The Effect of the Medium on the Thermocapillary Force of a Heated Droplet Drifting in a Viscous Liquid in the Field of External Temperature Gradient

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Abstract—The Stokes approximation is used to theoretically investigate the effect of the medium on the thermocapillary drift of a heated droplet inside which nonuniformly distributed heat sources are operative. The droplet drift proceeds in a viscous incompressible liquid under conditions of arbitrary temperature differences between the particle surface and a region away from it. The problem is solved in view of the exponential-power form of the temperature dependence of viscosity. In the process of solving the hydrodynamic equations, analytical expressions are derived for the thermocapillary force acting on the droplet. It is demonstrated that, in the approximation applied, the droplet retains its spherical shape. Numerical estimates indicate that the motion of the medium affects considerably the magnitude of the thermocapillary force.

FORMULATION OF THE PROBLEM

We treat a steady-state motion of a liquid droplet with the viscosity μ_i, inside which nonuniformly distributed heat sources (sinks) of power q_i are operative. The droplet is suspended in another viscous liquid which is immiscible with the droplet and fills the entire space. At infinity, the liquid is at rest; a constant temperature gradient (∇T) is preassigned.

Following are the differences between our formulation of the problem and that made in [1–6]: (1) the motion is treated under conditions of arbitrary temperature differences between the droplet surface and the region away from it, (2) internal heat sources nonuniformly distributed over the droplet volume are taken into account, and (3) the temperature dependence of viscosity has the exponential-power form.

It is assumed that the density, thermal conductivity coefficient, and the heat capacity of liquids outside and inside the droplet are constant; the thermal conductivity coefficient of the droplet exceeds that of the surrounding liquid, and the surface tension coefficient is an arbitrary function of temperature ($\sigma = \sigma(T)$). A fairly slow droplet motion is treated, when the values of the Reynolds and Peclet numbers are small. The droplet retains the spherical shape (the droplet deformation will be treated below). The problem is axisymmetric relative to the z-axis passing through the droplet center in parallel with the external temperature gradient.

The presence of heat sources (sinks) inside the droplet brings about a considerable difference between the average temperature of the droplet surface and the temperature of the surrounding liquid away from the droplet. The heating of the droplet surface may affect the thermal characteristics of the surrounding liquid and the distribution of the velocity and pressure fields in its neighborhood. Because it is only the viscosity coefficient of liquid that depends on temperature [7], we will take this dependence into account using the exponential-power formula which enables one to describe the variation of viscosity in a wide temperature range with any desired accuracy,

$$\mu_e = \mu_\infty\left[1 + \sum_{n=1}^{\infty} F_n\left(\frac{T_e}{T_\infty} - 1\right)^n\right]\exp\left\{-A\left(\frac{T_e}{T_\infty} - 1\right)\right\}, \quad (1)$$

where A and F_n are constants, T_∞ is the temperature of liquid away from the heated droplet, and $\mu_\infty = \mu_e(T_\infty)$. Here and below, the subscripts e and i indicate the external liquid and the droplet, respectively. At $F_n = 0$, this formula may be reduced to the known Reynolds relation [7].

If the coefficients F_n are disregarded, the relative error of determination of viscosity by Eq. (1) may be as high as 40%. Given in Tables 1 and 2 for illustration are the values of F_n for two liquids, namely, glycerin and water. The coefficients F_n were calculated using the Maple V mathematical software package (the dynamic viscosity μ_{calc} was calculated by formula (1)) and compared with the experimentally obtained values of dynamic viscosity μ_{exp}. One can see that the relative error does not exceed 3%.

