ON THE EFFECT OF ANOMALOUS PHOTOABSORPTION IN PARAMETRIC X RADIATION

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The parametric X radiation of relativistic electrons passing through a monocrystal is considered. It is demonstrated that, contrary to widespread opinion, conditions for the effect of anomalous photoabsorption to show up can be realized in the Laue scattering geometry. It is expected that the predicted effect will increase the spectral-angular density of the radiation several times.

INTRODUCTION

One of the most brilliant effects of the dynamic scattering of free X rays in a crystal is the effect of anomalously low-intense photoabsorption resulting from a special disposition of the electric field of the X-ray wave relative to the crystal atoms [1]. Of considerable importance is the question of whether a similar effect takes place in the parametric X radiation (PXR) that results from the Bragg diffraction of the pseudophotons of the Coulomb field of a fast microparticle moving in a crystal [2–4]. Since the effect of anomalous photoabsorption shows up only under the conditions of the Bragg resonance for a photon scattered by a crystal, which may not be fulfilled precisely in the process of emission of PXR, it is believed that the effect under discussion does not take place in PXR [5, 6]. However, if photoabsorption could be suppressed, this would make it possible to enhance substantially the luminosity of quasimonochromatic X-radiation sources based on the PXR mechanism. Therefore, a detailed analysis of the feasibility of the effect of anomalous photoabsorption in PXR is quite reasonable.

Earlier it has been shown that the additional contribution of the transition radiation generated by a fast particle at the inlet surface of a crystal, which diffracts at the atomic planes that are responsible for the appearance of PXR, to the PXR yield can increase abruptly due to the effect of anomalous photoabsorption which is associated with the free photons of the transition radiation [7]. In the work [8] where PXR in the Bragg scattering geometry was considered, the effect of anomalous photoabsorption was predicted directly for the scattering of the pseudophoton field of a radiating particle. It has been shown that for the case of a fast particle having a rather high energy, such that the PXR intensity peaks near the anomalous dispersion region (region of complete outward reflection of X rays from the crystal), the spectral-angular density of PXR increases by more than an order of magnitude if the imaginary parts of the corresponding coefficients of the Fourrier series for the dielectric susceptibility appear to be close in value (this condition is identical to that under which the effect of anomalous photoabsorption shows up for free X-radiation waves in a crystal).

In this work, the PXR of a relativistic charge particle in the Laue geometry is investigated. It is shown that the effect of anomalous photoabsorption is not realized in the region of low energies of the radiating particle for which the kinematic approximation of the theory of diffraction of X rays in a crystal is valid. However, for the high energy region where the effects of dynamic diffraction show up, the anomalous photoabsorption has a considerable effect on the PXR properties although the condition of sharp Bragg resonance for the radiated wave is not fulfilled. Also, the contribution of the diffracted transition radiation to the total radiation yield and the interference of this radiation with PXR are investigated.

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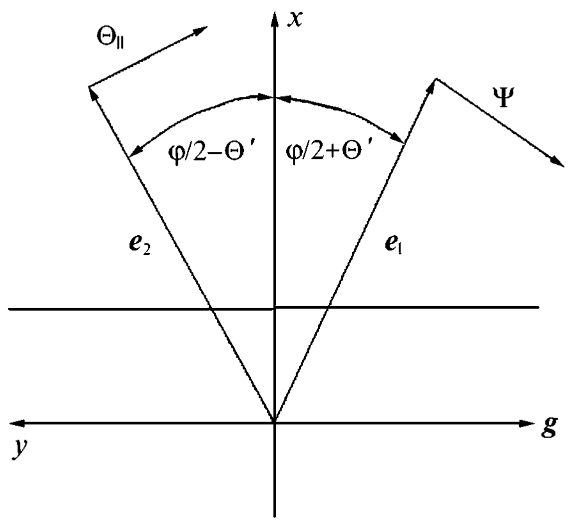

Fig. 1. Geometry of emission of parametric radiation: e_1 and e_2 are the respective axes of the electromagnetic beam and the radiation detector; Θ' is the orientation angle, and $\boldsymbol{g}$ is the inverse lattice vector.

