Coherent X-radiation along the velocity of a relativistic electron in a bounded periodic multilayer medium

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Abstract. In the present work a theory of the coherent X-radiation generated by the relativistic electron crossing an artificial periodic layered structure in Laue scattering geometry is built. The expressions are derived for spectral-angular distribution of the radiation in direction of the particle velocity. The comparative analysis of the spectral-angular density of the radiation is carried out for an artificial periodic structure and a single crystal in similar conditions. It is shown, that the spectral peak of the parametric X-radiation in the artificial periodic layered structure in the forward direction (FPXR) will be considerably wider than the one in a single crystal target which can make its experimental detection and investigation appreciably easier.

1. Introduction

Traditionally the radiation of the relativistic particle in periodical multilayer structure is considered only in the Bragg scattering geometry for the case, when the reflecting layers are situated across the target entrance surface, i.e. for the case of symmetric reflection. This radiation is usually considered as the resonance transition radiation [1-2]. In the work [3] this radiation was represented as a sum of the diffracted transition radiation (DTR) and parametric X-radiation (PXR). Recently in our work [4] a theory of coherent X-radiation in Bragg direction generated by a relativistic electron crossing the artificial periodic layered structure has been constructed in Laue scattering geometry for general asymmetry of the electron coulomb field reflection, when the reflecting layered structure is situated at any angle to target surface.

The dynamic theory of relativistic particle in the crystal predicts the radiation not only in Bragg scattering direction, but also in the direction along of incident particle velocity [5-7] (FPXR). This radiation is a result of the manifestation of dynamic diffraction in PXR. The detailed theoretical decryption of the dynamical effect of FPXR together with accompanying background of transition radiation in the case of symmetric reflection was given in works [8-10]. Later, in our woks [11-12] the coherent X-radiation along the velocity of the relativistic electron crossing a crystal was considered both in the Laue geometry and in the Bragg geometry in the general case of asymmetric reflection, i.e. in the case when the atomic planes in the crystal can be situated at any angular δ to the target entrance surface.

In the present wok the dynamic theory of coherent radiation generated by relativistic electron crossing artificial periodic structure bounded by two parallel planes is constructed in Laue scattering geometry for radiation directed along the particle velocity (FPXR). Based on the two-wave approach of dynamic theory of diffraction the expressions of spectral-angular density of the radiation are derived. The calculated values of the spectral-angular densities of the FPXR in an artificial periodic structure and in a single crystal target are compared in similar radiation conditions.

2. Radiation amplitude

Let us consider the radiation of a relativistic electron crossing with the velocity $\mathbf{V}$ a multilayer structure consisted of the periodically situated amorphous layers with thicknesses a and b ($T = a + b$ – period of the structure) of dielectric susceptibility χ_a and χ_b, respectively.

To derive the expressions describing the spectral and angular characteristics FPXR and transition radiation (TR), we used the notation and techniques similar to those shown in [13].

The solution of the combined equations (3) (see [13]) gives us the relativistic particle field in the periodic structure in such a form:

$$E_0^{(s)medium} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{-\omega^2\beta - 2\omega\frac{\gamma_g}{\gamma_0}\lambda_0}{4\frac{\gamma_g}{\gamma_0}\left(\lambda_0 - \lambda^{(1)}\right)\left(\lambda_0 - \lambda^{(2)}\right)} \delta\left(\lambda_0 - \lambda_0^*\right) + E_0^{(s)(1)}\delta\left(\lambda_0 - \lambda^{(1)}\right) + E_0^{(s)(2)}\delta\left(\lambda_0 - \lambda^{(2)}\right), \quad (1)$$

where $\lambda_0^* = \omega\left(\frac{\gamma^{-2} + \theta^2 - \chi_0}{2}\right)$, $\gamma = 1/\sqrt{1-V^2}$ is Lorentz factor of the electron, $E_0^{(s)(1)}$, $E_0^{(s)(2)}$ are free incident fields in the considered medium.

The solution of the combined equations (see [13]) gives us the relativistic particle field in the vacuum in front of the radiator as

$$E_0^{(s)vacI} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{1}{-\chi_0 - \frac{2}{\omega}\lambda_0} \delta\left(\lambda_0 - \lambda_0^*\right). \quad (2)$$

The expression for the field in the vacuum behind the target we write as

$$E_0^{(s)vacII} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{1}{-\chi_0 - \frac{2\lambda_0}{\omega}} \delta(\lambda_0 - \lambda_0^*) + E_0^{(s)Rad}\delta\left(\lambda_0 + \frac{\omega\chi_0}{2}\right), \quad (3)$$

where $E_0^{(s)Rad}$ is the amplitude of coherent field along the electron velocity.

