Noise reduction in the nondegenerate parametric oscillator with direct detection feedback

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A quantum analysis of the above-threshold intensity fluctuations in a nondegenerate parametric oscillator with direct-detection feedback onto the pump amplitude is presented. We derive a master equation for the signal (in-loop) and idler (out-of-loop) modes by adiabatically eliminating the pump mode and incorporating a feedback term, using the Wiseman-Milburn quantum feedback theory [Phys. Rev. Lett. **70**, 548 (1993)]. In the absence of feedback and far above threshold, we find that both beams are 50% intensity squeezed. For small negative (positive) feedback, the intensity fluctuations in the out-of-loop (in-loop) beam are reduced further. For larger values of feedback, the fluctuations grow, the fields eventually becoming unsqueezed.

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I. INTRODUCTION

For many years experimentalists have used feedback to stabilize optical systems by measuring the output of one part of the system and using this information to alter the input. For perfect measuring devices and negligible time delay in the feedback loop, classically one might expect to gain complete control over the properties of the output. Such techniques have been shown to work even in chaotic regimes. Quantum mechanically, however, mostly because of the limitations imposed by the Heisenberg uncertainty principle, such control may no longer be expected.

Recently, Wiseman and Milburn have presented a theory of quantum feedback [1]. In the Markovian limit, their theory gives a prescription for incorporating most types of feedback into the nonfeedback master equation of any system. The resulting master equation may then be solved by standard methods. One major conclusion derived from their work is that in a classical system, feedback cannot produce nonclassical (e.g., squeezed) field states from classical ones; although, if the field is initially nonclassical (without feedback), then feedback may enhance its nonclassicality (e.g., more squeezed).

In this paper, we consider the possible reduction of quantum noise in the nondegenerate parametric oscillator (NDPO) with feedback. In the NDPO a pump photon is destroyed, simultaneously creating one photon in the signal field and another in the idler field. Thus, there is a high correlation between the fields. On resonance, this implies that the difference intensity is perfectly squeezed, even though the individual fields are generally not squeezed. However, we show that in the large pump damping limit, each field is intensity squeezed by 50%. Thus, in lieu of this fact and the preceding discussion, we expect feedback to enhance this result.

Experimentally, Tapster *et al.* [4] have used negative, direct-detection feedback to generate sub-Poissonian light in the out-of-loop beam of a parametric amplifier arrangement. They and Plimak [5] have given theoretical arguments indicating that generally the greater the negative feedback, the greater the noise reduction; assuming perfect detection efficiency. Similar claims have been made by Mertz, Heidmann, and Fabre [4]. Our analysis indicates, however, that only small amounts of negative (positive) feedback can reduce quantum fluctuations in the out of loop (in loop); at least above threshold. For larger values of feedback, the increasing noise introduced by feedback appears to overwhelm the noise reduction. Below threshold, feedback has no effect, except to alter the stability region.

II. MODEL

The nondegenerate parametric oscillator (NDPO) consists of three optical field modes interacting with a $\chi^{(2)}$ crystal within a cavity, see Fig. 1. The pump mode with frequency ω_c is driven by a resonant external coherent field through mirror 1. The idler and signal modes a and b with frequencies ω_a and ω_b, respectively, are damped by mirror 2. The resonance condition for these modes is $\omega_a + \omega_b = \omega_c$. The standard Hamiltonian modeling this system is ($\hbar = 1$)

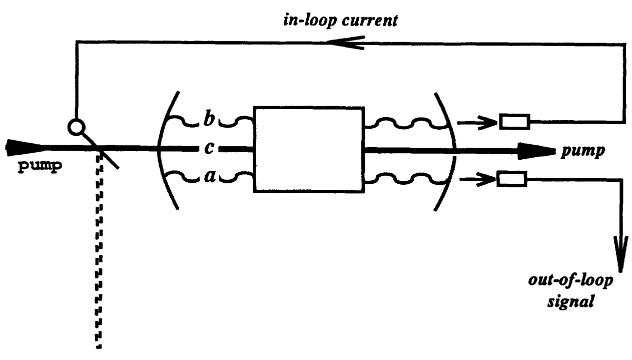

FIG. 1. A schematic representation of a NDPO with direct-detection feedback of the b mode. The photocurrent is fed back onto the pump modulating the pump amplitude.

$$H = H_{\rm rev} + H_{\rm irrev} , \tag{2.1}$$

where

$$H_{\rm rev} = \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_c c^\dagger c + g(c^\dagger ab + ca^\dagger b^\dagger) + E[c^\dagger e^{-i\omega_c t} + c\, e^{i\omega_c t}] , \tag{2.2a}$$

and

$$H_{\rm irrev} = a\Gamma_a^\dagger + a^\dagger \Gamma_a + b\Gamma_b^\dagger + b^\dagger\Gamma_b + c\Gamma_c^\dagger + c^\dagger \Gamma_c . \tag{2.2b}$$

Here $\{a^\dagger, a\}$, $\{b^\dagger, b\}$, and $\{c^\dagger, c\}$ are the creation and annihilation operators for the idler, signal, and pump modes, respectively. The parameters E and g are assumed real and represent the external field amplitude and the nonlinear coupling due to the medium, respectively. The damping of the cavity modes to the external modes are described by the reservoirs Γ_a, Γ_b, and Γ_c. This decay gives rise to the damping constants γ_a, γ_b, and γ_c for the modes a, b, and c, respectively.

