On the Thermophoresis of a Spheroidal Solid Aerosol Particle

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Abstract—The steady motion of a spheroidal aerosol particle with inner nonuniformly distributed heat sources (sinks) that is placed in an external temperature gradient is theoretically studied in the Stokes approximation. The mean temperature of the particle surface is assumed to differ slightly from that of the gaseous environment. An analytic expression for the force and rate of thermophoresis are found by solving the gas-dynamic equations in view of the motion of the environment.

1. FORMULATION OF THE PROBLEM

To date, the thermophoresis of spherical aerosol particles has been studied in great detail [1–4]. Many particles occurring both in nature and in industrial plants are nonspherical, e.g., spheroidal. The problem of thermophoresis of a spheroidal aerosol particle has been considered in [5–7]. However, the convective terms in the heat conduction equation were neglected. Praudman and Pearson [8] for the hydrodynamic problem, as well as Acrivos and Taylor [9] for the heat problem, showed that, away from the sphere, the inertial and convective terms became of the same order of magnitude as the molecular-transport terms. Therefore, normal expansion in a small parameter introduces an error, since it fails to rigorously satisfy the boundary conditions at infinity and find a unique exact solution that is valid throughout the flow region even in a second approximation. From the above, it follows that the issue of how the motion of the medium affects the force and rate of thermophoresis of a spheroidal particle is of theoretical and practical interest.

Consider the steady motion of a spheroidal (oblate spheroid) solid aerosol particle with a velocity $\mathbf{U}$ in the negative direction of the $0z$ axis. The particle contains nonuniformly distributed heat sources with a density q_i. The gas is at rest at infinity, and a small temperature gradient ∇T is provided by external sources. The temperature drop in the neighborhood of the particle is assumed to be small; i.e., $(T_s - T_\infty)/T_\infty \ll 1$, where T_s is the mean temperature of the particle surface and T_∞ is the gas temperature away from the particle. In this case, the thermal conductivity, as well as the dynamic and kinematic viscosity, can be considered as constants and the gas, as an incompressible medium. The particle size is considerably larger than the free paths of gas mixture molecules; therefore, corrections in Knudsen number will be neglected [3]. Hereafter, the subscripts e and i correspond to the environment and spheroid, respectively.

We will describe the thermophoresis of a particle in the spheroidal coordinate system $(\varepsilon, \eta, \varphi)$ with the origin at the center of the spheroid; i.e., the origin of the fixed coordinate system coincides with the instantaneous position of the center of the particle. The curvilinear coordinates ε, η, and φ are related to the Cartesian coordinates by the relations [10]

$$x = c\sinh\varepsilon\sin\eta\cos\varphi, \quad y = c\sinh\varepsilon\sin\eta\sin\varphi, \quad z = c\cosh\varepsilon\cos\eta, \tag{1.1}$$

$$x = c\cosh\varepsilon\sin\eta\cos\varphi, \quad y = c\cosh\varepsilon\sin\eta\sin\varphi, \quad z = c\sinh\varepsilon\cos\eta, \tag{1.2}$$

where $c = \sqrt{b^2 - a^2}$ in the case of a prolate spheroid ($a < b$, formula (1.1)) or $c = \sqrt{a^2 - b^2}$ for an oblate spheroid ($a > b$, formula (1.2)) and a and b are the spheroid semiaxes. The $0z$ axis of the Cartesian coordinate system coincides with the symmetry axis of the spheroid.

In view of the above assumptions, the distributions of the velocity $\mathbf{U}_e$, pressure P_e, and temperatures T_e and T_i are described by the set of equations (1.3) and (1.4) with boundary conditions (1.5)–(1.7):

$$\nabla P_e = \mu_e \Delta \mathbf{U}_e, \quad \operatorname{div}\mathbf{U}_e = 0, \tag{1.3}$$

$$\rho_e c_{pe}(\mathbf{U}_e \cdot \nabla)T_e = \lambda_e \Delta T_e, \quad \Delta T_i = -q_i/\lambda_i, \tag{1.4}$$

$$U_\varepsilon = -\frac{cU\cosh\varepsilon}{H_\varepsilon}\cos\eta, \quad U_\eta = \frac{cU\sinh\varepsilon}{H_\varepsilon}\sin\eta - K_{tc}\frac{\nu_e}{T_e}(\nabla T_e \cdot \mathbf{e}_\eta), \tag{1.5}$$

