Dynamical-Diffraction Effects in Parametric Radiation

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Abstract—An analytic theory is developed for dynamical-diffraction effects in x-ray radiation from a relativistic electron traversing a thin single crystal. It is shown that such dynamical effects may be responsible for a glaring discrepancy between recent experimental data and the traditional theory of parametric x-ray radiation.

1. INTRODUCTION

Parametric x-ray radiation [1–4] arises owing to the scattering of the Coulomb field of a fast particle moving in a medium characterized by a periodic dielectric permittivity, $\varepsilon(\omega, \mathbf{r}) = 1 + \chi_0(\omega) + \sum_{\mathbf{g}} \chi_{\mathbf{g}}(\omega) e^{i\mathbf{g}\cdot\mathbf{r}}$, where ω is the photon energy, while $\mathbf{g}$ is a reciprocal-lattice vector. The dynamical theory of x-ray diffraction in a crystal [5] or its simplified version, kinematical theory (perturbation theory), is usually used to describe this radiation. Previous investigations of the spectral and angular distributions of relevant x-ray photons propagating along the direction of Bragg scattering revealed good agreement between experimental data on parametric radiation and kinematical theory. Here, the absence of dynamical effects is due to the fact that, in the case of parametric x-ray radiation, the necessary condition of synchronism between the emitted photon and the radiating particle, $\omega = \mathbf{k} \cdot \mathbf{v}$ ($\mathbf{k}$ is the photon wave vector, and $\mathbf{v}$ is the velocity of the radiating particle), is satisfied only off the region of the dynamical maximum in the Bragg scattering of the pseudophoton field of a particle on the set of atomic crystal planes.

Of particular interest in connection with the aforesaid is the experimental result reported recently by Freudenberger *et al.* [6], who measured, in Bragg geometry, the orientation dependence of the yield of collimated parametric x-ray radiation generated by 87-MeV electrons in a Si crystal (111). At the maximum of this orientation dependence, the measured yield is eight times as great as the theoretical prediction, the orientation angle corresponding to the experimental maximum being 2.8 times smaller than that which follows from the theory of parametric radiation.

The objective of the present study is to develop an analytic theory of the dynamical scattering of the electromagnetic field of a relativistic electron traversing a thin single crystal. On the basis of our results, we will attempt to explain the experimental results presented in [6] and propose a new scheme for generating x-ray radiation.

2. GENERAL RELATIONS

Let us consider the structure of the electromagnetic field excited by a relativistic electron traversing a thin crystal whose reflecting crystallographic plane (which is specified by a reciprocal-lattice vector $\mathbf{g}$) is parallel to the crystal surface (see Fig. 1). Within the two-wave approximation of the dynamical theory of diffraction [5], the Fourier amplitude of the electric field, $\mathbf{E}_{\mathbf{k}\omega} = \frac{1}{(2\pi)^4}\int d^3r dt\, \mathbf{E}(\mathbf{r}, t) e^{-i\mathbf{k}\cdot\mathbf{r} + i\omega t}$, is sought in the form of the sum of the direct and diffracted waves, $\mathbf{E}_0 = \sum_{\lambda=1}^{2} \mathbf{e}_{\lambda 0} E_{\lambda 0}$ and $\mathbf{E}_{\mathbf{g}} = \sum_{\lambda=1}^{2} \mathbf{e}_{\lambda\mathbf{g}} E_{\lambda\mathbf{g}}$, respectively, where the polarization vectors are given by $\mathbf{e}_{10} = \mathbf{e}_{1\mathbf{g}} \approx \mathbf{k} \times \mathbf{g}$, $\mathbf{e}_{20} \approx \mathbf{k} \times \mathbf{e}_{10}$, $\mathbf{e}_{2\mathbf{g}} \approx \mathbf{k}_g \times \mathbf{e}_{10}$, and $\mathbf{k}_g = \mathbf{k} + \mathbf{g}$. Determining the free and forced solutions to the Maxwell equations for the Fourier amplitudes of the relevant fields within and outside the crystal and finding unknown coefficients from the boundary conditions at

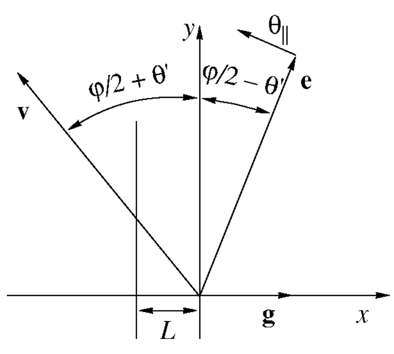

Fig. 1. Geometry of the parametric-radiation process. The following notation is adopted in this figure: $\mathbf{v}$ is the velocity of the radiating particle, $\mathbf{n} = \mathbf{e}\left(1 - \frac{1}{2}\theta^2\right) + \boldsymbol{\theta}$ is a unit vector in the direction of radiation ($\mathbf{e} \cdot \boldsymbol{\theta} = 0$), θ' is the orientation angle measured with respect to the position of the exact Bragg resonance, L is the crystal thickness, $\mathbf{g}$ is a reciprocal-lattice vector, and $\theta_{\parallel}$ is the absolute value of the projection of the vector $\boldsymbol{\theta}$ onto a direction parallel to the reaction plane.

the crystal surface, one obtains the conventional (rather cumbersome) expression for the distribution of the radiation with respect to spectral and angular variables in Bragg geometry (see, for example, [3, 4]).

