Enhancement of Spectral-Angular Density of Parametric X-rays in Laue Geometry due to Change in the Angle between a Target Surface and Reflecting Atomic Planes

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Abstract—Coherent X-rays of a relativistic electron crossing a single crystal with a uniform velocity in the Laue scattering geometry are considered in the two-wave approximation of dynamic diffraction theory [1]. Analytical expressions for the spectral-angular distribution of parametric X-rays (PXR) and diffracted transition radiation (DTR) have been obtained. The case when the system of diffracting atomic planes of a crystal is located at an arbitrary angle δ to a crystal surface (asymmetric reflection) is considered. The value $\delta = \pi/2$ corresponds to the symmetric reflection in the given scattering geometry. The dependence of the PXR and DTR spectral-angular density on the angle δ has been investigated. It has been shown that the PXR spectrum width depends substantially on the given angle, which, in particular, allows one to increase significantly the PXR angular density by decreasing the angle δ.

INTRODUCTION

When a fast charged particle is crossing a single crystal, its Coulomb field is scattered by the system of parallel atomic planes of a crystal generating parametric X-rays (PXR) [2–4]. When a charged particle is crossing a plate surface, transition radiation (TR) arises [5], which then is diffracted by the system of parallel atomic planes of a crystal forming diffracted transition radiation (DTR) [6]. The scheme of asymmetric reflection for PXR and DTR has been considered in [7, 8] for the case when a charged particle is crossing a semi-infinite crystal in the Bragg scattering geometry. It has been shown that the asymmetry affects substantially the spectral-angular radiation characteristics. For the crystal of a finite thickness, the effect of the angle δ on the PXR and DTR spectral-angular characteristics has been considered in the Bragg geometry in [9, 10].

The effect of asymmetry on the PXR and DTR spectral-angular characteristics in the Laue geometry has been studied in this work. The expressions for the PXR and DTR spectral-angular distribution and for the term describing interference of these two radiation mechanisms were obtained for a general case of asymmetric reflection, when the atomic planes of a crystal form the angle δ with a surface. It has been shown that, at the fixed angle of incidence θ_B of the relativistic electron onto the system of parallel atomic planes of a crystal, a decrease in the angle δ results in the broadening of the PXR spectrum and consequently in the increase in the integral in frequency radiation yield. It should be noted that this effect is not related to the radiation absorption in a crystal. It has been shown that the DTR angular density also depends substantially on the reflection asymmetry, and the DTR and PXR interference could result in the angular density oscillations of the total radiation.

SPECTRAL-ANGULAR RADIATION DISTRIBUTION

Consider the radiation of a fast charged particle crossing a single crystal plate with a uniform velocity $\mathbf{V}$ (Fig. 1). When solving the task, we will consider the equation of Fourier transform of electromagnetic field

$$\mathbf{E}(\mathbf{k}, \omega) = \int dt d^3\mathbf{r}\mathbf{E}(\mathbf{r}, t)\exp(i\omega t - i\mathbf{k}\mathbf{r}). \qquad (1)$$

Since a relativistic particle field may be considered with good accuracy to be transverse, the incident $\mathbf{E}_0(\mathbf{k}, \omega)$ and diffracted $\mathbf{E}_\mathbf{g}(\mathbf{k}, \omega)$ electromagnetic waves are determined by two amplitudes with different values of transverse polarization

$$\begin{aligned} \mathbf{E}_0(\mathbf{k}, \omega) &= E_0^{(1)}(\mathbf{k}, \omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k}, \omega)\mathbf{e}_0^{(2)}, \\ \mathbf{E}_\mathbf{g}(\mathbf{k}, \omega) &= E_\mathbf{g}^{(1)}(\mathbf{k}, \omega)\mathbf{e}_1^{(1)} + E_\mathbf{g}^{(2)}(\mathbf{k}, \omega)\mathbf{e}_1^{(2)}. \end{aligned} \qquad (2)$$

The unit polarization vectors $\mathbf{e}_0^{(1)}$, $\mathbf{e}_0^{(2)}$, $\mathbf{e}_1^{(1)}$, and $\mathbf{e}_1^{(2)}$ are selected as follows. The vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to the vector $\mathbf{k}$ and the vectors $\mathbf{e}_1^{(1)}$

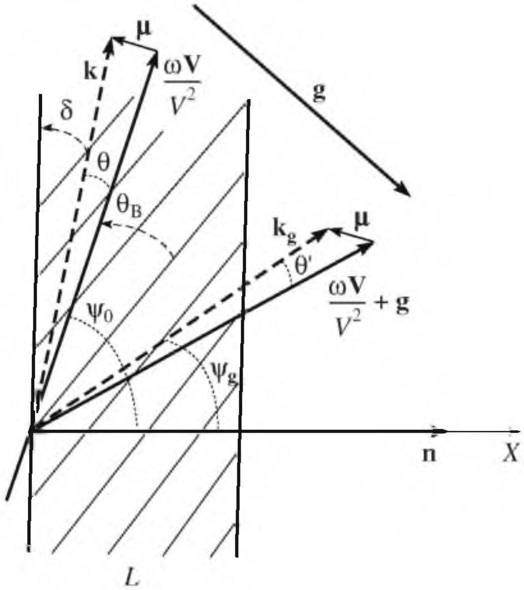

Fig. 1. Geometry of radiation process: θ' is the radiation angle; θ_B is the Bragg angle; δ is the angle between a surface and the considered atomic planes of a crystal; and **k** and $\mathbf{k_g}$ are the wave vectors of incident and diffracted photon.