At $\lambda_i > \lambda_e$, the droplet motion will proceed with small temperature differences in the droplet volume, and the viscosity coefficient of the droplet may be assumed to be constant. The boundary conditions (given by Eq. (5) below) on the droplet surface ($r = R$) include the equality of the temperatures, the continuity of the heat fluxes, the difference between the tangential

Table 1. Comparison of the calculated and experimentally obtained values of the coefficient of dynamic viscosity for glycerin

$A = 17.29,\ F_1 = 1.228,\ F_2 = 7.022,\ T_\infty = 303$ K			
T, °C	μ_{calc} (Pa s)	μ_{exp} (Pa s)	$\frac{\lvert\mu_{calc} - \mu_{exp}\rvert}{\mu_{exp}} \times 100\%$
30	0.600000	0.600	0.00
40	0.327979	0.330	0.61
50	0.182001	0.180	1.11
60	0.102619	0.102	0.60
70	0.058797	0.059	0.34
80	0.034212	0.035	2.25
90	0.020189	0.021	3.86

Table 2. Comparison of the calculated and experimentally obtained values of the coefficient of dynamic viscosity for water

$A = 5.779,\ F_1 = -2.318,\ F_2 = 9.118,\ T_\infty = 273$ K			
T, °C	μ_{calc} (Pa s)	μ_{exp} (Pa s)	$\frac{\lvert\mu_{calc} - \mu_{exp}\rvert}{\mu_{exp}} \times 100\%$
0	0.0017525	0.0017525	0.00
10	0.0013151	0.0012992	1.22
20	0.0010089	0.0010015	0.74
30	0.0007943	0.0007971	0.35
40	0.0006433	0.0006513	1.22
50	0.0005359	0.0005441	1.51
60	0.0004581	0.0004630	1.06
70	0.0004002	0.0004005	0.07
80	0.0003556	0.0003509	1.35
90	0.0003199	0.0003113	2.76

velocities for the internal and external media, and the continuity of the tangential components of the stress tensor.

We will select the origin of fixed coordinates in an instantaneous position of the center of a spherical droplet of radius R. Within the assumptions made, the equations and boundary conditions for velocity and temperature in spherical coordinates will be written as [8, 9]

$$\frac{\partial P_e}{\partial x_k} = \frac{\partial}{\partial x_j}\left\{\mu_e\left(\frac{\partial U_k^e}{\partial x_j} + \frac{\partial U_j^e}{\partial x_k}\right)\right\}, \quad \operatorname{div} U_e = 0, \tag{2}$$

$$\mu_i \Delta U_i = \nabla P_i, \quad \operatorname{div} U_i = 0, \tag{3}$$

$$\rho_e c_{pe}(U_e \nabla) T_e = \lambda_e \Delta T_e, \quad \Delta T_i = -q_i/\lambda_i, \tag{4}$$

$$\begin{gathered} r = R, \quad T_e = T_i, \quad \lambda_e \frac{\partial T_e}{\partial r} = \lambda_i \frac{\partial T_i}{\partial r}, \\ U_r^e = U_r^i = -U\cos\theta, \quad U_\theta^e = U_\vartheta^i, \\ \mu_e\left[r\frac{\partial}{\partial r}\left(\frac{U_\theta^e}{r}\right) + \frac{1}{r}\frac{\partial U_r^e}{\partial \theta}\right] + \frac{1}{r}\frac{\partial \sigma}{\partial T_i}\frac{\partial T_i}{\partial \theta} \\ = \mu_i\left[r\frac{\partial}{\partial r}\left(\frac{U_\theta^i}{r}\right) + \frac{1}{r}\frac{\partial U_r^i}{\partial \theta}\right], \end{gathered} \tag{5}$$

$$\begin{gathered} r \longrightarrow \infty, \quad U_e \longrightarrow 0, \quad P_e \longrightarrow P_\infty, \\ T_e \longrightarrow T_{\infty +} |\nabla T| r\cos\theta, \end{gathered} \tag{6}$$

$$r \longrightarrow 0, \quad |U_i| \neq \infty, \quad P_i \neq \infty, \quad T_i \neq \infty. \tag{7}$$

Here, P is the pressure; R is the droplet radius; ρ is the density; T is the temperature; λ is the thermal conductivity coefficient; x_k denotes Cartesian coordinates; U_k denotes components of the mass velocity U; q_i is the density of heat sources inside the droplet, depending on the spherical coordinates r and θ ($0 \le \theta \le \pi$); $|\nabla T|$ is the preassigned constant temperature gradient at a large distance from the droplet, parallel to the axis $0z$; and c_{pe} is the heat capacity at constant pressure.

