On the Gravitational Motion of a Nonuniformly Heated Solid Particle in a Gaseous Medium

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Abstract—The steady motion of a nonuniformly heated spherical aerosol particle through a viscous gaseous medium is theoretically studied in the Stokes approximation. It is assumed that the mean temperature of the particle surface may differ appreciably from the ambient temperature. The solution of gasdynamic equations yields an analytical expression for the drag of the medium and the gravitational fall velocity of the nonuniformly heated spherical solid particle with allowance for the temperature dependence of the density of the medium and molecular transfer coefficients (viscosity and thermal conductivity). Numerical estimates show that heating of the particle surface considerably influences the drag force and gravitational fall velocity.

INTRODUCTION

Aerosol particles entering into an aerodispersed system may experience forces of various kinds. The gravitational motion, i.e., the motion of aerosol particles in the gravity field due to the difference between the specific weights of the particles and ambient medium, is the most common one. The gravitational motion is fundamental for many operating procedures, such as floatation, sizing analysis of aerodispersed systems, and fine gas cleaning.

When designing experimental setups in which the directional motion of particles must be provided, developing techniques for fine cleaning of gases from aerosol particles, mathematically simulating the precipitation of particles in a plane-parallel channel kept at different temperatures, etc., as well as because of the deteriorating ecological situation, one should know the drag of the ambient medium to the motion of particles.

The drag can be effectively controlled by heating the particle surface, e.g., using laser radiation. In this case, particles move under the conditions of considerable relative temperature difference in their vicinity. The relative temperature difference is the difference between the temperatures near and far from the particle surface divided by the latter. It is considered tangible if $(t_{iS} - t_{e\infty})/t_{e\infty} \sim 0(1)$, where t_{iS} is the mean temperature of the particle surface and $t_{e\infty}$ is the temperature of the gaseous medium far from the particle.

The heated surface of an aerosol particle considerably influences the thermophysical characteristics of the gaseous environment and eventually the drag.

The problem of drag to the motion of a heated solid particle was first solved in [1]. However, the analytical results obtained in [1] are inapplicable to large temperature differences, because the method selected for solving gasdynamic equations turned out to be inappropriate. In addition, the authors of [1] considered only the linear temperature dependences of the thermal conductivity and dynamic viscosity.

Molecular transfer coefficients in a gas are known to depend on temperature by a power law [2]. With regard to such a dependence, the problem of drag to the motion of a heated spherical particle was first analytically solved in [3–6]. The formulas derived in those works enable one to make estimates at large temperature differences. In [3–6], the equations of gas dynamics were solved with a method proposed by Shchukin, in which they are expanded into power series in parameter $l(y) = \Gamma_0/(y + \Gamma_0)$, where $\gamma_0 = t_S^{1+\alpha} - 1$, $t_S = T_S/T_{e\infty}$, and $y = r/R$ is the dimensionless radial coordinate.

In a number of works, e.g., in [7], it was assumed that the Barnett temperature stresses should also be taken into consideration in solving problems with nonisothermal flows. In [8], the problem of flow about a strongly heated sphere is solved with allowance for the Barnett stresses. The same problem was analytically solved in [9]. The Barnett stresses may strongly influence the motion of particles at Mach numbers $M \longrightarrow 0$ [8]. In this paper, we consider the motion of a particle at sufficiently small Knudsen numbers and moderate Mach numbers. Under these conditions, the temperature stresses may be neglected even at a temperature difference of the order of unity. It should be noted that in works published earlier solutions to differential equations describing the velocity and pressure fields were sought in the form of power series by the order reduction method, which yielded awkward final expressions.

In this work, a solution to equations of gas dynamics is sought immediately in the form of generalized power series. Such an approach made it possible to significantly simplify the final expressions.

1. STATEMENT OF THE PROBLEM

The object of consideration is the gravitational motion of a spherical solid particle in a viscous nonisothermal gaseous medium. Inside the particle, heat sources (heat sinks) with density q_i are nonuniformly distributed.