$$T_e = T_i, \quad \lambda_e(\nabla T_e \cdot \mathbf{e}_\varepsilon) = \lambda_i(\nabla T_i \cdot \mathbf{e}_\varepsilon) \quad \text{for} \quad \varepsilon = \varepsilon_0,$$

$$\mathbf{U}_e \longrightarrow 0, \quad T_e \longrightarrow T_\infty + |\nabla T| c\sinh\varepsilon\cos\eta, \quad P_e \longrightarrow P_\infty \quad \text{for} \quad \varepsilon \longrightarrow \infty, \tag{1.6}$$

$$T_i \neq \infty \quad \text{for} \quad \varepsilon \longrightarrow 0. \tag{1.7}$$

Here, $\mathbf{e}_\varepsilon$ and $\mathbf{e}_\eta$ are the unit vectors of the spheroidal coordinate system; λ is the thermal conductivity; $U = |\mathbf{U}|$; $H_\varepsilon = \sqrt{\cosh^2\varepsilon - \sin^2\eta}$ is the Lamé coefficient; c_{pe} is the specific heat; K_{tc} is the thermal creep coefficient, which is calculated from the kinetic theory of gases; and ∇ is the Laplacian. Today, the most rigorous expression for the coefficient K_{tc} is known for a spherical particle [3]. The gas-kinetic coefficient $K_{tc} = 1.152$ when the accommodation coefficients of tangential momentum, α_τ, and energy, α_E, are equal to unity [3, 4]. In numerical calculations, we assume that the coefficient K_{tc} for a spheroid differs insignificantly from that for a sphere [6].

Boundary conditions (1.5) on the particle surface ($\varepsilon = \varepsilon_0$) allow for creep for the tangent component of the mass velocity, temperature equality, and the continuity of heat fluxes on the particle surface. Away from the particle ($\varepsilon \longrightarrow \infty$), boundary conditions (1.6) are valid, and the finiteness of the physical quantities characterizing the particle at $\varepsilon \longrightarrow 0$ is taken into account in (1.7).

The resultant force acting on a spheroidal particle from the environment is given by the formula [11]

$$F_z = \int_S \left(-P_e\cos\eta + \sigma_{\varepsilon\varepsilon}\cos\eta - \frac{\sinh\varepsilon}{\cosh\varepsilon}\sigma_{\varepsilon\eta}\sin\eta\right)dS, \tag{1.8}$$

where $dS = c^2\cosh^2\varepsilon\sin\eta d\eta d\varphi$ is a differential element of area, and $\sigma_{\varepsilon\varepsilon}$ and $\sigma_{\varepsilon\eta}$ are the strain tensor components in the spheroidal coordinate system.

2. TEMPERATURE DISTRIBUTION IN THE VICINITY OF THE PARTICLE, FORCE AND VELOCITY OF THERMOPHORESIS

We make Eqs. (1.3) and (1.4) and boundary conditions (1.5)–(1.7) dimensionless by introducing the dimensionless velocity, temperature, and pressure: $\mathbf{V}_e = \mathbf{U}_e/U$, $t_k = T_k/T_\infty$, and $p_k = P_k/P_\infty$ ($k = e, i$). Here, the spheroid major semiaxis is taken as the unit length; U, as the unit velocity; $P_\infty = \mu_e U/a$, as the unit pressure; and T_∞, as the unit temperature ($U \sim \mu_e|\nabla T|/(\rho_e T_\infty)$).

Expressions (1.3)–(1.7) have the single controllable small parameter $\xi = a|\nabla T|/T_\infty \ll 1$. Therefore, we will look for a solution to the boundary-value problem in the form of expansion in powers of ξ:

$$\mathbf{V}_e = \mathbf{V}_e^{(0)} + \xi\mathbf{V}_e^{(1)} + \ldots, \quad t = t^{(0)} + \xi t^{(1)} + \ldots, \quad p_e = p_e^{(0)} + \xi p_e^{(1)} + \ldots. \tag{2.1}$$

