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# On the Possibility of Using Parametric X-ray Radiation to Study Anisotropy of a Crystal Mosaic Structure

**A. S. Kubankin and N. N. Nasonov**

*Laboratory of Radiation Physics, Belgorod State University, Belgorod, Russia*

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**Abstract**—Parametric X-ray radiation (PXR) of relativistic electrons moving in a mosaic crystal is considered. A strong dependence of the PXR angular spectral distribution on the mosaic structure is shown. The effect of mosaic structure on the PXR spectrum strongly increases in the longitudinal plane specified by the radiating electron velocity vector and the averaged reciprocal lattice vector and decreases in the transverse plane with a decrease in the angle of incidence of the radiating electron onto a crystal plane, which allows one to study the anisotropy of mosaic structure of crystal planes.

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## INTRODUCTION

There are many techniques for the investigation of solid structures, which can be separated into two basic groups. The first one uses X-ray structural analysis measuring a distribution of the diffracted X-ray quanta. In the second group, the spatial distribution of electrons traversing a sample is studied. In this work a radiative response of a medium arising in passing a relativistic electron through it is considered.

Structurally perfect crystals have found wide utility in physics of X-rays, where they are mostly used as radiant elements and crystal monochromators. It is impossible to grow a perfect crystal; that is why the study of mosaic structure is of importance. The mosaic structure is mostly anisotropic, which results in different outcomes when using these crystals with equal Bragg angles with respect to the direction set by the averaged reciprocal lattice vector.

It is well known that, while a fast particle traverses a crystal, the parametric X-ray radiation depending on the crystal structure is generated. By now parametric X-ray radiation, generated due to coherent scattering of the Coulomb field of a moving charged particle by the atomic planes of a crystal, has been much studied both theoretically and experimentally [1–6]. The PXR angular spectrum characteristics are very sensitive to the position of a reciprocal lattice vector with respect to the trajectory of a radiating particle; this allows one to suppose the existence of a dependence of radiation characteristics on the crystal mosaic structure.

## THEORETICAL ANALYSIS

Consider the radiation generated in a crystal during rectilinear motion of a relativistic electron in the geometry presented in Fig. 1, where **V** is the radiating electron velocity vector; **g** is the reciprocal lattice vector of a crystallographic reflection plane; $\eta_{\parallel}$ and $\eta_{\perp}$ are the angular variables defining the position of **g** in space; **n** is the direction of a radiated quantum propagation; and $\Theta_{\parallel}$ and $\Theta_{\perp}$ are the angular variables defining the position of **n** in space.

To describe PXR properties in a mosaic crystal we use the result obtained in the kinematical approximation of the theory of X-ray diffraction in crystals [1, 4]. The PXR angular spectral distribution for the fixed