The determining parameters of the problem include the constant quantities ρ_e, μ_∞, λ_e, and c_{pe} and the values of R, $|\nabla T|$, T_∞, and U. Three dimensionless combinations may be made up of these parameters: $\varepsilon = R|\nabla T|/T_\infty \ll 1$ characterizing the temperature difference over the particle diameter and the Reynolds and Peclet numbers [9].

We will render Eqs. (2)–(4) and boundary conditions (5)–(7) dimensionless by introducing the dimensionless velocity, temperature, and pressure as follows: $V_k = U_k/U$, $t_k = T_k/T_\infty$, and $p_k = P_k/P_\infty$ ($P_\infty = (\mu_\infty U)/R$, $k = e, i$). Here, the droplet radius R serves as the unit of distance variation, T_∞ as the unit of temperature, P_∞ as the unit of pressure, and U as the unit of velocity where $U \sim (|\mu_\infty|\nabla T||/(\rho_e T_\infty))$.

The motion of nonuniformly heated droplets in a viscous medium under the effect of the external temperature gradient brings about the emergence of tangential stresses on the droplet surface as a result of variation of the surface tension coefficient with temperature. Note that this is the first ever attempt at estimating the effect of the motion of the medium on the thermocapillary force acting on a nonuniformly heated droplet in a viscous liquid.

TEMPERATURE FIELDS OUTSIDE AND INSIDE A HEATED DROPLET

In order to find the force acting on a nonuniformly heated droplet and the velocity of its motion, one must determine the temperature fields outside and inside the droplet. For this purpose, we will solve Eq. (4) with relevant boundary conditions. The distribution of temperature inside a nonuniformly heated droplet will be sought in the form

$$t_i(y,\theta) = \sum_{n=0}^{\infty} \varepsilon^n t_{in}(y) P_n(\cos\theta), \tag{8}$$

where $y = r/R$, $t_{in}(y)$ is a function dependent on the radial coordinate, and $P_n(\cos\theta)$ denotes Legendre polynomials.

We substitute Eq. (8) into the second one of Eqs. (4), divide the variables, and use the properties of orthogonality of Legendre polynomials to derive the following solution for the function $t_{in}(y)$ satisfying the condition of finiteness of solution at $r \longrightarrow 0$:

$$\begin{aligned} t_{in}(y) = \Bigg\{ & B_n y^n + \frac{1}{(2n+1)y^{n+1}} \int_1^0 \psi_n(y) y^n dy \\ & + \frac{1}{2n+1}\left[y^n \int_1^y \frac{\psi_n(y)}{y^{n+1}} dy - \frac{1}{y^{n+1}} \int_1^y \psi_n(y) y^n dy \right] \Bigg\}. \end{aligned} \tag{9}$$

Here, $\psi_n(y) = -\dfrac{R^2}{\lambda_i T_\infty} y^2 \dfrac{2n+1}{2} \int_{-1}^{+1} q_i P_n(\cos\theta) d(\cos\theta)$.

The force acting on a nonuniformly heated droplet is determined by integration of the stress tensor over the particle surface [9]. Therefore, in what follows, we use expressions for the functions $t_{i0}(y)$ and $t_{i1}(y)$ which have the form

$$\begin{aligned} t_{i0}(y) = \Bigg\{ & B_0 + \frac{1}{4\pi R T_\infty \lambda_i y} \int_V q_i dV \\ & + \int_1^y \frac{\psi_0}{y} dy - \frac{1}{y} \int_1^y \psi_0 dy \Bigg\}, \end{aligned} \tag{10}$$

$$\begin{aligned} t_{i1}(y) = \Bigg\{ & B_1 y + \frac{1}{4\pi R^2 T_\infty \lambda_i y^2} \int_V q_i z dV \\ & + \frac{1}{3}\left[y \int_1^y \frac{\psi_1}{y^2} dy - \frac{1}{y^2} \int_1^y \psi_1 y dy \right] \Bigg\}. \end{aligned} \tag{11}$$

In Eqs. (10) and (11), the integration is performed over the entire volume of the heated droplet, $z = r\cos\theta$.