In the theory of parametric x-ray radiation, a traditional approach relies on the asymptotic formula for the above distribution. This formula describes the yield of parametric x-ray radiation associated with the scattering of the equilibrium Coulomb field of a fast particle over the entire crystal thickness. Because of the screening of the equilibrium field due to the polarization of medium electrons, the yield of this radiation is saturated fast with increasing energy of the radiating particle (density effect in parametric x-ray radiation [7]).

By analyzing the general formula for the yield of parametric x-ray radiation generated by a relativistic electron traversing the surface of a crystal, it was shown in [8] that there is additional radiation due to the dynamical scattering of transition radiation from this electron in the crystal. A generalization of the problem considered in [8] to the case of a finite-thickness crystal was given in [9, 10], where the relevant results were obtained from a numerical analysis. In the present study, we describe the process analytically on the basis of an asymptotic approach, which is opposite, in a sense, to that which is adopted in the conventional theory of parametric x-ray radiation.

In the general expression for the distribution of the radiation with respect to the spectral and angular variables, we first single out terms representing the contribution of transition radiation (such terms are always discarded in the asymptotic formula for parametric x-ray radiation). After some simple algebra, we obtain

$$\omega\frac{dN_\lambda}{d\omega d^2\theta} = \frac{e^2}{\pi^2}\frac{\Omega_\lambda^2}{(\gamma^{-2}+\Omega^2)^2}R(\zeta_\lambda, \delta_\lambda),$$
$$R = \frac{\left|\sinh^2(\delta_\lambda\sqrt{1-\zeta_\lambda^2})\right|}{\left|1-\zeta_\lambda^2\right| + \left|\sinh^2(\delta_\lambda\sqrt{1-\zeta_\lambda^2})\right|}, \qquad (1)$$

where λ is the polarization subscript, $\Omega_1 = \theta_\perp$, $\theta_\| = 2\theta' + \theta_\|$, γ is the Lorentz factor, $\zeta_\lambda = (\omega_b'/\omega_b - 1)/\beta_\lambda$, $\beta_\lambda = 2\omega^2|\chi_g||\alpha_\lambda|/g^2$, $\alpha_1 = 1$, $\alpha_2 = \cos\varphi$, $\omega_b' = \omega_b(1 + (\theta' + \theta_\|)\cot(\varphi/2))$, $\omega_b = g/2\sin(\varphi/2)$, and $\delta_\lambda = \beta_\lambda gL/2$.

The ensuing analysis will rely on expression (1), which differs markedly from that traditionally used in the theory of parametric x-ray radiation. This expression describes radiation correctly for $\gamma^2|\chi_0| = \gamma^2\omega_0^2/\omega^2 \gg 1$ since, under the conditions being considered, the angular distribution in (1) ($\Delta\Omega \approx \gamma^{-1}$) is concentrated almost completely in the region of a dip in the angular distribution of conventional parametric x-ray radiation ($\Omega \le \omega_0/\omega$, $\omega_0/\omega \gg \gamma^{-1}$, where ω_0 is the plasmon frequency in the medium) that is generated by a particle over the entire crystal thickness and which is suppressed by the density effect.

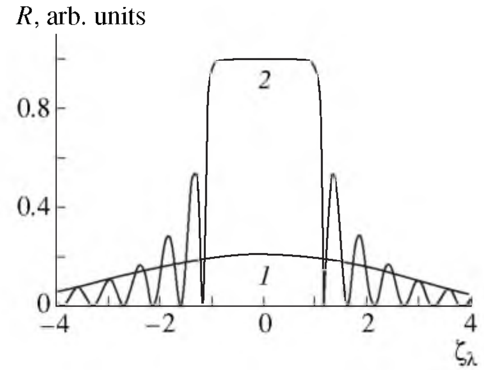

Fig. 2. Universal frequency dependence of the coefficient of the dynamical reflection of the field of a fast particle from a crystal at δ_λ = (*1*) 0.5 and (*2*) 5.

We note that the factor $e^2\Omega_\lambda^2\pi^{-2}(\gamma^{-2} + \Omega^2)^{-2}$ in (1) describes the angular distribution of the vacuum Coulomb field of a relativistic electron and that the quantity $R(\zeta_\lambda, \delta_\lambda)$ corresponds to the coefficient of the dynamical reflection of the field from the crystal. The reason why the dynamical regime of reflection is realized here is that, in the present case, the process involves free transition-radiation photons (rather than bound pseudophotons, as in conventional parametric x-ray radiation), whose distribution with respect to spectral and angular variables differs only slightly from the distribution of pseudophotons in a vacuum for $\omega \ll \gamma\omega_0$ (recall that we consider precisely this region of frequencies). The dependence $R(\zeta_\lambda, \delta_\lambda)$, which characterizes the intrinsic linewidth of the radiation being discussed, is illustrated in Fig. 2.