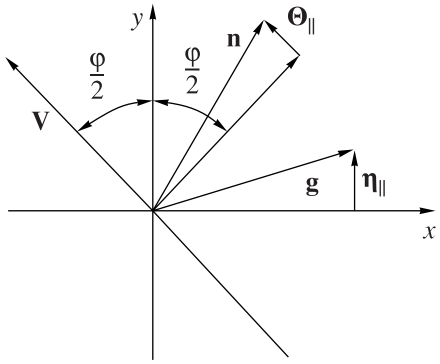

**Fig. 1.** Geometry of radiation process at the rectilinear motion of a relativistic electron.

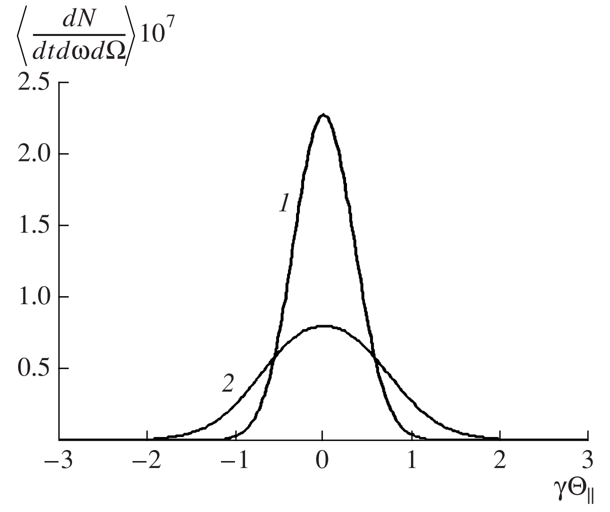

**Fig. 2.** PXR angular spectral distributions at a fixed radiation energy: (*1*) $\sigma_\parallel = 0.5$ and $\sigma_\perp = 1$, (*2*) $\sigma_\parallel = 1$ and $\sigma_\perp = 0.5$; $\gamma = 100$; $\omega = \omega_B = 5$ keV; $\varphi/2 = \pi/3$, $\Theta_\perp = 0$.

reciprocal lattice vector may be presented in the considered case by the following form:

$$\frac{dN}{dtd\omega d\Omega} = \frac{e^2}{\pi}\frac{\omega_{\mathbf{g}}^4}{\omega g^2}F(\omega, \varphi, \Theta_\parallel, \Theta_\perp, \eta_\parallel, \eta_\perp)\delta(\omega - \omega'_B),$$

$$F(\omega, \Theta_\parallel, \Theta_\perp, \eta_\parallel, \eta_\perp)$$

$$= \frac{(\Theta_\parallel - 2\eta_\parallel)^2\cos^2(\varphi) + (\Theta_\perp - 2\eta_\perp\sin(\varphi/2))^2}{\left(\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + (\Theta_\parallel - 2\eta_\parallel)^2 + (\Theta_\perp - 2\eta_\perp\sin(\varphi/2))^2\right)^2}, \quad (1)$$

$$\omega'_B = \omega_B(1 + (\Theta_\parallel - \eta_\parallel)\cot(\varphi/2)), \quad \omega_B = \frac{g}{2\sin(\varphi/2)},$$

$$\omega_{\mathbf{g}}^2 = \omega_0^2 F(g)S(\mathbf{g})\exp\left(-\frac{1}{2}g^2u_T^2\right).$$

Here, $\omega_0$ is the plasma frequency; $\gamma$ is the Lorentz factor of a radiating electron; $F(g)$ is the atomic form factor normalized to a number of electrons in an atom; and $S(\mathbf{g})$ is the structural factor normalized to a number of atoms in a unit cell (in what follows $F(g)S(\mathbf{g})\exp(-g^2u_T^2/2) = 1$). It is important that the energy of a radiated quantum $\omega'_B$ depends only on the longitudinal component of the mosaic structure. This dependence allows study of the mosaic structure in the reaction plane only without the effect of the second component of a crystal plane misorientation on the radiation spectrum. No less important a fact is the increase in the mosaic structure effect in the reaction plane on the PXR spectrum with a decrease in the angle of incidence of a particle onto the plane ($\cot(\varphi) \longrightarrow \infty$ at $\varphi \longrightarrow 0$).

The expression for the PXR angular spectral distribution in the mosaic crystal can be obtained by the averaging Eq. (1) over possible orientations of the reciprocal lattice vector. We use the conventional Gaussian distribution as the distribution function

$$\left\langle\frac{dB}{dtd\omega d\Omega}\right\rangle = \frac{e^2}{\pi^2}\frac{\omega_{\mathbf{g}}^4}{\pi g^2}\frac{1}{\sigma_\parallel\sigma_\perp}$$

$$\times \int_{-\infty}^{\infty} d\eta_\parallel d\eta_\perp \exp(-\eta_\parallel^2/\sigma_\parallel^2)\exp(-\eta_\perp^2/\sigma_\perp^2) \quad (2)$$

$$\times F(\omega, \varphi, \Theta_\parallel, \Theta_\perp, \eta_\perp)\delta(\omega - \omega'_B),$$

where the coefficients $\sigma_\parallel$ and $\sigma_\perp$ specify the magnitude of plane misorientation in the corresponding directions. One can see that $\eta_\perp$ enters the function $F(\omega, \varphi, \Theta_\parallel, \Theta_\perp, \eta_\parallel, \eta_\perp)$ only multiplied by $\sin(\varphi/2)$, which weakens the dependence of the PXR angular spectral distribution in the mosaic crystal on the crystal plane misorientation in the transverse plane with a decrease in the angle of incidence of a radiating particle onto the considered plane. On the other hand, the dependence of a spectrum on the longitudinal component of misorientation increases with a decrease in the angle of incidence. Thus, it is preferable to use small angles of incidence of the radiating particle onto the considered plane to study the anisotropy of a crystal mosaic structure.