GENERAL RELATIONSHIPS

Let us consider the radiation emitted by a fast particle moving with a constant velocity v in a crystal with a permittivity $\varepsilon(\omega,\boldsymbol{r}) = 1+\chi_0(\omega)+\sum_{\boldsymbol{g}}'\chi_{\boldsymbol{g}}(\omega)e^{i\boldsymbol{g}\boldsymbol{r}}$ whose inlet and outlet surfaces are parallel to the yz-plane and the reflecting crystallographic plane specified by a fixed reciprocal lattice vector $\boldsymbol{g}$ is parallel to the $\boldsymbol{xz}$-plane (Fig. 1). We find the Fourier transform of the electric field $\boldsymbol{E}_{\omega k} = (2\pi)^{-4}\int dt d^3 r \boldsymbol{E}(\boldsymbol{r},t)e^{i\omega t - i\boldsymbol{k}\boldsymbol{r}}$ from Maxwell's equation

$$\left(k^2-\omega^2(1+\chi_0)\right)\boldsymbol{E}_{\omega k} - k(kE_{\omega k}) - \omega^2\sum_{\boldsymbol{g}}'\chi_{-\boldsymbol{g}}\boldsymbol{E}_{\beta\omega k+g} = \frac{i\omega e}{2\pi^2}\boldsymbol{v}\delta(\omega-\boldsymbol{k}v). \quad (1)$$

The excited magnetic field is essentially transverse in the X-ray frequency range ($\boldsymbol{E}_{\omega\boldsymbol{k}} \cong \sum_{\lambda=1}^{2}\boldsymbol{e}_{\lambda\boldsymbol{k}}E_{\lambda 0}$ and $\boldsymbol{E}_{\omega\boldsymbol{k}+\boldsymbol{g}} \cong \sum_{\lambda=1}^{2}\boldsymbol{e}_{\lambda\boldsymbol{k}+\boldsymbol{g}}E_{\lambda g}$ are the polarization vectors of $\boldsymbol{e}_{\lambda\boldsymbol{k}}$ and $\boldsymbol{e}_{\lambda\boldsymbol{k}+\boldsymbol{g}}$); therefore, in terms of the two-wave approximation of the dynamic theory of diffraction [9], Eq. (1) is reduced to the well-known system of equations

$$\left(k^2-\omega^2(1+\chi_0)\right)\boldsymbol{E}_{\lambda 0} - \omega^2\chi_{-\boldsymbol{g}}\alpha_\lambda E_{\lambda\boldsymbol{g}} = \frac{i\omega e}{2\pi^2}\boldsymbol{e}_{\lambda\boldsymbol{k}}\boldsymbol{v}\delta(\omega-\boldsymbol{k}v), \quad (2a)$$

$$\left((\boldsymbol{k}+\boldsymbol{g})^2-\omega^2(1+\chi_0)\right)\boldsymbol{E}_{\lambda\boldsymbol{g}} - \omega^2\chi_{\boldsymbol{g}}\alpha_\lambda E_{\lambda 0} = 0. \quad (2b)$$

The equations for the field in vacuum outside the crystal follow from (2) in the limit $\chi_0 = \chi_g = 0$. The quantities α_λ in (2) are defined by the relations $\alpha_1 = 1$ and $\alpha_2 = \boldsymbol{k}(\boldsymbol{k}+\boldsymbol{g})/\mathrm{k}|\boldsymbol{k}+\boldsymbol{g}|$ that follow from the definitions $\boldsymbol{e}_{1\boldsymbol{k}} = \boldsymbol{e}_{1\boldsymbol{k}+\boldsymbol{g}} = [\boldsymbol{k}\boldsymbol{g}]/\sqrt{k^2g^2-(\boldsymbol{k}\boldsymbol{g})^2}$, $\boldsymbol{e}_{2\boldsymbol{k}} = [\boldsymbol{k}e_{1\boldsymbol{k}}]/k$, $\boldsymbol{e}_{2\boldsymbol{k}+\boldsymbol{g}} = [\boldsymbol{k}+g, e_{1\boldsymbol{k}}]/|\boldsymbol{k}+g|$.

The solutions of the system of equations (2) for the diffracted field in the crystal

$$E_{\lambda \boldsymbol{g}}^{\mathrm{cr}} = a_\lambda \delta(k_{gx} - k_1) + b_\lambda \delta(k_{gx} - k_2) + \frac{i\omega e}{2\pi^2 v_x} \frac{e_{\lambda k} v \omega^2 \chi_g \alpha_\lambda}{(k_{gx}^2 - k_1^2)(k_{gx}^2 - k_2^2)} \delta(k_{gx} - k_*) \tag{3a}$$

and in vacuum behind the crystal ($x > L$ with L being the thickness of the crystal)

$$E_{\lambda \boldsymbol{g}}^{\mathrm{vac}} = C_\lambda \delta(k_{gx} - p) \tag{3b}$$

contain unknown coefficients a_λ, b_λ, and C_λ. To determine these coefficients, it is necessary to use Eqs. (2) and conventional boundary conditions for the fields at the inlet and outlet surfaces of the crystal. In relations (3), the following notations are used:

$$k_{1,2}^2 = p^2 + \omega^2 \chi_0 - g\left(\frac{g}{2} + k_{gy}\right) \pm \sqrt{g^2 \left(\frac{g}{2} + k_{gy}\right)^2 + \omega^4 \chi_{\boldsymbol{g}} \chi_{-\boldsymbol{g}} \alpha_\lambda^2}\ ,$$