A feedback loop is initiated by a photodetector lying outside mirror 2 measuring the intensity of the external signal beam. The output photoelectron current is fed back to the driving field modifying the amplitude E. Using the feedback formalism of Wiseman and Milburn [1] and in an interaction picture, the master equation for the combined system may be written as

$$\dot\rho = -i[H_{\rm rev}, \rho] + \frac{\gamma_a}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \frac{\gamma_b}{2}(2b\rho b^\dagger - b^\dagger b\rho - \rho b^\dagger b) + \frac{\gamma_c}{2}(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c) + \eta[e^{\mathcal{K}} - 1](b\rho b^\dagger) , \tag{2.3}$$

where we have assumed the reservoirs are at zero temperature. The last term on the right hand side (rhs) of Eq. (2.3) represents the effect of nonselective feedback via the direct detection of the b mode, where $\mathcal{K}$ is the feedback superoperator and η is the photodetector efficiency. In this paper, we assume that the feedback photocurrent is fed back to the pump, modulating the driving amplitude. Thus, the action of $\mathcal{K}$ on ρ is defined as

$$\mathcal{K}\rho = -i\kappa[c + c^\dagger, \rho] , \tag{2.4}$$

where κ is a real parameter. Here feedback is assumed to act instantaneously, i.e., the time delay in the feedback loop is negligible. This property is not only experimentally desirable, as feedback time delays can lead to instabilities, but theoretically necessary in order to formulate the feedback process into a valid Markovian master equation. For effective feedback control, it is also important for the pump to respond quickly to the feedback. This ensures that any intensity fluctuations measured by the photodetector are instantaneously fed back to the signal-idler modes. Thus, if γ_c is large enough, the pump mode is slaved to the signal-idler modes. We proceed with the adiabatic elimination of the pump mode in a manner similar to Refs. [2] and [3]. First, we displace the pump mode to a state near the vacuum. Thus, each term of Eq. (2.3) is transformed by letting $c \to c + \delta$ and $c^\dagger \to c^\dagger + \delta^*$, where δ is a c number. Transforming the pump mode damping term

$$\frac{\gamma_c}{2}[2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c] \to \frac{\gamma_c}{2}[2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c] + \frac{\gamma_c}{2}[\delta^* c - \delta c^\dagger, \rho],$$

we pick up an extra term proportional to the displacement. If we let $\delta = 2iE\gamma_c$, this term exactly cancels the driving term $-iE[c + c^\dagger, \rho]$ from the master equation (2.3). Likewise, transforming the $\chi^{(2)}$ interaction term, we find

$$-ig[c^\dagger ab + ca^\dagger b^\dagger, \rho] \to -ig[c^\dagger ab + ca^\dagger b^\dagger, \rho] + \epsilon[a^\dagger b^\dagger - ab, \rho],$$

where $\epsilon = 2Eg/\gamma_c$ is now the effective driving constant. If the pump damping constant is larger than any other parameter

$$\left|\frac{\gamma_a}{\gamma_c}\right| \sim \left|\frac{\gamma_b}{\gamma_c}\right| \sim \left|\frac{\kappa}{\gamma_c}\right| \sim \left|\frac{g}{\gamma_c}\right| = \varepsilon \ll 1 , \tag{2.5}$$

then the density operator may be expanded in powers of ε. Thus, we put

$$\rho = \rho_0 \otimes |0\rangle_c\langle 0| + [\rho_1 \otimes |1\rangle_c\langle 0| + \text{H.c.}] + \rho_2 \otimes |1\rangle_c\langle 1| + O(\varepsilon^3) , \tag{2.6}$$

where the density operators $\rho_i, i = 0, 1, 2$ describe the combined state of the signal and idler modes and the subscript indicates orders of magnitude in ε. The two other terms of order $O(\varepsilon^2)$ in Eq. (2.6) are inconsequential to further calculations and are omitted. Substituting this expression into the master equation (2.3) and expanding the feedback term to first order in κ, we find the following set of differential equations

$$\dot\rho_0 = \gamma_c\rho_2 - ig(a^\dagger b^\dagger \rho_1 - \rho_1^\dagger ab) + i\kappa(\rho_1 - \rho_1^\dagger) , \tag{2.7a}$$

$$\dot\rho_1 = -\frac{\gamma_c}{2}\rho_1 - ig(ab\rho_0 - \rho_2 ab) + i\kappa(\rho_0 - \rho_2), \tag{2.7b}$$

$$\dot\rho_2 = -\gamma_c\rho_2 - ig(ab\rho_1^\dagger - \rho_1 a^\dagger b^\dagger) + i\kappa(\rho_1^\dagger - \rho_1) . \tag{2.7c}$$

For large γ_c, ρ_1 and ρ_2 damp rapidly. The signal-idler density operator is $\rho = \rho_0 + \rho_2$. From Eqs. (2.7), we find

$$\dot\rho = -ig([ab, \rho_1^\dagger] + [a^\dagger b^\dagger, \rho_1]). \tag{2.8}$$

We proceed by setting the time derivative to zero in Eq. (2.7b) and solving for ρ_1, noting $\rho \simeq \rho_0$. Substituting for ρ_1 and $\rho_1^\dagger$ in Eq. (2.8), we find

$$\dot\rho = \Gamma[2ab\rho a^\dagger b^\dagger - \rho a^\dagger ab^\dagger a - a^\dagger ab^\dagger b\rho] + \chi[a^\dagger b^\dagger - ab, \rho], \tag{2.9}$$

where $\Gamma = 2g^2/\gamma_c$ is the two-photon damping constant and $\chi = 2\kappa g/\gamma_c$ is now the feedback parameter. The final form of the master equation for modes a and b can now be written as

$$\dot{\rho} = \epsilon[a^\dagger b^\dagger - ab, \rho] + \Gamma[2ab\rho a^\dagger b^\dagger - \rho a^\dagger ab^\dagger b - a^\dagger ab^\dagger b\rho] + \frac{\gamma_a}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \frac{\gamma_b}{2}(2b\rho b^\dagger - b^\dagger b\rho - \rho b^\dagger b)$$
$$+\chi[a^\dagger b^\dagger - ab, b\rho b^\dagger] + \frac{\chi^2}{2\eta}[a^\dagger b^\dagger - ab, [a^\dagger b^\dagger - ab, b\rho b^\dagger]] \;, \tag{2.10}$$

where we have expanded the feedback term to second order in χ. For $\chi \ll 1$, higher order terms can be ignored. The single commutator term linear in χ represents the direct feedback effect, while the double commutator term quadratic in χ is the necessary noise due to the feedback.