and $\mathbf{e}_1^{(2)}$ are perpendicular to the vector $\mathbf{k_g} = \mathbf{k} + \mathbf{g}$. The vectors $\mathbf{e}_0^{(2)}$ and $\mathbf{e}_1^{(2)}$ lie in the plane of the vectors **k** and $\mathbf{k_g}$ (π polarization) and the vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_1^{(1)}$ are normal to it (σ polarization); **g** is the reciprocal lattice vector corresponding to the system of reflecting atomic planes of a crystal. The system of equations for Fourier transform of electromagnetic field has the following form in the two-wave approximation of dynamic diffraction theory [11]:

$$\begin{cases} (\omega^2(1+\chi_0)-k^2)E_0^{(s)} + \omega^2\chi_{-\mathbf{g}}C^{(s)}E_{\mathbf{g}}^{(s)} \\ = 8\pi^2 ie\omega\theta VP^{(s)}\delta(\omega-\mathbf{kV}), \\ \omega^2\chi_{\mathbf{g}}C^{(s)}E_0^{(s)} + (\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)E_{\mathbf{g}}^{(s)} = 0, \end{cases} \tag{3}$$

where $\chi_{\mathbf{g}}$ and $\chi_{-\mathbf{g}}$ are the coefficients of Fourier expansion of dielectric susceptibility in terms of the reciprocal lattice vectors **g**

$$\chi(\omega, \mathbf{r}) = \sum_{\mathbf{g}}\chi_{\mathbf{g}}(\omega)\exp(i\mathbf{gr}) = \sum_{\mathbf{g}}(\chi'_{\mathbf{g}}(\omega) + i\chi''_{\mathbf{g}}(\omega))\exp i\mathbf{gr}. \tag{4}$$

We will consider a crystal with the symmetry $\chi_{\mathbf{g}} = \chi_{-\mathbf{g}}$, $\chi_{\mathbf{g}}$ is determined by the expression

$$\chi_{\mathbf{g}} = \chi_0(F(g)/Z)(S(\mathbf{g})/N_0)\exp\left(-\frac{1}{2}g^2u_\tau^2\right), \tag{5}$$

where $\chi_0 = \chi'_0 + i\chi''_0$ is the average dielectric susceptibility; $F(g)$ is the form factor of an atom having Z electrons; $S(\mathbf{g})$ is the structural factor of a unit cell containing N_0 atoms; and u_τ is the rms amplitude of temperature oscillations of crystal atoms. In this work the X-ray frequency range ($\chi'_{\mathbf{g}} < 0$, $\chi'_0 < 0$) is considered.

The $C^{(s)}$ and $P^{(s)}$ magnitudes are defined in system of equations (3) in the following way:

$$\begin{aligned} C^{(s)} &= \mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)}, \quad C^{(1)} = 1, \quad C^{(2)} = \cos 2\theta_B, \\ P^{(s)} &= \mathbf{e}_0^{(s)}(\boldsymbol{\mu}/\mu), \quad P^{(1)} = \sin\varphi, \quad P^{(2)} = \cos\varphi, \end{aligned} \tag{6}$$

where $\boldsymbol{\mu} = \mathbf{k} - \omega\mathbf{V}/V^2$ is the component of virtual photon momentum, normal to the particle velocity **V** ($\mu = \omega\theta/V$, $\theta \ll 1$ is the angle between the vectors **k** and **V**); θ_B is the angle between the electron velocity and the system of crystallographic planes (Bragg angle); and φ is the azimuthal angle of radiation counted off from the plane formed by the vectors **V** and **g**. The reciprocal lattice vector is determined by the expression $g = 2\omega_B\sin\theta_B/V$, where ω_B is the Bragg frequency. The angle between the vector $\frac{\omega\mathbf{V}}{V^2} + \mathbf{g}$ and the wave vector of diffracted wave $\mathbf{k_g}$ is denoted by θ'. System of equations (3) describes the σ polarized fields at $s = 1$ and the π polarized fields at $s = 2$.

We solve the dispersion equation following from system of equations (3)

$$(\omega^2(1+\chi_0)-k^2)(\omega^2(1+\chi_0)-k_{\mathbf{g}}^2) - \omega^4\chi_{-\mathbf{g}}\chi_{\mathbf{g}}C^{(s)^2} = 0, \tag{7}$$

for X-ray waves in a crystal using standard methods of dynamic theory [1].

We will seek for projections of wave vectors **k** and $\mathbf{k_g}$ onto the X-axis coinciding with the **n** vector (Fig. 1) in the form

$$\begin{aligned} k_x &= \omega\cos\psi_0 + \frac{\omega\chi_0}{2\cos\psi_0} + \frac{\lambda_0}{\cos\psi_0}, \\ k_{\mathbf{g}x} &= \omega\cos\psi_{\mathbf{g}} + \frac{\omega\chi_0}{2\cos\psi_{\mathbf{g}}} + \frac{\lambda_{\mathbf{g}}}{\cos\psi_{\mathbf{g}}}. \end{aligned} \tag{8}$$