$$k_* = \frac{1}{v_x}(\omega - g v_y - \boldsymbol{k}_{g\parallel} \boldsymbol{v}_\parallel), \quad \boldsymbol{k}_g = \boldsymbol{k} + g = \boldsymbol{e}_x k_{gx} + \boldsymbol{k}_{g\parallel}, \quad \boldsymbol{e}_x \boldsymbol{k}_{g\parallel} = 0\,, \tag{4}$$

$$p^2 = \omega^2 - k_{g\parallel}^2 .$$

Simple manipulations yield the following expression for the coefficient C_λ that describes the radiation field:

$$\begin{aligned} C_\lambda(\boldsymbol{k}_{g\parallel}) = {} & \frac{i\omega e}{2\pi^2 v_x} \boldsymbol{e}_\lambda v \frac{\omega^2 \chi_{\boldsymbol{g}} \alpha_\lambda}{2\sqrt{g^2 \left(\frac{g}{2} + k_{gy}\right)^2 + \omega^4 \chi_{\boldsymbol{g}} \chi_{-\boldsymbol{g}} \alpha_\lambda^2}} \\ & \times \left[\left(\frac{1}{k_*^2 - k_1^2} - \frac{1}{k_*^2 - p^2 + 2g\left(\frac{g}{2} + k_{gy}\right)} \right) \left(1 - e^{-i(k_* - k_1)L}\right) \right. \\ & \left. - \left(\frac{1}{k_*^2 - k_2^2} - \frac{1}{k_*^2 - p^2 + 2g\left(\frac{g}{2} + k_{gy}\right)} \right) \left(1 - e^{-i(k_* - k_2)L}\right) \right] e^{i(k_x - p)L} . \end{aligned} \tag{5}$$

To determine the spectral-angular distribution of the radiation, it is necessary to calculate the Fourier integral

$$E_\lambda^{\mathrm{rad}} = \int d^3 k_g e^{i k_g \boldsymbol{n} \boldsymbol{r}} E_{\lambda \boldsymbol{g}}^{\mathrm{vac}}\ , \tag{6}$$

where $\boldsymbol{n}$ is the unit vector directed in line with the radiation and $E_{\lambda \boldsymbol{g}}^{\mathrm{vac}}$ is determined by relation (3b). For the wave zone, the integral (6) can be calculated by the stationary phase method. The calculations yield

$$E_\lambda^{\mathrm{rad}} = A_\lambda \frac{e^{i\omega r}}{r}, \quad A_\lambda = -2\pi i \omega n_x C_\lambda(\omega \boldsymbol{n}_\parallel)\,. \tag{7}$$

Formulas (7) and (5) completely describe the spectral-angular and polarization properties of the radiation propagating in line with the Bragg scattering.

For further analysis it is convenient to bring into consideration angular variables $\boldsymbol{\Theta}$ and $\boldsymbol{\Psi}$ in accordance with the definitions

$$\boldsymbol{v} = \boldsymbol{e}_1\left(1-\frac{1}{2}\gamma^{-2}-\frac{1}{2}\Psi^2\right)+\boldsymbol{\Psi}, \quad \boldsymbol{e}_1\boldsymbol{\Psi}=0, \tag{8}$$

$$\boldsymbol{n} = \boldsymbol{e}_2\left(1-\frac{1}{2}\Theta^2\right)+\boldsymbol{\Theta}, \quad \boldsymbol{e}_2\boldsymbol{\Theta}=0, \quad \boldsymbol{e}_1\boldsymbol{e}_2=\cos\varphi .$$

We also will use the simplest expressions for the susceptibilities χ_0 and $\chi_{\pm g}$:

$$\chi_0 = -\frac{\omega_0^2}{\omega}+i\chi_0'', \quad \omega_0^2=\frac{4\pi Ze^2 n_0}{m}, \quad \frac{\omega^2}{\omega_0^2}\chi_0''=f''<<1,$$

$$\chi_g=\chi_{-g}=-\frac{\omega_g^2}{\omega^2}+i\chi_g'', \quad \omega_g^2=\omega_0^2\frac{F(\boldsymbol{g})}{Z}\frac{S(\boldsymbol{g})}{N_0}e^{-\frac{1}{2}g^2u^2}, \tag{9}$$

where $F(g)$ is the atomic form factor, Z is the number of electrons in the atom, $S(g)$ is the structure factor of an elementary crystal cell which contains N_0 atoms, n_0 is the density of atoms, and u is the root-mean-square amplitude of the thermal oscillations of the atoms.

