Chaos and coherence in an optical system subject to photon nondemolition measurement

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We consider the effect of quantum-nondemolition monitoring of the photon number on the quantum suppression of classical chaos in a recently proposed quantum optical model, the parametrically kicked nonlinear oscillator [G. J. Milburn, Phys. Rev. A **41**, 6567 (1990)]. Classically the effect of the quantum-nondemolition measurement is equivalent to a phase diffusion in the phase plane of the oscillator. A similar result holds in the quantum description, but in addition the measurement rapidly diagonalizes the system density operator in the photon-number basis. This has the effect of causing the evolution of the quantum moments to approach the corresponding classical moments and thus restores the classical dynamics for practical purposes.

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I. INTRODUCTION

It is now generally recognized that quantum mechanics greatly restricts the ability of kicked Hamiltonian systems to exhibit chaotic behavior [1–4]. This is due to discrete quasienergies and the ability of the Schrödinger equation to preserve the coherence between superposed quasienergy eigenstates. In a recent work [5] one of us (G.J.M.) proposed a quantum optical system which might enable an experimental test of the quantum limits to classical chaos in kicked Hamiltonian systems. This system comprises a single-mode cavity field interacting with a nonlinear refractive index (Kerr medium), and a parametric amplifier pumped by a pulsed pump field (see Fig. 1). This is equivalent to a nonlinear oscillator in which the frequency is proportional to the energy. The effect of the parametric kick is to momentarily turn the origin in phase space into a hyperbolic fixed point. A detailed discussion of the classical dynamics of this system, including dissipation, has already been given [5,6].

When this system is treated quantum mechanically the dynamical behavior is quite different to that predicted by classical mechanics. This is most easily seen in the behavior of the mean energy. For initial states located close to period-one fixed points in the classical picture, the mean energy as a function of kick number (that is, time) undergoes an initial oscillation before settling down to a steady value determined by the distance of the fixed points from the origin in phase space. The quantum mean follows the classical result for a short time but continues to oscillate, undergoing a collapse and revival sequence. If the corresponding classical system exhibits regular motion in this region of phase -space, the quantum moments undergo a regular collapse and revival sequence. If, however, the classical system is chaotic in this region of phase space, the mean energy oscillates in a highly irregular way, coming back close to the initial value at apparently random times. Such a change in the evolution of quantum moments has also been predicted in other systems [4].

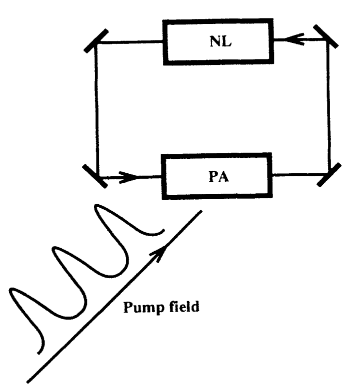

FIG. 1. Schematic representation of a parametrically kicked nonlinear oscillator realized as a cavity field interacting with a Kerr nonlinear medium and kicked by a pulse pumped parametric amplifier.

When dissipation is included the classical behavior is modified. Essentially what were fixed points become attractors. However, more complicated dynamical structures are also apparent for certain parameter values. The classical mean energy now settles down to a steady state as points are pulled onto the attractive points in the phase plane. The effect of dissipation on the quantum dynamics is to suppress coherent quantum features such as revival sequences, and for quite small values of the damping rate, the quantum and classical mean energy have practically the same time dependence. In Ref. [6] it was pointed out that the form of dissipation included could equivalently be interpreted as arising from a standard photon-counting measurement to monitor the energy of the system. Such measurements destroy photons and thus present a linear loss as far as the system is concerned. In this paper we consider a different kind of photon-counting measurement which does not absorb photons: a quantum nondemolition photon-counting measurement. Recently such measurements were suggested as a means of investigating the chaotic behavior of quantum systems [7].

Even though nondemolition photon counting does not introduce any systematic dissipation into the system, the measurement does have an effect on both the classical and quantum dynamics. In both cases the effect of the measurement is described in terms of phase diffusion in phase space. That is to say, the measurement adds a diffusive component to the oscillators nonlinear frequency. In classical mechanics we assume that the measurement may be so arranged that this effect can be made arbitrarily small without affecting the quality of the measurement. However, in quantum mechanics this is not possible. An accurate determination of the energy of the oscillator must add noise to the phase in accordance with a generalized uncertainty principle. In order to compare the quantum and classical dynamics we include the same level of phase diffusion in both descriptions.

The effect of the mean energy on the quantum evolution is similar to the effect of dissipation in that it causes the evolution of the mean energy to follow arbitrarily closely the classical result. If one regards a quantum-nondemolition measurement as the least disturbing way to monitor a system, then this result means that the observation of any coherent quantum features in nonlinear dynamics will be very difficult. A measurement, in accordance with the uncertainty principle, must at least add noise to a system, even if it is not accompanied by any systematic damping, and this is sufficient to restore apparently classical behavior.

II. PARAMETRICALLY KICKED NONLINEAR OSCILLATOR

Consider a single-mode cavity field interacting with a Kerr nonlinear medium. This interaction may be described by the Hamiltonian [5]

$$H_{\mathrm{NL}}=\frac{\chi}{2}(a^{\dagger})^2a^2\ , \tag{2.1}$$

where χ is proportional to the third-order nonlinear susceptibility of the medium and a is the boson annihilation operator for the intracavity field. This Hamiltonian is essentially proportional to the square of the Hamiltonian for a simple harmonic oscillator. It is thus easy to see that it will describe a nonlinear oscillator in which the frequency is proportional to the energy. This will induce a rotational sheer in the phase space. In fact, solving the Heisenberg equations of motion for a, we find

$$a(t)=e^{-i\mu a^{\dagger}a}a(0)\ , \tag{2.2}$$

where $\mu=\chi t$. We will assume that the parametric amplifier is turned on and off so rapidly compared to the free dynamics that the Hamiltonian describing this process may be approximated by

