Effect of Particle Surface Heating on the Sedimentation Rate

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Abstract—The Stokes and Hadamard–Rybchinsky formulas are generalized, making it possible to take into account the temperature dependence of viscosity in a wide range of temperatures and to calculate the force of resistance to motion and velocity of gravitational fall at arbitrary temperature differences between the particle surface and a remote region.

Determining the granulometric composition of disperse systems via their sedimentation is one of the most practicable and widely used methods of dispersion analysis [1]. In cleaning of liquids to remove contaminating particles and dressing of minerals, it is important to accelerate sedimentation. This is commonly achieved in various ways, depending on the kind of particles and their environment. Sedimentometry is based on the relationship between the velocity of particle motion in a viscous medium and the particle size. The velocity of particle motion can be markedly raised by heating the particle surface, since the viscosity of a fluid decreases exponentially with increasing temperature. Of practical and theoretical interest in this regard is description of the gravitational motion of a heated particle in a viscous fluid.

Let us consider the gravitational motion of a uniformly heated hydrosol particle in another, viscous incompressible fluid, which is immiscible with the former and fills the entire space. At infinity, the fluid is at rest. By heated (cooled) particle is understood a particle whose surface temperature differs from that at a distance from the particle. The temperature difference between the particle and the viscous fluid can be maintained steadily via, e.g., heat release in chemical reactions at the particle surface, radioactive decay of the drop substance, external irradiation, etc. In particular, if a flux of electromagnetic radiation (wavelength λ_0, intensity I_0) is incident on the drop, then the energy absorbed by the drop is $\pi R^2 I_0 K_a$, where R is the drop radius and K_a is the absorption factor [2]. If the heat conductivity of the drop much exceeds that of the external medium and $\lambda_0 \gg R$, then the absorbed energy is evenly distributed over the drop surface and, consequently, the drop can be considered uniformly heated.

The heated particle surface affects the heat-transfer properties of the ambient fluid and, in the end, the distribution of the velocity and pressure fields in the vicinity of the particle.

In contrast to previous investigations [3–7], the present study generalizes the Stokes and Hadamard–Rybchinsky formulas to the case of uniformly heated spherical drop in a viscous incompressible fluid at arbitrary temperature difference between the particle surface and remote region with account of the temperature dependence of viscosity.

Of all fluid transfer parameters, the viscosity coefficient depends on temperature most strongly [8]. To take into account the temperature dependence of the dynamic viscosity, we use a formula that describes changes in the viscosity of a fluid in wide range of temperatures with any required accuracy (at $F_n = 0$ this formula can be reduced to the known Reynolds relation [8]):

$$\mu_e = \mu_\infty \left[1 + \sum_{n=1}^{\infty} F_n (T_e/T_\infty - 1)^n\right] \exp\left[-A(T_e/T_\infty - 1)\right], \quad (1)$$

where A and F_n are constants, $\mu_\infty = \mu_e(T_\infty)$, and T_∞ is the fluid temperature far away from the particle.

Hereinafter the indices e and i refer, respectively, to viscous fluid and heated particle; index ∞ denotes fluid parameters at infinity in an unperturbed flow; and index s refers to values of physical quantities at average surface temperature T_s.

It is known that the fluid viscosity decreases exponentially with increasing temperature [8]. An analysis of the available semiempirical formulas demonstrated that expression (1) describes the variation of viscosity in a wide temperature range in the best way, with any required accuracy. For example, we have $A = 5.779$, $F_1 = -2.318$, $F_2 = 9.118$ for water in the temperature range 0–90°C, with relative error not exceeding 2% ($T_\infty = 273$ K).

It is assumed that the densities, heat conductivities, and specific heats of the fluid and particle are constant, the heat conductivity coefficient of the particle much exceeds that of the medium, and particle motion is rather slow (small particles by Reynolds and Peclet).

It is convenient to introduce a reference system associated with the center of a moving particle (the problem is reduced to analysis of how an infinite plane-parallel flow, whose velocity U_∞ is to be determined, moves past a particle). The distributions of velocity and temperature is axially symmetric with respect to the $0Z$ axis passing through the drop center in the direction of the velocity vector of the incident flow. Therefore, the analysis uses a spherical system of coordinates in which radius is reckoned from the drop center, and angle θ, from the velocity direction of the incident flow.