And we will use the known relation connecting dynamic additives λ_0 and $\lambda_{\mathbf{g}}$ for X-ray waves [1]

$$\lambda_{\mathbf{g}} = \frac{\omega\beta}{2} + \lambda_0\frac{\gamma_{\mathbf{g}}}{\gamma_0}, \tag{9}$$

where $\beta = \alpha - \chi_0\left(1 - \frac{\gamma_{\mathbf{g}}}{\gamma_0}\right)$, $\alpha = \frac{1}{\omega^2}(k_{\mathbf{g}}^2 - k^2)$; $\gamma_0 = \cos\psi_0$; $\gamma_{\mathbf{g}} = \cos\psi_{\mathbf{g}}$; ψ_0 is the angle between the wave vector of incident wave **k** and the normal vector to the plate surface **n**, and $\psi_{\mathbf{g}}$ is the angle between the wave vector $\mathbf{k_g}$

and the vector **n** (Fig. 1). The magnitudes of vectors **k** and $\mathbf{k_g}$ have the form

$$k = \omega\sqrt{1+\chi_0}+\lambda_0, \quad k_{\mathbf{g}} = \omega\sqrt{1+\chi_0}+\lambda_{\mathbf{g}}. \quad (10)$$

We substitute (8) into (7), taking into account (9) with $k_{\|} \approx \omega\sin\psi_0$ and $k_{\mathbf{g}\|} \approx \omega\sin\psi_{\mathbf{g}}$ to obtain the equations for dynamic additives

$$\lambda_{\mathbf{g}}^{(1,2)} = \frac{\omega}{4}\left(\beta \pm \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right),$$
$$\lambda_0^{(1,2)} = \omega\frac{\gamma_0}{4\gamma_{\mathbf{g}}}\left(-\beta \pm \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right). \quad (11)$$

Since $|\lambda_0| \ll \omega$ and $|\lambda_{\mathbf{g}}| \ll \omega$, it can be shown that $\theta \approx \theta'$ (Fig. 1); therefore, θ' will further be denoted as θ.

It is convenient to represent the solution of system of equations (3) for diffracted field in a crystal in the form

$$E_{\mathbf{g}}^{(s)\mathrm{cr}} = -\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{-\mathbf{g}}C^{(s)}}{4\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)})(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)})}$$
$$\times\delta\left(\frac{\omega\beta}{2}+\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^* - \lambda_{\mathbf{g}}\right) + E^{(s)^{(1)}}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)})$$
$$+ E^{(s)^2}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)}), \quad (12)$$

where $\lambda_0^* = \omega\left(\frac{\gamma^{-2}+\theta^2-\chi_0}{2}\right)$, $\gamma = \sqrt{1-V^2}$ is the Lorentz factor of a particle, and $E^{(s)^{(1)}}$ and $E^{(s)^{(2)}}$ are the free fields corresponding to two solutions (11) of dispersion equation (7).

The solution of system (3) for the field in vacuum in front of a crystal has the form

$$E_0^{(s)\mathrm{vac}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{-\chi_0-\frac{2}{\omega}\lambda_0}\delta(\lambda_0^*-\lambda_0)$$
$$= \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(-\chi_0-\frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}}+\beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}$$
$$\times\delta\left(\frac{\omega\beta}{2}+\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^*-\lambda_{\mathbf{g}}\right), \quad (13)$$

here the relation $\delta(\lambda_0^*-\lambda_0) = \frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}}\delta\left(\frac{\omega\beta}{2}+\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^*-\lambda_{\mathbf{g}}\right)$ resulting from (9) is used.

We obtain for the field in vacuum behind a crystal

$$E_{\mathbf{g}}^{(s)\mathrm{vac}} = E_{\mathbf{g}}^{(s)\mathrm{Rad}}\delta\left(\lambda_{\mathbf{g}}+\frac{\omega\chi_0}{2}\right), \quad (14)$$

where $E^{(s)\mathrm{Rad}}$ is the radiation field.

The expression relating the diffracted and incident fields in a crystal follows from the second equation of system (3),

$$E_0^{(s)\mathrm{cr}} = \frac{2\omega\lambda_{\mathbf{g}}}{\omega^2\chi_{\mathbf{g}}C^{(s)}}E_{\mathbf{g}}^{(s)\mathrm{cr}}. \quad (15)$$

Using general boundary conditions

$$\int E_0^{(s)\mathrm{vac}}d\lambda_{\mathbf{g}} = \int E_0^{(s)\mathrm{cr}}d\lambda_{\mathbf{g}}, \quad (16a)$$

$$\int E_{\mathbf{g}}^{(s)\mathrm{cr}}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}L}d\lambda_{\mathbf{g}} = \int E_{\mathbf{g}}^{(s)\mathrm{vac}}e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}L}d\lambda_{\mathbf{g}}, \quad (16b)$$

$$\int E_{\mathbf{g}}^{(s)\mathrm{cr}}d\lambda_{\mathbf{g}} = 0, \quad (16c)$$

we obtain the expression for radiation field

$$E_{\mathbf{g}}^{(s)\mathrm{rad}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}\exp\left(i\left(\frac{\omega\chi_0}{2}+\lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right)}{2\omega(\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^{(2)})}$$
$$\times\left[\left(\frac{\omega}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}(-\chi_0\omega-2\lambda_0^*)}+\frac{\omega}{2\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)})}\right)\right.$$
$$\times\left(1-\exp\left(-i\frac{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right) \quad (17)$$
$$-\left(\frac{\omega}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}(-\chi_0\omega-2\lambda_0^*)}+\frac{\omega}{2\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)})}\right)$$
$$\left.\times\left(1-\exp\left(-i\frac{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)\right)\right],$$

where $\lambda_{\mathbf{g}}^* = \frac{\omega\beta}{2}+\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^*$.