Substitution of (8) and (9) into formulas (7) and (5) yields the final expression for the radiation amplitude:

$$A_\lambda = A_\lambda^1 + a_\lambda^2, \tag{10a}$$

$$A_\lambda^{(1)}=\frac{e}{2\pi}\frac{\boldsymbol{e}_\lambda \boldsymbol{v}}{\sqrt{1+\tau_\lambda^2}}\frac{1}{\gamma^{-2}+\frac{\omega_0^2}{\omega^2}+(\Theta_\perp-\Psi_\perp)^2+\left(2\Theta'+\Theta_\parallel+\Psi_\parallel\right)^2}$$

$$\times\left[\frac{\tau_\lambda+\sqrt{1+\tau_\lambda^2}}{\sigma_\lambda-\tau_\lambda-\sqrt{1+\tau_\lambda^2}-i\beta_\lambda\left(1-k_\lambda/\sqrt{1+\tau_\lambda^2}\right)}\right.$$

$$\times\left(1-\exp\left(-\frac{\omega L\chi_0''}{2\cos\varphi/2}\left(1-\frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)-i\frac{\omega_g^2 L\alpha_\lambda}{2\omega\cos\varphi/2}\left(\sigma_\lambda-\tau_\lambda-\sqrt{1+\tau_\lambda^2}\right)\right)\right)$$

$$-\frac{\tau_\lambda-\sqrt{1+\tau_\lambda^2}}{\sigma_\lambda-\tau_\lambda+\sqrt{1+\tau_\lambda^2}-i\beta_\lambda\left(1+k_\lambda/\sqrt{1+\tau_\lambda^2}\right)}$$

$$\left.\times\left(1-\exp\left(-\frac{\omega L\chi_0''}{2\cos\varphi/2}\left(1+\frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)-i\frac{\omega_g^2 L\alpha_\lambda}{2\omega\cos\varphi/2}\left(\sigma_\lambda-\tau_\lambda+\sqrt{1+\tau_\lambda^2}\right)\right)\right)\right], \tag{10b}$$

$$A_\lambda^{(2)}=\frac{e}{2\pi}\frac{\boldsymbol{e}_\lambda \boldsymbol{v}}{\sqrt{1+\tau_\lambda^2}}\left(\gamma^{-2}+(\Theta_\perp-\Psi_\perp)^2+\left(2\Theta'+\Theta_\parallel+\Psi_\parallel\right)^2\right)^{-1}$$

$$-\left(\gamma^{-2}+\frac{\omega_0^2}{\omega^2}+(\Theta_\perp-\Psi_\perp)^2+\left(2\Theta'+\Theta_\parallel+\Psi_\parallel\right)^2\right)^{-1}$$

$$\times\left\{\exp\left(-\frac{\omega L\chi_0''}{2\cos\varphi/2}\left(1-\frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)-i\frac{\omega_g^2 L\alpha_\lambda}{2\omega\cos\varphi/2}\left(\sigma_\lambda-\tau_\lambda-\sqrt{1+\tau_\lambda^2}\right)\right)\right.$$

$$\left.-\exp\left(-\frac{\omega L\chi_0''}{2\cos\varphi/2}\left(1+\frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)-i\frac{\omega_g^2 L\alpha_\lambda}{2\omega\cos\varphi/2}\left(\sigma_\lambda-\tau_\lambda+\sqrt{1+\tau_\lambda^2}\right)\right)\right\}. \tag{10c}$$

The quantity $A_\lambda^{(1)}$ determined by formula (10b) describes the amplitude of the parametric X radiation and the quantity $A_\lambda^{(2)}$ corresponds to the transition radiation that appears at the inlet surface of the crystal and is diffracted in line with the Bragg scattering. In formulas (10), the following notations are used:

$$\tau_\lambda=\frac{g^2}{2\omega_g^2\alpha_\lambda}\left(\frac{\omega_B'}{\omega}-1\right),\qquad \omega_B'=\omega_B\left(1+\left(\Theta'+\Theta_\parallel\right)\operatorname{cotan}\varphi/2\right),\qquad \omega_B=\frac{g}{2\sin\varphi/2},$$

$$\sigma_\lambda=\frac{\omega^2}{\omega_g^2\alpha_\lambda}\left(\gamma^{-2}+\frac{\omega_0^2}{\omega^2}+(\Theta_\perp-\Psi_\perp)^2+\left(2\Theta'+\Theta_\parallel+\Psi_\parallel\right)^2\right), \tag{11}$$

$$\beta_\lambda=\frac{\omega^2\chi_0''}{\omega_g^2\alpha_\lambda}<<1,\qquad k_\lambda=\frac{\chi_g''}{\chi_0''}\alpha_\lambda.$$

Relations (10) and (11) form the basis for further analysis of the radiation properties.