In the theoretical description of the flow about an aerosol particle, we will assume that all processes in the particle–gas system are quasi-stationary because of a short thermal relaxation time of the system. The particle moves at Peclet and Reynolds numbers that are much less than unity. Since it is heated, one should take into account the temperature dependences of the molecular transfer coefficients (viscosity and thermal conductivity) and density of the gaseous medium. Here, we use the power form of these dependences [2],

$$\mu_e = \mu_{e\infty}\left(\frac{T_e}{T_{e\infty}}\right)^{\beta}, \quad \lambda_e = \lambda_{e\infty}\left(\frac{T_e}{T_{e\infty}}\right)^{\alpha}$$

$$(0 \le \alpha, \beta \le 1),$$

where $\mu_{e\infty} = \mu_e(T_{e\infty})$; $\lambda_{e\infty} = \lambda_e(T_{e\infty})$; μ_e and λ_e are the dynamic viscosity and thermal conductivity of the gaseous medium, respectively; and T_e is the gas (environmental) temperature. The particle is assumed to be uniform in composition and large (Knudsen number $\mathrm{Kn} = \lambda/R \ll 0.01$, where λ is the mean free path length of gas molecules). The phase transition on the particle's surface is absent. The radius of the particle is sufficiently small, so that the influence of gravitational convection on the temperature distribution can be neglected.

In the Stokes approximation with regard to the assumptions formulated above, the equations for mass velocity $\mathbf{U}_e$, pressure P_e, and temperature T outside and inside the heated particle are written in the form [10–12]

$$\frac{\partial P_e}{\partial x_k} = \frac{\partial}{\partial x_j}\left(\mu_e\left[\frac{\partial U_j^e}{\partial x_k} + \frac{\partial U_k^e}{\partial x_j} - \frac{2}{3}\delta_{jk}\frac{\partial U_m^e}{\partial x_m}\right]\right) + F_{mg}, \tag{1.1}$$

$$\mathrm{div}(\rho_e U_e) = 0;$$

$$\mathrm{div}(\lambda_e \nabla T_e) = 0, \quad \mathrm{div}(\lambda_i \nabla T_i) = -q_i. \tag{1.2}$$

To close this set of equations, it is necessary to add the equation of state.

It is convenient to consider the gravitational motion of the heated particle in the coordinate system related to the center of mass of the particle. In this case, the problem is reduced to the case of a plane-parallel flow with velocity $\mathbf{U}_\infty$ ($\mathbf{U}_\infty \parallel OZ$) about a spherical particle.

The problem stated by (1.1) and (1.2) is solved with boundary conditions that in the spherical coordinate system (r, θ, φ) are given by

$$r = R, \quad U_r^e = 0, \quad U_\theta^e = 0, \quad T_e = T_i,$$
$$\lambda_e\frac{\partial T_e}{\partial r} = \lambda_i\frac{\partial T_i}{\partial r} + \sigma_0\sigma_1(T_i^4 - T_{e\infty}^4); \tag{1.3}$$

$$r \longrightarrow \infty, \quad \mathbf{U}_e \longrightarrow U_\infty\cos\theta\mathbf{e}_r - U_\infty\sin\theta\mathbf{e}_\theta,$$
$$P_e \longrightarrow P_{e\infty}, \quad T_e \longrightarrow T_{e\infty}; \tag{1.4}$$

$$r \longrightarrow 0, \quad T_i \ne \infty. \tag{1.5}$$

Here, U_r^e and U_θ^e are, respectively, the radial and tangential components of mass velocity U_e; σ_0 is the Stefan–Boltzmann constant; σ_1 is the total emissivity; $U_\infty = |\mathbf{U}_\infty|$, $\mathbf{U}_\infty$ is the velocity of the incident flow, which is determined from the vanishing condition for the total force acting on the particle; $\lambda_i = \lambda_{i\infty}t_i^{\omega}$; $\lambda_{i\infty} = \lambda_i(T_{e\infty})$; $t_i = T_i/T_{e\infty}$; $-1 \le \omega \le 1$; and $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors of the spherical coordinate system. Hereinafter, subscript "e" refers to the gas environment and subscript ∞ refers to physical quantities characterizing the environment in the unperturbed flow.