The most convenient way for the investigation of anisotropy of mosaic structure is the circuit design using the measurement of the PXR structure when a crystal rotates about the normal to the investigated crystal plane. In this case the angular spectrum characteristics would depend on the ratio of $\sigma_\parallel$ to $\sigma_\perp$. The plots of the PXR angular spectral distribution for the fixed value of quantum energy and two different angles of incidence of a radiating particle onto the investigated plane are shown in Figs. 2 and 3. These curves correspond to different mosaic structure plane orientations differing in the rotation angle by 90 degrees about the normal. One can see a strong dependence of the angular distribution on the angle of incidence of a particle onto the plane.

As a practical matter, the spectral distribution of collimated radiation is most important. It is precisely this that is usually measured in experiment. We restrict the consideration to the PXR angular spectral distribution as a function of quantum energy with a fixed observation angle that corresponds to the spectral distribution of radiation into the pinhole collimator with a detector having an infinite resolution. The sensitivity of the PXR spectral distribution to the anisotropy of the mosaic structure is the most important characteristic. The PXR angular spectral distributions as a function of radiation energy with the fixed observation angle are plotted in Figs. 4–6. The curves of each figure correspond to the

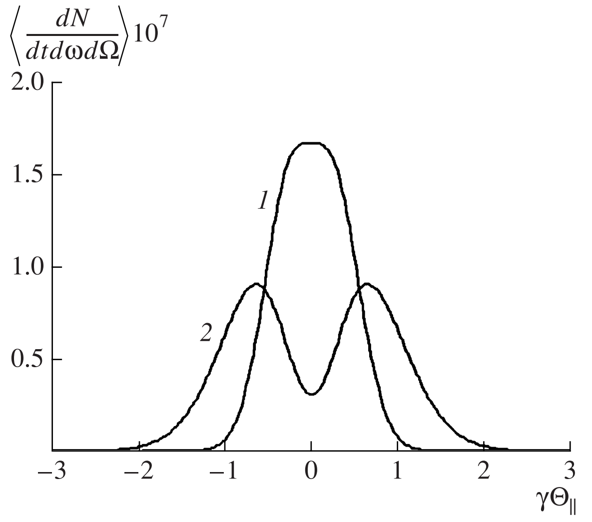

**Fig. 3.** PXR angular spectral distributions at a fixed radiation energy: (*1*) $\sigma_{\parallel} = 0.5$ and $\sigma_{\perp} = 1$, (*2*) $\sigma_{\parallel} = 1$, $\sigma_{\perp} = 0.5$. $\gamma = 100$, $\omega = \omega_B = 5$ keV; $\varphi/2 = \pi/10$, $\Theta_{\perp} = 0$.

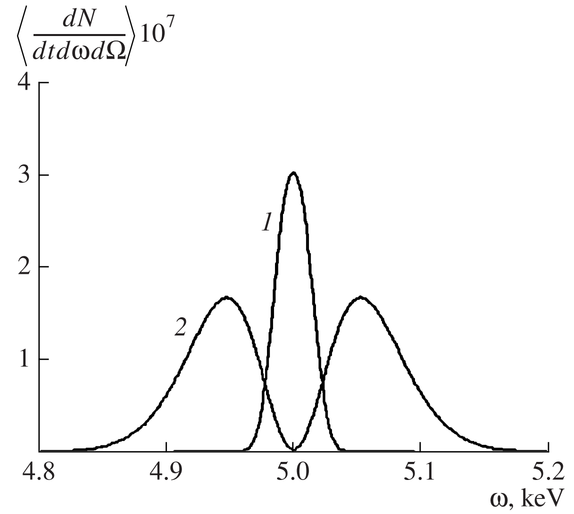

**Fig. 4.** PXR angular spectral distributions at a fixed observation angle: (*1*) $\sigma_{\parallel} = 0.1$ and $\sigma_{\perp} = 0.5$, (*2*) $\sigma_{\parallel} = 0.5$, $\sigma_{\perp} = 0.1$. $\gamma = 100$, $\omega = \omega_B = 5$ keV; $\Theta_{\parallel} = \Theta_{\perp} = 0$, $\varphi/2 = \pi/10$.