The expression connecting the incident and diffracted fields in the layered periodic medium follows from the second equation of the system (3):

$$E_0^{(s)medium} = \frac{2\omega\lambda_{\mathbf{g}}}{\omega^2\chi_{\mathbf{g}}C^{(s)}} E_{\mathbf{g}}^{(s)medium}. \quad (4)$$

Using the standard boundary conditions at entrance and exit surfaces of the target

$$\int E_0^{(s)vacI} d\lambda_0 = \int E_0^{(s)medium} d\lambda_0, \quad \int E_{\mathbf{g}}^{(s)medium} d\lambda_0 = 0,$$

$$\int E_0^{(s)medium} e^{i\frac{\lambda_0}{\gamma_0}L} d\lambda_0 = \int E_0^{(s)vacII} e^{i\frac{\lambda_0}{\gamma_0}L} d\lambda_0, \quad (5)$$

we obtain the expression for the radiation field amplitude:

$$E_{Rad}^{(s)} = -\frac{4\pi^2 ieV\theta P^{(s)}}{\omega} e^{i\frac{\lambda_0^* + \frac{\omega\chi_0}{2}}{\gamma_0}L} \left[\left(1 - \frac{\beta}{\sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}\right)\left(\frac{\omega}{-\omega\chi_0 - 2\lambda_0^*} + \frac{\omega}{2(\lambda_0^* - \lambda_0^{(2)})}\right)\left(1 - e^{i\frac{\lambda_0^{(2)} - \lambda_0^*}{\gamma_0}L}\right)\right.$$

$$+\left(1+\frac{\beta}{\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}\right)\left(\frac{\omega}{-\omega\chi_0-2\lambda_0^*}+\frac{\omega}{2(\lambda_0^*-\lambda_0^{(1)})}\right)\left(1-e^{i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L}\right)\Bigg] \quad (6)$$

The formula (6) allows the description of spectral and angular characteristics of the radiation. Prior to the beginning of the spectral-angular characteristics analysis it is necessary to note that three mechanisms make a contribution to the total yield of the relativistic electron coherent radiation in multilayer periodic structure in the forward direction: bremsstrahlung, transition radiation and parametric radiation (FPXR).

The radiation amplitude $E_{Rad}^{(s)}$ contains the contributions of the radiation mechanisms analogue to FPXR and TR in the single crystal target. So far as the transition radiation is the main underground noise in experimental studying of FPXR, let us represent $E_{Rad}^{(s)}$ as a sum of FPXR and TR amplitudes. Such a presentation allows the estimation of the FPXR and TR contributions to the total radiation yield.

So we write the expression for the radiation field (6) as

$$E_0^{(s)Rad}=E_0^{(s)FPXR}+E_0^{(s)TR}, \quad (7)$$

$$E_0^{(s)FPXR}=\frac{4\pi^2 ieV\theta P^{(s)}}{\omega}e^{i\frac{\lambda_0^*+\frac{\omega\chi_0}{2}}{\gamma_0}L}\frac{\omega^2\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}}{2\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}\frac{1}{\lambda_0^*}\left[\frac{1-e^{i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}}}{\lambda_0^*-\lambda_0^{(2)}}-\frac{1-e^{i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}}}{\lambda_0^*-\lambda_0^{(1)}}\right] \quad (8)$$

$$E_0^{(s)TR}=\frac{4\pi^2 ieV\theta P^{(s)}}{\omega}e^{i\frac{\lambda_0^*+\frac{\omega\chi_0}{2}}{\gamma_0}L}\left(\frac{\omega}{\omega\chi_0+2\lambda_0^*}-\frac{\omega}{2\lambda_0^*}\right)$$

$$\times\left[\left(1-\frac{\beta}{\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}\right)\left(1-e^{i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L}\right)+\left(1+\frac{\beta}{\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}\right)\left(1-e^{i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L}\right)\right]. \quad (9)$$

The dynamical corrections corresponding to the two branches of dispersion relation depend not only on photon frequency and target properties, but also on ε. Thus, the dispersion of real photons in the crystal depends on the asymmetry:

$$k^{(1,2)}=\omega\sqrt{1+\chi_0}+\lambda_0^{(1,2)}(\omega,\varepsilon). \quad (10)$$