III. ITO STOCHASTIC EQUATIONS

In this section we use standard techniques to transform the master equation (2.10) into a set of stochastic differential equations. The density operator ρ is expanded in the positive P representation [6]

$$\rho = \int \frac{|\theta\rangle\langle\theta^\dagger|}{\langle\theta^\dagger|\theta\rangle} P(\theta,\theta^\dagger) d^2\theta d^2\theta^\dagger \;, \tag{3.1}$$

where

$$\theta = (\alpha,\beta) \;, \qquad \theta^\dagger = (\alpha^\dagger,\beta^\dagger) \;. \tag{3.2}$$

This establishes a correspondence between independent complex variables $\alpha, \alpha^\dagger, \beta$, and $\beta^\dagger$ and the mode operators $a, a^\dagger, b$, and $b^\dagger$, respectively. Hence, the master equation (2.10) is converted into a Fokker-Planck equation for P. However, to obtain a valid Fokker-Planck equation, third- and forth-order derivatives resulting from the double-commutator term are ignored. Proceeding by using the equivalence between Fokker-Planck equations and stochastic equations [7], we find to second order in χ the following set of Ito stochastic differential equations

$$\begin{aligned}
\dot{\alpha} &= \epsilon\beta^\dagger - \Gamma\alpha\beta\beta^\dagger - \gamma_a\alpha/2 + \chi(\beta^\dagger + \chi\alpha)\beta\beta^\dagger + \xi_\alpha(t) \;, \\
\dot{\alpha^\dagger} &= \epsilon\beta - \Gamma\alpha^\dagger\beta\beta^\dagger - \gamma_a\alpha^\dagger/2 + \chi(\beta + \chi\alpha^\dagger)\beta\beta^\dagger + \xi_\alpha^\dagger(t) \;, \\
\dot{\beta} &= \epsilon\alpha^\dagger - \Gamma\beta\alpha\alpha^\dagger - \gamma_b\beta/2 + \chi(\alpha^\dagger + \chi\beta)\beta\beta^\dagger + \xi_\beta(t) \;, \\
\dot{\beta^\dagger} &= \epsilon\alpha - \Gamma\beta^\dagger\alpha\alpha^\dagger - \gamma_b\beta^\dagger/2 + \chi(\alpha + \chi\beta^\dagger)\beta\beta^\dagger + \xi_\beta^\dagger(t) \;.
\end{aligned} \tag{3.3}$$

The correlations in the noise terms $\xi_\alpha(t), \xi_\alpha^\dagger(t), \xi_\beta(t)$ and $\xi_\beta^\dagger(t)$ are

$$\begin{aligned}
\langle\xi_\alpha(t)\xi_\alpha(t')\rangle &= \langle\xi_\alpha^\dagger(t)\xi_\alpha^\dagger(t')\rangle^\dagger \\
&= \chi^2\beta\beta^\dagger(\beta^\dagger)^2\delta(t-t') \;, \\
\langle\xi_\beta(t)\xi_\beta(t')\rangle &= \langle\xi_\beta^\dagger(t)\xi_\beta^\dagger(t')\rangle^\dagger \\
&= \chi^2\beta\beta^\dagger(\alpha^\dagger)^2\delta(t-t') \;, \\
\langle\xi_\alpha(t)\xi_\alpha^\dagger(t')\rangle &= \chi^2\beta\beta^\dagger(\beta\beta^\dagger + 1)\delta(t-t') \;, \\
\langle\xi_\beta(t)\xi_\beta^\dagger(t')\rangle &= \chi^2\beta\beta^\dagger(\alpha\alpha^\dagger + 1)\delta(t-t') \;, \\
\langle\xi_\alpha(t)\xi_\beta^\dagger(t')\rangle &= \langle\xi_\alpha^\dagger(t)\xi_\beta(t')\rangle^\dagger \\
&= \chi^2\beta\beta^\dagger\alpha\beta^\dagger\delta(t-t') \;, \\
\langle\xi_\alpha^\dagger(t)\xi_\beta^\dagger(t')\rangle &= \langle\xi_\alpha(t)\xi_\beta(t')\rangle^\dagger \\
&= [\chi(\chi\alpha\beta + 1)\beta\beta^\dagger \\
&\quad -\Gamma\alpha^\dagger\beta^\dagger + \epsilon]\delta(t-t') \;.
\end{aligned} \tag{3.4}$$

Here we have scaled all variables to η, so that

$$\bar{\epsilon} = \epsilon/\eta, \quad \bar{\chi} = \chi/\eta, \quad \bar{\Gamma} = \Gamma/\eta, \quad \bar{\gamma} = \gamma/\eta \;, \tag{3.5}$$

and dropped the bar notation for simplicity. The equations of motion for the classical amplitudes are found from Eqs. (3.3) by ignoring the noise terms and setting $\alpha^\dagger = \alpha^*$ and $\beta^\dagger = \beta^*$. The steady-state solutions are

$$\alpha^0 = \beta^0 = 0 \qquad \text{for} \qquad |\epsilon| < \gamma/2 \tag{3.6}$$

and

$$|\alpha^0|^2 = |\beta^0|^2 = \frac{2\epsilon - \gamma}{2\Gamma - \chi(\chi+2)} \tag{3.7a}$$

for

$$|\epsilon| > \gamma/2 \qquad \text{and} \qquad \chi(\chi+2) < 2\Gamma \;. \tag{3.7b}$$

For simplicity, we have set $\gamma_a = \gamma_b = \gamma$, giving equal steady-state intensities to both modes. The stability of these solutions can be checked by linearizing Eqs. (3.3) around the steady-state values and requiring the eigenvalues of the resulting drift matrix to be positive.