In terms of the assumptions made, the equations and boundary conditions for the velocity, pressure, and temperature inside and outside a drop can be written in Stokes approximation in the spherical system of coordinates r, θ, η in the form [9]

$$\nabla P_e = \mu_e \nabla^2 U_e + 2(\nabla\mu_e)\nabla U_e + (\nabla\mu_e)\times(\nabla U_e),\ \operatorname{div} U_e = 0, \tag{2}$$

$$\mu_i \Delta \mathbf{U}_i = \nabla P_i, \qquad \operatorname{div} U_i = 0, \tag{3}$$

$$\Delta T_e = 0, \tag{4}$$

$$r = R, \quad U_r^e = U_r^i = 0, \quad U_\theta^e = U_\theta^i, \quad T_e = T_s, \tag{5}$$

$$r \to \infty,\ \mathbf{U}_e \to U_\infty \cos\theta \mathbf{e}_r - U_\infty \sin\theta \mathbf{e}_\theta,\ T_e \to T_\infty,\ P_e \to P_\infty, \tag{6}$$

$$r \to 0, \quad |\mathbf{U}_i| \neq \infty, \quad P_i \neq \infty, \tag{7}$$

where μ and λ are, respectively, the dynamic viscosity and heat conductivity; P is the pressure; U_r and U_θ are the radial and tangential components of the mass velocity of the fluid, $\mathbf{U}$, in the spherical system of coordinates; $U_\infty = |\mathbf{U}_\infty|$, U_∞ is the velocity of the incident flow, which is to be determined from the condition of vanishing total force acting on the particle ($U_\infty > 0$ if this velocity is directed along the z axis, and $U_\infty < 0$ otherwise); $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors in the spherical system of coordinates; and T_s is the mean temperature of the drop surface.

Conditions (5), assuming impermeability and continuity of the normal and tangential components of the mass velocity and constant particle surface temperature, are taken for the particle surface. As boundary conditions at infinity, i.e., far away from the particle, are taken conditions (6), and the finiteness of the physical quantities characterizing the drop at $r \to 0$ is accounted for by conditions (7).

To obtain a closed problem, the boundary conditions on the surface of a uniformly heated drop are supplemented with the condition of continuity of the stress tensor components (normal and tangential) on the drop surface [9]:

$$-P_e + 2\mu_e \frac{\partial U_r^e}{\partial r} = -P_i + 2\mu_i \frac{\partial U_r^i}{\partial r}, \tag{8}$$

$$\mu_e\left(\frac{1}{r}\frac{\partial U_r^e}{\partial \theta} + \frac{\partial U_\theta^e}{\partial r} - \frac{U_\theta^e}{r}\right) = \mu_i\left(\frac{1}{r}\frac{\partial U_r^i}{\partial \theta} + \frac{\partial U_\theta^i}{\partial r} - \frac{U_\theta^i}{r}\right). \tag{9}$$

The boundary conditions (5)–(9) make it possible to separate variables and reduce the system for perturbed quantities to a system of ordinary differential equations. Solving Eqs. (2)–(4) with boundary conditions (5)–(9) gives the distribution of velocities, pressures, and temperature in the fluid and the drop. After these distributions are found, we can calculate the force acting on a uniformly heated drop and the velocity of its drift.

At $\mathrm{Re}_\infty = \pi_e R U_\infty/\mu_\infty \ll 1$, the incident flow exerts only perturbing influence, and, therefore, the solution to the equations of hydrodynamics and heat transfer is to be sought for as an expansion in the Reynolds number Re_∞. In determining the force acting on a uniformly heated drop and its drift velocity, we restrict the consideration to corrections of the first order of smallness. To find these corrections, it is necessary to know the fields of velocity, pressure, and temperature in the vicinity of the particle. The general solution to the heat conduction equation (4), satisfying the relevant boundary conditions, has the form

$$t_e = 1 + \gamma/y, \qquad \gamma = (T_s - T_\infty)/T_\infty, \tag{10}$$

where γ is a dimensionless parameter characterizing the temperature difference between the particle surface and remote region; $t_e = T_e/T_\infty$, $y = r/R$.

Substituting (10) in (1), we obtain

$$\mu_e = \mu_\infty[1 + \sum_{n=1}^{\infty} F_n(\gamma^n/y^n)]\exp(-A\gamma/y). \quad (11)$$

Formula (11) is further used for determining the velocity and pressure fields in the vicinity of a uniformly heated drop.