The expression for the radiation field of the rectilinearly moving electron (17) can be divided into two terms

$$E_{\mathbf{g}}^{(s)\mathrm{rad}} = E_{\mathbf{g}}^{(s)\mathrm{PXR}} + E_{\mathbf{g}}^{(s)\mathrm{DTR}}, \quad (18a)$$

$$E_{\mathbf{g}}^{(s)\mathrm{PXR}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega(\lambda_{\mathbf{g}}^{(1)}-\lambda_{\mathbf{g}}^{(2)})\frac{\gamma_0}{\gamma_{\mathbf{g}}}}$$

$$\times\left[\left(\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)})} - \frac{\omega}{2\lambda_0^*}\right)\left(1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right)\right.$$

$$\left.-\left(\frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)})} - \frac{\omega}{2\lambda_0^*}\right)\left(1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)\right)\right] \tag{18b}$$

$$\times\exp\left(i\left(\frac{\omega\chi_0}{2} + \lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right),$$

$$E_{\mathbf{g}}^{(s)\mathrm{DTR}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega(\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^{(2)})\frac{\gamma_0}{\gamma_{\mathbf{g}}}}$$

$$\times\left(\frac{\omega}{-\omega\chi_0 - 2\lambda_0^*} + \frac{\omega}{2\lambda_0^*}\right)\left(\exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)\right. \tag{18c}$$

$$\left.-\exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right)\exp\left(i\left(\frac{\omega\chi_0}{2} + \lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right).$$

Eq. (18b) describes the PXR field. It is seen from this expression that the first PXR branch yields the more significant contribution, since the real part of denominator of this branch can become zero ($\mathrm{Re}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}) = 0$), and the one of the second PXR branch can not ($\mathrm{Re}(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}) \neq 0$).

Eq. (18c) describes the DTR field which arises as a consequence of diffraction of the transition radiation, formed at the input surface, at the system of atomic planes of a crystal (at the same surface at which PXR is also formed).

Substituting Eqs. (11) into (18b) and (18c) and taking only the first PXR branch, we represent them in the form

$$E_{\mathbf{g}}^{(s)\mathrm{PXR}} = -\frac{4\pi^2 ieV}{\omega}\frac{\theta P^{(s)}}{\theta^2 + \gamma^{-2} - \chi_0} \tag{19a}$$

$$\times\frac{\exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right]}{K^{(s)}}\frac{\xi^{(s)}(\omega) - \frac{i\rho^{(s)}(1-\varepsilon)}{2} - K^{(s)}}{\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega) - K^{(s)}}{\varepsilon}}$$

$$\times\left(1 - \exp\left[-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon} + \frac{\xi^{(s)}(\omega) - K^{(s)}}{\varepsilon}\right)\right]\right),$$

$$E_{\mathbf{g}}^{(s)\mathrm{DTR}} = -\frac{4\pi^2 ieV}{\omega}\theta P^{(s)}\left(\frac{1}{\theta^2 + \gamma^{-2}} - \frac{1}{\theta^2 + \gamma^{-2} - \chi_0}\right)$$

$$\times\frac{\exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_{\mathbf{g}}^*\right)\frac{L}{\gamma_{\mathbf{g}}}\right]}{\frac{K^{(s)}}{\varepsilon}}\left(\exp\left[-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}\right.\right.\right. \tag{19b}$$

$$\left.\left.+\frac{\xi^{(s)}(\omega) - K^{(s)}}{\varepsilon}\right)\right] - \exp\left[-ib^{(s)}\left(\sigma^{(s)} - \frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}\right.\right.$$

$$\left.\left.\left.+\frac{\xi^{(s)}(\omega) + K^{(s)}}{\varepsilon}\right)\right]\right),$$

where

$$K^{(s)} = \sqrt{\xi^{(s)}(\omega)^2 + \varepsilon - 2i\rho^{(s)}\left(\frac{(1-\varepsilon)}{2}\xi^{(s)}(\omega) + \kappa^{(s)}\varepsilon\right) - \rho^{(s)^2}\left(\frac{(1-\varepsilon)^2}{4} + \kappa^{(s)^2}\varepsilon\right)},$$

$$\xi^{(s)}(\omega) = \frac{\alpha}{2|\chi'_{\mathbf{g}}|C^{(s)}} - \frac{\chi'_0(1-\varepsilon)}{2|\chi'_{\mathbf{g}}|C^{(s)}} = \eta^{(s)}(\omega) + \frac{(1-\varepsilon)}{2\nu^{(s)}},\quad \nu^{(s)} = \frac{|\chi'_{\mathbf{g}}|C^{(s)}}{|\chi'_0|},\quad \rho^{(s)} = \frac{\chi''_0}{|\chi'_{\mathbf{g}}|C^{(s)}},$$

$$\eta^{(s)}(\omega) = \frac{\alpha}{2|\chi'_{\mathbf{g}}|C^{(s)}} = \frac{2\sin^2\theta_B}{V^2|\chi'_{\mathbf{g}}|C^{(s)}}\left(1 - \frac{\omega(1 - \theta\cos\varphi\cot\theta_B)}{\omega_B}\right),\quad \varepsilon = \frac{\gamma_{\mathbf{g}}}{\gamma_0},\quad \kappa^{(s)} = \frac{\chi''_{\mathbf{g}}C^{(s)}}{\chi''_0}, \tag{20}$$

$$\sigma^{(s)} = \frac{1}{|\chi'_{\mathbf{g}}|C^{(s)}}(\theta^2 + \gamma^{-2} - \chi'_0),\quad b^{(s)} = \frac{\omega|\chi'_{\mathbf{g}}|C^{(s)}}{2}\frac{L}{\gamma_0}.$$

The parameter $b^{(s)}$ determines the role of the electron path length L/γ_0 in a crystal under radiation. Since the inequality $2\sin^2\theta_B/V^2|\chi'_{\mathbf{g}}|C^{(s)} \gg 1$ holds in the range of X-ray frequencies, $\eta^{(s)}(\omega)$ is the fast function of ω. To further analyze the PXR and DTR spectra, it is very convenient to consider $\eta^{(s)}(\omega)$ as a spectral variable characterizing the frequency ω.