$$H_{\mathrm{PA}}=H_K\sum_{n=-\infty}^{\infty}\delta(t-n\tau)\ , \tag{2.3}$$

where τ is the period of free evolution between each pump pulse and the Hamiltonian H_K is given by

$$H_K=i\hbar\frac{\kappa}{2}[(a^{\dagger})^2-a^2]\ . \tag{2.4}$$

The effect of the parametric kicks is then given by

$$a(t_n^+)=a(t_n^-)\cosh r+a^{\dagger}(t_n^-)\sinh r\ , \tag{2.5}$$

where t_n^+ (t_n^-) is the time just after (before) the passage of the nth pulse and r is the effective parametric constant for the kick. In terms of dimensionless position and momentum variables $\hat{X}_1$ and $\hat{X}_2$ defined by

$$a=\hat{X}_1+i\hat{X}_2\ , \tag{2.6}$$

Eq. (2.5) may be written

$$\hat{X}_1(t_n^+)=e^{r}\hat{X}_1(t_n^-)\equiv g\hat{X}_1(t_n^-)\ , \tag{2.7}$$

$$\hat{X}_2(t_n^+)=e^{-r}\hat{X}_2(t_n^-)\equiv\frac{1}{g}\hat{X}_2(t_n^-)\ , \tag{2.8}$$

where $g\equiv e^r$ is the parametric gain. The effect of the parametric pump can be seen to expand the X_1 coordinate by g and compress the X_2 coordinate by $1/g$. The net effect is to move the phase-space point along a rectangular hyperbola.

To define the corresponding classical equations we replace the operators $(a,a^{\dagger})$ and $(\hat{X}_1,\hat{X}_2)$ by classical commuting phase-space variables (α,α^*) and (X_1,X_2). The classical map describing the system after each kick in terms of the state just after a previous kick is

$$X_1'=g[\cos(\mu R^2)X_1+\sin(\mu R^2)X_2]\ , \tag{2.9}$$

$$X_2'=\frac{1}{g}[-\sin(\mu R^2)X_1+\cos(\mu R^2)X_2]\ , \tag{2.10}$$

where $R^2=X_1^2+X_2^2$.

In Figs. 2(a) and 2(b) phase portraits of the classical dynamics are given for two values of the gain parameter. In Fig. 2(a) two period-one fixed points near the origin are quite evident. In Fig. 2(b) there is a large region of chaotic motion close to the origin, but period-one fixed points remain. Detailed discussions of these structures will be found in Ref. [6].

The quantum map is more easily written in terms of the state vector rather than the operators. This is easily done by solving for the state vector over the period of free evolution and then applying the unitary transformation corresponding to the kick transformations in Eq. (2.7). If the system is completely isolated from the environment during the period of free evolution, the quantum state map is

$$|\psi'\rangle=U|\psi\rangle\ , \tag{2.11}$$

where

$$U=\exp\left[\frac{r}{2}[(a^{\dagger})^2-a^2]\right]\exp\left[-i\frac{\mu}{2}(a^{\dagger})^2a^2\right]\ . \tag{2.12}$$

In order to compare the quantum and classical dynamics we consider the evolution of the mean energy. In the classical case we consider an initial, uniformly filled, circular distribution of points [6]. In the quantum case we consider an initial coherent state. The radius of the classical distribution is chosen to replicate the initial uncertainties in the phase-space variables for a coherent state. In Figs. 3(a), 4(a), 5(a), and 6(a) we plot the mean energy in both the quantum and classical case for the same parameters as used in the classical phase portraits shown in Figs. 2(a) and 2(b). Figure 4(a) depicts the mean energy versus kick number for an initial vacuum state when the phase space near the origin is dominated by regular trajectories. Whereas the classical prediction for the mean energy [Fig. 3(a)] in the cavity approaches a steady state, the quantum result deviates from this except on a very short times scale, giving a regular sequence of collapse and revival of oscillations. The revivals are a peculiarly quantum feature and reflect the underlying quantum coherence between quasienergy eigenstates preserved under unitary evolution. In Figs. 5(a) and 6(a) we change the parametric gain to access a region where, classically, there is a chaotic behavior near the origin. The initial state was taken to be $\alpha=1$, which is localized in the chaotic region. Once again the quantum prediction for the mean intensity only follows the classical mean for short times [cf. Fig. 5(a)]. Now the departure from the classical result is characterized by a highly irregular recurrence sequence. Such an irregular revival sequence was also observed in the chaotic region of the kicked top [4]. The transition from a regular revival sequence to an irregular sequence is a quantum reflection of the corresponding transition to chaotic behavior in the classical model.

The above results all assume that the system is isolated from any outside influence between the kicks, thus ensuring that the dynamics is completely unitary. This is, of course, a very difficult thing to arrange. Even for highly