The crystal-thickness dependence of the radiation yield is a very important characteristic. Integrating expression (1) with respect to ω, we obtain

$$\frac{dN_\lambda}{d^2\theta} = \frac{e^2}{\pi^2}\beta_\lambda\frac{\Omega_\lambda^2}{(\gamma^{-2}+\Omega^2)^2}F(\delta_\lambda),$$
$$F(\delta_\lambda) = \frac{2}{\pi}\int_0^\infty\frac{dx\left|\sinh^2(\delta_\lambda\sqrt{1-x^2})\right|}{\left|1-x^2\right| + \left|\sinh^2(\delta_\lambda\sqrt{1-x^2})\right|}. \qquad (2)$$

According to the curve in Fig. 3, the yield of the radiation being considered is saturated at a crystal thickness approximately equal to the extinction length for Bragg scattering.

By comparing the theoretical result in (2) with the experimental data from [6], we find that there is agreement for the values of the orientation angle θ' that correspond to the maximum of the orientation dependence of the radiation yield; in addition, it turns out that the calculated radiation yield at the maximum of the radiation dependence is approximately twice as large as the

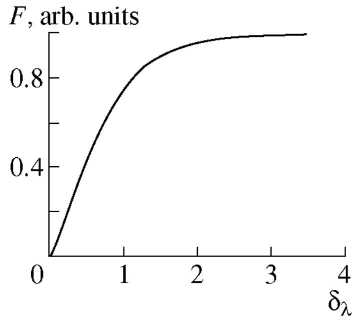

Fig. 3. Universal crystal-thickness dependence of the radiation yield.

measured value. The latter may be due to a sizable interference contribution of conventional parametric x-ray radiation {in the experiment reported in [6], the crystal thickness was comparatively large, while the coefficient $\gamma\omega_0/\omega$ exceeded unity insignificantly (it was about 2.5)}.

Performing a two-dimensional integration of expression (1) with respect to θ, we find the spectrum of a noncollimated radiation in the form

$$\frac{dN_\lambda}{d\omega} = \frac{e^2}{2}\beta_\lambda F(\delta_\lambda)T_\lambda(\omega),$$

$$T_1 = \left[\omega_b^2\gamma^{-2}\cot^2\frac{\varphi}{2} + \left(\omega - \omega_b\left(1 - \theta'\cot\frac{\varphi}{2}\right)\right)^2\right]^{-1/2}, \quad (3)$$

$$T_2 = \frac{\left(\omega - \omega_b\left(1 - \theta'\cot\frac{\varphi}{2}\right)\right)^2}{\left[\omega_b^2\gamma^{-2}\cot^2\frac{\varphi}{2} + \left(\omega - \omega_b\left(1 - \theta'\cot\frac{\varphi}{2}\right)\right)^2\right]^{3/2}}.$$

Expressions (2) and (3) show that, under the condition $\omega \ll \gamma\omega_0$, both the angular width $\Delta\theta \approx \gamma^{-1}$ and the relative spectral width $\Delta\omega/\omega \approx \gamma^{-1}$ of the radiation being considered are much less than the corresponding quantities for conventional parametric x-ray radiation.

From (2), it follows that the total number of emitted photons is

$$N_\lambda = \frac{e^2}{4}\beta_\lambda F(\delta_\lambda)\left[\ln(1+\gamma^2\theta_d^2) - \frac{\gamma^2\theta_d^2}{1+\gamma^2\theta_d^2}\right], \quad (4)$$

where θ_d is the angular dimension of the collimator. On the basis of expression (4), it can be concluded that, in the crystal-thickness region $L < 4/(g\beta_\lambda)$, where the effect of saturation is not yet observed, the total yield of the radiation being considered is on the same order of magnitude as the total yield of conventional parametric x-ray radiation.

A considerable excess of the distribution of our radiation with respect to spectral and angular variables over the analogous quantity for the conventional parametric x-ray radiation (by a factor of about $\gamma^2\omega_0^2/\omega_b^2 \gg 1$ at $L \approx L_{opt} = 4/g\beta_\lambda$) can be used to develop an efficient source of quasimonochromatic pencil-like x-ray radiation.

The proposed source consists of a set of thin crystals (of thickness $L \approx L_{opt}$) positioned in a vacuum along the trajectory of a beam of radiating electrons, the distance T between the neighboring crystals being greater than the radiation-formation length in a vacuum, also known as the coherence length $l_{coh} = 2\gamma^2/\omega_b$. In this case, photons are emitted in each crystal independently and propagate in a vacuum at a large angle with respect to the electron trajectory without undergoing photoabsorption (here, the total thickness of the system, $L_{tot} = NL_{opt}$, N being the number of crystals, may considerably exceed the photoabsorption length, which restricts the yield of radiation in conventional x-ray sources, where photons propagate along the trajectory of particles radiating in a medium).

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