of the radiation collimator is sufficiently small. In the following, we will study the collimator-size effect on the spectrum of the detected radiation.

We consider the case of a rectangular collimator:

$$-\frac{1}{2}\Delta\theta_\parallel < \theta_\parallel < \frac{1}{2}\Delta\theta_\parallel, \quad -\frac{1}{2}\Delta\theta_\perp < \theta_\perp < \frac{1}{2}\Delta\theta_\perp.$$

Performing a two-dimensional integration in (3) with respect to θ, we obtain the observed radiation spectrum in the eventual form

$$\omega\frac{dN}{d\omega} = N_0 F(x, y, \lambda_\perp, \lambda_\parallel),$$

$$N_0 = \frac{2e^2\omega_0^4}{\pi g^3}\frac{|S(\mathbf{g})|^2 e^{-g^2u^2}\gamma^* L\sin^2(\varphi/2)}{(1+g^2R^2)^2\cos(\varphi/2)},$$

$$F = \frac{1}{\sqrt{1+(x+y)^2}}\left\{\left[1 + \frac{(x+y)^2\cos^2\varphi}{1+(x+y)^2}\right] \times \arctan\left(\frac{\lambda_\perp}{\sqrt{1+(x+y)^2}}\right) - \left[1 - \frac{(x+y)^2\cos^2\varphi}{1+(x+y)^2}\right] \times \frac{\lambda_\perp(1+(x+y)^2)^{-1/2}}{1+\lambda_\perp(1+(x+y)^2)^{-1}}\right\}\sigma(x - y + \lambda_\parallel)\sigma(y + \lambda_\parallel - x), \quad (4)$$

where $x = \gamma^*\tan(\varphi/2)(\omega/\omega_b - 1)$, $y = \gamma^*\theta'$, $\lambda_\perp = \frac{1}{2}\gamma^*\Delta\theta_\perp$, $\lambda_\parallel = \frac{1}{2}\gamma^*\Delta\theta_\parallel$, $\sigma(Z)$ is the Heaviside function, and L is the crystal thickness assumed to be much smaller than the photoabsorption length.

Below, the universal function F is used in our analysis of the effect of the parameters $\lambda_\parallel$ and $\lambda_\perp$ on the spectrum and on the orientation dependence of the radiation yield.

3. DISCUSSION

Let us first consider the properties of the radiation in the limiting case of $\lambda_\perp \ll 1$, $\varphi \neq \pi/2$. Formula (4) is then simplified considerably to become

$$F \approx 2\lambda_\perp\cos^2\varphi\frac{(x+y)^2}{[1+(x+y)^2]^2} \times \sigma(x - y + \lambda_\parallel)\sigma(y + \lambda_\parallel - x), \quad (5)$$

whence we can see that, under the conditions being considered, the shape and the amplitude of the spectrum depend substantially on the parameters $\lambda_\parallel$ and y. For $\lambda_\parallel \ll 1$, the spectrum represents a narrow line ($\Delta x = 2\lambda_\parallel \ll 1$) centered at the point $x = y$. The corresponding energy of the emitted photon is given by

$$\omega = \omega_b(1 + \theta'\cot(\varphi/2)). \quad (6)$$

This expression is usually used in the theory of parametric x-ray radiation.

The amplitude of the spectral line is determined by the value of the orientation parameter y in accordance with the factor $4y^2/(1 + 4y^2)^2$ following from (5) for $\lambda_\parallel \ll 1$. This factor describes the orientation dependence of the radiation yield with two peaks at $y = \pm 1/2$ (it is typical of parametric x-ray radiation).