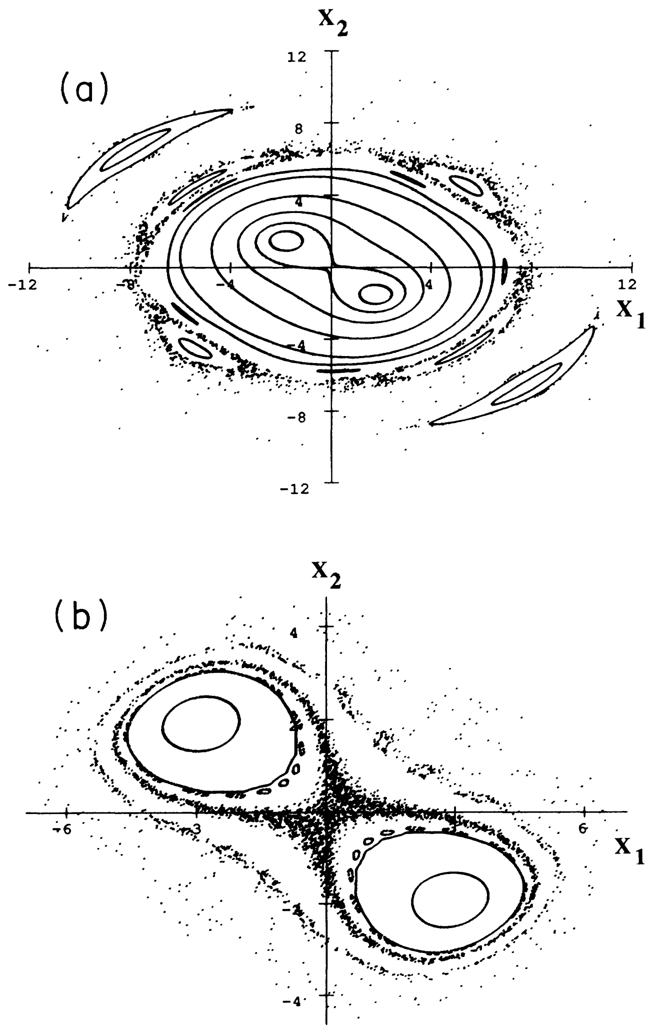

FIG. 2. The classical phase-space portrait for the model considered here. In (a) $g=1.2$ while in (b) $g=1.5$. In both cases $\chi=0.01\pi$.

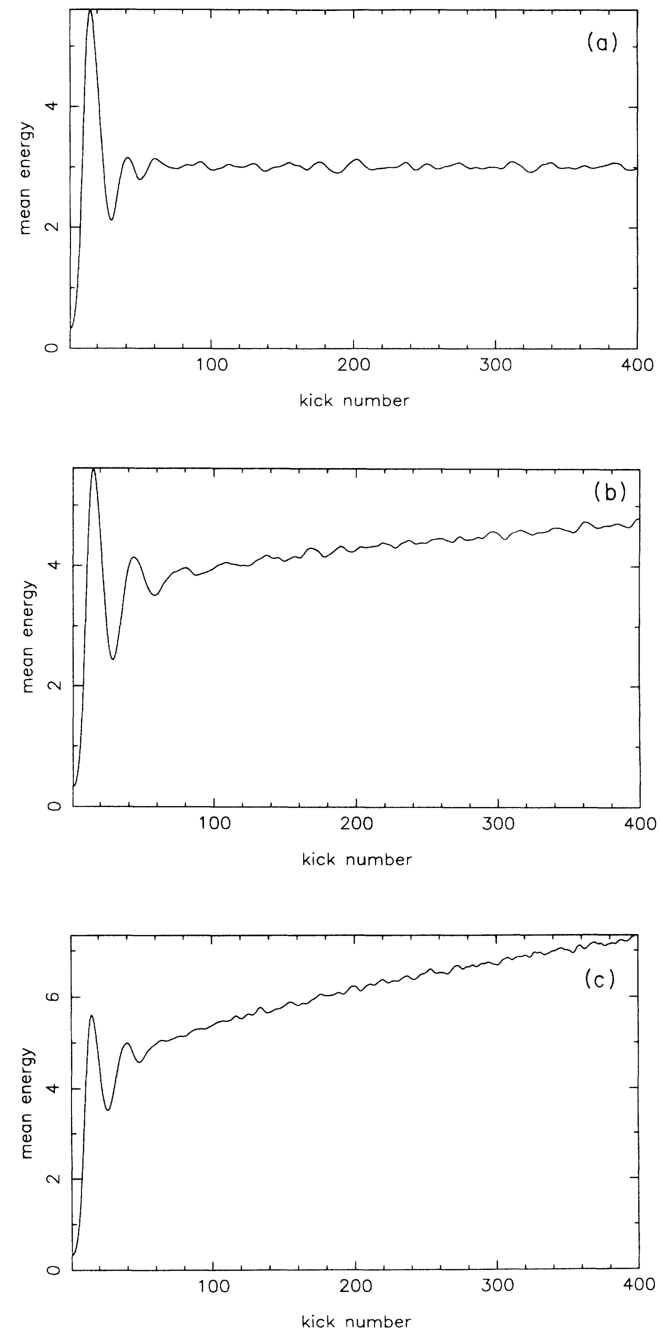

FIG. 3. Plot of the mean energy (in units of photon energy) vs kick number for the classical description; (a) $\gamma=0.0$, (b) $\gamma=0.0001$, and (c) $\gamma=0.001$. In all cases $\chi=0.01\pi, g=1.2$.

reflecting mirrors there will be some loss of energy from the cavity. The effect of damping on the quantum and classical dynamics was discussed at some length in Ref. [6]. At the very least we must couple the system to a measurement device if any of the above predictions are to be tested. What is the absolute minimum disturbance that we need to introduce into the system in order to extract information from it? In Sec. III we argue that this is answered by considering quantum-nondemolition measurement of the intracavity photon number, that is, the energy.

Dittrich and Graham [8] consider a similar measurement model to the one of this paper applied to the kicked rotor. In contrast to the chaotic system considered in this paper, the kicked rotor exhibits dynamic localization. This refers to a suppression of the classical diffusive growth in momentum in the absence of measurement. Dittrich and Graham show that measurement may restore this classical diffusive growth in momentum. In the model of this paper the departure of quantum from classical dynamics is manifest as regular or irregular partial revivals of the initial state, as opposed to the almost steady-state behavior of the classical Liouville dynamics.