The dispersion of virtual photons associated with the Coulomb field is described by the formula

$$k^*=\omega\sqrt{1+\chi_0}+\lambda_0^*=\omega+\frac{\omega}{2}\left(\theta^2+\gamma^{-2}\right). \quad (11)$$

A FPXR reflex can be observed only if at least one denominator in the bracketed expression in (7b) vanishes:

$$\mathrm{Re}\left(\lambda_0^*-\lambda_0^{(1)}\right)=0, \qquad \mathrm{Re}\left(\lambda_0^*-\lambda_0^{(2)}\right)=0. \quad (12)$$

Substituting (8) into (7b) and (7c), we represent the FPXR and TR field amplitudes as

$$E_0^{(s)FPXR} = \frac{4\pi^2 ieV\theta P^{(s)}}{\omega} \frac{e^{i\frac{\omega(\gamma^{-2}+\theta^2)}{2\gamma_0}L}}{\gamma^{-2}+\theta^2-\chi_0} \frac{1}{\sqrt{\xi^{(s)2}+\varepsilon}} \times$$

$$\times \left(\frac{1-\exp(-ib^{(s)}\Omega^{(2)}(\omega)-b^{(s)}\rho^{(s)}\Delta^{(2)})}{\Omega^{(2)}(\omega)-i\rho^{(s)}\Delta^{(2)}} - \frac{1-\exp\left(-ib^{(s)}\Omega^{(1)}(\omega)-b^{(s)}\rho^{(s)}\Delta^{(1)}\right)}{\Omega^{(1)}(\omega)-i\rho^{(s)}\Delta^{(1)}} \right), \quad (13a)$$

$$E_0^{(s)TR} = \frac{4\pi^2 ieV\theta P^{(s)}}{\omega} e^{i\frac{\omega(\gamma^{-2}+\theta^2)}{2\gamma_0}L} \left(\frac{1}{\gamma^{-2}+\theta^2} - \frac{1}{\gamma^{-2}+\theta^2-\chi_0} \right) \times$$

$$\left(\left(1-\xi^{(s)}/\sqrt{\xi^{(s)^2}+\varepsilon} \right) \left(1-\exp\left(-ib^{(s)}\Omega^{(2)}(\omega)-b^{(s)}\rho^{(s)}\Delta^{(2)}\right)\right) + \right.$$

$$\left. + \left(1+\xi^{(s)}/\sqrt{\xi^{(s)^2}+\varepsilon} \right) \left(1-\exp(-ib^{(s)}\Omega^{(1)}(\omega)-b^{(s)}\rho^{(s)}\Delta^{(1)}) \right) \right), \quad (13b)$$

where

$$\Delta^{(1)} = \frac{\varepsilon+1}{2\varepsilon} - \frac{1-\varepsilon}{2\varepsilon} \frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}} - \frac{\kappa^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}, \quad \Delta^{(2)} = \frac{\varepsilon+1}{2\varepsilon} + \frac{1-\varepsilon}{2\varepsilon} \frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}} + \frac{\kappa^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}},$$

$$\sigma^{(s)} = \frac{1}{\nu^{(s)}} \left(\frac{\theta^2}{|\chi'_0|} + \frac{1}{\gamma^2|\chi'_0|} + 1 \right), \quad b^{(s)} = \frac{\omega|\chi'_{\mathbf{g}}|C^{(s)}}{2} \frac{L}{\gamma_0}, \quad \Omega^{(1,2)}(\omega) = \sigma^{(s)} + \left(\xi(\omega) \mp \sqrt{\xi(\omega)^2+\varepsilon} \right)/\varepsilon . \quad (14)$$

The parameter $b^{(s)}$ can be represented as

$$b^{(s)} = \frac{1}{2\sin(\delta-\theta_B)} \frac{L}{L_{ext}^{(s)}}, \quad (15)$$

i.e., $b^{(s)}$ is half the distance traveled by an electron in the plate, measured in units of extinction length

$$L_{ext}^{(s)} = \frac{1}{2C^{(s)}\omega} \frac{gT}{\left|\sin\left(\frac{ga}{2}\right)\right| |\chi'_b - \chi'_a|}. \quad (16)$$

Accordingly, the equation

$$\sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)2}+\varepsilon}}{\varepsilon} = 0, \quad (17)$$

determines the center frequency ω_* of the spectrum of FPXR photons emitted at a particular observation angle.