Equation (3.6) represents the below-threshold solution. Mode damping dominates over driving, giving a steady-state intensity of zero. Equation (3.7) is the above-threshold result. Here the steady-state intensity is nonzero and modulated by the feedback. For positive Γ, $\chi < 0$ lowers the steady-state intensity relative to the $\chi = 0$ (no feedback) case and $\chi > 0$ increases it.

Upon linearization of Eqs. (3.3) about the above-threshold intensity (3.7), one eigenvalue is found to be zero. This eigenvalue is associated with the phase diffusion in the signal and idler modes. Thus, any assumption about small fluctuations in the amplitudes would be unwarranted here. For this reason and because we are interested in intensity fluctuations, we transform Eqs. (3.3) and (3.4) for the amplitudes to equations for the intensities and phase sum and difference variables. Following the Ito rules for changing variables, we find the following intensity-phase stochastic equations

$$\begin{aligned}
\dot{I}_a &= 2\sqrt{I_aI_b}(\epsilon + \chi I_b)\cos\phi_+ + I_aI_b(\chi^2 - 2\Gamma) \\
&\quad -\gamma I_a + \chi^2 I_b(I_b+1) + \xi_{I_a} \;,
\end{aligned} \tag{3.8a}$$

$$\begin{aligned}
\dot{I}_b &= 2\sqrt{I_aI_b}(\epsilon + \chi I_b)\cos\phi_+ + I_aI_b(\chi^2 - 2\Gamma) \\
&\quad -\gamma I_b + \chi^2 I_b(I_b+1) + \xi_{I_b} \;,
\end{aligned} \tag{3.8b}$$

$$\begin{aligned}
\dot{\phi}_+ &= -(I_b + I_a)\sin(\phi_+)(\epsilon + \chi I_b)/\sqrt{I_aI_b} \\
&\quad +\chi^2\sin(2\phi_+)(I_b^2 + I_a^2)/I_a + \xi_{\phi_+} \;,
\end{aligned} \tag{3.8c}$$

$$\begin{aligned}
\dot{\phi}_- &= -(I_b - I_a)\sin(\phi_+)(\epsilon + \chi I_b)/\sqrt{I_aI_b} \\
&\quad +\chi^2\sin(2\phi_+)(I_b^2 - I_a^2)/I_a + \xi_{\phi_-} \;.
\end{aligned} \tag{3.8d}$$

Here I_a and I_b, are the intensities of the idler and signal modes, respectively, and ϕ_+ and ϕ_- are the phase sum and phase difference of these modes. In terms of the amplitudes these variables are defined as

$$I_a = \alpha\alpha^\dagger \ , \quad I_b = \beta\beta^\dagger \ , \tag{3.9a}$$

and

$$\phi_+ = \ln(\alpha\beta/\alpha^\dagger\beta^\dagger)/2i \ , \quad \phi_- = \ln(\alpha\beta^\dagger/\alpha^\dagger\beta)/2i \ . \tag{3.9b}$$

The correlations to the noise terms in (3.8) are formally given by

$$\begin{aligned} \xi_{I_j} &= j\xi_j^\dagger + j^\dagger\xi_j \ , \qquad j = \alpha,\ \beta \\ \xi_{\phi_\pm} &= [(\xi_\alpha/\alpha - \xi_\alpha^\dagger/\alpha^\dagger) \pm (\xi_\beta/\beta - \xi_\beta^\dagger/\beta^\dagger)]/2i \ . \end{aligned} \tag{3.10}$$

The intensity equations (3.8a) and (3.8b) demonstrate that the driving amplitude is reduced if χ is negative and increased if χ is positive. Thus, the effective feedback, χI_b, is negative or positive according to the sign of χ. Notice also that Eqs. (3.8) are not symmetric in the intensities I_a and I_b. Since the effective feedback is proportional to the intensity of the signal mode, the symmetry between the modes is broken. This asymmetry ultimately leads to the two modes having different spectra (see Sec. V).

IV. LINEARIZATION AND STABILITY ANALYSIS

Assuming small fluctuations, the intensity and phase sum equations can be linearized about $I_a^0 = I_b^0 = I$ and ϕ_+^0 and put in the form

$$\Delta\mathbf{x} = -A\Delta\mathbf{x} + \boldsymbol{\xi}_\mathbf{x}^0 \ , \tag{4.1}$$

where $\mathbf{x} = (I_a, I_b, \phi_+)^\dagger$ and A is the linearized drift matrix. The eigenvalues of A must be positive for the system to be stable against small perturbations. The vector $\Delta\mathbf{x}$ contains the difference variables ΔI_α, ΔI_β, and $\Delta\phi_+$, defined as

$$\begin{aligned} \Delta I_j &= I_j - I_j^0, \qquad j = \alpha,\ \beta \\ \Delta\phi_+ &= \phi_+ - \phi_+^0 \ . \end{aligned} \tag{4.2}$$

Using Eqs. (3.8a) and (3.8c), we find

$$\begin{aligned} \Delta\dot{I}_a = {} & [(\epsilon + \chi I)\cos\phi_+^0 + (\chi^2 - 2\Gamma)I - \gamma]\Delta I_a \\ & + [(\epsilon + 3\chi I)\cos\phi_+^0 + (3\chi^2 - 2\Gamma)I + 2\chi^2]\Delta I_b \\ & - 2I\sin(\phi_+^0)(\epsilon + \chi I)\Delta\phi_+ + \xi_{I_a}^0 \ , \end{aligned} \tag{4.3a}$$

$$\begin{aligned} \Delta\dot{I}_b = {} & [(\epsilon + \chi I)\cos\phi_+^0 + (\chi^2 - 2\Gamma)I]\Delta I_a \\ & + [(\epsilon + 3\chi I)\cos\phi_+^0 + (3\chi^2 - 2\Gamma)I + 2\chi^2 - \gamma]\Delta I_b \\ & - 2I\sin(\phi_+^0)(\epsilon + \chi I)\Delta\phi_+ + \xi_{I_b}^0 \ , \end{aligned} \tag{4.3b}$$

$$\Delta\dot{\phi_+} = 2\chi\sin(\phi_+^0)(\chi\cos\phi_+^0 - 1)\Delta I_b - \gamma_+\Delta\phi_+ + \xi_{\phi_+}^0 \ , \tag{4.3c}$$

$$\dot{\phi_-} = \xi_{\phi_-}^0 \ , \tag{4.3d}$$

where $\gamma_+ = 2[\epsilon + I\chi(1 - 2\chi)]$.