Let us now consider the case of $\lambda_\parallel \longrightarrow \infty$ (slit collimator). The spectrum is then described by the formula

$$F \approx 2\lambda_\perp\cos^2\varphi\frac{(x+y)^2}{[1+(x+y)^2]^2}, \quad (7)$$

which follows from (5) and which predicts two substantial modifications to the spectrum in relation to the preceding case. From Eq. (7), we can see that, in the case being considered, the spectrum represents a symmetric curve having two peaks at $x = \pm 1 - y$. The width of the spectral distribution is large, $\Delta x \geq 1$, and it is centered at $x = -y$, which corresponds to the emitted-photon energy

$$\omega = \omega_b(1 - \theta'\cot(\varphi/2)). \quad (8)$$

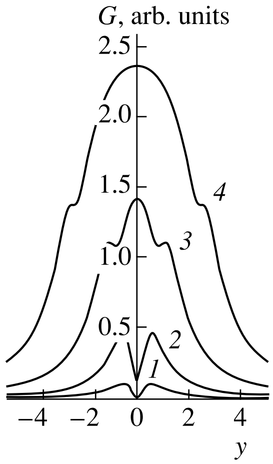

Fig. 2. Universal orientation dependence of the radiation yield for $\lambda_\perp \ll 1$ at $\lambda_\parallel$ = (*1*) 0.1, (*2*) 0.5, (*3*) 2, and (*4*) 5.

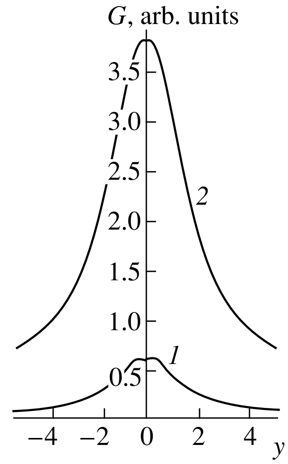

Fig. 3. Universal orientation dependence of the radiation yield for $\lambda_\perp \longrightarrow \infty$ and $\varphi = \pi/8$ at $\lambda_\parallel$ = (*1*) 0.3 and (*2*) 2.

It follows that, when we go over from a pointlike to a slit collimator, the spectrum of parametric x-ray radiation is split; concurrently, line broadening and a shift of the spectral-distribution center occur, the latter being dependent on the orientation angle. For this splitting, $\Delta\omega_p$, and the shift of the distribution center, $\Delta\omega_c$, we have

$$\Delta\omega_p = 2\omega_b \gamma^{*-1} \cot\frac{\varphi}{2}, \quad \Delta\omega_c = 2\omega_b \theta' \cot\frac{\varphi}{2}. \quad (9)$$

Integrating the function F in (5) with respect to x, we obtain the orientation dependence of the radiation yield in the form

$$G(y, \lambda_\parallel) = \int dx F = \lambda_\perp \cos^2\varphi \Bigg[\arctan(2y + \lambda_\parallel) - \arctan(2y - \lambda_\parallel) - \frac{(2y + \lambda_\parallel)}{1 + (2y + \lambda_\parallel)^2} + \frac{(2y - \lambda_\parallel)}{1 + (2y - \lambda_\parallel)^2}\Bigg]. \quad (10)$$

The function G shown in Fig. 2 for various values of the parameter $\lambda_\parallel$ exhibits a qualitative variation in the orientation dependence of the yield of parametric x-ray radiation with increasing angular size of the collimator in the reaction plane. With increasing $\lambda_\parallel$, the orientation curve that is typical of parametric x-ray radiation and which has two peaks at small values of $\lambda_\parallel$ transforms into an orientationally independent constant.

Formulas (5)–(9) are not valid in the particular case of $\varphi = \pi/2$. For this value of the emission angle φ, it follows from (4) that

$$F = \frac{2}{3} \frac{\lambda_\perp^3}{[1 + (x + y)^2]^2} \sigma(x - y + \lambda_\parallel) \sigma(y + \lambda_\parallel - x). \quad (11)$$

An analysis of this formula by a method similar to that used above leads to results that are by and large analogous to those in the preceding case, but the spectral distribution of radiation does not undergo splitting here.