Since the viscosity depends only on the coordinate y, the equations of hydrodynamics are solved by separation of variables, with the velocity and pressure fields expanded in Legendre and Gegenbauer polynomials [10]. In particular, the following expressions were derived for the components of the mass velocity U:

$$U_r^e(y,\theta) = U_\infty\cos\theta[1 + A_1G_1(y) + A_2G_2(y)], \quad (12)$$

$$U_\theta^e(y,\theta) = -U_\infty\sin\theta[1 + A_1G_3(y) + A_2G_4(y)], \quad (13)$$

$$U_r^i(y,\theta) = U_\infty\cos\theta(A_3 + A_4y^2), \quad U_\theta^i(y,\theta) = -U_\infty\sin\theta(A_3 + 2A_4y^2), \quad (14)$$

where

$$G_1(y) = -y^{-3}\sum_{n=0}^{\infty}\Delta_n^{(1)}[(n+3)y^n]^{-1},$$

$$G_1^{\mathrm{I}} = \mathrm{d}G_1/\mathrm{d}y, \quad G_2^{\mathrm{I}} = \mathrm{d}G_2/\mathrm{d}y,$$

$$G_2(y) = -y^{-1}\sum_{n=0}^{\infty}\Delta_n^{(2)}[(n+1)y^n]^{-1} - \alpha y^{-3}\sum_{n=0}^{\infty}[(n+3)\ln y^{-1} - 1]\Delta_n^{(1)}[(n+3)^2y^n]^{-1},$$

$$\Delta_n^{(1)} = [n(n+5)]^{-1}\sum_{k=1}^{n}\{(n+4-k)[\alpha_k^{(1)}(n+5-k) - \alpha_k^{(2)}] + \alpha_k^{(3)}\}\gamma^k\Delta_{n-k}^{(1)} \quad (n \geq 1). \quad (15)$$

$$G_3(y) = G_1(y) + 0.5yG_1^{\mathrm{I}}, \quad G_4(y) = G_2(y) + 0.5yG_2^{\mathrm{I}},$$

$$\Delta_n^{(2)} = [(n+3)(n-2)]^{-1}[-6\alpha_n^{(4)}\gamma^n + \sum_{k=1}^{n}\{(n+2-k) \times [\alpha_k^{(1)}(n+3-k) - \alpha_k^{(2)}] + \alpha_k^{(3)}\}\gamma^k\Delta_{n-k}^{(2)} + \alpha\sum_{k=0}^{n}[(2n+5 - 2k)\alpha_k^{(1)} - \alpha_k^{(2)}]\gamma^k\Delta_{n-k-2}^{(1)} \quad (n \geq 3). \quad (16)$$

In calculating the coefficients $\Delta_n^{(1)}$, $\Delta_n^{(2)}$ by means of recurrence formulas (15) and (16), account should be taken that

$$\Delta_0^{(1)} = -3, \ \Delta_0^{(2)} = -1, \ \Delta_2^{(2)} = -1, \ \alpha_0^{(1)} = \alpha_0^{(4)} = -1,$$

$$\alpha_0^{(2)} = 4, \ \alpha_0^{(3)} = -4, \ \alpha_n^{(1)} = F_n, \ \alpha_n^{(2)} = (4-n)F_n + AF_{n-1},$$

$$\alpha_n^{(3)} = 2AF_{n-1} - 2(2+n)AF_n, \ \alpha_n^{(4)} = A^n/n!,$$

$$\alpha = -\gamma\{6\gamma^2\alpha_2^{(4)} - [3(4\alpha_1^{(1)} - \alpha_1^{(2)}) + \alpha_1^{(3)}]\Delta_1^{(2)} + [2(3\alpha_2^{(1)} - \alpha_2^{(2)}) + \alpha_2^{(3)}]\gamma\}/15,$$

$$\Delta_1^{(2)} = -\gamma[6\alpha_1^{(4)} + 2(3\alpha_1^{(1)} - \alpha_1^{(2)}) + \alpha_1^{(3)}]/4.$$