In spherical coordinates, the equation describing the temperature distribution outside a heated particle has the form

$$\varepsilon \Pr_\infty \left(V_r^e \frac{\partial t_e}{\partial y} + \frac{V_\theta^e}{y} \frac{\partial t_e}{\partial \theta} \right) = \Delta t_e, \tag{12}$$

where $\Pr_\infty = \mu_\infty c_{pe}/\lambda_e$ is the Prandtl number.

The expression for t_e in the general case may be represented in the form of series expansion in ε. We have performed the solution of the problem in a first approximation in ε; therefore, we have

$$t_e(y,\theta) = t_{e0}(y) + \varepsilon t_{e1}(y,\theta), \quad t_{e1} = \tau(y)\cos\theta. \tag{13}$$

We substitute expressions (13) into Eq. (12) to derive the following set of equations:

$$\Delta t_{e0} = 0, \tag{14}$$

$$y = 1, \quad t_{e0} = t_{i0}, \quad \lambda_e \frac{\partial t_{e0}}{\partial y} = \lambda_i \frac{\partial t_{i0}}{\partial y}, \tag{15}$$

$$y \longrightarrow \infty, \quad t_{e0} \longrightarrow 1 \tag{16}$$

and, accordingly,

$$\Pr_\infty V_r^e \frac{\partial t_{e0}}{\partial y} = \Delta t_{e1}, \tag{17}$$

$$y = 1, \quad t_{e1} = t_{i1}, \quad \lambda_e \frac{\partial t_{e1}}{\partial y} = \lambda_i \frac{\partial t_{i1}}{\partial y}, \tag{18}$$

$$y \longrightarrow \infty, \quad t_{e1} \longrightarrow y\cos\theta. \tag{19}$$

The general solution of Eq. (14) satisfying the boundary conditions given by Eqs. (15) and (16) in view of Eq. (10) will have the form

$$t_{e0}(y) = \left(1 + \frac{\gamma}{y}\right), \tag{20}$$

where $\gamma = t_s - 1$ is the dimensionless parameter characterizing the temperature difference between the droplet surface and the region away from it, and $t_s = T_s/T_\infty$, where T_s is the average temperature of the droplet surface defined by the formula

$$\frac{T_s}{T_\infty} = 1 + \frac{1}{4\pi R \lambda_e T_\infty} \int_V q_i dV. \tag{21}$$

In Eq. (21), the integration is performed over the entire volume of the heated droplet.

At $\lambda_e < \lambda_i$ in the dynamic viscosity coefficient, one can ignore the dependence with respect to the angle θ in a droplet–liquid medium system and assume that $\mu_e(t_e) = \mu_e(t_{e0})$. In view of this, expression (1) takes the form

$$\mu_e = \mu_\infty \exp\left\{-A\frac{\gamma}{y}\right\}\left[1 + \sum_{n=1}^{\infty} F_n \frac{\gamma^n}{y^n}\right]. \tag{22}$$

In what follows, formula (22) is used in finding the fields of velocity and pressure in the neighborhood of the heated droplet.

One can see in Eq. (17) that, in order to determine t_{e1} and, accordingly, the temperature field outside the droplet, one must first find the velocity field.

DETERMINING THE FORCE AND VELOCITY OF DRIFT OF HEATED DROPLET

The form of the boundary conditions at infinity given by Eqs. (6) makes possible the search of expressions for V_r, V_θ, and p in the form

$$V_r = G(y)\cos\theta, \quad V_\theta = -g(y)\sin\theta, \qquad p = p_0 + h(y)\cos\theta, \tag{23}$$

where $G(y)$, $g(y)$, and $h(y)$ are functions dependent on y. After the substitution of Eqs. (23) into (2) and (3), inclusion of Eq. (22), and subsequent elimination of pressure, the set of equations in partial derivatives (2) and (3) may be reduced to a set of ordinary differential equations analogous to that in [10]. As a result, we have the following expression for the components of mass velocity and pressure:

$$V_r^e(y,\theta) = \cos\theta(A_1G_1 + A_2G_2),$$

$$V_\theta^e(y,\theta) = -\sin\theta(A_1G_3 + A_2G_4), \tag{24}$$

$$p_e(y,\theta) = 1 + \eta_e\cos\theta(A_1G_5 + A_2G_6) \quad (\eta = \mu/\mu_\infty),$$

$$V_r^i(y,\theta) = \cos\theta(A_3 + A_4y^2),$$

$$V_\theta^i(y,\theta) = -\sin\theta(A_3 + 2A_4y^2), \tag{25}$$

$$p_i(y,\theta) = p_0 + 10\eta_i\cos\theta y^2 A_4,$$

where

$$G_1 = -\frac{1}{y^3}\sum_{n=0}^{\infty}\frac{\Delta_n^{(1)}}{(n+3)y^n},$$

$$G_3 = G_1 + \frac{y}{2}G_1^{\mathrm{I}}, \quad G_4 = G_2 + \frac{y}{2}G_2^{\mathrm{I}},$$

$$G_2 = -\frac{1}{y}\sum_{n=0}^{\infty}\frac{\Delta_n^{(2)}}{(n+1)y^n}$$

$$-\frac{\alpha}{y^3}\sum_{n=0}^{\infty}\left[(n+3)\ln\left(\frac{1}{y}-1\right)\right]\frac{\Delta_n^{(1)}}{(n+3)^2y^n},$$

$$G_5 = \frac{y^2}{2}G_1^{\mathrm{III}} + y\left(3 + \frac{1}{2}\sum_{n=0}^{\infty}s_n\frac{\gamma^n}{y^n}\right)G_1^{\mathrm{II}} + \left(2 + \sum_{n=0}^{\infty}s_n\frac{\gamma^n}{y^n}\right)G_1^{\mathrm{I}}, \tag{26}$$

$$G_6 = \frac{y^2}{2}G_2^{\mathrm{III}} + y\left(3 + \frac{1}{2}\sum_{n=0}^{\infty}s_n\frac{\gamma^n}{y^n}\right)G_2^{\mathrm{II}} + \left(2 + \sum_{n=0}^{\infty}s_n\frac{\gamma^n}{y^n}\right)G_2^{\mathrm{I}},$$

$$s_n = AF_{n-1} - nF_n - \sum_{k=1}^{n}s_{n-k}F_k,$$

$$F_0 = 1,\ F_n \text{ at } n<0 \text{ are zero.}$$

In Eq. (25), G_k^{I}, G_k^{II}, and G_k^{III} are the first, second, and third derivatives with respect to y of the respective functions ($k = 1.2$). The values of the coefficients $\Delta_n^{(1)}$ and $\Delta_n^{(2)}$ are found using the recurrent relations

$$\Delta_n^{(1)} = -\frac{1}{n(n+5)}\sum_{k=1}^{n}[(n+4-k) \times\{\alpha_k^{(1)}(n+5-k) - \alpha_k^{(2)}\} + \alpha_k^{(3)}]\gamma^k\Delta_{n-k}^{(1)} \quad (n \ge 1),$$

$$\Delta_n^{(2)} = -\frac{1}{(n+3)(n-2)}\Bigg[6\alpha_n^{(4)}\gamma^n + \sum_{k=1}^{n}\{(n+2-k) \times[(n+3-k)\alpha_k^{(1)} - \alpha_k^{(2)}] + \alpha_k^{(3)}\}\gamma^k\Delta_{n-k}^{(2)} + \alpha\sum_{k=0}^{n}\{(2n+5-2k)\alpha_k^{(1)} - \alpha_k^{(2)}\}\gamma^k\Delta_{n-k-2}^{(1)}\Bigg] \quad (n \ge 3). \tag{27}$$