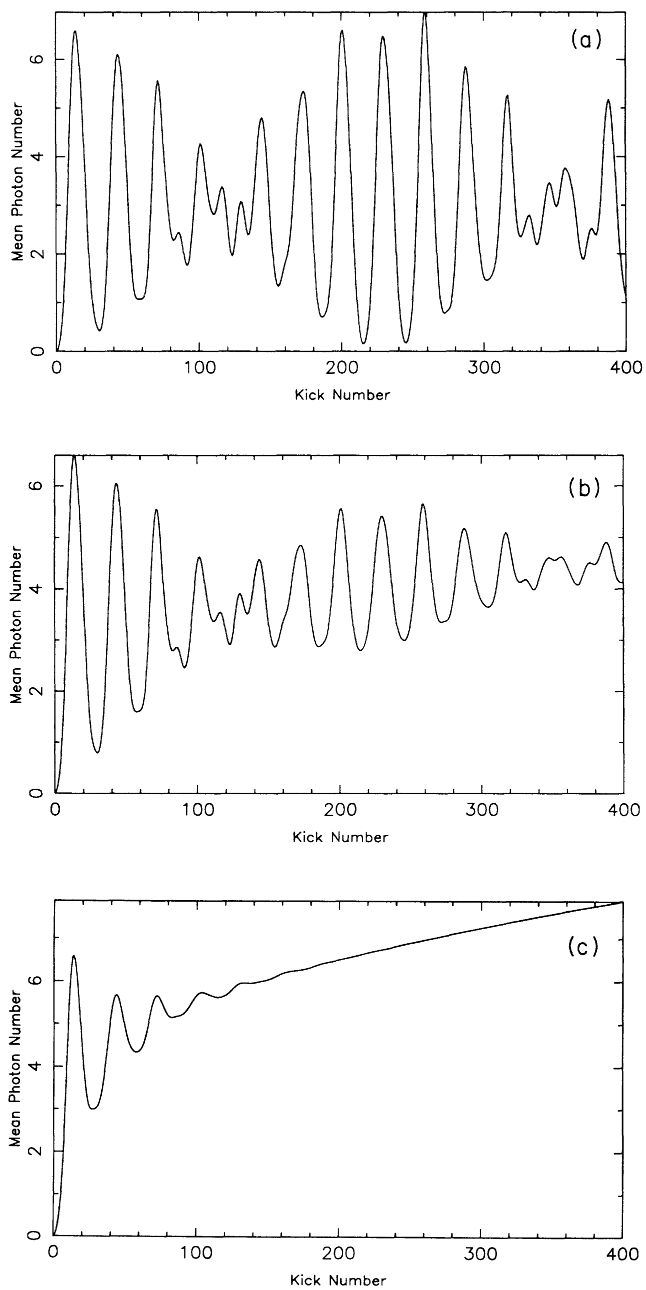

FIG. 4. Plot of the mean photon number in the quantum case vs kick number for (a) $\gamma=0.0$, (b) $\gamma=0.0001$, and (c) $\gamma=0.001$. In all cases $\chi=0.01\pi, g=1.2$.

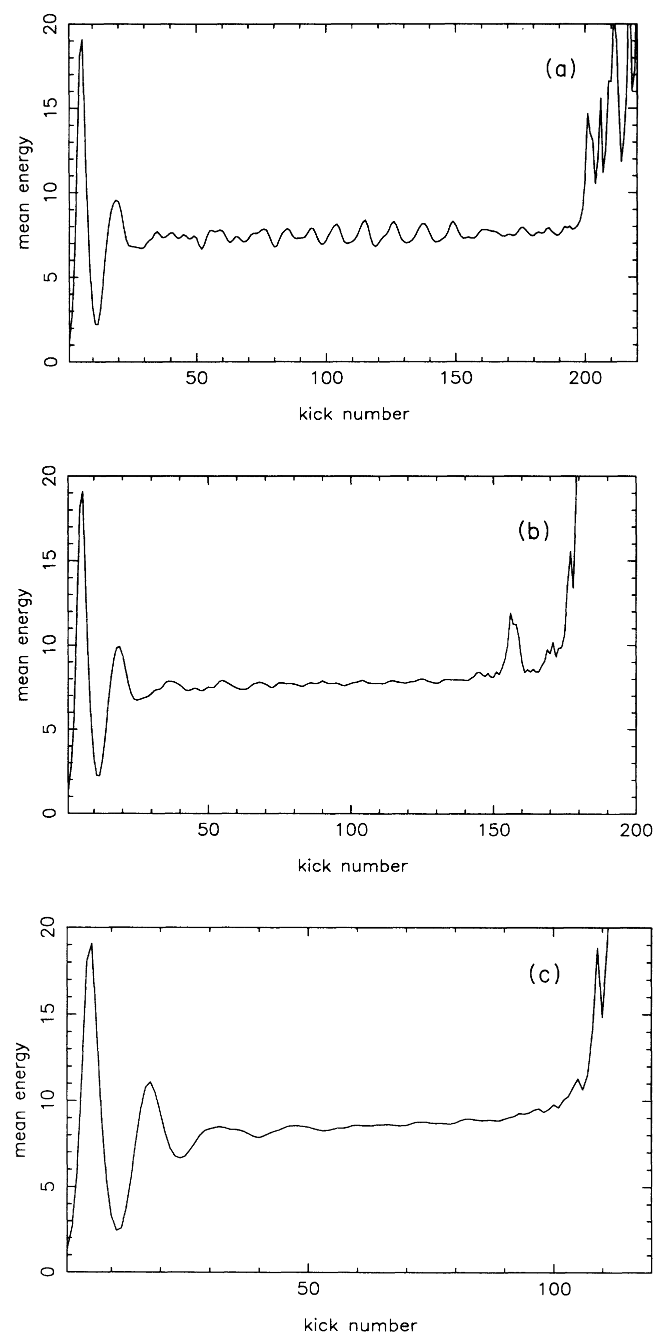

FIG. 5. Plot of the mean energy (in units of photon energy) vs kick number for the classical description; (a) $\gamma=0.0$, (b) $\gamma=0.00001$, and (c) $\gamma=0.0001$. In all cases $\chi=0.01\pi, g=1.5$.