We will restrict our consideration to the first-order terms in ξ when calculating the force acting on the particle and the velocity of its thermophoretic motion in the given external temperature gradient field. In order to find these quantities, one has to know the distributions of the velocity, pressure, and temperature both outside and inside the spheroid. Substituting (2.1) into (1.4), leaving terms $\sim\xi$, and solving the sets of equations found by the method of separation of variables, we will finally find in the zero approximation ($\xi = 0$)

$$t_e^{(0)}(\lambda) = 1 + \gamma\lambda_0\operatorname{arccot}\lambda \quad (\lambda = \sinh\varepsilon), \tag{2.2}$$

$$t_i^{(0)}(\lambda) = D + \frac{\lambda_e}{\lambda_i}\gamma\lambda_0\operatorname{arccot}\lambda + \int_{\lambda_0}^{\lambda}\operatorname{arccot}\lambda f d\lambda - \operatorname{arccot}\lambda\int_{\lambda_0}^{\lambda} f d\lambda. \tag{2.3}$$

Here, $\lambda_0 = \sinh\varepsilon_0$, $\gamma = t_s - 1$ is the dimensionless parameter, $t_s = T_s/T_\infty$, and T_s is the mean temperature of the spheroid surface given by

$$\frac{T_s}{T_\infty} = 1 + \frac{1}{4\pi\lambda_e c\lambda_0 T_\infty}\int_V q_i dV,$$

$$D = 1 + \left(1 - \frac{\lambda_e}{\lambda_i}\right)\gamma\lambda_0\operatorname{arccot}\lambda_0, \tag{2.4}$$

$$f = -\frac{c^2}{2\lambda_i T_\infty}\int_{-1}^{+1} q_i(\lambda^2 + x^2)dx, \quad x = \cos\eta.$$

In (2.4), the integral is taken over the entire particle volume. In the first approximation ($\sim\xi$),

$$t_e^{(1)}(\lambda, x) = \cos\eta\left\{\frac{c\lambda}{a} + \Gamma c(\lambda\operatorname{arccot}\lambda - 1) + \omega\left[A_2\left(\operatorname{arccot}\lambda - \frac{\lambda}{2}\operatorname{arccot}^2\lambda\right) + \frac{A_1}{2}(\operatorname{arccot}\lambda - \lambda\operatorname{arccot}^2\lambda)\right]\right\}, \tag{2.5}$$

$$t_i^{(1)}(\lambda, x) = \cos\eta\left\{Bc\lambda + \frac{3(1 - \lambda\operatorname{arccot}\lambda)}{4\pi c^2\lambda_i T_\infty}\int_V q_i z dV - \lambda\int_{\lambda_0}^{\lambda}(\lambda\operatorname{arccot}\lambda - 1)f_1 d\lambda + (\lambda\operatorname{arccot}\lambda - 1)\int_{\lambda_0}^{\lambda}\lambda f_1 d\lambda\right\}. \tag{2.6}$$

Here, $\omega = \mathrm{Pr}\gamma\lambda_0/(ac)$ and Pr is the Prandtl number. The constants of integration A_1 and A_2 appear in expressions for the components of the mass velocity and pressure. These expressions are found by solving Stokes equations (1.3) in the oblate coordinate system and have the form [10]

$$U_\varepsilon(\varepsilon,\eta) = \frac{U}{c\cosh\varepsilon H_\varepsilon}\cos\eta \tag{2.7}$$
$$\times\{\lambda A_2 + [\lambda - (1+\lambda^2)\operatorname{arccot}\lambda]A_1 + c^2(1+\lambda^2)\},$$

$$U_\eta(\varepsilon\eta) = -\frac{U}{cH_\varepsilon}\sin\eta\left\{\frac{A_2}{2} + (1-\lambda\operatorname{arccot}\lambda)A_1 + c^2\lambda\right\}, \tag{2.8}$$

$$P_e(\varepsilon,\eta) = P_\infty + c\frac{\mu_e U}{H_\varepsilon^4}x(\lambda^2+x^2)A_2. \tag{2.9}$$