Boundary conditions (1.3) on the surface of a drop include the impermeability conditions for the normal and tangential components of the mass velocity, equality condition for the temperatures, and continuity conditions for heat fluxes. Away from the drop, conditions (1.4) are valid and the finiteness of physical quantities is taken into account in (1.5).

The force with which the flow acts on the particle is given by [9, 10]

$$F_z = \int_{(S)}(-P_e\cos\theta + \sigma_{rr}\cos\theta - \sigma_{r\theta}\sin\theta)r^2\sin\theta d\theta d\varphi. \tag{1.6}$$

Here,

$$\sigma_{rr} = \mu_e\left(2\frac{\partial U_r^e}{\partial r} - \frac{2}{3}\mathrm{div}U_e\right),$$

$$\sigma_{r\theta} = \mu_e\left(\frac{\partial U_\theta^e}{\partial r} + \frac{1}{r}\frac{\partial U_r^e}{\partial \theta} - \frac{U_\theta^e}{r}\right)$$

are the components of the total stress tensor in the spherical coordinate system. The incident flow has only a perturbing effect; therefore, a solution to the set of equations of hydrodynamics and heat transfer should be sought in the form of an expansion in small parameter $\varepsilon = \mathrm{Re}_\infty = (\rho_{e\infty}U_\infty R)/\mu_{e\infty}$.

The form of boundary conditions (1.3)–(1.5) makes it possible to seek expressions for the velocity, pressure, and temperature fields by the method of separation of variables,

$$U_r^e(r,\theta) = U_\infty G(y)\cos\theta,$$
$$U_\theta^e(r,\theta) = -U_\infty g(y)\sin\theta, \qquad (1.7)$$
$$P_e(r,\theta) = h(y)\cos\theta,$$

where $G(y)$, $g(y)$, and $h(y)$ are arbitrary functions depending on coordinate y, and then reduce the set of equations for the perturbed quantities to a set of ordinary differential equations in functions $G(y)$, $g(y)$, and $h(y)$.

2. VELOCITY AND TEMPERATURE FIELDS. GRAVITATIONAL FALL VELOCITY OF THE PARTICLE

To find the force acting on a uniformly heated aerosol particle and the velocity of its gravitational fall, it is necessary to know the temperature, velocity, and pressure distributions in its vicinity. A general solution to the heat conduction equation that satisfies the corresponding boundary conditions has the form

$$t_e(y,\theta) = t_{e0}(y) = \left(1+\frac{\Gamma_0}{y}\right)^{1/(1+\alpha)}, \qquad (2.1)$$

$$t_i(y,\theta) = t_{i0}(y)$$
$$= \left(B_0 + \frac{D_0}{y} + \frac{1}{y}\int_y^1 \psi_0 dy - \int_y^1 \frac{\psi_0}{y} dy\right)^{1/(1+\omega)}. \qquad (2.2)$$

Here, B_0 and D_0 are constants determined from boundary conditions (1.3) on the particle's surface,

$$\psi_0 = -\frac{R^2}{2\lambda_{i\infty}} y^2 \frac{1+\omega}{T_{e\infty}} \int_{-1}^{+1} q_i dx,$$

$x = \cos\theta$, $\Gamma_0 = t_{eS}^{+\alpha} - 1$ is a dimensionless parameter characterizing the heating of the particle's surface, and $t_{eS} = T_{eS}/T_{e\infty}$.