Let us now consider another limiting case, that of $\lambda_\perp \longrightarrow \infty$. From (4), we then obtain

$$F = \frac{\pi}{2} \frac{1}{\sqrt{1 + (x + y)^2}} \left[1 + \frac{(x + y)^2 \cos^2\varphi}{1 + (x + y)^2}\right] \times \sigma(x - y + \lambda_\parallel) \sigma(y + \lambda_\parallel - x). \quad (12)$$

It can easily be verified that, in the case being considered, the radiation spectrum transforms from a narrow line of width $2\lambda_\parallel$ for $\lambda_\parallel \ll 1$ into a bell-like curve of width $\Delta x \geq 1$ for $\lambda_\parallel \longrightarrow \infty$. However, only in the emission-angle region $\varphi < \pi/4$ does the spectral curve split into two peaks, but these peaks are rather weak. In response to an increase in the collimator size, the center of the spectral distribution of the radiation shows a shift of the same magnitude as in the preceding case of $\lambda_\perp \ll 1$.

The orientation dependence of the radiation yield is given by the formula

$$G(y, \lambda_\parallel) = \int dx F = (1 + \cos^2\varphi)[\operatorname{arcsinh}(2y + \lambda_\parallel) - \operatorname{arcsinh}(2y - \lambda_\parallel)] - \cos^2\varphi \left[\frac{(2y + \lambda_\parallel)}{\sqrt{1 + (2y + \lambda_\parallel)^2}} - \frac{(2y - \lambda_\parallel)}{\sqrt{1 + (2y - \lambda_\parallel)^2}}\right], \quad (13)$$

which follows from (11). The curves that represent the dependence $G(y)$ for various values of $\lambda_\parallel$ and φ are shown in Figs. 3 and 4.

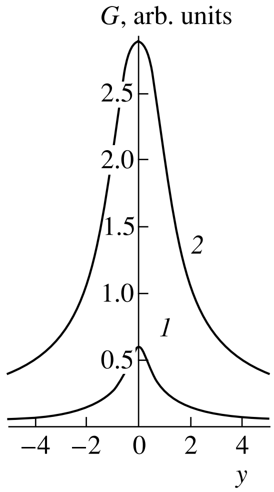

Fig. 4. As in Fig. 3, but for $\varphi = \pi/2$.

4. CONCLUSION

The spectral and orientation characteristics of parametric x-ray radiation depend substantially on the angular sizes of the radiation collimator.

With increasing collimator size in the reaction plane, the spectral distribution of the radiation broadens, while its center undergoes a shift.

If a slit collimator, with the slit being oriented along the reaction plane, is used, the observed radiation spectrum splits into two isolated peaks.

An increase in the angular size of the collimator can radically change the orientation dependence of the yield of parametric x-ray radiation (a trend toward weakening of this dependence is observed).

The above effects are of a simple geometric origin. The angular distribution (3) of the reflex of parametric x-ray radiation is bell-shaped and has a dip at its center (this is the angular structure of transverse pseudophotons of the Coulomb field of a relativistic particle that are reflected by a crystal). Because of the presence of a delta function in (3), there is a tight correlation here between the energy of the emitted photon, on one hand, and the values of the angle of radiation observation in the reaction plane, $\theta_{\|}$, and the orientation angle θ', on the other hand. Owing to the above features, the shape of the observed spectrum and its orientation dependence are determined by the degree of overlap of the reflex of parametric x-ray radiation and the collimator. If the aperture of the collimator is small in relation to the intrinsic width of the angular distribution of parametric x-ray radiation, the observed spectrum is narrow, and the orientation dependence is similar to the angular distribution of parametric x-ray radiation. On the other hand, no orientation dependence is observed if the collimator is open completely; as to the corresponding spectrum, which features all possible photon energies, it can have a dip (at specific transverse sizes of the collimator, $\Delta\theta_{\perp}$, and specific values of the emission angle φ) caused by the absence of photons emitted through the mechanism of parametric x-ray radiation in the direction of Bragg scattering.

ACKNOWLEDGMENTS

This work was supported in part by the Ministry for Higher Education of the Russian Federation (grant no. 97-07.2-155).

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Translated by V. Bukhanov