Note that not $\eta^{(s)}(\omega)$, but $\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{(1-\varepsilon)}{2\nu^{(s)}}$, where the second term arises as a consequence of the radiation refraction effect at asymmetrical reflection, enters into (19). It is equal to zero for the symmetrical reflection ($\varepsilon = 1$).

Let us represent the parameter ε in the form of $\varepsilon = \sin(\delta + \theta_B)/\sin(\delta - \theta_B)$, where δ is the angle between the input target surface and the crystallographic plane. As the angle δ decreases, the parameter ε increases, and vice versa.

Substituting (19a) and (19b) into the known [11] expression for the spectral-angular density of X-rays,

$$\frac{d^2W}{d\omega d\Omega} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E_{\mathrm{rad}}^{(s)}\right|^2, \tag{21}$$

we obtain the formulae for the PXR and DTR spectral-angular density and for the term being a result of interference of these radiation types

$$\omega\frac{d^2N_{\mathrm{PXR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}\frac{\theta^2 P^{(s)^2}}{(\theta^2+\gamma^{-2}-\chi_0')^2}$$

$$\times\left|\frac{1}{K^{(s)}}\frac{\xi^{(s)}(\omega)-\frac{i\rho^{(s)}(1-\varepsilon)}{2}-K^{(s)}}{\sigma^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}}\left(1-\exp\left[-ib^{(s)}\left(\sigma^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right]\right)\right|^2, \tag{22a}$$

$$\omega\frac{d^2N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}\theta^2 P^{(s)^2}\left(\frac{1}{\theta^2+\gamma^{-2}}-\frac{1}{\theta^2+\gamma^{-2}-\chi_0'}\right)^2$$

$$\times\left|\frac{\exp\left[-ib^{(s)}\left(\sigma^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right]-\exp\left[-ib^{(s)}\left(\sigma^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)+K^{(s)}}{\varepsilon}\right)\right]}{\frac{K^{(s)}}{\varepsilon}}\right|^2, \tag{22b}$$

$$\omega\frac{d^2N_{\mathrm{interf}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{2\pi^2}\frac{\theta^2 P^{(s)^2}}{(\theta^2+\gamma^{-2}-\chi_0')}\left(\frac{1}{\theta^2+\gamma^{-2}}-\frac{1}{\theta^2+\gamma^{-2}-\chi_0'}\right)\frac{\varepsilon}{|K^{(s)}|^2}$$

$$\times\mathrm{Re}\left(\frac{\xi^{(s)}(\omega)-\frac{i\rho^{(s)}(1-\varepsilon)}{2}-K^{(s)}}{\sigma^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}}\left(1-\exp\left[-ib^{(s)}\left(\sigma^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right]\right)\right. \tag{22c}$$

$$\left.\times\left(\exp\left[ib^{(s)}\left(\sigma^{(s)}+\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)*}}{\varepsilon}\right)\right]-\exp\left[ib^{(s)}\left(\sigma^{(s)}+\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)+K^{(s)*}}{\varepsilon}\right)\right]\right)\right),$$

where $K^{(s)*}$ is the complex conjugate of $K^{(s)}$.

The expressions for the PXR and DTR spectral-angular density are the principal result of the given work. They were obtained on the base of two-wave approximation of dynamic diffraction theory taking into account the radiation absorption in medium and the possibility of different orientations of diffracting atomic planes in a crystal relative to the crystalline plate surface.

For the particular case of symmetrical reflection, when the atomic crystal planes are perpendicular to the input surface ($\delta = \pi/2$ and $\varepsilon = 1$), Eqs. (22a) and (22b) rearrange to the expressions obtained in [12].

EFFECT OF REFLECTION ASYMMETRY ON SPECTRAL-ANGULAR RADIATION DISTRIBUTION

Consider a thin nonabsorbing crystal. Assuming that $\rho^{(s)} = \dfrac{\chi_0''}{|\chi_g'|C^{(s)}} = 0$ in (22), we obtain the expressions for the PXR and DTR spectral-angular density and their interference term for this case

$$\omega\frac{d^2N_{\mathrm{PXR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{2\pi^2}\frac{\theta^2 P^{(s)^2}}{(\theta^2+\gamma^{-2}-\chi_0')^2}\left(\frac{\xi^{(s)}(\omega)}{\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}-1\right)^2$$
$$\times\frac{1-\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}(\omega)-\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi^{(s)}(\omega)-\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\varepsilon}\right)^2}, \tag{23a}$$

$$\omega\frac{d^2N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{2\pi^2}\theta^2 P^{(s)^2}\left(\frac{1}{\theta^2+\gamma^{-2}}-\frac{1}{\theta^2+\gamma^{-2}-\chi_0'}\right)^2$$
$$\times\frac{1-\cos\left(\frac{2b^{(s)}\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\varepsilon}\right)}{\frac{\xi^{(s)}(\omega)^2+\varepsilon}{\varepsilon^2}}, \tag{23b}$$

$$\omega\frac{d^2N_{\mathrm{interf}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{\pi^2}\frac{\theta^2 P^{(s)^2}}{(\theta^2+\gamma^{-2}-\chi_0')}$$
$$\times\left(\frac{1}{\theta^2+\gamma^{-2}}-\frac{1}{\theta^2+\gamma^{-2}-\chi_0'}\right)\varepsilon\frac{\xi^{(s)}(\omega)-\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\xi^{(s)}(\omega)^2+\varepsilon}$$
$$\times\sin\left(\frac{b^{(s)}\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\varepsilon}\right) \tag{23c}$$
$$\times\frac{\sin\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}(\omega)}{\varepsilon}\right)\right)-\sin\left(\frac{b^{(s)}\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\varepsilon}\right)}{\sigma^{(s)}+\frac{\xi^{(s)}(\omega)-\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\varepsilon}}.$$