In calculating the coefficients $\Delta_n^{(1)}$ and $\Delta_n^{(2)}$ by formulas (27), one must take into account the fact that

$$\Delta_0^{(1)} = -3, \quad \Delta_0^{(2)} = -1, \quad \Delta_2^{(2)} = 1, \quad \alpha_0^{(1)} = 1,$$

$$\alpha_n^{(3)} = 2AF_{n-1} - 2(2+n)F_n, \quad \alpha_0^{(3)} = -4,$$

$$\alpha_n^{(1)} = F_n, \quad \alpha_n^{(2)} = (4-n)F_n + AF_{n-1},$$

$$\Delta_1^{(2)} = -\gamma[6\alpha_1^{(4)} + 2(3\alpha_1^{(1)} - \alpha_1^{(2)}) + \alpha_1^{(3)}],$$

$$\alpha_n^{(4)} = A^n/n!, \quad \alpha_0^{(4)} = 1, \quad \alpha_0^{(2)} = 4,$$

$$\alpha = \frac{\gamma}{15}\{-6\gamma\alpha_2^{(4)} + [3(4\alpha_1^{(1)} - \alpha_1^{(2)}) + \alpha_1^{(3)}]\Delta_1^{(2)} - [2(3\alpha_2^{(1)} - \alpha_2^{(2)}) + \alpha_2^{(3)}]\gamma\}.$$

We substitute Eq. (24) into (17) to derive the following equation for the function t_{e1}:

$$\Delta t_{e1} = -\frac{\omega}{y^2}f(y)\cos\theta. \tag{28}$$

Here, $f(y) = (A_1G_1 + A_2G_2)$ and $\omega = \mathrm{Pr}_\infty\gamma$.

The general solution of Eq. (28) satisfying the boundary condition at $r \longrightarrow \infty$ has the form

$$t_{e1}(y,\theta) = \left\{y + \frac{\Gamma}{y^2} + \omega\sum_{k=1}^{2} A_k\tau_k\right\}\cos\theta,$$

$$\tau_1(y) = \frac{1}{y^3}\sum_{n=0}^{\infty}\frac{\Delta_n^{(1)}}{(n+1)(n+3)(n+4)y^n},$$

$$\tau_2(y) = -\frac{1}{y}\left\{-\frac{1}{2} + \frac{\Delta_1^{(2)}}{6y}\ln y - \sum_{n=2}^{\infty}\frac{\Delta_n^{(2)}}{(n^2-1)(n+2)y^n} - \frac{\alpha}{y^2}\sum_{n=0}^{\infty}\left[(n+1)(n+3)(n+4)\ln\frac{1}{y} - (3n^2+16n+19)\right]\frac{\Delta_n^{(1)}}{(n+1)^2(n+3)^2(n+4)^2y^n}\right\}. \quad (29)$$

The integration constant Γ is determined from the boundary conditions on the surface of a nonuniformly heated droplet given by Eq. (18),

$$\Gamma = \frac{1}{\delta}\left\{\frac{3}{4\pi R^2\lambda_i T_\infty}\int_V q_i z dV - \left(1 - \frac{\lambda_e}{\lambda_i}\right) - \omega\sum_{k=1}^{2}A_k\varphi_k\right\}.$$

Here, $\delta = 1 + 2\frac{\lambda_e}{\lambda_i}$ and $\varphi_k = \tau_k - \frac{\lambda_e}{\lambda_i}\tau_k^{\mathrm{I}}$, where the superscript I indicates the first derivative in y of the function τ_k.

As a result, the temperature fields outside and inside a nonuniformly heated droplet are determined. Consequently, one can determine the integration constants A_1, A_2, A_3, and A_4 entering the expressions for velocity (24) and (25). After this, we can find the force acting on the nonuniformly heated droplet and the velocity of its motion. The force acting on the droplet is determined by integration of the stress tensor over the particle surface [9] and will be made up of the force F_t proportional to the external temperature gradient and the force F_q due to the nonunformity of distribution of the density of heat sources over the droplet volume and proportional to J,

$$F = \varepsilon(F_t + F_q),$$
$$F_t = -6\pi R\mu_\infty f_t n_z, \quad F_q = -6\pi R\mu_\infty f_q J n_z. \quad (30)$$