As we shall show, the measurement tends to restore the classical behavior in the quantum problem. More analogous to the model of this paper is the discussion of the kicked top in Ref. [9], which is also free of dynamical localization.

It is important to note that the measurement changes both the quantum and classical dynamics. Thus in comparing the quantum and classical dynamics when the measurement is included, we must be careful to correctly define the relevant classical model. This important point was also noted by Dittrich and Graham [8]. A quite different measurement model is discussed by Adachi, Toda, and Ikeda [11]. Nonetheless, the effect of the measurement is to restore the classical Liouville dynamics.

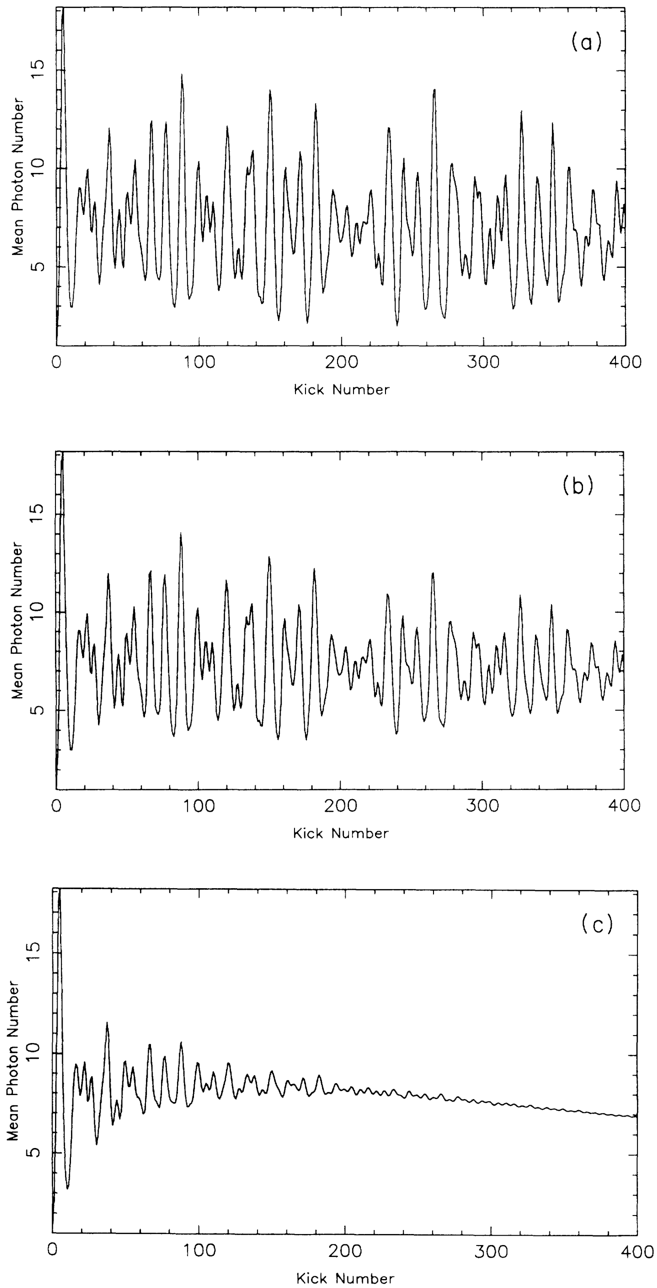

FIG. 6. Plot of the mean photon number in the quantum case vs kick number for (a) $\gamma=0.0$, (b) $\gamma=0.00001$, and (c) $\gamma=0.0001$. In all cases $\chi=0.01\pi, g=1.5$.

III. QUANTUM NONDEMOLITION PHOTON-NUMBER MEASUREMENTS

In quantum mechanics, unlike classical mechanics, measurement necessarily adds noise to the measured system. This is a direct result of the noncommutativity of the operators which represent physical quantities. For example, if the position of a free particle is measured to a high accuracy, the uncertainty in the momentum is greatly increased. This would not cause a problem if the momentum was isolated from the position variable. However, for a free particle the uncertainty in the momentum is coupled into the position. Thus a sequence of highly accurate position measurements made on a single free particle would produce practically a random sequence of numbers. The essential idea in a quantum-nondemolition measurement is to find quantities which may be measured over and over again yielding a near-determinate sequence of results [12]. In order to do this we must find an observable, a quantum-nondemolition (QND) observable, which is isolated from the "back-reaction" of the measurement. Any quantity that is a constant of the motion, and remains a constant when the interaction with the measuring device is included, will clearly be a QND observable.

For a simple harmonic oscillator (or a single-mode field) the energy is clearly a QND variable. Thus we need to couple the photon-number operator to a measuring device in such a way that it remains a constant of the motion. This is accomplished by an interaction of the form

$$H_M=\hbar E a^\dagger a \Gamma(t) , \tag{3.1}$$

where Γ is some operator of the measuring device and E is a coupling constant. There are a number of ways this can be done in an optical context [13–15]. The details need not concern us here. What we need is a means to describe the effect of the measurement on the cavity field state in the period between the kicks. This may be done following the general theory of continuous measurement presented in Ref. [12]. This approach has been used to determine the effect of measurement on other chaotic systems [8–10].

In the period between the kicks we assume that the effect of a QND measurement of the photon number may be described by the master equation,

$$\frac{d\rho}{dt}=\frac{-i}{\hbar}[H_{NL},\rho]-\gamma[a^\dagger a,[a^\dagger a,\rho]] , \tag{3.2}$$

where γ is a parameter which is determined by the noise added by the measuring device and the bandwidth of the measurement response [16]. There are a number of important features in Eq. (3.2). First, it is clear that the photon number remains a constant of the motion as required for a QND variable. Second, the double commutator term ensures that there is a decay of the off-diagonal elements of ρ in the photon-number basis. This ensures that the density operator becomes diagonal in the basis which diagonalizes the measured quantity, a manifestation of state reduction. Finally, the double commutator term drives a phase diffusion process in the nonlinear oscillator. This last point is essential in order to find the classical model corresponding to Eq. (3.2). We now describe in some detail how this phase diffusion comes about.