The constants Γ and B enter into expressions (2.5) and (2.6) for the temperature fields inside and outside the particle from the corresponding boundary conditions on the spheroid surface. Since an expression for the coefficient Γ will be of interest to us, we write it in explicit form:

$$\Gamma = -\frac{1-\delta}{\Delta a} + \frac{3}{4\pi c^3\lambda_i T_\infty\Delta\lambda_0(1+\lambda_0^2)}\int_V q_i z dV$$
$$+\frac{\omega}{c\Delta}\left\{A_2\left[-\frac{\delta}{1+\lambda_0^2} + \left(\delta\frac{\lambda_0^2}{1+\lambda_0^2} - \frac{1}{\lambda_0}\right)\operatorname{arccot}\lambda_0\right.\right.$$
$$\left.+\frac{1-\delta}{2}\operatorname{arccot}^2\lambda_0\right] + \frac{A_1}{2}\left[-\frac{\delta}{1+\lambda_0^2} + (1-\delta)\operatorname{arccot}^2\lambda_0\right. \tag{2.10}$$
$$\left.\left.+\left(\delta\frac{2\lambda_0}{1+\lambda_0^2} - \frac{1}{\lambda_0}\right)\operatorname{arccot}\lambda_0\right]\right\},$$

$$\Delta = (1-\delta)\operatorname{arccot}\lambda_0 + \delta\frac{\lambda_0}{1+\lambda_0^2} - \frac{1}{\lambda_0}, \qquad \delta = \frac{\lambda_e^s}{\lambda_i^s}.$$

Hereafter, the superscript s denotes the values of physical quantities at the mean temperature T_s of the spheroid surface, which is given by formula (2.4).

Substituting (2.7)–(2.9) into (1.8) and integrating, we arrive at

$$F_z = 4\pi\frac{\mu_e U}{c}A_2. \tag{2.11}$$

The coefficient A_2 is found from boundary conditions (1.5) in view of expressions (2.7), (2.8), and (2.10):

$$A_2 = -\frac{2c^2}{\beta[\lambda_0 + (1-\lambda_0^2)\operatorname{arccot}\lambda_0]}$$
$$-2K_{tc}\frac{\nu_e^s c^2}{t_s U}\frac{\delta}{1+\lambda_0^2}\frac{\lambda_0-(1+\lambda_0^2)\operatorname{arccot}\lambda_0}{\beta[\lambda_0+(1-\lambda_0^2)\operatorname{arccot}\lambda_0]}\frac{|\nabla T|}{T_\infty}$$
$$\times\left\{1 - \frac{3a(\lambda_0\operatorname{arccot}\lambda_0 - 1)}{4\pi c^3\lambda_e T_\infty\lambda_0}\int_V q_i z dV\right.$$
$$\left.+\Pr\frac{1+\lambda_0^2}{2}\lambda_0\gamma\frac{1-\lambda_0\operatorname{arccot}\lambda_0(2-\lambda_0\operatorname{arccot}\lambda_0)}{\lambda_0-(1+\lambda_0^2)\operatorname{arccot}\lambda_0}\right\},$$

where

$$\beta = 1 - 2K_{tc}\frac{\lambda_0}{t_s}\gamma\frac{\Pr}{\Delta}\frac{\delta}{1+\lambda_0^2}\frac{\lambda_0-(1+\lambda_0^2)\operatorname{arccot}\lambda_0}{\lambda_0+(1-\lambda_0^2)\operatorname{arccot}\lambda_0}$$
$$\times\left[1 - \left(\lambda_0 + \frac{1}{2}\operatorname{arccot}\lambda_0\right)\operatorname{arccot}\lambda_0\right.$$
$$\left.-\frac{\lambda_0 - (2-\lambda_0\operatorname{arccot}\lambda_0)\lambda_0^2\operatorname{arccot}\lambda_0}{\lambda_0-(1+\lambda_0^2)\operatorname{arccot}\lambda_0}\right].$$

In view of the explicit form of the coefficient A_2, we find a general expression for the force acting on a spheroidal particle. This force is the sum of the viscous force F_μ and the force $F^{(1)}$

$$F = F_\mu + F^{(1)}, \tag{2.12}$$

where

$$F_\mu = -8\pi\mu_e^s U\frac{c^2}{\beta[\lambda_0+(1-\lambda_0^2)\operatorname{arccot}\lambda_0]}, \tag{2.13}$$

$$F^{(1)} = -8\pi\mu_e^s cK_{tc}\frac{\nu_e^s}{t_s}\frac{\lambda_0-(1+\lambda_0^2)\operatorname{arccot}\lambda_0}{\beta[\lambda_0(1-\lambda_0^2)\operatorname{arccot}\lambda_0]}\frac{\delta}{(1+\lambda_0^2)\Delta}$$
$$\times\frac{|\nabla T|}{T_\infty}\left[1 - \frac{3a(\lambda_0\operatorname{arccot}\lambda_0-1)}{4\pi c^3\lambda_e T_\infty\lambda_0}\int_V q_i z dV\right. \tag{2.14}$$
$$\left.+\Pr\frac{1+\lambda_0^2}{2}\lambda_0\gamma\frac{1-\lambda_0\operatorname{arccot}\lambda_0(2-\lambda_0\operatorname{arccot}\lambda_0)}{\lambda_0-(1+\lambda_0^2)\operatorname{arccot}\lambda_0}\right].$$