The values of the coefficients f_t and f_q may be estimated by the formulas

$$\delta = 1 + 2\frac{\lambda_e^s}{\lambda_i^s},$$

$$f_t = \frac{4}{3\Delta}G_1\exp\{-A\gamma\}\frac{\lambda_e^s}{\delta\lambda_i^s}\frac{1}{\mu_i^s}\frac{\partial\sigma}{\partial t_i}\left[1 - \frac{\omega}{3G_1}(2\tau_1 + \tau_1^{\mathrm{I}})\right],$$

$$J = \frac{1}{V}\int_V q_i z dV, \quad V = \frac{4}{3}\pi R^3, \quad (31)$$

$$f_q = \frac{4}{9\Delta}\exp\{-A\gamma\}\frac{G_1}{\delta\lambda_1^s T_\infty}\frac{R}{\mu_i^s}\frac{\partial\sigma}{\partial t_i},$$

$$\Delta = N_1 + \frac{\mu_e^s}{3\mu_i^s}N_2 + \frac{2\rho_e R\omega}{3\mu_\infty}\frac{\lambda_e^s}{\delta\lambda_i^s\mu_i^s}\frac{\partial\sigma}{\partial t_i}(G_2\Phi_1 - G_1\Phi_2),$$

$$\Phi_k = 2\tau_k - \tau_k^{\mathrm{I}}.$$

Here, $k = 1, 2$; n_z is a unit vector in the direction of the z-axis; $\int_V q_i z dV$ is the dipole moment of the density of heat sources; and the functions Φ_1, Φ_2, G_1, G_2, N_1, N_2, N_3, and N_4 are determined at $y = 1$ ($N_1 = (G_1G_2^{\mathrm{I}} - G_2G_1^{\mathrm{I}})$, $N_4 = (2G_1^{\mathrm{I}} + G_1^{\mathrm{II}})$ $N_3 = -G_1^{\mathrm{I}}$, $N_2 = [G_2(2G_1^{\mathrm{I}} + G_1^{\mathrm{II}}) - G_1(2G_2^{\mathrm{I}} + G_2^{\mathrm{II}})]$). In estimating the coefficients f_t and f_q, one must take into account that the values of the physical quantities with the subscript s are determined for the average temperature of the droplet surface T_s calculated by formula (21).

Expression (30) indicates that the magnitude of the thermocapillary force in the field of the external preassigned temperature gradient will be affected by the variation of the surface tension coefficient of the droplet with temperature and the nonuniform distribution of the density of heat sources in the particle volume and of the motion of the medium (the expression proportional to $\omega = \mathrm{Pr}_\infty\gamma$). The convection term depends on the average temperature of the droplet surface and on the Prandtl number, i.e., is proportional to the product of the Prandtl number by the relative temperature difference between the droplet surface and the region away from it. In view of the fact that the problem may be solved for considerable differences in liquid temperature and that the Prandtl number may take high values, the inclusion of convective terms in the heat equation may have a considerable effect on the magnitude of the thermocapillary force. In the case of gases, the inclusion of the motion of the medium cannot have a qualitative effect on the force, because the Prandtl number in gas is of the order of unity.

In the case when the heating of the droplet surface is fairly low, i.e., the average temperature of the droplet

Table 3. Comparison of the φ_t and φ_t^* functions calculated in view of and disregarding the motion of the medium

T_s, °C	φ_t	φ_t^*
0	1	1
10	0.7765287	0.8070467
20	0.5898698	0.6373528
30	0.4427460	0.4976188
40	0.3298598	0.3859649
50	0.2455074	0.2993985
60	0.1822527	0.2319948
70	0.1352527	0.1800407
80	0.1001674	0.1397424
90	0.0740130	0.1084872

surface differs little from the ambient temperature at infinity ($\gamma \longrightarrow 0$), the temperature dependence of the viscosity coefficient may be ignored and, consequently, $G_1 = 1$, $G_1^{\mathrm{I}} = -3$, $G_1^{\mathrm{II}} = 12$, $G_2 = 1$, $G_2^{\mathrm{I}} = -1$, $G_2^{\mathrm{II}} = 2$, $N_1 = 2$, $N_2 = 6$, $N_3 = 3$, and $N_4 = 6$. With $q_i = 0$ and $\omega = 0$, formula (30) transforms to the expression for thermocapillary force obtained in [1, 5].