The nonzero steady-state noise correlations are

$$\begin{aligned} \langle\xi_{I_a}^0(t)\xi_{I_a}^0(t')\rangle &= \langle\xi_{I_b}^0(t)\xi_{I_b}^0(t')\rangle \\ &= 2\chi^2 I^2(1 + I)\delta(t - t') \ , \\ \langle\xi_{I_a}^0(t)\xi_{I_b}^0(t')\rangle &= 2I[\chi I(2\chi I + 1) + (\epsilon - \Gamma I)]\delta(t - t') \ , \\ \langle\xi_{\phi_+}^0(t)\xi_{\phi_+}^0(t')\rangle &= [\chi(\chi - 1) + \Gamma - \epsilon/I]\delta(t - t') \ , \\ \langle\xi_{\phi_-}^0(t)\xi_{\phi_-}^0(t')\rangle &= [\chi(\chi + 1) - \Gamma + \epsilon/I]\delta(t - t') \ . \end{aligned} \tag{4.4}$$

The equation for the phase difference (4.3d) shows that ϕ_- diffuses and can attain a continuum of values. This result follows from the before mentioned fact that above threshold the signal and idler amplitudes diffuse and have no unique steady state. Thus, no small fluctuation assumption can be made for ϕ_- and its evolution equation (3.9d) is treated exactly.

When $\sin\phi_+^0 = 0$, the equation for the phase sum decouples from the intensity equations and we find the following steady-state intensities and associated stability regions

$$I = 0 \quad \text{for} \quad 0 < 2|\epsilon| < (\gamma - \chi^2) \ , \tag{4.5}$$

and

$$I_a^0 = I_b^0 = \frac{\epsilon + \chi^2/2 - \gamma/2}{\Gamma - \chi(1 + \chi)} \tag{4.6a}$$

for

$$2\epsilon > (\gamma - \chi^2) > 0, \ \ \gamma_+ > 0, \ \ \Gamma > \chi(1 + \chi) \ , \tag{4.6b}$$

where we have set $\cos\phi_+^0 = 1$. Equation (4.5) represents the below-threshold, steady-state intensity and stability region, while Eqs. (4.6) are the above-threshold results. The last condition in Eq. (4.6b) comes from the requirement that the intensity be positive. The solutions for $\cos\phi_+^0 = -1$ can be found from the above results by letting $\epsilon \mapsto -\epsilon$ and $\chi \mapsto -\chi$.

These stability regions are shown in Fig. 2. The below-threshold solution is stable in the half-oval region between the x axis ($\epsilon = 0$) and the curve [$2\epsilon = (1 - \chi^2)$]. Here, we have let $\gamma = 1$ and $\Gamma = 0.5$. Notice that it is the Ito correction term ($\chi^2/2$) that constrains χ to be less than one. The above-threshold, stability region is shaded and lies above the below-threshold region. This solution is restricted from large negative values of χ by the phase sum stability condition ($\gamma_+ > 0$) and from large positive χ by the $\Gamma = \chi(1 + \chi)$ boundary (dot-dashed line). For smaller values of Γ, both side boundaries of the above threshold stability region converge on the y axis further restricting the size of χ; whereas the below-threshold region is not affected.

One can see from the phase sum Eq. (3.9c), that $\sin\phi_+^0 = 0$ is the most obvious solution in steady state, but not the only one. An analysis of solutions with $\sin\phi_+^0 \neq 0$ reveals a narrow stability region in the large negative χ regime of Fig. 2. Here, a continuum of possible values of $\cos\phi_+^0$ between 0 and 1 exist. However, as we will see in Sec. V, the most important effects of feedback in terms of squeezing occur for small values of $|\chi|$. Thus, we ignore these solutions for the rest of the paper and concentrate on the $\cos\phi_+^0 = 1$ solutions.

Comparing of solutions (4.5) and (4.6) with the solutions found from the amplitude equations (3.7) and (3.8),

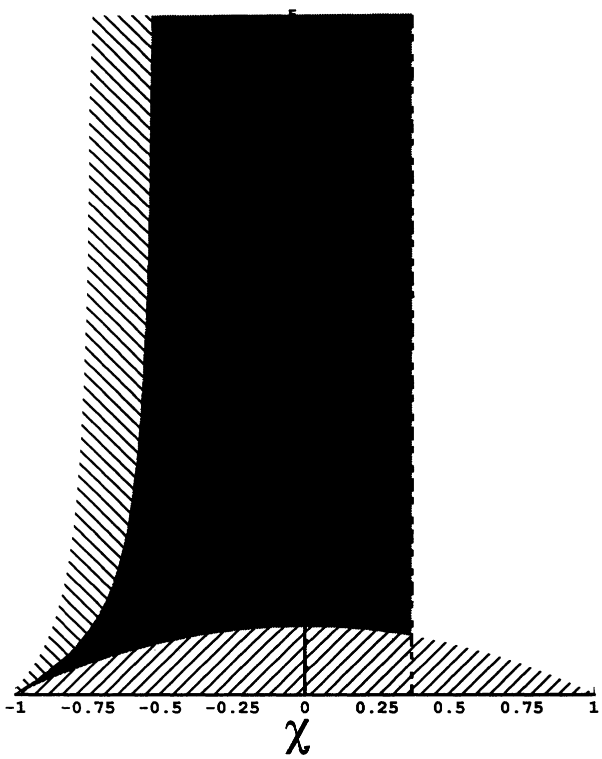

FIG. 2. Above- and below-threshold stability regions with $\Gamma = 0.5$ and $\gamma = 1.0$.

one finds that they are different. Below threshold, the stability region as determined by the intensity equations is dependent on χ and above threshold, both the intensities and stability regions are different. These differences arise from the Ito correction terms involved in the nonlinear transformation of variables. When solving for the steady-state amplitudes, all operators are factored and the noise terms are ignored. However, these noise terms contain the Ito corrections, which in the intensity equations become part of the deterministic evolution.