The mean value of temperature t_{iS} on the particle's surface is found by solving the set of transcendental equations

$$\begin{cases} t_{iS} = t_{eS}, \\ \dfrac{l^{(S)}}{1+\alpha}\dfrac{\lambda_{eS}}{\lambda_{iS}} t_{eS} = \dfrac{R^2}{3\lambda_{iS}T_{e\infty}} J_0 - \sigma_0\sigma_1 \dfrac{RT_{e\infty}^3}{\lambda_{iS}}(t_{iS}^4 - 1), \end{cases} \qquad (2.3)$$

where $\lambda_{eS} = \lambda_{e\infty} t_{eS}^{\alpha}$, $\lambda_{iS} = \lambda_{i\infty} t_{iS}^{\omega}$, $t_{iS} = t_{i0}|_{y=1}$, $t_{eS} = t_{e0}|_{y=1}$, $t_{iS} = T_{iS}/T_{e\infty}$, $J_0 = \frac{1}{V}\int_V q_i(r,\theta)dV$, $V = \frac{4}{3}\pi R^3$, and $l^{(S)} = \frac{\Gamma_0}{1+\Gamma_0}$. In (2.3), integration is over the particle's volume.

Subject to (2.1), the expression for the dynamic viscosity can be represented in the form

$$\mu_e = \mu_{e\infty} t_{e0}^{\beta}. \qquad (2.4)$$

Formula (2.4) will be subsequently used to find the velocity and pressure fields in the vicinity of the heated drop.

Having substituted (2.4) and (1.7) into the Navier–Stokes equation linearized in velocity and having separated the variables, we obtain an inhomogeneous differential equation in function $G(y)$,

$$y^3\frac{d^3G}{dy^3} + y^2(4+\gamma_1 l)\frac{d^2G}{dy^2} - y(4+\gamma_2 l - \gamma_3 l^2)\frac{dG}{dt} - (2-l)\gamma_3 l^2 G = -\frac{D}{y t_{e0}^{\beta}}, \qquad (2.5)$$

where

$$\gamma_1 = \frac{1-\beta}{1+\alpha}, \quad \gamma_2 = 2\frac{1+\beta}{1+\alpha}, \quad \gamma_3 = \frac{2+2\alpha-\beta}{(1+\alpha)^2},$$

$$D = \text{const}, \quad l(y) = \frac{\Gamma_0}{y+\Gamma_0}.$$

A solution to Eq. (2.5) will be sought in the form of a generalized power series [13, 14]. First, let us find a solution to homogeneous equation (2.5),

$$y^3\frac{d^3G}{dy^3} + y^2(4+\gamma_1 l)\frac{d^2G}{dy^2} - y(4+\gamma_2 l - \gamma_3 l^2)\frac{dG}{dt} - (2-l)\gamma_3 l^2 G = 0. \qquad (2.6)$$

For homogeneous equation (2.6), the point $y = 0$ is a regular point [13], so that its solution is sought in the form

$$G = y^{\rho}\sum_{n=0}^{\infty} C_n l^n, \quad C_0 \neq 0. \qquad (2.7)$$

Substituting (2.7) into (2.6) yields the determining equation $\rho(\rho+3)(\rho-2) = 0$, the roots of which are $\rho_1 = -3$, $\rho_3 = 0$, and $\rho_2 = 2$.

The largest of the roots corresponds to the solution

$$G_1 = \frac{1}{y^3}\sum_{n=0}^{\infty} C_n^{(1)} l^n, \quad C_0 = 1.$$

The second solution to homogeneous equation (2.6), which satisfies the finiteness condition at $y \longrightarrow \infty$, and a particular solution to Eq. (2.5) are sought in the form

$$G_3 = \sum_{n=0}^{\infty} C_n^{(3)} l^n + \omega_3 \ln y \frac{1}{y^3}\sum_{n=0}^{\infty} C_n^{(1)} l^n,$$

$$G_2 = \frac{1}{y}\sum_{n=0}^{\infty} C_n^{(2)} l^n + \frac{\omega_2}{y^3}\ln y \sum_{n=0}^{\infty} C_n^{(1)} l^n,$$

respectively.