To find the classical version of the master equation we resort to a c-number representation of the density operator based on the Q function [17]. This function is defined by

$$Q(\alpha,\alpha^*)=\langle\alpha|\rho|\alpha\rangle \; , \tag{3.3}$$

where $|\alpha\rangle$ is a Glauber oscillator coherent state [18]. An evolution equation for this function may be found by computing the coherent-state matrix elements of the right-hand side of Eq. (3.2). We will only calculate the contribution from the double commutator with the number operator which describes the effect of the measurement. The contribution from the nonlinear term is given in Ref. [15]. The result is

$$\frac{\partial Q}{\partial t}=\gamma\left[\frac{\partial}{\partial\alpha}\alpha Q+\frac{\partial}{\partial\alpha^*}\alpha^* Q+2\frac{\partial^2}{\partial\alpha\partial\alpha^*}|\alpha|^2 Q -\frac{\partial^2}{\partial\alpha^2}\alpha^2 Q-\frac{\partial^2}{\partial\alpha^{*2}}\alpha^{*2} Q\right] . \tag{3.4}$$

This equation is in the form of a Fokker-Planck equation and thus we are able to write down the corresponding Ito stochastic differential equations [19],

$$\frac{d\alpha(t)}{dt}=-\gamma\alpha(t)+i\sqrt{2\gamma}\alpha(t)\xi(t) \; , \tag{3.5}$$

where $\xi(t)$ is a δ-correlated complex Gaussian noise source satisfying

$$\langle\xi(t)\xi^*(t')\rangle=\delta(t-t') \; . \tag{3.6}$$

To see that this describes phase diffusion we transform to polar coordinates (r,θ), where $\alpha=re^{i\theta}$, using the Ito change of variable rules [19],

$$\frac{dr}{dt}=0 \; , \tag{3.7}$$

$$\frac{d\theta}{dt}=\sqrt{2\gamma}\xi_\theta(t) \; , \tag{3.8}$$

where $\xi_\theta(t)$ is a real δ-correlated noise term. Equation (3.8) describes diffusion in the phase with diffusion constant of 2γ.

We are now in a position to define the classical model equivalent to the quantum evolution equation in Eq. (3.2). Adding the nonlinear phase shift arising from the Kerr nonlinearity to the Ito stochastic differential equation [Eqs. (3.7) and (3.8)], we obtain

$$\frac{dr}{dt}=0 \; , \tag{3.9}$$

$$\frac{d\theta}{dt}=-\chi r^2+\sqrt{2\gamma}\xi_\theta(t) \tag{3.10}$$

as the defining equation for the classical dynamics in the period between the kicks. The effect of the measurement is to add to the nonlinear phase shift a random phase shift described by a diffusion process. Dittrich and Graham [8] also find a phase diffusion process describes the effect of the measurement on the classical dynamics.

Is this the correct classical model to describe the effect of measurement? It has been demonstrated that the Q function does indeed give the classical Liouville dynamics in the semiclassical limit [17]. This limit must leave the phase diffusion terms unchanged as they describe a true classical diffusion process independent of Planck's constant. It is possible that the nonlinear dynamics may introduce multiplicative noise, or worse, nonpositive definite diffusion, into the evolution equation. However, as shown in [17], such terms are not significant in the semiclassical limit except on very long time scales.

IV. QUANTUM AND CLASSICAL DYNAMICS

To compare the quantum and classical dynamics we compute the mean energy (mean photon number) in the cavity for an initial uniform circular distribution of points in the classical case and an initial coherent state in the quantum case. In the classical case this is done by simulating the stochastic trajectory for each point based on the stochastic differential equations (3.9) and (3.10), and then applying the kick. In the quantum case we solve the master equation (3.2) in the photon-number basis and compute the average photon number after each kick. The kick transformation in the photon-number basis is given in Ref. [5]. In Figs. 3(a)–3(c) we plot the mean photon number for the classical problem with $g=1.2$, corresponding to the phase portrait shown in Fig. 2(a). The initial distribution was centered on the origin. We see that the effect of phase diffusion on the classical model is to cause a slow diffusion of the energy. In the absence of measurement the initial distribution moves onto two annular regions centered on each of the period-one fixed points. This is demonstrated in Fig. 7(a), where we depict the successive evolution of an initial cloud of points. When the measurement is included, each of these annular distributions then slowly diffuses in width [Fig. 7(b)]. It would appear that around the fixed points there is some coupling between the diffusing phase and the radius relative to the fixed points. The rate of diffusion increases as the measurement constant γ increases. In Figs. 4(a)–4(c) we plot the quantum mean photon number for an initial coherent state centered on the origin (this is the ground state of the cavity mode), and for a gain parameter $g=1.2$. For even a very small value of the measurement parameter, the time dependence of the quantum average becomes very similar to the classical distribution. The quantum recurrence features are destroyed and a classical diffusion of the energy is apparent.