Such anomalies between the steady states in the amplitude and intensity equations do not normally occur in the standard analysis of the NDPO. Here, these differences arise from the double commutator feedback term. For $|\chi| \ll 1$, their effect is expected to be small. For larger values of χ, our analysis breaks down since we have truncated the feedback term to second order. However, in the Appendix we carry out this same analysis without truncation of the feedback term. Those results indicate that the analysis here is qualitatively correct even for large χ.

Above threshold, the fluctuations in the intensity difference of the two beams $\Delta I_- = \Delta I_a - \Delta I_b$ is found to obey

$$\Delta \dot{I_-} = -\gamma I_- + \xi^0_{I_-} , \tag{4.7}$$

with noise correlations

$$\langle \xi^0_{I_-}(t)\xi^0_{I_-}(t')\rangle = 4I^2[\chi(\chi-1) + \Gamma - \epsilon/I]\delta(t-t') . \tag{4.8}$$

Examining Eqs. (4.7) and (4.3c), we can write the solutions for ΔI_- and ϕ_-

$$\Delta I_-(t) = \Delta I_-(t_0)e^{-\gamma(t-t_0)} + \int_{t_0}^{t} e^{\gamma(t-t_0)}\xi_{I_-}(t')dt' , \tag{4.9}$$

$$\phi_-(t) = \phi_-(t_0) + \int_{t_0}^{t} \xi_{\phi_-}(t')dt' , \tag{4.10}$$

where t_0 is the initial time. The fluctuations in intensity difference damp to zero with time constant, γ; whereas, the phase difference is not damped but continuously diffuses. This implies, for instance,

$$\langle[\phi_-(\tau) - \phi_-(0)]^2\rangle = (\gamma - \chi^2)|\tau|/2I^0 . \tag{4.11}$$

From the stability conditions (4.6b), this diffusion must remain positive. However, for a given intensity it is smaller than the $\chi = 0$ result, originally obtained by Graham and Haken [8].

V. INTENSITY SPECTRA

Above-threshold analysis and with $\cos\phi^0_+ = 1$, the fluctuations in the intensity difference represent a Ornstein-Uhlenbeck process. Thus, from (4.9) its stationary, two-time correlation function is

$$\langle \Delta I_-(t)\Delta I_-(t+\tau)\rangle = -I\frac{\Gamma - \chi + \chi^2(1-\Gamma-4\epsilon)}{\Gamma - \chi(1+\chi)}e^{-\gamma|\tau|} \tag{5.1}$$

where, for simplicity, we now measure time in inverse units of γ. Except for the denominator, the above expression has been expanded to second order in χ. This is to ensure that the spectrum is qualitatively similar to the exact result found in the Appendix. The spectrum is given by

$$S_{I_-}(\omega) = 2\gamma I + \int_{-\infty}^{\infty} d\tau e^{-i\omega\tau}\langle \Delta I_-(t)\Delta I_-(t+\tau)\rangle . \tag{5.2}$$

Substituting (5.1) into (5.2) and normalizing with the shot noise, we find the normalized spectrum

$$\begin{aligned} \bar{S}_{I_-}(\omega) &= \frac{S_{I_-}(\omega)}{2I} - 1 \\ &= -\frac{\Gamma - \chi + \chi^2(1-\Gamma-4\epsilon)}{(\Gamma - \chi(1+\chi))(1+\omega^2)} . \end{aligned} \tag{5.3}$$

On resonance ($\omega = 0$) and with no feedback ($\chi = 0$), Eq. (5.3) reduces to the expected result of $\bar{S}_{I_-}(0) = -1$; representing perfect correlation between the two beams.

The spectrum of the intensity fluctuations in the in-loop and out-of-loop beams is most easily obtained by applying the formula [7]

$$S_{I_a,I_b} = \mathrm{diag}_{1,2}[(A' - i\omega)D(A' + i\omega)] , \tag{5.4}$$

where A' is the linearized 2×2 drift submatrix of A in Eq. (4.3) and D is the 2×2 intensity diffusion matrix

$$D = \begin{pmatrix} \langle \xi_{I_a}\xi_{I_a}\rangle & \langle \xi_{I_a}\xi_{I_b}\rangle \\ \langle \xi_{I_b}\xi_{I_a}\rangle & \langle \xi_{I_b}\xi_{I_b}\rangle \end{pmatrix} . \tag{5.5}$$

The notation in Eq. (5.4) implies that S_{I_a} is the $(1,1)$ element of the resulting matrix and S_{I_b} is the $(2,2)$ element. The resulting normalized spectra $\bar{S} = S/I - 1$ for I_a and I_b are

$$\bar{S}_{I_a} = \frac{A_1(\epsilon,\chi,\Gamma)}{1+\omega^2} + \frac{A_2(\epsilon,\chi,\Gamma)}{(-1+2\epsilon+\chi^2)^2+\omega^2}\,, \tag{5.6a}$$

$$\bar{S}_{I_b} = \frac{B_1(\epsilon,\chi,\Gamma)}{1+\omega^2} + \frac{B_2(\epsilon,\chi,\Gamma)}{(-1+2\epsilon+\chi^2)^2+\omega^2}\,. \tag{5.6b}$$