Coefficients $C_n^{(1)}$ $(n \geq 1)$, $C_n^{(3)}$ $(n \geq 4)$, and $C_n^{(2)}$ $(n \geq 3)$ are determined by the method of undetermined coefficients,

$$C_n^{(1)} = \frac{1}{n(n+3)(n+5)}\{[(n-1)(3n^2+13n+8) + \gamma_1(n+2)(n+3) + \gamma_2(n+2)]C_{n-1}^{(1)} - [(n-1)(n-2)(3n+5) + 2\gamma_1(n^2-4) + \gamma_2(n-2) + \gamma_3(n+3)]C_{n-2}^{(1)} + (n-2)[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3]C_{n-3}^{(1)}\},$$

$$C_n^{(2)} = \frac{1}{(n+1)(n+3)(n-2)} \times \Bigg\{[(n-1)(3n^2+n-6) + \gamma_1 n(n+1) + n\gamma_2]C_{n-1}^{(2)} - [\gamma_3(n+1) + (n-1)(n-2)(3n-1) + 2\gamma_1 n(n-2) + \gamma_2(n-2)]C_{n-2}^{(2)} + (n-2)[(n-1)(n-3) + \gamma_3 + \gamma_1(n-3)]C_{n-3}^{(2)} + \frac{\omega_2}{\Gamma_0^2}\sum_{k=0}^{n-2}(n-k-1)\Delta_k - 6(-1)^n\frac{\omega_0!}{n!(\omega_0-n)!}\Bigg\},$$

$$C_n^{(3)} = \frac{1}{n(n+2)(n-3)} \times \Bigg\{(n-1)[3n^2-5n-4+\gamma_1 n+\gamma_2]C_{n-1}^{(3)} - [(n-1)(n-2)(3n-4) + 2\gamma_1(n-1)(n-2) + \gamma_2(n-2) + n\gamma_3]C_{n-2}^{(3)} + (n-2)[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3]C_{n-3}^{(3)} + \frac{\omega_3}{2\Gamma_0^3}\sum_{k=0}^{n-3}(n-k-2)(n-k-1)\Delta_k\Bigg\},$$

$$\Delta_k = (3k^2+16k+15)C_k^{(1)} - ((k-1)(6k+13) + \gamma_1(2k+5) + \gamma_2)C_{k-1}^{(1)} + (3(k-1)(k-2) + 2\gamma_1(k-2) + \gamma_3)C_{k-2}^{(1)}.$$

To calculate coefficients $C_n^{(1)}$, $C_n^{(2)}$, and $C_n^{(3)}$ using recurrent formulas, it is necessary to take into account that

$$C_0^{(1)} = 1, \quad C_0^{(3)} = 1, \quad C_1^{(3)} = 0, \quad C_2^{(3)} = \frac{1}{4}\gamma_3, \quad C_3^{(3)} = 1,$$

$$\frac{\omega_3}{2\Gamma_0^3} = -\frac{\gamma_3}{60}(10+3\gamma_1+\gamma_2), \quad C_0^{(2)} = 1, \quad C_2^{(2)} = 1,$$

$$\frac{\omega_2}{\Gamma_0^2} = \frac{1}{15}\left[\frac{1}{4}(2\gamma_1+\gamma_2+6\omega_0)(4+3\gamma_1+\gamma_2) + 3\gamma_3 + 3\omega_0(\omega_0-1)\right], \quad C_1^{(2)} = -\frac{1}{8}(2\gamma_1+\gamma_2+6\omega_0),$$

$$\omega_0 = \frac{\beta}{1+\alpha}.$$

Also, $C_n^{(k)}$ $(k = 1, 2, 3)$ equal zero at $n < 0$.