3. Spectral-angular density of the radiation

Substituting (7a), (13a), and (13b) into a well known expression for the spectral-angular density of X-ray radiation [14],

$$\omega \frac{d^2N}{d\omega d\Omega} = \omega^2 (2\pi)^{-6} \sum_{s=1}^{2} \left| E_0^{(s)Rad} \right|^2, \quad (18)$$

we find the contributions to the spectral-angular density due to FPXR, TR, and their interference:

$$\omega \frac{d^2N_{FPXR}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} P^{(s)^2} \frac{\theta^2}{(\theta^2+\gamma^{-2}-\chi'_0)^2} R_{FPXR}^{(s)}, \quad (19a)$$

$$R^{(s)}_{FPXR} = \frac{1}{\xi^{(s)2}+\varepsilon}\,\frac{1+e^{-2b^{(s)}\rho^{(s)}\Delta^{(1)}} - 2e^{-b^{(s)}\rho^{(s)}\Delta^{(1)}}\cos\left(b^{(s)}\Omega^{(1)}(\omega)\right)}{\Omega^{(1)}(\omega)^2+\left(\rho^{(s)}\Delta^{(1)}\right)^2}, \tag{19b}$$

$$\omega\frac{d^2N^{(s)}_{TR}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}P^{(s)^2}\theta^2\left(\frac{1}{\theta^2+\gamma^{-2}} - \frac{1}{\theta^2+\gamma^{-2}-\chi'_0}\right)^2 R^{(s)}_{TR}, \tag{20a}$$

$$R^{(s)}_{TR} = \left(1-\xi^{(s)}/\sqrt{\xi^{(s)^2}+\varepsilon}\right)^2\left(1+e^{-2b^{(s)}\rho^{(s)}\Delta^{(2)}} - 2e^{-b^{(s)}\rho^{(s)}\Delta^{(2)}}\cos\left(b^{(s)}\Omega^{(2)}(\omega)\right)\right)$$
$$+\left(1+\xi^{(s)}/\sqrt{\xi^{(s)^2}+\varepsilon}\right)^2\left(1+e^{-2b^{(s)}\rho^{(s)}\Delta^{(1)}} - 2e^{-b^{(s)}\rho^{(s)}\Delta^{(1)}}\cos\left(b^{(s)}\Omega^{(1)}(\omega)\right)\right)$$
$$+\frac{2\varepsilon}{\xi^{(s)^2}+\varepsilon}\left(1+e^{-b^{(s)}\rho^{(s)}\frac{\varepsilon+1}{\varepsilon}}\cdot\cos\left(\frac{2b^{(s)}\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right) - e^{-b^{(s)}\rho^{(s)}\Delta^{(2)}}\cos\left(b^{(s)}\Omega^{(1)}(\omega)\right) - e^{-b^{(s)}\rho^{(s)}\Delta^{(1)}}\cos\left(b^{(s)}\Omega^{(1)}(\omega)\right)\right), \tag{20b}$$

$$\omega\frac{d^2N^{(s)}_{INT}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}P^{(s)^2}\theta^2\left(\frac{1}{\theta^2+\gamma^{-2}} - \frac{1}{\theta^2+\gamma^{-2}-\chi'_0}\right)\frac{1}{\theta^2+\gamma^{-2}-\chi'_0}R^{(s)}_{INT}, \tag{21a}$$

$$R^{(s)}_{INT} = \frac{-2}{\sqrt{\xi^{(s)2}+\varepsilon}}\mathrm{Re}\left(\frac{1-e^{-ib^{(s)}\Omega^{(1)}(\omega)-b^{(s)}\rho^{(s)}\Delta^{(1)}}}{\Omega^{(1)}(\omega)-i\rho^{(s)}\Delta^{(1)}}\right.$$
$$\left.\times\left(\left(1-\xi^{(s)}/\sqrt{\xi^{(s)^2}+\varepsilon}\right)\left(1-e^{ib^{(s)}\Omega^{(2)}(\omega)-b^{(s)}\rho^{(s)}\Delta^{(2)}}\right)+\left(1+\xi^{(s)}/\sqrt{\xi^{(s)^2}+\varepsilon}\right)\left(1-e^{ib^{(s)}\Omega^{(1)}(\omega)-b^{(s)}\rho^{(s)}\Delta^{(1)}}\right)\right)\right). \tag{21b}$$

Expressions (19)–(21) are the main result of this study. They are obtained in the two-beam approximation of dynamical diffraction theory with taking into account the radiation absorption for an arbitrary orientation of the diffracting planes with respect to the surface of an absorbing crystal. Using the above formulas we will analyze the dynamic effect of reflection asymmetry on spectral-angular characteristics of the radiation.