The integration constants A_1–A_4, appearing in Eqs. (12)–(14), are found upon their substitution in the corresponding boundary conditions on the drop surface. With this done, the force acting upon the particle is found by integration of the stress tensor over its surface [9] and has the following form:

$$\mathbf{F} = -4\pi R\mu_\infty U_\infty A_2\exp\{-A\gamma\}\mathbf{n}_z, \quad (17)$$

where

$$A_2 = -(N_3 + N_4\mu_e^s/3\mu_i^s)/\Delta, \quad \Delta\Big|_{y=1} = N_1 + N_2\mu_e^s/\mu_i^s,$$

$$N_4\Big|_{y=1} = 2G_1^{\mathrm{I}} + G_1^{\mathrm{II}},$$

$$N_2\Big|_{y=1} = G_2(2G_1^{\mathrm{I}} + G_1^{\mathrm{II}}) - G_1(2G_2^{\mathrm{I}} + G_2^{\mathrm{II}}),$$

$$N_1\Big|_{y=1} = G_1G_2^{\mathrm{I}} - G_2G_1^{\mathrm{I}}, \quad N_3\Big|_{y=1} = G_1^{\mathrm{I}},$$

The index s denotes the values of the physical quantities for the mean surface temperature of a uniformly heated drop, T_S; $\mathbf{n}_z$ is the unit vector in the direction of the z axis.

It is noteworthy that the force **F** is calculated on the assumption of a uniform drop motion, which is only possible if the total force acting upon the particle is zero. Since the force (17) is proportional to the velocity and vanishes together with the latter, then, for uniform motion of a heated drop to occur, it is necessary to assume the presence of a certain extraneous force counterbalancing the force (17).

Substituting the explicit form of coefficient A_2 in (17), we obtain the expression

$$\mathbf{F} = 6\pi R\mu_\infty U_\infty f_\mu\mathbf{n}_z, \quad (18)$$

$$f_\mu = 2\exp\{-A\gamma[N_3 + N_4\mu_e^s/(3\mu_i^s)]\}/[3(N_1 + N_2\mu_e^s/\mu_i^s)].$$

Let us consider as an example the gravitational fall of a uniformly heated drop. A spherical drop falling under the action of gravitational force in a viscous medium ultimately starts to move at a constant velocity at which the action of the gravitational force is counterbalanced by hydrodynamic forces. The gravitational force acting upon a particle is given, with account of the buoyancy force, by

$$\mathbf{F} = (\rho_i - \rho_e)g\pi R^3\mathbf{n}_z\times 4/3, \qquad (19)$$

where g is the gravitational acceleration.

Equating expressions (18) and (19), we obtain the fall velocity of a uniformly heated spherical particle (analog of the Hadamard–Rybchinsky formula):

$$\mathbf{U} = h_\mu\mathbf{n}_z, \qquad (20)$$

where

$$h_\mu = \frac{2}{9}R^2\frac{\rho_i - \rho_e}{\mu_\infty f_\mu}g.$$

If $\mu_e^s/\mu_i^s \to 0$ in expressions (18) and (20), then we obtain a formula for the resistance force and the velocity of gravitational fall of a solid uniformly heated particle (analog of the Stokes formula).

Thus, formulas (18) and (20) allow evaluation of the force acting upon a uniformly heated drop and the velocity of its gravitational fall with account of the temperature dependence of viscosity, represented as an exponential-power series at arbitrary temperature differences between the particle surface and a remote region.

In the case when the drop surface is heated to a relatively low extent, i.e., the mean surface temperature differs only slightly from the temperature of the ambient at infinity [$\gamma = (T_s - T_\infty)/T_\infty \to 0$], the temperature dependence of the dynamic viscosity coefficient can be neglected and $G_1 = 1$, $G_1^I = -3$, $G_1^{II} = 12$, $G_2 = 1$, $G_2^I = -1$, $G_2^{II} = 2$, $N_1 = 2$, $N_2 = 6$, $N_3 = 3$, and $N_4 = 6$. In this case, formulas (18) and (20) are transformed into the known expression for a sphere [9, 10].

Mention should be made of some problems concerning the motion of a uniformly heated spherical drop, whose solutions can be found directly from the results obtained here. Let us consider motion of a particle containing uniformly distributed heat sources

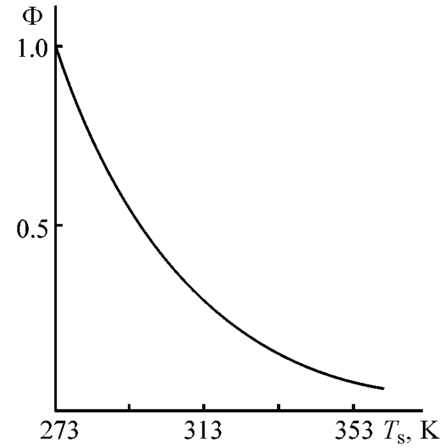

Fig. 1. Contribution to the resistance force, $\Phi = f_\mu/f_\mu|_{T_s = 273\text{ K}}$, vs. the mean temperature T_s of the mercury drop surface.