Function $g(y)$ entering into the expression for U_θ^e is related to function $G(y)$ by a functional relationship that is obtained from the continuity equation (the second equation in (1.1)) with regard to the temperature dependence of the density of the gaseous medium ($\rho_e = 1/t_{e0}$),

$$g(y) = G(y) + \frac{1}{2}y\left(\frac{dG(y)}{dy} - fG(y)\right),$$

$$f = \frac{1}{t_{e0}}\frac{dt_{e0}}{dy} = -\frac{l}{y(1+\alpha)}.$$

Thus, for the components of the mass velocity, we have

$$U_r^e = U_\infty\cos\theta(A_1G_1 + A_2G_2 + G_3), \tag{2.8}$$

$$U_\theta^e = -U_\infty\sin\theta(A_1G_4 + A_2G_5 + G_6), \tag{2.9}$$

where

$$G_k = \left(1 + \frac{l}{2(1+\alpha)}\right)G_{k-3} + \frac{1}{2}yG_{k-3}^1 \quad (k = 4, 5, 6),$$

and G_1^I, G_2^I, and G_3^I are the first derivatives of functions G_1, G_2, and G_3 with respect to y.

Constants of integration A_1 and A_2 are determined by substituting expressions (2.8) and (2.9) into the corresponding boundary conditions on the particle's surface. Having determined them and having integrated (1.6), we arrive at a formula for the drag of the medium to a heated spherical solid particle with nonuniformly distributed thermal sources with density q_i inside,

$$\mathbf{F}_\mu = 6\pi R\mu_{e\infty}f_\mu U_\infty\mathbf{n}_z, \tag{2.10}$$

where $f_\mu = \frac{2N_2}{3N_1}$, $N_1|_{y=1} = G_1G_2^I - G_2G_1^I$, $N_2|_{y=1} = G_1G_3^I - G_3G_1^I$, and $\mathbf{n}_z$ is the unit vector along the z axis.

A spherical particle falling under the action of the gravity force in a viscous medium starts to move with a constant velocity, since the gravity force is counterbal-

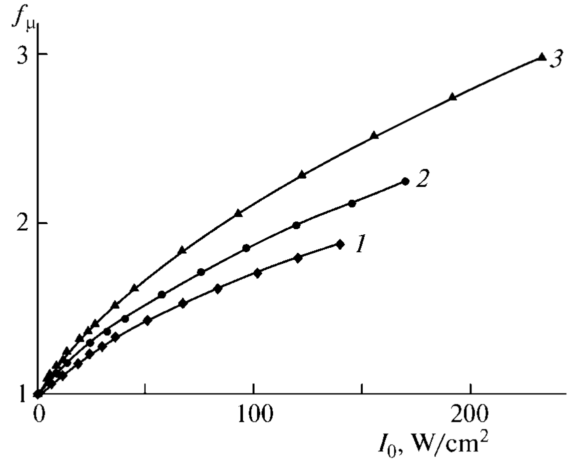

Fig. 1. Function f_μ vs. incident radiation intensity I_0 for $\alpha = \beta =$ (*1*) 0.5, (*2*) 0.7, and (*3*) 1.0.

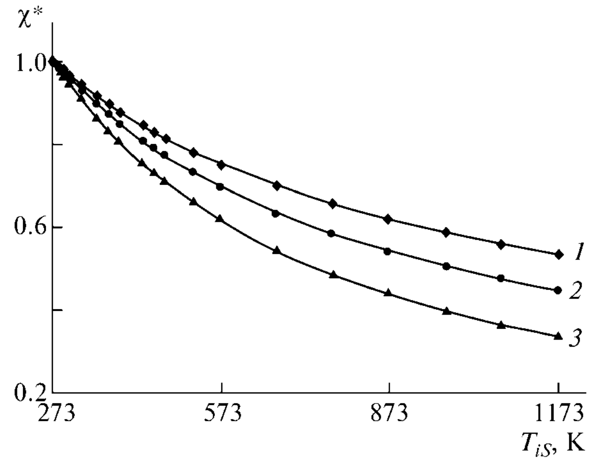

Fig. 2. Function χ^* vs. mean temperature T_{iS} of the particle's surface for $\alpha = \beta =$ (*1*) 0.5, (*2*) 0.7, and (*3*) 1.0.

anced by hydrodynamic forces. With regard to the buoyancy force, the gravity force acting on a particle takes the form

$$\mathbf{F} = (\rho_i - \rho_e)g\frac{4}{3}\pi R^3\mathbf{n}_z, \qquad (2.11)$$

where g is the free-fall acceleration. Here, subscript "i" marks the particle.