Consider the dependence of the PXR spectral density on the orientation of the crystalline plate surface relative to the system of parallel diffracting atomic planes (determined by the parameter ε) at the fixed angle between the electron velocity and reflecting planes (θ_B) and at the fixed path covered by an electron in the plate (L/γ_0). Three of a set of possible orientations of the crystalline plate surface relative to the system of parallel diffracting atomic planes corresponding to the specified length of rectilinear trajectory of the relativistic electron are shown in Fig. 2. It is important to note that the plate thickness should change as the parameter ε changes, so that the path covered by an electron in the plate (L/γ_0) was unchanged.

Let us extract the part describing the PXR spectrum from expression (23a)

$$P_{\mathrm{PXR}}^{(s)} = \left(\frac{\xi^{(s)}(\omega)}{\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}-1\right)^2$$
$$\times\frac{1-\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}(\omega)-\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi^{(s)}(\omega)-\sqrt{\xi^{(s)}(\omega)^2+\varepsilon}}{\varepsilon}\right)^2}, \tag{24}$$
$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega)+\frac{(1-\varepsilon)}{2\nu^{(s)}},$$
$$\sigma^{(s)} = \frac{1}{\nu^{(s)}}\left(\frac{\theta^2}{|\chi_0'|}+\frac{1}{\gamma^2|\chi_0'|}+1\right).$$

It follows from (24) that an increase in the parameter ε (a decrease in δ) results in the broadening of the PXR spectrum at the fixed electron energy and observation angle (fixed $\sigma^{(s)}$), since the denominator of expression (24) varies less strongly with $\xi^{(s)}(\omega)$ variation at larger ε. The curves describing the PXR spectrum and constructed by Eq. (24) for the fixed electron energy γ and observation angle θ are presented in Fig. 3. It can be seen that the spectrum width depends substantially on the parameter ε.

Consider the PXR angular density. For this purpose let us integrate Eq. (23a) over the frequency function $\eta^{(s)}(\omega)$:

$$\frac{dN_{\mathrm{PXR}}^{(s)}}{d\Omega} = \frac{e^2\nu^{(s)}P^{(s)^2}}{4\pi^2\sin^2\theta_B}R_{\mathrm{PXR}}^{(s)},$$
$$R_{\mathrm{PXR}}^{(s)} = \int_{-\infty}^{+\infty}F_{\mathrm{PXR}}^{(s)}d\eta^{(s)}(\omega), \tag{25a}$$

$$F_{\mathrm{PXR}}^{(s)} = \frac{\frac{\theta^2}{|\chi_0'|}}{\left(\frac{\theta^2}{|\chi_0'|}+\frac{1}{\gamma^2|\chi_0'|}+1\right)^2}\left(\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}-1\right)^2$$
$$\times\frac{1-\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)^2}, \tag{25b}$$

The angular dependences $R_{\mathrm{PXR}}^{(s)}$ presented in Fig. 4 and constructed for the specified crystal parameters and the energy of incident electron demonstrate a substantial increase in the PXR angular density with increasing the ε parameter.

It should be noted that this effect is stronger in the case of a thick absorbing crystal (Fig. 5) due to a decrease in the path length of diffracted photon in a crystal at the ε parameter increase with the constant trajectory of relativistic electron length (Fig. 2). The curves $F^{(s)}$ presented in Fig. 5 are constructed by the formula following from (22a)

$$\omega\frac{d^2N^{(s)}_{\mathrm{PXR}}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}\frac{P^{(s)^2}}{|\chi'_0|}F^{(s)}_{\mathrm{PXR}},$$

$$F^{(s)}_{\mathrm{PXR}} = \frac{\dfrac{\theta^2}{|\chi'_0|}}{\left(\dfrac{\theta^2}{|\chi'_0|}+\dfrac{1}{\gamma^2|\chi'_0|}+1\right)^2}\left|\frac{1}{K^{(s)}}\frac{\xi^{(s)}(\omega)-\dfrac{i\rho^{(s)}(1-\varepsilon)}{2}-K^{(s)}}{\sigma^{(s)}-\dfrac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\dfrac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}}\right. \tag{26}$$

$$\left.\times\left(1-\exp\left[-ib^{(s)}\left(\sigma^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2\varepsilon}+\frac{\xi^{(s)}(\omega)-K^{(s)}}{\varepsilon}\right)\right]\right)\right|^2.$$