Given the distribution of heat sources over the volume, formula (30) enables one to include the effect of the motion of the medium on the magnitude of the thermocapillary force acting on a heated droplet under conditions of arbitrary temperature differences between the particle surface and the region away from it, with due regard for the exponential-power form of the temperature dependence of viscosity in the external field of the temperature gradient.

Formulas (30) and (31) further indicate that the thermocapillary force will be affected by the magnitude and direction of the dipole moment of the density of heat sources $\int_V q_i z dV$.

For example, if the heating of the droplet surface occurs due to the absorption of electromagnetic radiation, the dipole moment may be both positive (most of the thermal energy released in the particle part facing the radiation flux) and negative (most of the thermal energy released in the shadow part of the particle), which depends on the optical properties of the droplet. In view of the fact that, for the majority of liquids, the surface tension decreases with temperature, i.e., $\partial\sigma/\partial t_i < 0$, the quantity $\int_V q_i z dV$ may be both positive and negative. Consequently, the overall thermocapillary force will also vary.

In addition, one can see from formulas (30) and (31) that the latter force depends considerably on the thermal conductivity of the droplet as well. With λ_i tending to infinity, this force (with the fixed magnitude of the dipole moment of the density of heat sources) tends to zero.

DEFORMATION OF THE DROPLET SURFACE SHAPE

The shape of the droplet surface is not known beforehand and must be determined from the solution; therefore, the boundary conditions given by Eqs. (5)–(7) for the problem being treated are preassigned on the unknown boundary. Because the problem is solved with corrections of the first order of smallness, one can write

$$\sigma = \sigma_0 + \varepsilon\sigma^{(1)}, \tag{32}$$

where σ_0 is zero term in the expansion of the function $\sigma(x)$ in terms of Legendre polynomials $P_n(x)$, $x = \cos\theta$.

We seek the shape of the droplet surface in the form [8]

$$r = R[1 + \varepsilon\xi]. \tag{33}$$

We will expand the sought quantities $\sigma(\theta)$ and $\xi(\theta)$ in series in terms of Legendre polynomials,

$$\sigma = \sum_{n=0}^{\infty} \sigma_n P_n(\cos\theta), \quad \xi = \sum_{n=0}^{\infty} \xi_n P_n(\cos\theta). \tag{34}$$

It follows from the condition of constancy of the droplet volume that $\xi_0 = 0$. In view of the fact that the origin of coordinates is placed at the center of mass of the heated particle, we have

$$\int_0^{\pi} \xi \sin^2\theta d\theta = 0, \tag{35}$$

$$\xi_1 \equiv 0. \tag{36}$$

The problem was solved disregarding the boundary condition for the normal components of the stress tensor. Within the terms proportional to ε, the boundary condition for normal stresses on the droplet surface may be written as [9]

$$\sigma_n^{e(1)} - \sigma_n^{i(1)} = \sigma_0 H^{(1)} + 2\frac{\sigma^{(1)}}{R}. \tag{37}$$

Here, $2H = \frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R} + \varepsilon H^{(1)}$; R_1 and R_2 are the principal radii of curvature of the droplet surface; and H is the average surface curvature which, in the axisymmetric case, is [9]

$$H^{(1)} = -\frac{2}{R}\xi - \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\xi}{\partial\theta}\right). \tag{38}$$

We use Eqs. (34) and (36) to transform expression (38) to

$$H^{(1)} = \sum_{n=2}^{\infty} \frac{(n+2)(n-1)}{R} \xi_n P_n(\cos\theta). \qquad (39)$$

With due regard for Eq. (39), Eq. (37) indicates that, in the approximation being treated, a nonuniformly heated droplet during its motion retains the spherical shape.

The calculation results characterizing the effect of the motion of the medium on the thermocapillary force are given in Table 3. The numerical estimates relate the values of $\varphi_t = f_t/f_t|_{T_s = 273\ \mathrm{K}}$ to the average surface temperature T_s of large mercury droplets of radius $R = 10^{-5}$ m moving in water at $T_\infty = 273$ K. The φ_t^* function is constructed disregarding the motion of the medium ($\omega = 0$).

The numerical estimates indicate that the motion of the medium may have a considerable effect on the magnitude and direction of the thermocapillary force.

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