Equating expression (2.10) to (2.11), we obtain a formula for the gravitational fall velocity of a nonuniformly heated spherical particle (analogue of the Stokes formula),

$$\mathbf{U}_p = h_\mu \mathbf{n}_z\left(h_\mu = \frac{2}{9}R^2\frac{\rho_i - \rho_e}{\mu_{e\infty}f_\mu}g\right). \qquad (2.12)$$

3. RESULTS AND ANALYSIS

Thus, formulas (2.10) and (2.11) enable us to evaluate the force acting on a nonuniformly heated sphere and the velocity of its gravitational fall with allowance for the temperature dependences of the density of the gaseous medium and the molecular transfer coefficients (viscosity and thermal conductivity) at an arbitrary difference between the temperatures on and far from the particle's surface.

If the heating of the particle's surface is weak, i.e., if the mean temperature of the surface is close to the ambient temperature far from it ($\Gamma_0 \longrightarrow 0$), the temperature dependences of the density and molecular transfer coefficients can be neglected; then, $G_1 = 1$, $G_1^1 = -3$, $G_2 = 1$, $G_2^1 = -1$, $G_3 = 1$, $G_3^1 = 0$, $N_1 = 2$, and $N_2 = 3$. In this case, formulas (2.10) and (2.11) turn into the well-known Stokes expression for a sphere [11].

Constant Γ_0 depends on mean relative surface temperature t_{eS}, which, in the case of the nonuniformly heated surface, is determined by solving transcendental equation (2.3) and, consequently, depends on the density of heat sources nonuniformly distributed over the particle's volume. It then follows that functions G_1, G_2, etc., also depend on the density of heat sources, since they contain parameter $l = \dfrac{\Gamma_0}{y + \Gamma_0}$.

To estimate the contribution of internal heat sources (i.e., heating of the surface) to the gravitational fall velocity of a spherical aerosol particle, let us consider the simplest case when the particle absorbs like a black body. In this case, absorption occurs in a thin layer of thickness $\delta R \ll R$ that adjoins the heated part of the surface. The density of heat sources inside a layer δR thick is given by

$$q_i(r, \theta) = \begin{cases} -\dfrac{I_0}{\delta R}\cos\theta, & \dfrac{\pi}{2} \le \theta \le \pi, \quad R - \delta R \le r \le R \\ 0, & 0 \le \theta \le \dfrac{\pi}{2}, \end{cases}$$

where I_0 is the incident radiation intensity.

In this case, integral $\int_V q_i dV$ is taken easily: $\int_V q_i dV \approx \pi R^2 I_0$. Thus, setting the incident radiation intensity, one can estimate the mean relative temperature of the particle's surface by formula (2.3),

$$T_{eS} \approx T_{e\infty}\left(1 + \frac{1+\alpha}{4\lambda_{e\infty}T_{e\infty}}RI_0\right)^{1/(1+\alpha)}.$$

It is seen that the relative surface temperature of a spherical nonuniformly heated particle depends on its radius and the incident radiation intensity.

Curves depicted in Fig. 1 relate the values of function f_μ on incident radiation intensity I_0, while those in Fig. 2 relate the values of function

$$h_\mu(\chi^* = h_\mu/h_\mu|_{T_{iS} = 273\text{ K}})$$

with T_{iS}. Numerical estimates were conducted for copper particles with a radius of 100 μm suspended in air under normal conditions. As seen in the curves, heating of the particle's surface appreciably influences the drag to and velocity of the gravitation motion of the particle. These theoretical conclusions are confirmed by experimental data (see, for example, [15]).

CONCLUSIONS

The Stokes formula describing the steady gravitational fall of a nonuniformly heated spherical solid particle in a viscous incompressible gas is generalized for the case of the temperature dependences of the ambient gas density and molecular transfer coefficients (viscosity and thermal conductivity) and an arbitrary difference between the temperatures on and away from the particle's surface.

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