In the general case, the force $F^{(1)}$ is the sum of three forces: the thermophoretic force, the force proportional to the dipole moment of the density of heat sources nonuniformly distributed over the particle volume, and the third term due to the motion of the medium (i.e., the force component taking into account the convective terms in the heat conduction equation).

Equating the resultant force F to zero, we arrive at a general expression for the drift (thermophoretic) velocity of a solid oblate spheroidal particle in the external temperature gradient field:

$$U_{th} = -\frac{b}{a}K_{tc}\frac{\nu_e^s}{t_s}\delta$$

$$\times \frac{1-(\lambda_0+1/\lambda_0)\operatorname{arccot}\lambda_0}{\sqrt{1+\lambda_0^2}\left[(1-\delta)\operatorname{arccot}\lambda_0+\delta\frac{\lambda_0}{1+\lambda_0^2}-\frac{1}{\lambda_0}\right]} \tag{2.15}$$

$$\times\left[1-\frac{3a}{4\pi c^3\lambda_e T_\infty}\left(\operatorname{arccot}\lambda_0-\frac{1}{\lambda_0}\right)\int_V q_i z dV\right.$$

$$\left.+\mathrm{Pr}\frac{1+\lambda_0^2}{2}\lambda_0\gamma\frac{1-\lambda_0\operatorname{arccot}\lambda_0(2-\lambda_0\operatorname{arccot}\lambda_0)}{\lambda_0-(1+\lambda_0^2)\operatorname{arccot}\lambda_0}\right]\frac{|\nabla T|}{T_\infty}.$$

In order to find the rate of thermophoresis for a prolate spheroid, one has to substitute $i\lambda$ for λ and ic for c (i is the imaginary unit) in (2.15).

Thus, formulas (2.12) and (2.15) have the most general form and make it possible to estimate the resultant force acting on a solid spheroidal aerosol particle and its drift velocity in the external temperature gradient field for the case when heat sources (sinks) are nonuniformly distributed inside the particle. In this approach, the motion of the environment is taken into account for small temperature differences in the vicinity of the particle.

3. RESULTS AND DISCUSSION

If one does not take into account the motion of the environment and internal heat sources, (2.15) is reduced to an expression for the purely thermophoretic velocity of a spheroidal particle:

$$U_{th}^b = K_{tc}\nu_e\delta f_{th}^{(b)}\frac{|\nabla T|}{T_\infty}$$

$$\left(f_{th}^{(b)} = -\frac{b}{a}\frac{1-(\lambda_0+1/\lambda_0)\operatorname{arccot}\lambda_0}{\sqrt{1+\lambda_0^2}\left[(1-\delta)\operatorname{arccot}\lambda_0+\delta\frac{\lambda_0}{1+\lambda_0^2}-\frac{1}{\lambda_0}\right]}\right), \tag{3.1}$$

which coincides with formula (9) in [5].

In the case of a sphere, (2.15) turns into an expression for the thermophoretic velocity of a solid spherical particle of radius R that includes the flow of the environment and internal heat sources:

$$U^{(\mathrm{sph})}(a=b=R) = K_{tc}\nu_e^s\delta f^{(\mathrm{sph})}\frac{|\nabla T|}{T_\infty}, \tag{3.2}$$

where

$$\gamma_0 = \frac{1}{4\pi R\lambda_e T_\infty}\int_V q_i dV,$$

$$f^{(\mathrm{sph})} = -\frac{2}{t_e^s(1+2\delta)}\left\{1+\frac{1}{4\pi R^2\lambda_e T_\infty}\int_V q_i dV-\frac{\mathrm{Pr}}{12}\gamma_0\right\}.$$