Each is the sum of two Lorentzians: one component with a width of 1 and far above threshold (large ϵ); the second component much broader with a width $\sim 2\epsilon$. To second order in χ, A_i and B_i are

$$\begin{aligned} A_1 = {} & [4\epsilon\Gamma^2(1-\epsilon) + 2\Gamma\chi(1-4\epsilon+4\epsilon^2) \\ & +\chi^2(1-4\epsilon+4\epsilon^2+2\Gamma-16\epsilon^2\Gamma+16\epsilon^3\Gamma \\ & -2\Gamma^2+4\epsilon^2\Gamma^2)]/8\epsilon\Gamma^2(\epsilon-1), \end{aligned} \tag{5.7a}$$

$$\begin{aligned} A_2 = {} & [4\epsilon\Gamma^2(\epsilon-1) - 2\Gamma\chi(1-4\epsilon+4\epsilon^2) \\ & +\chi^2(-1-4\epsilon+36\epsilon^2-64\epsilon^3+32\epsilon^4 \\ & -2\Gamma+8\epsilon\Gamma-8\epsilon^2\Gamma+2\Gamma^2-4\epsilon^2R^2)]/8\epsilon\Gamma^2(\epsilon-1), \end{aligned} \tag{5.7b}$$

$$\begin{aligned} B_1 = {} & [4\epsilon\Gamma^2(1-\epsilon) - 2\Gamma\chi(1-4\epsilon+4\epsilon^2) \\ & +\chi^2(-3+12\epsilon-12\epsilon^2-2\Gamma+16\epsilon\Gamma-32\epsilon^2\Gamma+16\epsilon^3\Gamma \\ & +2\Gamma^2-8\epsilon\Gamma^2+4\epsilon^2\Gamma^2)]/8\epsilon\Gamma^2(\epsilon-1), \end{aligned} \tag{5.7c}$$

$$\begin{aligned} B_2 = {} & [4\epsilon\Gamma^2(\epsilon-1) + 2\Gamma\chi(1-4\epsilon+4\epsilon^2) \\ & +\chi^2(3-20\epsilon+52\epsilon^2-64\epsilon^3+32\epsilon^4+2\Gamma-8\epsilon\Gamma \\ & +8\epsilon^2\Gamma-2\Gamma^2+8\epsilon\Gamma^2-4\epsilon^2\Gamma^2)]/8\epsilon\Gamma^2(\epsilon-1). \end{aligned} \tag{5.7d}$$

In the limit of $\chi \to 0$, the above coefficients reduce to $A_1 = B_1 = -1/2$ and $A_2 = B_2 = 1/2$. Thus, on resonance and far above threshold the second Lorentzian in (5.6) can be ignored and the normalized spectra reduce to $\bar{S}_{I_a} = \bar{S}_{I_b} = -1/2$.

In Fig. 3, we show the on resonance spectra of the out-of-loop (solid line), in-loop (dashed line), and difference (dotted line) beams versus the feedback parameter χ. We have set $\epsilon = 5.0$ and $\Gamma = 0.5$. With no feedback, the spectrum of the intensity difference shows perfect correlation. By including feedback, the spectrum eventually becomes classical. On the other hand, the in-loop spectrum reaches its minimum with small positive feedback and the out-of-loop spectrum with small negative feedback. In the large driving limit, these minima in the spectral curves are $\chi_{\min} \sim -1/4\epsilon$ for the out-of-loop beam and $\chi_{\min} \sim 1/4\epsilon$ for the in-loop beam. When these values are substituted into Eqs. (5.6), one finds the maximum squeezing to be $\sim -1/2 - 1/8\epsilon\Gamma$ for both beams.

However, for smaller values of driving, feedback may be much more influential in reducing quantum noise. In Fig. 4, we show the same curves as in Fig. 3, but now with $\epsilon = 1$. With no feedback, the in-loop and out-of-loop beams are just unsqueezed. Small negative feedback gives the out-of-loop beam a maximum squeezing of $\sim 15\%$ and small positive feedback produces $\sim 35\%$ squeezing in the in-loop beam.

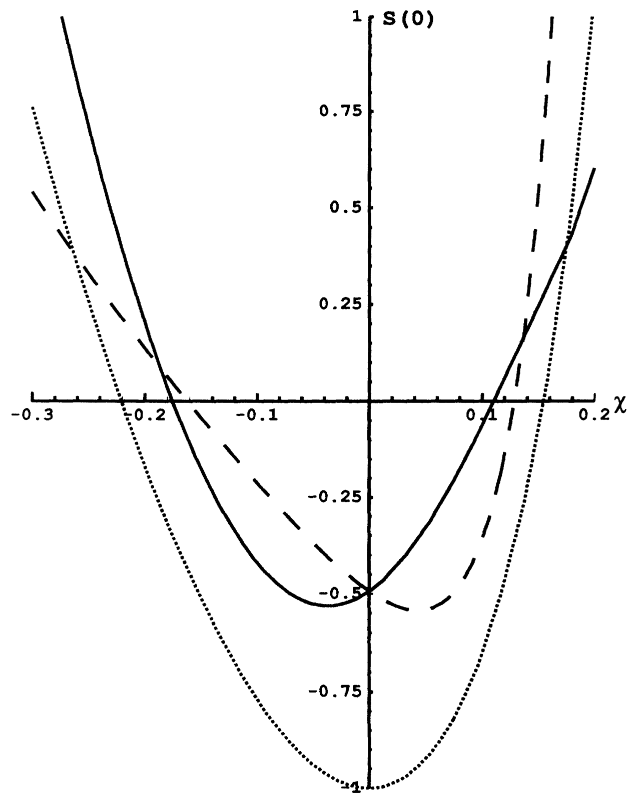

FIG. 3. Above-threshold intensity spectra for the out-of-loop (solid line), in-loop (dashed line), and difference (dotted line) beams with $\epsilon = 5.0$ and $\Gamma = 0.5$.