We should consider further the effect of asymmetry on the DTR angular density and the PXR and DTR interference. Let us represent Eqs. (23b) and (23c) in the convenient for the analysis of angular density form, analogous to (25a) and (25b)

$$\frac{dN^{(s)}_{\mathrm{DTR}}}{d\Omega} = \frac{e^2\nu^{(s)}P^{(s)^2}}{4\pi^2\sin^2\theta_B}R^{(s)}_{\mathrm{DTR}},$$
$$R^{(s)}_{\mathrm{DTR}} = \int_{-\infty}^{+\infty}F^{(s)}_{\mathrm{DTR}}d\eta^{(s)}(\omega), \tag{27a}$$

$$F^{(s)}_{\mathrm{DTR}} = \frac{\dfrac{\theta^2}{|\chi'_0|}}{\left(\dfrac{\theta^2}{|\chi'_0|}+\dfrac{1}{\gamma^2|\chi'_0|}+1\right)^2\left(\dfrac{\theta^2}{|\chi'_0|}+\dfrac{1}{\gamma^2|\chi'_0|}\right)^2} \tag{27b}$$
$$\times\frac{1-\cos\left(\dfrac{2b^{(s)}\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)}{\dfrac{\xi^{(s)^2}+\varepsilon}{\varepsilon^2}},$$

$$\frac{dN^{(s)}_{\mathrm{interf}}}{d\Omega} = \frac{e^2\nu^{(s)}P^{(s)^2}}{4\pi^2\sin^2\theta_B}R^{(s)}_{\mathrm{interf}},$$
$$R^{(s)}_{\mathrm{interf}} = \int_{-\infty}^{+\infty}F^{(s)}_{\mathrm{interf}}d\eta^{(s)}(\omega), \tag{28a}$$

$$F^{(s)}_{\mathrm{interf}} = \frac{2\dfrac{\theta^2}{|\chi'_0|}}{\left(\dfrac{\theta^2}{|\chi'_0|}+\dfrac{1}{\gamma^2|\chi'_0|}+1\right)^2\left(\dfrac{\theta^2}{|\chi'_0|}+\dfrac{1}{\gamma^2|\chi'_0|}\right)}$$
$$\times\varepsilon\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\xi^{(s)^2}+\varepsilon}\sin\left(\frac{b^{(s)}\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right) \tag{28b}$$
$$\times\frac{\sin\left(b^{(s)}\left(\sigma^{(s)}+\dfrac{\xi^{(s)}}{\varepsilon}\right)\right)-\sin\left(\dfrac{b^{(s)}\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)}{\sigma^{(s)}+\dfrac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}}.$$

The angular dependences $R^{(s)}_{\mathrm{DTR}}$ presented in Fig. 6 and constructed by Eqs. (27a) and (27b) for the same values of the crystal and electron energy parameters demonstrate a substantial increase in the DTR angular density as the angle between the crystalline plate surface and the system of diffracting atomic planes of a crystal decreases. The curves of the angular dependence $R^{(s)}_{\mathrm{DTR}}$, $R^{(s)}_{\mathrm{PXR}}$, $R^{(s)}_{\mathrm{interf}}$ constructed by Eqs. (25), (27), and (28) and the PXR and DTR total angular density $R^{(s)}_{\mathrm{SUM}}$ taking into account the interference of these radiation mechanisms are presented in Fig. 7. It follows from Fig. 7 that the interference conditions for these mechanisms resulting in the appearance of oscillations in the angular radiation density can be created by reflection asymmetry at rather high electron energy when the

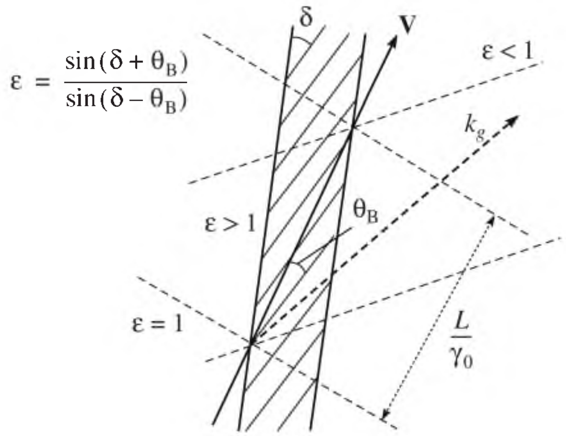

Fig. 2. Symmetrical (ε = 1) and asymmetrical (ε > 1 and ε < 1) reflections of particle field.

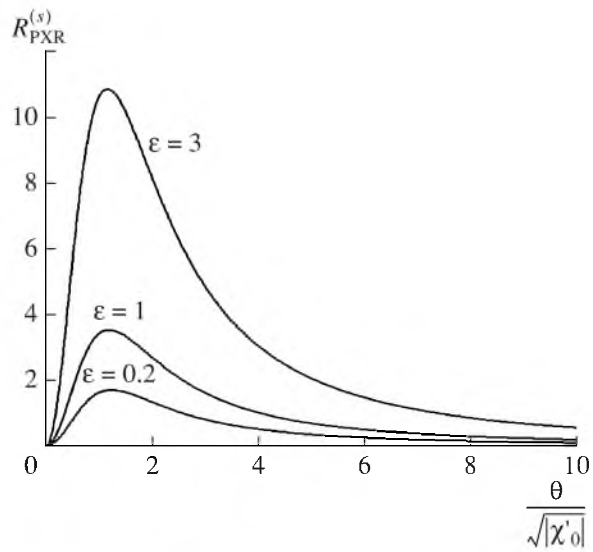

Fig. 4. Effect of asymmetry on the PXR angular density at the parameters $b^{(s)} = 3$, $\nu^{(s)} = 0.5$, and $1/\gamma^2|\chi'_0| = 0.25$.