Disregarding the flow of the environment and internal heat sources yields the conventional formula for the thermophoretic velocity of a large spherical particle [1, 2]

$$U_{th}(a=b=R) = -2K_{tc}\frac{\nu_e^s}{t_e^s}\frac{\delta}{1+2\delta}\frac{|\nabla T|}{T_\infty}. \tag{3.3}$$

In order to estimate how the motion of the environment affects the thermophoretic velocity of a spheroidal particle, one has to specify the nature of heat sources nonuniformly distributed over its volume. As an example, let us consider the simplest case when the particle absorbs radiation as a black body. In this case, radiation is absorbed in a thin layer of depth $\delta\varepsilon \ll \varepsilon_0$ that is adjacent to the heated particle surface. The density of heat sources inside the layer of depth $\delta\varepsilon$ is equal to [12, 13]

$$q_i(\varepsilon,\eta) = \begin{cases} -\dfrac{\cosh\varepsilon\cos\eta}{c(\cosh^2\varepsilon-\sin^2\eta)\delta\varepsilon}I_0, & \dfrac{\pi}{2}\le\eta\le\pi, \\ \varepsilon_0-\delta\varepsilon\le\varepsilon\le\varepsilon_0 & \\ 0, \quad 0\le\eta\le\dfrac{\pi}{2}, & \end{cases} \tag{3.4}$$

where I_0 is the intensity of an incident radiation.

The integrals $\int_V q_i dV$ and $\int_V q_i z dV$ appear in the expression for the thermophoretic velocity. Substituting (3.4) into these integrals in view of the fact that $\delta\varepsilon \ll \varepsilon_0$ and performing integration, we find

$$\int_V q_i dV = \pi I_0 c^2\lambda_0^2\left(1+\frac{1}{\lambda_0^2}\right),$$

$$\int_V q_i z dV = -\frac{2}{3}\pi I_0 c^3\lambda_0^3\left(1+\frac{1}{\lambda_0^2}\right). \tag{3.5}$$

In view of (3.5), expression (2.15) takes the form

$$U_{th}^* = K_{tc}\nu_e^s\delta f_{th}^*\frac{|\nabla T|}{T_\infty}, \tag{3.6}$$

where

$$f_{th}^* = -\frac{b}{a}\frac{1-(\lambda_0+1/\lambda_0)\operatorname{arccot}\lambda_0}{\sqrt{1+\lambda_0^2}t_e^s\left[(1-\delta)\operatorname{arccot}\lambda_0+\delta\frac{\lambda_0}{1+\lambda_0^2}-\frac{1}{\lambda_0}\right]}$$

$$\times\left\{1+\frac{\lambda_0^2 a}{2\lambda_e T_\infty}I_0\left(1+\frac{1}{\lambda_0^2}\right)\left[\lambda_0\operatorname{arccot}\lambda_0-1\right.\right. \tag{3.7}$$

$$\left.\left.+\frac{\mathrm{Pr}}{4}\sqrt{1+\lambda_0^2}\frac{1-\lambda_0\operatorname{arccot}\lambda_0(2-\lambda_0\operatorname{arccot}\lambda_0)}{\lambda_0-(1+\lambda_0^2)\operatorname{arccot}\lambda_0}\right]\right\}.$$

Table 1

a, μm	$b/a = 0.1$							
	$I_0 \times 10^2$, W/m^2							
	0.5		2		5		10	
	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$
15	0.52	0.32	2.08	1.30	5.20	3.29	10.40	6.71
20	0.69	0.43	2.77	1.74	6.93	4.42	13.87	9.08
25	0.87	0.54	3.47	2.18	8.67	5.56	17.33	11.51

Table 2

a, μm	$b/a = 0.3$							
	$I_0 \times 10^2$, W/m^2							
	0.5		2		5		10	
	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$
15	0.39	0.20	1.59	0.81	3.97	2.04	7.95	4.16
20	0.59	0.27	2.12	1.08	5.30	2.74	10.60	5.63
25	0.66	0.33	2.65	1.35	6.62	3.44	13.25	7.13

Table 3

a, μm	$b/a = 0.5$							
	$I_0 \times 10^2$, W/m^2							
	0.5		2		5		10	
	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$
15	0.32	0.12	1.26	0.48	3.15	1.20	6.31	2.45
20	0.42	0.16	1.68	0.64	4.21	1.61	8.41	3.32
25	0.53	0.20	2.10	0.80	5.26	2.03	10.51	4.20