4. Dynamic scattering parameters of the X-waves

As it was mentioned above, the X-ray scattering in artificial periodic structure is studied traditionally in the Bragg scattering geometry in symmetric reflection case, when the layers of the structure are situated along its entrance surface, while the radiation in crystal target is investigated mainly in the Laue scattering geometry because of well known conditions of the experiment.

The obtained expressions (19–21) allow studying the dependence of radiation characteristics on the values of layer thicknesses a and b for different amorphous substances (with the corresponding dielectric susceptibility χ_a and χ_b) and on the parameter of asymmetry ε, defining the angle between the reflecting layers and the target surface.

The parameter $\nu^{(s)}$ possessing the value $0 \le \nu^{(s)} \le 1$ characterizes the reflecting power of a set of the atomic planes as determined by the interference between waves scattered by atoms in different layers ($\nu^{(s)} \approx 1$ and 0 when the interference is constructive and destructive, respectively). Dynamical diffraction phenomena, such as FPXR, can be observed only when the interference is constructive. Under $g = \frac{2\pi}{T}$ it can be presented as

$$\nu^{(s)} = \frac{2C^{(s)}\left|\sin\left(\frac{\pi a}{T}\right)\right|}{g}\left|\frac{\chi'_b-\chi'_a}{a\chi'_a+b\chi'_b}\right|. \tag{22}$$

The parameter $\rho^{(s)} = \dfrac{L_{ext}^{(s)}}{L_{abs}}$ is the ratio between the extinction length $L_{ext}^{(s)}$ (16) and the absorption length $L_{abs} = \dfrac{T}{\omega\left|a\chi_a'' + b\chi_b''\right|}$ for X-rays in the layered periodic structure. Note that the energy of the primary wave is completely transferred to the secondary wave propagating in the Bragg direction at the depth equal to the extinction length.

The parameter $\kappa^{(s)}$ quantifies the anomalous transmission effect (Borrmann effect) for X-rays passing through an artificial multilayer periodic structure which is well known in the physics of X-ray scattering in crystal [15]. The Borrmann effect manifestation in a crystal for different asymmetry of scattering was studied in [16]. The necessary condition of the Borrman effect manifestation both in crystal and artificial periodic structures is $\kappa^{(s)} \approx 1$. Under this condition the value of $\Delta^{(1)}$ in (14)

$$\Delta^{(1)} = \frac{\varepsilon+1}{2\varepsilon} - \frac{1-\varepsilon}{2\varepsilon}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)2}+\varepsilon}} - \frac{\kappa^{(s)}}{\sqrt{\xi^{(s)2}+\varepsilon}}, \tag{23}$$

becomes minimal which leads to minimal value of $\rho^{(s)}\Delta^{(1)}$ characterizing absorption of the radiation.

So the Borrmann effect can manifest itself in the coherent radiation of the relativistic electron in a multilayer artificial periodic structure.

5. Radiation amplitude. Comparision of the spectral-angular distributions

To simplify the formulas and to visualize the results we consider the case, when the layers of the periodic structure have the equal thickness, i.e. $T = 2a = 2b$. In this case the formula (19) for the σ-polarized waves takes on the following form:

$$\omega\frac{d^2N_{FPXR}^{(1)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}\frac{\theta_\perp^2}{\left(\theta_\perp^2+\gamma^{-2}+\dfrac{\left|\chi_a'+\chi_b'\right|}{2}\right)^2}R_{FPXR}, \tag{24a}$$

$$R_{FPXR} = \frac{1}{\xi(\omega)^2+\varepsilon}\frac{1+e^{-2b^{(1)}\rho^{(1)}\Delta^{(1)}} - 2e^{-b^{(1)}\rho^{(1)}\Delta^{(1)}}\cos\left(b^{(1)}\Omega^{(1)}(\omega)\right)}{\Omega^{(1)}(\omega)^2+\left(\rho^{(s)}\Delta^{(1)}\right)^2}, \tag{24b}$$