THE EFFECT OF ANOMALOUS PHOTOABSORPTION IN PXR

Let us first consider the properties of PXR. According to (10b), the two branches of the solution of the dispersion relation $k_{gx}=k_{1,2}(\omega,\boldsymbol{n})$ (see (4)) make contribution to the PXR yield. However, it can readily be seen that for a sufficiently thick crystal, such that the range of a radiating particle in the target, $L/(\cos\varphi/2)$, is substantially greater than the extinction length of X rays in the crystal, $\omega/(\omega_g^2\alpha_\lambda)$ (it is this case that is of practical interest from the viewpoint of the creation of efficient PXR-based sources of quasimonochromatic X radiation), the radiation yield is formed in the main only by one branch that is associated with the first term in (10c). Indeed, one can immediately check that it is only in this term that the real part of the denominator vanishes. The solution of the corresponding equation

$$\sigma_\lambda-\tau_\lambda(\omega)-\sqrt{1+\tau_\lambda^2(\omega)}=0 \tag{12}$$

defines the frequency ω_* in the neighborhood of which the spectrum of the PXR photons radiated at a fixed observation angle is localized.

From (12) and the respective formula (11) that determines the quantity $\tau_\lambda(\omega)$ we get the relation

$$\frac{\omega_B'}{\omega_*}-1=\frac{\omega_g^2\alpha_\gamma}{g^2}\frac{\sigma_\lambda^2-1}{\sigma_\lambda}<<1, \tag{13}$$

from which it follows that the quantity ω_* is very close in value to the Bragg frequency ω'_B.

It is of critical importance that the wave making the main contribution to the formation of PXR is a weakly damped wave, that is, the coefficient of absorption for this wave, $\omega\chi_0'' = \left(1 - k_\lambda / \sqrt{1+\tau_\lambda^2}\right)$, is lower than the respective coefficient of absorption for a wave propagating in the crystal far away from the region where the Bragg resonance condition is fulfilled. To analyze the effect of photoabsorption, we consider the spectral-angular distribution of PXR:

$$\omega \frac{dN_\lambda^{(1)}}{d\omega d^2\Theta} = A_\lambda^{(1)^2} = \frac{e^2}{4\pi^2} \frac{(\boldsymbol{e}_\lambda \boldsymbol{v})^2}{\left(\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + (\Theta_\perp - \Psi_\perp)^2 + \left(2\Theta' + \Theta_\parallel + \Psi_\parallel\right)^2\right)^2}$$

$$\times \left(1 + \frac{\tau_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)^2 \frac{\mathcal{J} - \mathcal{G}}{\left(\sigma_\lambda - \tau_\lambda - \sqrt{1+\tau_\lambda^2}\right)^2 + \beta_\lambda^2 \left(1 - k_\lambda / \sqrt{1+\tau_\lambda^2}\right)} \tag{14}$$

with the following notations:

$$\mathcal{J} = 1 + \exp\left\{ -\frac{\omega L \chi_0''}{\cos\varphi/2} \left(1 - \frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)\right\},$$

$$\mathcal{G} = 2\exp\left\{ -\frac{\omega L \chi_0''}{\cos\varphi/2} \left(1 - \frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)\right\} \cos\left[\frac{\omega_g^2 L \alpha_\lambda}{2\omega\cos\varphi/2} \left(\sigma_\alpha - \tau_\lambda - \sqrt{1+\tau_\lambda^2}\right)\right],$$

$e_1 v = \Theta_\perp - \Psi_\perp$, $e_2 v = \Theta'' + \Theta_\parallel + \Psi_\parallel$, $\alpha_1 = 1$, and $\alpha_2 = \cos\varphi$. The dependence of the distribution (14) on photon energy ω is almost completely provided by the universal variable $\tau_\lambda(\omega)$.

The distribution (14) is substantially different from the traditional distribution predicted by the dynamic theory of PXR; however, in the limit

$$\frac{\omega L \chi_0''}{2\cos\varphi/2} << 1 \quad \text{and} \quad \frac{\omega_g^2 L \alpha_\lambda}{2\omega\cos\varphi/2} << 1,$$

in view of the dependence $\tau_\lambda(\omega)$ from (11) and the passage to the limit $(1 - \cos nx)/x^2 \to \pi n \delta(x)$ $(n >> 1)$, we obtain from (14) the well-known formula

$$\omega \frac{dN_\lambda^{(1)}}{d\omega d^2\Theta} = \frac{e^2 \omega_g^4}{\pi g^2} \frac{(\boldsymbol{e}_\lambda \boldsymbol{v})^2 \alpha_\lambda^2}{\left\{\gamma^{-2} + \frac{\omega_0^2}{\omega^2} + (\Theta_\perp - \Psi_\perp)^2 + \left(2\Theta' + \Theta_\parallel + \Psi_\parallel\right)^2\right\}^2 + \alpha_\lambda^2 \omega_g^4 / \omega^4} \frac{L}{\cos\varphi/2} \delta(\omega - \omega'_B). \tag{15}$$