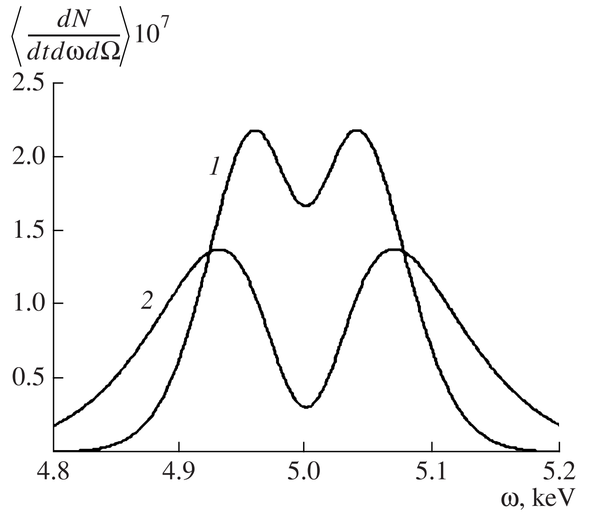

**Fig. 5.** PXR angular spectral distributions at a fixed observation angle: (*1*) $\sigma_{\parallel} = 0.5$ and $\sigma_{\perp} = 1$, (*2*) $\sigma_{\parallel} = 1$, $\sigma_{\perp} = 0.5$. $\gamma = 100$, $\omega = \omega_B = 5$ keV; $\Theta_{\parallel} = \Theta_{\perp} = 0$, $\varphi/2 = \pi/10$.

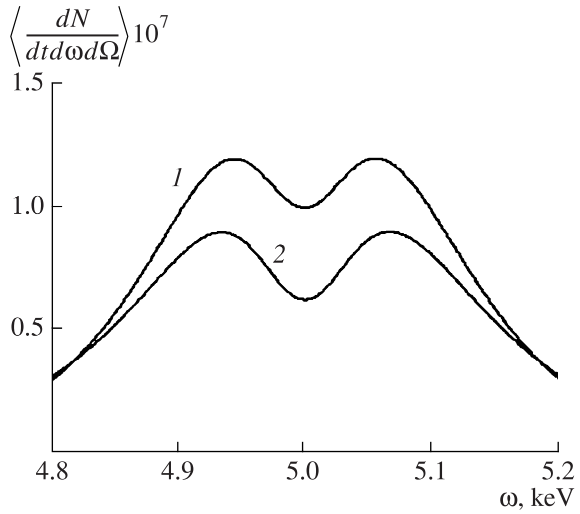

**Fig. 6.** PXR angular spectral distributions at a fixed observation angle: (*1*) $\sigma_{\parallel} = 1.5$ and $\sigma_{\perp} = 2$, (*2*) $\sigma_{\parallel} = 2$, $\sigma_{\perp} = 1.5$. $\gamma = 100$, $\omega = \omega_B = 5$ keV; $\Theta_{\parallel} = \Theta_{\perp} = 0$, $\varphi/2 = \pi/10$.

different orientations of plane mosaic structure differing by the rotation angle by 90 degrees about the normal. The curves demonstrate the strong dependence of the radiation spectrum on the anisotropy of the mosaic structure, which is most pronounced at small values of the coefficients $\sigma_{\parallel}$ and $\sigma_{\perp}$.

## CONCLUSIONS

The calculations performed demonstrate a strong dependence of PXR characteristics on the anisotropy of the crystal mosaic structure. The advantage of measurements at small angles of incidence of the radiating particle onto a target is shown. This is related to a decrease in sensitivity of radiation characteristics to a mosaic structure in the transverse direction and to a simultaneous increase in sensitivity to the misorientation of blocks in the longitudinal plane. The PXR angular spectrum nature is mostly determined by the longitudinal component of mosaic structure, which allows study of the anisotropy of mosaic structure by a simple rotation of the crystal about the normal to the investigated crystal plane.

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