where

$$\Omega^{(1)}(\omega) = \sigma(\theta,\gamma) + \left(\xi(\omega) - \sqrt{\xi(\omega)^2+\varepsilon}\right)/\varepsilon, \quad \sigma(\theta,\gamma) = \frac{\pi}{\left|\chi_b'-\chi_a'\right|}\cdot\left(\theta_\perp^2+\gamma^{-2}+\frac{\left|\chi_a'+\chi_b'\right|}{2}\right)$$

$$\Delta^{(1)} = \frac{\varepsilon+1}{2\varepsilon} - \frac{1-\varepsilon}{2\varepsilon}\frac{\xi}{\sqrt{\xi^2+\varepsilon}} - \frac{\kappa^{(1)}}{\sqrt{\xi^2+\varepsilon}}, \quad \xi(\omega) = \frac{2\pi\sin^2(\theta_B)}{\left|\chi_b'-\chi_a'\right|}\cdot\left(1-\frac{\omega}{\omega_B}\right) + \frac{1-\varepsilon}{2\nu^{(1)}},$$

$$\kappa^{(1)} = \frac{2}{\pi}\cdot\left|\frac{\chi_b''-\chi_a''}{\chi_b''+\chi_a''}\right|, \quad \rho^{(1)} = \frac{\pi}{2}\cdot\left|\frac{\chi_b''+\chi_a''}{\chi_b'-\chi_a'}\right|, \quad \nu^{(1)} = \frac{2}{\pi}\cdot\left|\frac{\chi_b'-\chi_a'}{\chi_b'+\chi_a'}\right|, \quad \theta_\perp = \theta\sin\varphi. \tag{25}$$

To compare the spectral-angular density of the radiation in the artificial periodic and crystal structures in similar conditions let us write the expression of FPXR in the crystal [12] for σ-polarized waves:

$$\omega\frac{d^2N_{FPXR}^{(1)Cr}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}\frac{\theta_\perp^2}{\left(\theta_\perp^2+\gamma^{-2}-\chi_0'\right)^2}R_{FPXR}^{Cr}, \tag{26a}$$

$$R_{FPXR}^{Cr} = \frac{1}{\xi(\omega)^2+\varepsilon}\frac{1+e^{-2b^{(1)}\rho^{(1)}\Delta^{(1)}} - 2e^{-b^{(1)}\rho^{(1)}\Delta^{(1)}}\cos\left(b^{(1)}\Omega^{(1)}(\omega)\right)}{\Omega^{(1)}(\omega)^2+\left(\rho^{(s)}\Delta^{(1)}\right)^2}. \tag{26b}$$

Here the notations corresponding to (38) will take on the following view

$$\kappa^{(1)} = \frac{\chi''_g}{\chi''_0}, \quad \rho^{(1)} = \frac{\chi''_0}{|\chi'_g|}, \quad \nu^{(1)} = \frac{\chi'_g}{\chi'_0}, \quad \sigma(\theta,\gamma) = \frac{1}{|\chi'_g|}\cdot\left(\theta_\perp^2 + \gamma^{-2} - \chi'_0\right),$$

$$\xi(\omega) = \frac{2\sin^2(\theta_B)}{|\chi'_g|}\cdot\left(1 - \frac{\omega}{\omega_B}\right) + \frac{1-\varepsilon}{2\nu^{(1)}}. \qquad (27)$$

Using the formulas (26 б) and (24 б) we charted the curves of the spectral-angular density of FPXR ($\omega_B = 8$ кeV) in crystal target of W (see figure 1) and FPXR in multilayer periodic structure composed of amorphous layers of Be and W (see figure 2). The path of the electron in the target $L_e = 56$ μm, asymmetry parameter $\varepsilon = 3$ and observation angle $\theta_\perp = 5$ мрад are chosen as the same for both the cases.

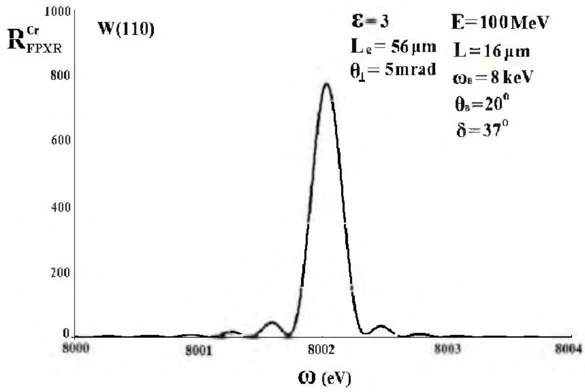

Figure 1. The spectrum of the relativistic electron FPXR in a single crystal (W) medium.