Returning to the analysis of the effect of photoabsorption, we consider relation (14) for the spectral distribution maximum at a fixed angle of observation of the radiation. Using (12), we get

$$\left(\omega \frac{dN_\lambda^{(1)}}{d\omega d^2\Theta}\right)_{\max} = \frac{e^2}{\pi^2} \frac{\sigma_\lambda^2 (\boldsymbol{e}_\lambda \boldsymbol{v})}{\left(\sigma_\lambda^2 + 1 - 2\sigma_\lambda k_\lambda\right)^2 (\chi_0'')^2} \left(1 - \exp\left(-\frac{\omega L \chi_0''}{2\cos\varphi/2} \frac{\sigma_\lambda^2 + 1 - 2\sigma_\lambda k_\lambda}{\sigma_\lambda^2 + 1}\right)\right)^2. \tag{16}$$

According to (16), as the target thickness is increased, the PXR yield tends to saturation and the role of photoabsorption becomes more and more pronounced. Let us consider the dependence of the amplitude of the PXR spectrum on the energy of the radiating particle for the case of greatest practical interest where the detector of the radiation is placed on the direction that coincides with the maximum of the angular distribution of PXR $\left((\Theta_\perp-\Psi_\perp)^2+(2\Theta'+\Theta_\parallel+\Psi_\parallel)^2=\gamma^{-2}+\dfrac{\omega_0^2}{\omega^2}\right)$. Assuming that the crystal thickness is greater than the saturation thickness, we get from (16) the following formula:

$$\left(\omega\frac{dN_\lambda^{(1)}}{d\omega d^2\Theta}\right)_{\max}=\frac{e^2}{\pi^2}\left(\frac{\delta_\lambda}{\gamma_*\chi_0''}\right)^2F(k_\lambda,\gamma/\gamma_*),$$

$$F=\frac{\left(1+\gamma^2/\gamma_*^2\right)^3\gamma^2/\gamma_*^2}{\left(1+\gamma^2/\gamma_*^2\right)^2+(\delta_\lambda/2)^2\gamma^4/\gamma_*^4-\delta_\lambda k_\lambda\left(1+\gamma^2/\gamma_*^2\right)\gamma^2/\gamma_*^2}. \tag{17}$$

Here

$$\delta_\lambda=\frac{\omega_g^2}{\omega_0^2}\alpha_\lambda\sim1,\qquad \gamma_*=\frac{g}{2\omega_0\sin\varphi/2}>>1. \tag{18}$$

Formula (17) (the principal result of this work) predicts the feasibility of the effect of anomalous absorption in PXR. As with the diffraction of free X rays, the degree of impact of this effect is determined by the value of the parameter k_λ. However, for the emission process under consideration, the range of the stipulated effect also depends on the parameter δ_λ and on the ratio γ/γ_*. It can readily be seen that for weak reflections ($\delta_\lambda << 1$) the effect of anomalous photoabsorption does not show up. The function $F\approx\left(\gamma^2/\gamma_*^2\right)\left(1+\gamma^2/\gamma_*^2\right)^{-1}$ describes in this case the saturation of the PXR yield with increasing γ due to the density effect [10]. For strong reflections ($\delta_\lambda\approx1$), the dependence of the function F on the parameter k_λ is highly different in character in the region of low particle energies ($\gamma << \gamma_*$) and in the region of high energies ($\gamma >> \gamma_*$). In the former region (region of kinematic PXR), the function $F\approx\left(\gamma^2/\gamma_*^2\right)$ is independent of the parameter k_λ, that is, the effect of anomalous photoabsorption does not show up in the PXR of low-energy particles with $\gamma << \gamma_*$ where the kinematic approximation of the PXR theory is valid. However, in the region of high energies of the radiating particle with $\gamma >> \gamma_*$, for which the dynamic PXR theory should be used, the function $F\approx\left(1+(\gamma_\lambda/2)^2-\delta_\lambda k_\lambda\right)^{-2}$ depends considerably on the parameter k_λ. It can readily be noticed that the ratio $F(k_\lambda\approx1)/F(k_\lambda\approx0)$ can reach a value of the order of 25; that is, the predicted effect of anomalous photoabsorption can substantially increase the spectral-angular density of PXR. The curves of the dependence $F\left(\gamma^2/\gamma_*^2\right)$ plotted by formula (17) for a fixed value of the parameter δ_λ and different values of the parameter k_λ are given in Fig. 2.