In Figs. 5(a)–5(c) mean energy versus kick number is plotted for the classical case with $g=1.5$ and an initial distribution centered on (1,0). The paths around this region are chaotic. The effect of diffusion is apparent in Fig. 5(c) for most of the evolution and is similar to the nonchaotic case. However, towards the end of the simu-

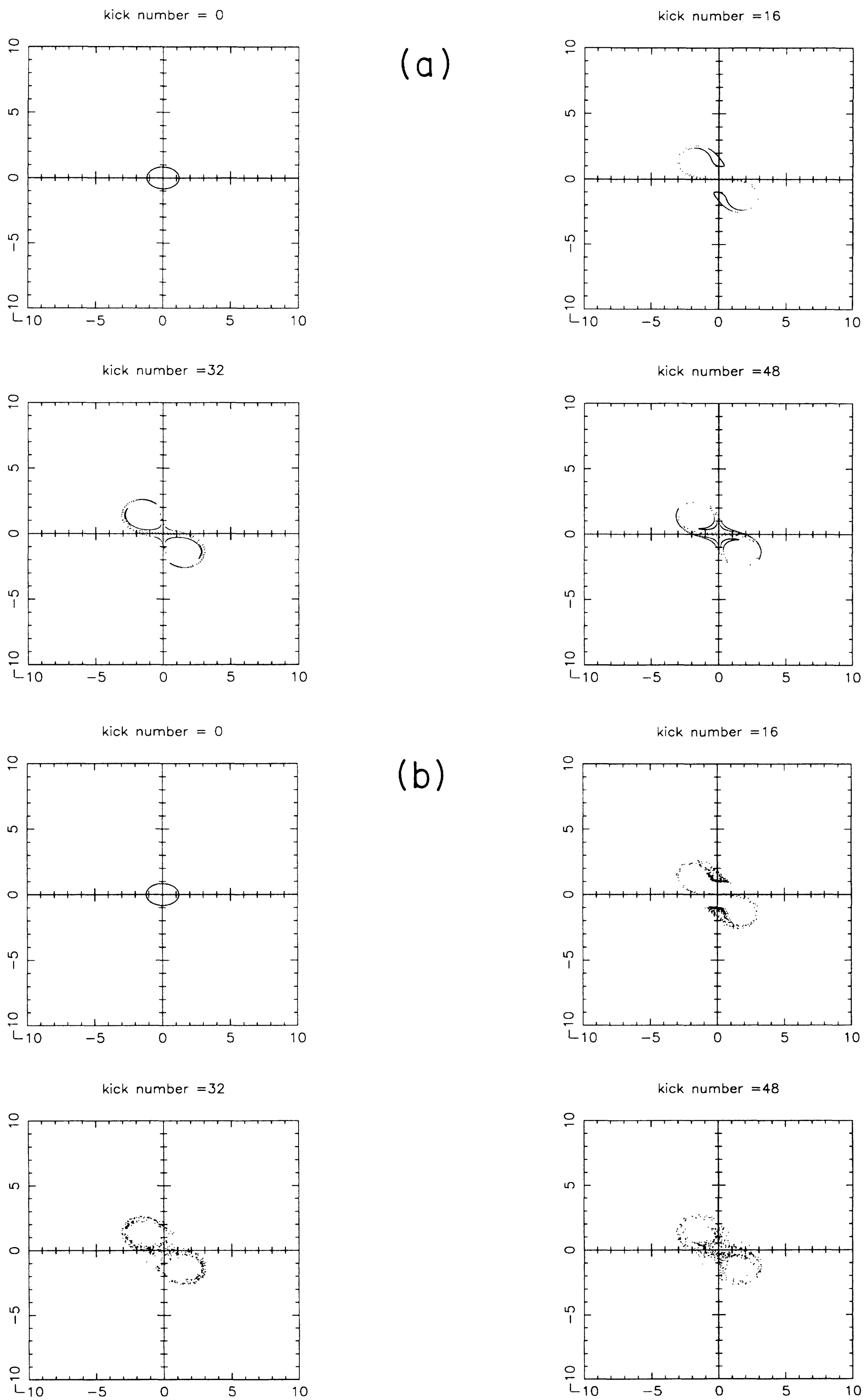

FIG. 7. Frames showing the evolution of a contour in classical phase space. In (a) $\gamma=0.0$ while in (b) $\gamma=0.0001$.

lation we see a very rapid growth in the mean energy. This rapid growth occurs at shorter times for larger diffusion constants. This feature is an artifact of our numerical simulation as we now explain.

The initial state was taken as a uniform circular distribution of points which weights all points equally. For $g=1.5$ the initial state overlaps a large number of chaotic trajectories which can eventually take a particular point to very large energies; energies so large in fact that the contribution of a few points totally dominates the mean energy. To avoid this one could try and take a larger number of initial points so that the weights of each point, including those points which grow to large energies, is sufficiently small. However, it is inherent in the very nature of chaos that the number of initial points would grow exponentially fast the longer we require an accurate simulation. It is no accident that these energy spikes do not occur in the regular region. (Another approach, more like the quantum case, would be to take an initial Gaussian distribution of points.) It is thus not surprising that increased diffusion causes a rapid energy growth to occur at smaller times. The diffusion increases the rate at which points move into regions which can move far from the origin.

In Fig. 8(a) we plot the energy of the point with maximum energy in the ensemble as it enters this region of escaping trajectories. The mean energy shown in Fig. 8(b) diverges at roughly the same time as the time at which this point makes its escape. Thus we feel confident in identifying the cause of the rapid growth of the mean energy as due to the escape of some of the points of the initial density into globally chaotic regions.

In Fig. (6) we use the same values of γ in the quantum case. As discussed in the Introduction, the quantum revivals are now more irregular; nevertheless, the effect of the measurement is to suppress these recurrences and restore classical behavior. Our quantum simulation is also subject to numerical limitations, though of quite another kind to that discussed above for the classical case. This limitation is indicated by the steady decline of the mean photon number at longer times. If the mean photon number becomes large, the number of basis states must be increased significantly. This of course requires very long computation times. If sudden growth does occur and the number of basis states is not increased there is a rapid breakdown of the validity of the simulation, the effect of which is to produce a fall in the value of the simulated photon number. This effect is demonstrated in Fig. 9. The parameters of the model used to generate both Figs. 9(a) and 9(b) are identical, the only change between the two being that (a) uses basis states up to $n=41$, as do all