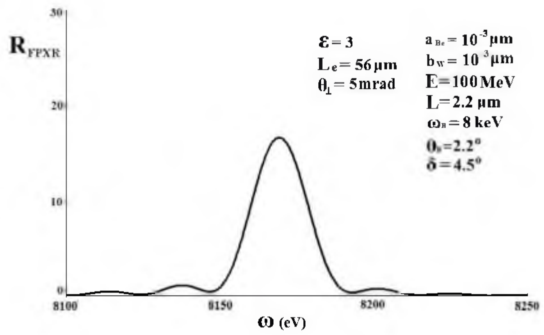

Figure 2. The spectrum of the relativistic electron FPXR in the artificial periodic multilayer (Be-W) medium in the similar figure 3 conditions.

As it followed from figure 1 and figure 2., the spectral amplitude in the crystal is considerably higher than it in a multilayer structure, but the width of the spectral peak of radiation in artificial is much more than in a crystal target because the coherent radiation in this case forms on less number of the medium obstacles. In this connection the experimental investigation of FPXR peak in multilayered periodic structure is interest and easier than in a crystal medium. Let us consider the ratio of the radiation angular densities in these two media. The expressions for angular density of FPXR in artificial periodic structure and crystal medium take on the following view:

$$\frac{dN^{(1)}_{FPXR}}{d\Omega} = \frac{e^2}{4\pi^2}\frac{\theta_\perp^2}{\left(\theta_\perp^2 + \gamma^{-2} + \dfrac{|\chi'_a + \chi'_b|}{2}\right)^2}\int R_{FPXR}\frac{d\omega}{\omega}, \quad \frac{dN^{(1)Cr}_{FPXR}}{d\Omega} = \frac{e^2}{4\pi^2}\frac{\theta_\perp^2}{\left(\theta_\perp^2 + \gamma^{-2} - \chi'_0\right)^2}\int R^{Cr}_{FPXR}\frac{d\omega}{\omega} \qquad (28)$$

The curve in the Figure3 and Figure 4 charted by formulas (28) show that the angular density of FPXR in artificial periodic structure as well as the total photons yield are much more than they are in the crystal medium.

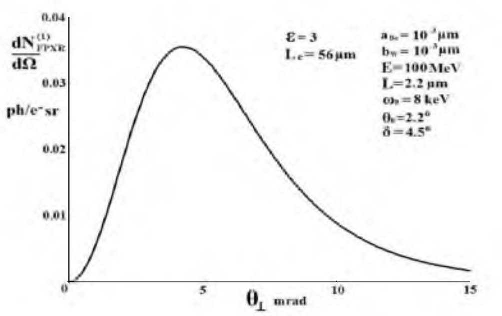

Figure 3. The angular density of relativistic electron FPXR in the artificial periodic multilayer (Be-W) medium

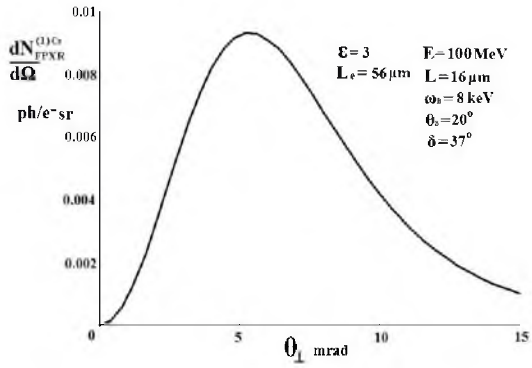

Figure 4. The angular density of the relativistic electron FPXR in the single crystal (W) medium in the similar figure 5 conditions.

6. Conclusion

In the present work the dynamic theory of the coherent X-radiation along the relativistic charged particle in artificial periodic structure are constructed for the Laue scattering geometry and a general case of the asymmetry of the reflection relative to the structure entry surface. In the frame of the two-wave approximation of the diffraction theory the expressions for spectral-angular characteristics of the radiation are obtained based on two different radiation mechanisms: parametric and transition.

It is shown that FPXR spectral peak in artificial periodic structure must be much wider than the analogues peak in the single crystal medium that allows simplifying its experimental investigation. It is shown, that the FPXR angular density in artificial periodic structure expected to be more than an order higher than in a single crystal radiator, under similar conditions.

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