CONTRIBUTION OF THE DIFFRACTED TRANSITION RADIATION

Now we turn to the analysis of the contribution of the transition radiation diffracted in line with the Bragg scattering. Using (10c), we obtain the following expression for the spectral-angular distribution of the radiation:

$$\omega\frac{dN_\lambda^{(2)}}{d\omega d^2\Theta}=\frac{e^2}{4\pi^2}\left(\frac{1}{\gamma^{-2}+(\Theta_\perp-\Psi_\perp)^2+(2\Theta'+\Theta_\parallel+\Psi_\parallel)^2}\right.$$

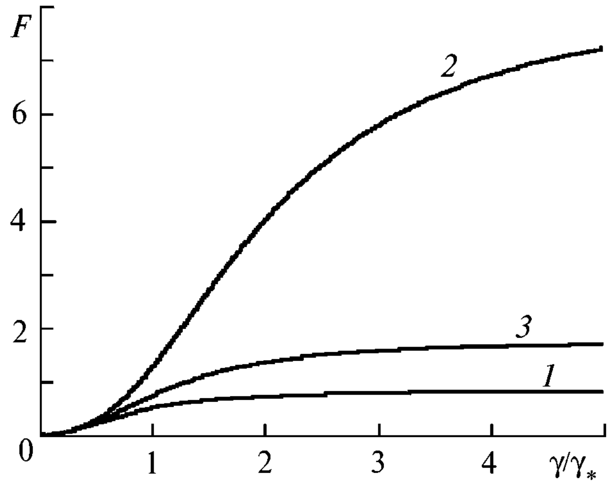

Fig. 2. Maximum radiation density versus the energy of the radiating particle. The curves are plotted for a fixed δ_λ and k_λ = 0 (*1*), 1 (*2*), and 0.9 (*3*).

$$
\left.-\frac{1}{\gamma^{-2}+\dfrac{\omega_0^2}{\omega^2}+(\Theta_\perp-\Psi_\perp)^2+(2\Theta'+\Theta_\parallel+\Psi_\parallel)^2}\right)^2\cdot\frac{(\boldsymbol{e}_\lambda\boldsymbol{v})^2}{1+\tau_\lambda^2}
$$

$$
\times\left[\exp\left(-\frac{\omega L\chi_0''}{\cos\varphi/2}\left(1-\frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)\right)+\exp\left(-\frac{\omega L\chi_0''}{\cos\varphi/2}\left(1+\frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)\right)\right.
$$

$$
\left.-2\exp\left(-\frac{\omega L\chi_0''}{\cos\varphi/2}\right)\cos\left(\frac{\omega_g^2 L\alpha_\lambda}{\omega\cos\varphi/2}\sqrt{1+\tau_\lambda^2}\right)\right]. \tag{19}
$$

Interference of waves corresponding to different branches of the solution of the dispersion relation (4) gives rise to oscillations in the cross section of the radiation described by (19). However, it can readily be seen that these oscillations may show up only in the case of a rather thin crystal with thickness $L < \cos\varphi/2/\omega\chi_0''$. For large L, the oscillations die out and the radiation at the inlet of the crystal can be observed only under conditions where anomalous photoabsorption shows up. In accordance with (19) and the definition of the function $\tau_\lambda(\omega)$ (11), the spectrum of the diffracted transition radiation consists of a narrow peak around the Bragg frequency ω'_B (the spectrum width is determined by the condition $|\tau_\lambda(\omega)| \le 1$, and under the conditions of anomalous photoabsorption, $k_\lambda \approx 1$, the spectrum width additionally decreases due to the abrupt increase in absorption with increasing τ_λ).

Since formula (19) describes the scattered transition radiation, the maximum in the angular distribution of the radiation coincides with that of the transition radiation ($(\Theta_\perp - \Psi_\perp)^2 + (2\Theta' + \Theta_{||} + \Psi_{||})^2 \approx \gamma^{-2}$). From (19) it can readily be seen that the radiation yield can be considerable only in the region of sufficiently high energies of the radiating particle ($\gamma \gg \gamma_*$) in complete compliance with the nature of the transition radiation. Assuming that the condition $\gamma \gg \gamma_*$ is satisfied, we consider the relative contribution of the diffracted transition radiation to the yield of the total radiation in the range of observation angles corresponding to the PXR maximum (see formula (17)). For the conditions of anomalous photoabsorption, from the relation

$$\left(\omega\frac{dN_\lambda^{(1)}}{d\omega d^2\Theta}\right)\Bigg/\left(\omega\frac{dN_\lambda^{(2)}}{d\omega d^2\Theta}\right)\approx\left(\frac{2}{f''}\frac{1+\left(\delta_\lambda/L^2\right)}{1+\left(\delta_\lambda/L^2\right)-\delta_\lambda k_\lambda}\right)^2>>1, \tag{20}$$

where the quantity f'' is defined in (9), it can be seen that it is possible to study experimentally the effect of anomalous photoabsorption in PXR not taking into account the contribution of the diffracted transition radiation.