(drains) with constant power and density q_1, i.e., the motion of a particle with uniform internal heat release. In this case, it is necessary to supplement Eqs. (2)–(4) with an equation describing the distribution of temperature within the drop ($\Delta T_i = q_i/\lambda_i$) and to take into account the equality of temperatures and heat fluxes in the boundary conditions. Then the mean surface temperature of a uniformly heated drop is given by the relation

$$T_s/T_\infty = 1 + q_i R^2/(\lambda_e T_\infty). \qquad (21)$$

It is noteworthy that, in formulating the problem, the boundary condition (8) for normal stresses, unnecessary in analyzing the flow around a uniformly heated drop, was left out. It can be shown that, upon substitution of the obtained solution for velocity and pressure components, the given boundary condition reduces to identity. This means that, in the approximation employed, the drop remains strictly spherical and its shape should be analyzed in terms of higher-order approximations.

To illustrate the contribution of particle surface heating to the resistance force and the fall velocity of a drop, i.e., to take into account the temperature dependence of viscosity, represented by formula (1), Figs. 1 and 2 show curves relating the values $\Phi = f_\mu/f_\mu|_{T_s = 273\text{ K}}$ and $\Phi_1 = h_\mu/h_\mu|_{T_s = 273\text{ K}}$ to T_s values for large mercury drops with radius $R = 2\times 10^{-5}$ m, moving in water at $T_\infty = 273$ K. The curve Φ_1^* is plotted using the formula for small relative temperature differences ($\gamma \to 0$) [9, 10], but with the molecular transfer coefficients taken for $T_e = T_s$. As seen from the presented curves, the drop surface heating

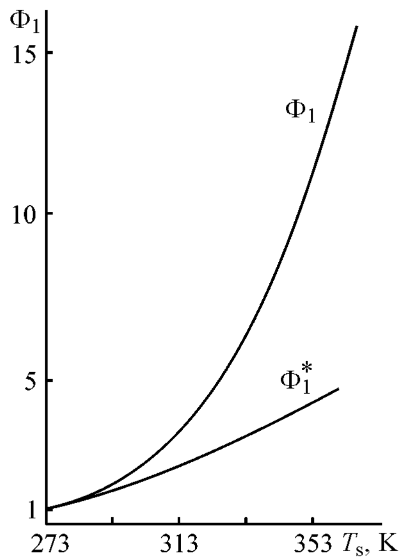

Fig. 2. Fall velocity of mercury drop, $\Phi_1 = h_\mu/h_\mu|_{T_s = 273\,\text{K}}$, vs. the mean temperature T_s of the mercury drop surface.

strongly affects the resistance force and the velocity of gravitational fall of the drop. This result can be used in practice for accelerating the sedimentation process [11]. Let us consider selective sedimentation of coal particles in a water flow with temperature of 20°C in a rectangular chamber with length L = 100 cm, width d = 80 cm, and height h = 5 cm. In the absence of heating, the particle velocity in this chamber is 1.109×10^{-2} cm s^{-1}. To this velocity corresponds the volumetric flow rate of cleaned water Q = 0.319 m^3 h^{-1} [11]. The formulas derived in the present study yield Q = 0.356 m^3 s^{-1}, which corresponds to sedimentation accelerated by a factor of 1.12, and, with the particle surface heated to 50°C, we obtain Q = 0.588 m^3 s^{-1}, which already gives an acceleration by a factor of 1.84. Consequently, the use of the derived formulas enables acceleration of particle sedimentation.

CONCLUSIONS

(1) An analytical solution of the problem of a fluid flow around a uniformly heated particle at small Reynolds numbers is described. In solving the hydrodynamics equations, account is taken of the temperature dependence of dynamic viscosity, represented as an exponential–power-law series. The Stokes and Hadamard–Rybchinsky formulas are generalized, which makes it possible to calculate the resistance force and the velocity of gravitational fall at arbitrary temperature differences between the particle surface and a remote region. It is shown that particle surface heating markedly affects the resistance force.

(2) The results obtained can be used in evaluating the settling velocity of spherical hydrosol particles in channels, in designing experimental installations in which a directed motion of hydrosol particles is to be ensured, in developing methods for fine cleaning of fluids to remove hydrosol particles, etc.

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