In the case of a sphere, (3.6) is recast as

$$U_{th}^{(\mathrm{sph})} = K_{\mathrm{tc}} \nu_{\mathrm{e}}^{\mathrm{s}} \delta f_{th}^{(\mathrm{sph})} \frac{|\nabla T|}{T_\infty}, \tag{3.8}$$

where

$$f^{(\mathrm{sph})} = -\frac{2}{t_{\mathrm{e}}^{\mathrm{s}}(1+2\delta)}\left[1 - \frac{RI_0}{6\lambda_{\mathrm{e}} T_\infty}\left(1 + \frac{\mathrm{Pr}}{8}\right)\right]. \tag{3.9}$$

The mean temperature of the spheroid surface is related to the incident radiation intensity I_0 as

$$T_{\mathrm{s}} = T_\infty + \frac{c\lambda_0}{4\lambda_{\mathrm{e}}} I_0 \left(1 + \frac{1}{\lambda_0^2}\right). \tag{3.10}$$

In order to illustrate the contributions of the form-factor (ratio of the spheroid semiaxes), flow of the environment, and internal heat release (nonuniform distribution of heat sources over the particle volume) to the thermophoretic velocity (3.6), Tables 1–4 list the numerical estimations for particles of borated graphite ($\lambda_{\mathrm{i}}^{\mathrm{s}} = 55$ W/(m K)) suspended in air at $T_\infty = 280$ K and $P_{\mathrm{e}} = 10^5$ Pa.

The numerical analysis showed that, at a given ratio between the semiaxes, the relative contribution of the other factors increases with increasing incident radiation intensity I_0. This effect depends significantly on the equatorial radius of the spheroid (a). For instance, in Table 1 ($a = 15$ μm), $f^{(1)} = 0.52$ at $I_0 = 0.5 \times 10^2$ W/m^2 and $f^{(1)} = 10.40$ at $I_0 = 10 \times 10^2$ W/m^2 ($f^{(1)} = (|f_{th}^{*} - f_{th}^{(b)}|/f_{th}^{(b)}) \times 100\%$). Such behavior of the function $f^{(1)}$ is due to the fact that, as follows from (3.10), (3.7), and the numerical estimations, the major contribution is from the terms proportional to the dipole moment of the density of heat sources nonuniformly distributed over the particle volume. In (3.7), this is the term

Table 4

a, μm	$b/a = 0.8$							
	$I_0 \times 10^2$, W/m^2							
	0.5		2		5		10	
	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$	$f^{(1)}$	$f^{(2)}$
15	0.24	0.04	0.94	0.15	2.35	0.38	4.69	0.77
20	0.31	0.05	1.25	0.20	3.13	0.51	6.26	1.04
25	0.39	0.06	1.56	0.25	3.91	0.64	7.82	1.32

$\lambda_0 \operatorname{arccot}\lambda_0 - 1$. The dimensionless term related to the motion of the environment (see the heat conduction equation) is proportional to the Prandtl number. In a gas, this number is on the order of unity; therefore, the contribution of this term differs from that of the first one by one order of magnitude. This fact may be used to separate particles by size, finely purify gases from aerosol particles, estimate translucent zones appearing in clouds and fogs when they are probed by laser radiation, etc. The influence of the factors mentioned above will increase with increasing radiation intensity. However, the mean temperature of the spheroid surface will also increase (see (3.10)). In this case, we cannot consider the coefficients of molecular transport to be constant. Therefore, expressions (2.15) and (3.6) must involve the mean values of the physical quantities at a given temperature of the particle surface, which is determined by (2.4) and (3.10), to avoid large errors. It is also of interest to compare the thermophoretic velocity with that for a spherical particle with a radius equal to the equatorial radius of a spheroid, i.e., with formula (3.8). The numerical analysis showed that in this case, too, the relative error increases with increasing incident radiation intensity and equatorial radius. For example, in Table 1 ($a = 15$ μm), $f^{(2)} = 0.32$ at $I_0 = 0.5 \times 10^2$ W/m^2 and $f^{(2)} = 6.71$ at $I_0 = 10 \times 10^2$ W/m^2 ($f^{(2)} = (|f_{th}^{\mathrm{sph}} - f_{th}^{(b)}|/f_{th}^{(b)}) \times 100\%$). However, this increase is approximately 1.5 times smaller than that in the former case.

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