We also dwell on the impact of the interference between PXR and the diffracted transition radiation. From formulas (10) we get the relation

$$\omega\frac{dN_\lambda^{\text{int}}}{d\omega d^2\Theta}\approx-\frac{e^2}{2\pi^2}\frac{(\boldsymbol{e}_\lambda\boldsymbol{v})^2}{\gamma^{-2}+\frac{\omega_0^2}{\omega^2}+(\Theta_\perp-\Psi_\perp)^2+\left(2\Theta'+\Theta_\parallel+\Psi_\parallel\right)^2}$$

$$\times\left(\frac{1}{\gamma^{-2}+(\Theta_\perp-\Psi_\perp)^2+\left(2\Theta'+\Theta_\parallel+\Psi_\parallel\right)^2}-\frac{1}{\gamma^{-2}+\frac{\omega_0^2}{\omega^2}+(\Theta_\perp-\Psi_\perp)^2+\left(2\Theta'+\Theta_\parallel+\Psi_\parallel\right)^2}\right)$$

$$\times\frac{\tau_\lambda+\sqrt{1+\tau_\lambda^2}}{1+\tau_\lambda^2}\frac{\sigma_\lambda-\tau_\lambda-\sqrt{1+\tau_\lambda^2}}{\left(\sigma_\lambda-\tau_\lambda-\sqrt{1+\tau_\lambda^2}\right)^2+\beta_\lambda^2\left(1-k_\lambda/\sqrt{1+\tau_\lambda^2}\right)^2} \tag{21}$$

$$\times\left[\exp\left(-\frac{\omega L\chi_0''}{2\cos\varphi/2}\left(1-\frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)\right)-\cos\left(\frac{\omega_g^2 L\alpha_\lambda}{2\omega\cos\varphi/2}\left(\sigma_\lambda-\tau_\lambda-\sqrt{1+\tau_\lambda^2}\right)\right)\right.$$

$$\left.\times\exp\left(-\frac{\omega L\chi_0''}{2\cos\varphi/2}\left(1-\frac{k_\lambda}{\sqrt{1+\tau_\lambda^2}}\right)\right)\right]$$

that describes the interference contribution to the total radiation yield. Formula (19) indicates that the interference term is small for the most interesting region of frequencies and observation angles that corresponds to the contribution of PXR (see condition (12)).

It should be noted that the analysis performed in this work does not take into account the effect of repeated scattering of radiating particles in a target. However, the effect of a considerably increasing yield of PXR is based on the possibility of a substantial increase in thickness of the crystal under the conditions of anomalous photoabsorption. In this connection, it should be noted that the formulas derived in this work are convenient for a numerical analysis of the mentioned effect. It can be shown that expressions (14), (19), and (21) are replaced, in view of repeated scattering, by

$$\omega\overline{\frac{dN_\lambda^{(j)}}{d\omega d^2\theta}}=\frac{1}{\pi\Psi_s^2}\int d\Psi_\perp d\Psi_\parallel\left\{E_1\left(\frac{\Psi_2}{\Psi_0^2+\Psi_s^2 L/\cos\varphi/2}\right)-E_1\left(\frac{\Psi^2}{\Psi_0^2}\right)\right\}\omega\frac{dN_\lambda^{(j)}}{d\omega d^2\theta}, \tag{22}$$

where Ψ_0 is the initial divergence of the radiating particle beam; $\Psi_s^2=\varepsilon_k^2/m^2\gamma^2 L_R$, $\varepsilon_k\approx 21$ MeV, and L_R is the radiation length of the target material.

CONCLUSION

Thus, in the course of the emission of the parametric X radiation of electrons in an absorbing crystal of finite thickness, the effect of anomalous photoabsorption may show up, which is characterized by the following features revealed in this work:

– as in the case of the diffraction of free X rays, the necessary condition for the effect of anomalous photoabsorption in PXR to show up is that the imaginary parts of the respective coefficients of the expansion of the dielectric susceptibility in a Fourier series over the reciprocal lattice vectors are close in value to one another;

– the PXR effect under consideration is dynamic in nature and can show up only in the region of sufficiently high radiating electron energies that has been determined in this work;

– anomalously profound photoabsorption in PXR is possible only for strong reflections where the real parts of the corresponding structural amplitudes of the crystal are close in value;

– under conditions where the effect of anomalous photoabsorption shows up, the spectral-angular density of PXR can increase by more than an order of magnitude.

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