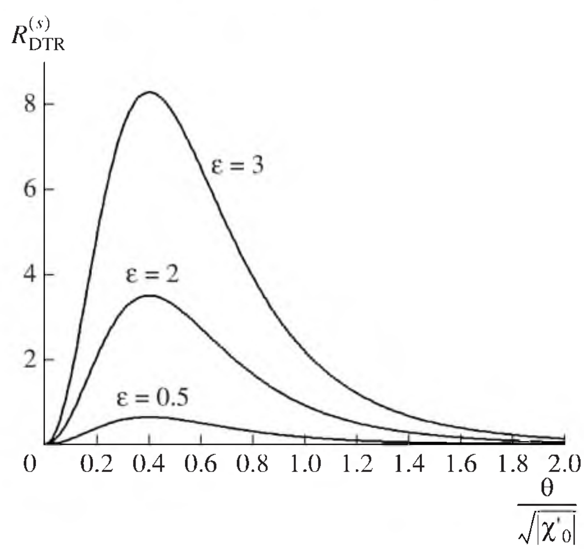

Fig. 6. Effect of asymmetry on the DTR angular density at the parameters $b^{(s)} = 3$, $\nu^{(s)} = 0.5$, and $1/\gamma^2|\chi'_0| = 0.25$.

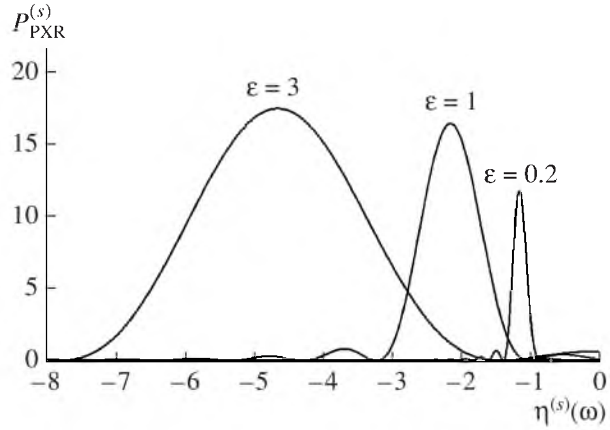

Fig. 3. Effect of reflection asymmetry on the PXR spectrum width at the following crystal parameters: $b^{(s)} = 3$, $\nu^{(s)} = 0.5$, electron energy $1/\gamma^2|\chi'_0| = 0.25$, and observation angle $\theta/\sqrt{|\chi'_0|} = 1$.

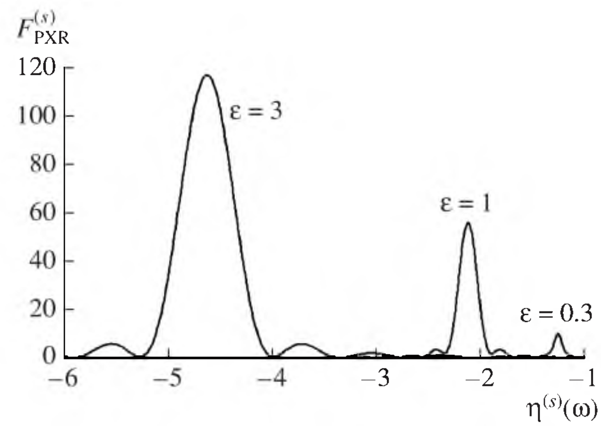

Fig. 5. Effect of asymmetry on the PXR spectrum in a thick absorbing crystal at the parameters $b^{(s)} = 15$, $\nu^{(s)} = 0.5$, $\rho^{(s)} = 0.1$, $\kappa^{(s)} = 0.5$, $1/\gamma^2|\chi'_0| = 0.25$, and $\theta/\sqrt{|\chi'_0|} = 1$.

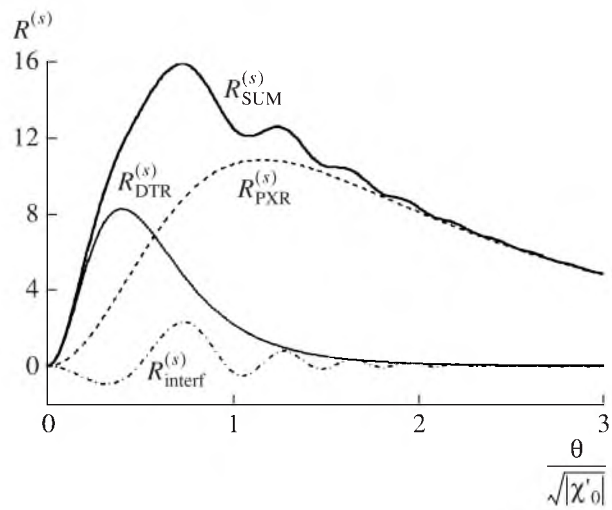

Fig. 7. Appearance of interference under asymmetry conditions (ε = 3) at the parameters $b^{(s)} = 3$, $\nu^{(s)} = 0.5$, and $1/\gamma^2|\chi'_0| = 0.25$.

DTR and PXR angular densities become comparable in value.

CONCLUSIONS

The analytical expressions for the PXR and DTR spectral-angular distribution of the relativistic electron crossing a crystalline plate of arbitrary thickness have been obtained in the Laue scattering geometry using the two-wave approximation of dynamic diffraction theory for the general case of asymmetrical reflection (for an arbitrary value of the angle δ between the crystalline plate surface and the system of diffracting atomic planes in a crystal). It has been shown that the PXR spectrum for the thin nonabsorbing crystal broadens with the decrease in angle δ that results in the substantial increase in the angular radiation density. The accounting of radiation absorption in a crystal results in an additional strengthening of the given effect due to optimization of the relation between the electron trajectories and the emitted photon in a crystal in going from symmetrical reflection to asymmetrical one. It has been shown that the DTR angular density also substantially decreases with the indicated angle decreasing. The effect of interference on the total angular density has been investigated. It has been shown that interference can result in oscillations in the angular radiation distribution.

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