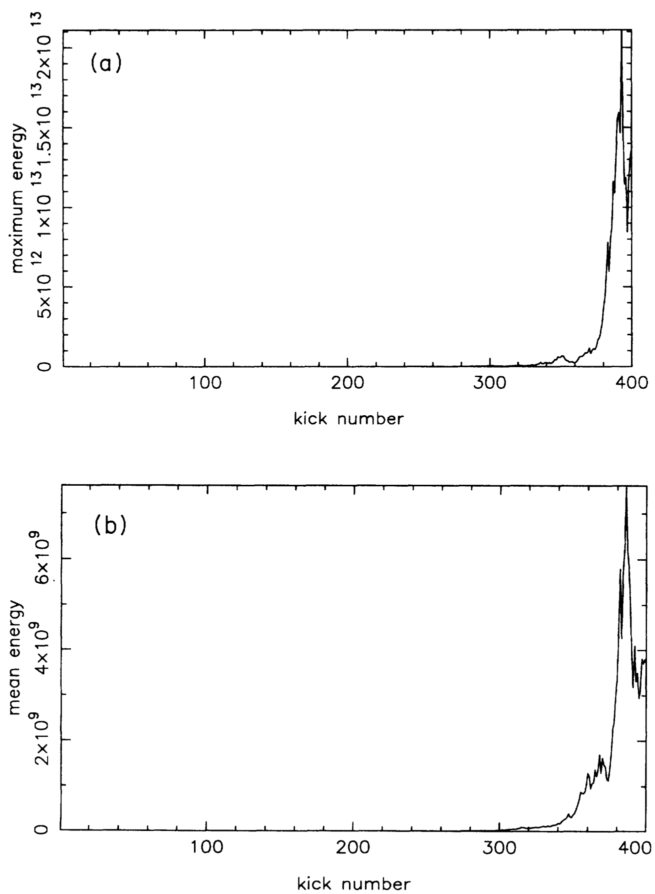

FIG. 8. Plot comparing the energy of the point with (a) maximum energy and (b) mean energy (in units of photon energy) in the region of classical divergence. In both cases $\gamma=0.1, g=1.2, \chi=0.01\pi$.

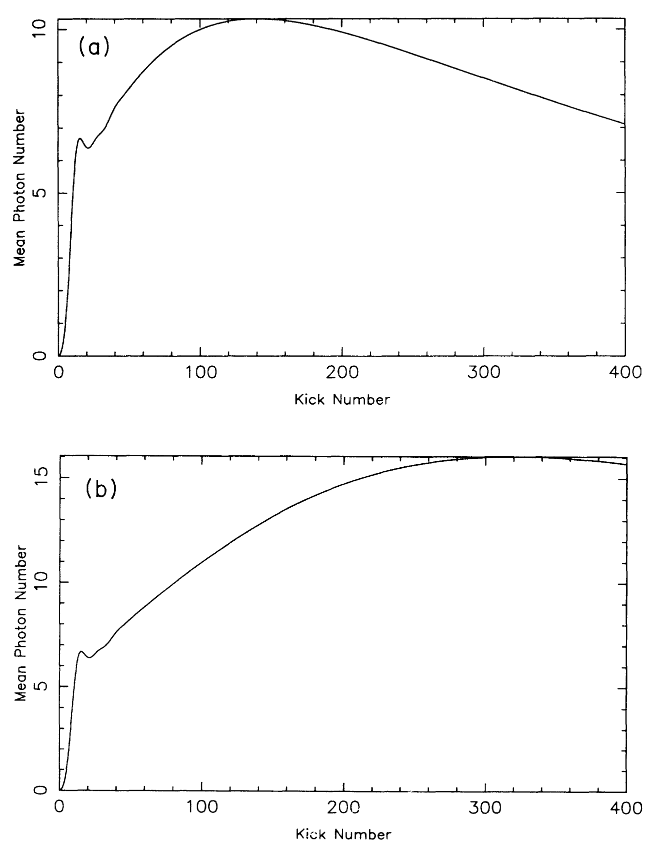

FIG. 9. Plot of the mean photon number vs kick number in the quantum case for a truncation of the density matrix of (a) $n=41$ and (b) $n=90$. In both cases $\gamma=0.01, g=1.2, \chi=0.01\pi$.

the other simulations in this paper, while (b) uses basis states of up to $n = 90$. It is seen that the suppression of quantum recurrences occurs at the same time and in the same way in both figures; however, while in the $n = 41$ model the photon number shows a decline after about 140 kicks, the $n = 90$ model shows a similar decrease only after kick number 320. So this apparent decrease in the mean photon number is seen to be due to the limited matrix size which must be used rather than a real feature of the theoretical model. Once the limitations of the numerical result are located and understood it is seen that they only become important at times after the destruction of quantum coherence has occurred. Over times for which the simulations are valid the quantum and classical dynamics do indeed coincide.

V. CONCLUSION

We have discussed the effect of photon-number quantum-nondemolition measurements on the nonlinear dynamics of a parametrically kicked nonlinear oscillator in both the quantum and classical case. In the absence of measurement the mean photon number in the quantum problem behaves quite differently to the average energy in the classical description, exhibiting quantum recurrences. When the measurement is included the effect on the classical dynamics is to cause a slow diffusion of the mean energy after an initial oscillatory period. The effect of the measurement on the quantum behavior is more dramatic. For even a very weak measurement coupling, quantum recurrences are destroyed and the classical diffusive behavior is restored. In Fig. 4(b), with $\gamma = 0.0001$, quantum recurrences are still apparent but are obviously damped. So in this case $\gamma = 0.0001$ is close to the minimum needed to restore classical behavior.

It has been argued that a quantum-nondemolition measurement is the very least disturbance of the quantum system that would permit the extraction of information needed to compare the quantum and classical dynamics. Such a measurement is not accompanied by any energy loss from the system but necessarily causes a phase diffusion in the phase plane. The above results indicate that this is sufficient to destroy the quantum coherence features responsible for the deviation of quantum and classical dynamics, even for very weak measurements. We conclude that searching for a breakdown of classical dynamics in nonlinear systems arising from underlying quantum coherence is going to be very difficult.

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