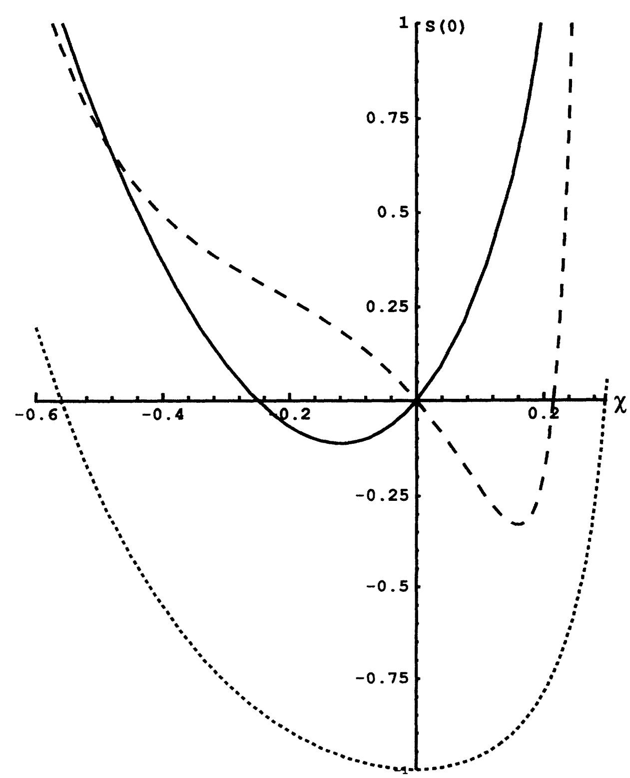

FIG. 4. Same as Fig. 3, except $\epsilon = 1.0$.

VI. CONCLUSIONS

We have presented a quantum analysis of the NDPO with direct detection feedback onto the pump amplitude. A master equation for the signal-idler density operator is derived in the limit that the damping of the pump mode is large and can be adiabatically eliminated. A feedback term representing the direct detection of the signal mode and subsequent modulation of the pump amplitude is included following the Wiseman-Milburn feedback theory. Using the positive P representation, we found the stochastic Ito equations for the semiclassical amplitudes, intensities, phase sum, and phase difference variables. Upon linearization of the intensity-phase equations, we then derived the steady-state intensities, stability regions, and spectra.

Below threshold, the steady-state intensity is zero and feedback is limited to $|\chi| < 1$. Above threshold, the stability region is bounded by the phase sum stability and the positive intensity conditions. Solutions to the steady-state intensity-phase equations exist for which $\sin\phi_+^0 \neq 0$. Such solutions become possible through the Ito correction term in the phase sum Ito equation (3.9c). However, the stability region for these solutions lies in the large negative feedback regime (for $0 < \cos\phi_+^0 < 1$) and their spectral qualities were ignored in this paper.

The intensity spectra show that far above threshold and in the absence of feedback, both the signal and idler are 50% squeezed. To our knowledge, this result has not been noticed by other authors. It can be understood as arising from the two-photon damping of the signal-idler modes, where it enters the intensity Ito equations quadratic in the intensity. Thus, acting as a nonlinear absorber. By comparison, the potential solution for the field amplitudes of the NDPO found by McNeil and Gardiner [9] shows that far above threshold the fields have Poissonian photon statistics *inside* the cavity.

The inclusion of small positive (negative) feedback far above threshold leads to modest improvements in the noise reduction of the out-of-loop (in-loop) beams. For smaller driving, but still above threshold, the beams are initially unsqueezed. Here feedback can have a much greater relative effect on reducing quantum fluctuations, but not below 50%. The Wiseman-Milburn no-go theorem, concerning creating squeezed states from unsqueezed states does not apply here as the NDPO is inherently a quantum system.

Still, in lieu of the traveling-wave results of Tapster *et al.*, it is somewhat disappointing that better noise reduction cannot be obtained in our model. In part, this may be due to the use of a cavity configuration. Our model feedback acts by measuring the intensity fluctuations in one beam (in loop) and instantly modulating the input to reduce fluctuations in the other highly correlated beam (out of loop). However, because photons randomly pass through the cavity mirror, the information gained by even a perfect photodetector is noisy. This randomness appears to severely limit the quantum noise reduction capabilities for this type of feedback.

Below threshold, feedback does not appear to affect the squeezing spectra. Evaluating either the amplitude or intensity noise correlation terms at $I = 0$, one finds that all terms proportional to χ are zero. In this regime, the photodetector is just detecting noise and no meaningful information is gained by which feedback can influence the spectra.

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APPENDIX

In the master equation (2.10) we carried out the direct detection feedback term

$$(e^{\hat{\chi}} - 1)(b\rho b^\dagger) \, , \tag{A1}$$

to second order in the feedback parameter, χ. Here $\hat{\chi}$ is the feedback superoperator

$$\hat{\chi}\rho = [a^\dagger b^\dagger - ab, \rho] \, . \tag{A2}$$

Beyond the double commutator term, the standard process of calculating the Fokker-Planck equation for the positive-P function becomes vitually impossible.

Instead, we find the deterministic Ito stochastic amplitude equations directly by calculating the expectation values of the time dervatives of the mode operators. Thus, for instance, $\langle\dot{a}\rangle = \mathrm{Tr}[a\dot{\rho}]$, where $\dot{\rho}$ is given by Eq. (2.10) using the full expansion of the feedback term (A1). Calculating the expectation values, $\langle\dot{a}\rangle, \langle\dot{a}^\dagger\rangle, \langle\dot{b}\rangle$, and $\langle\dot{b}^\dagger\rangle$ and summing the resulting series, we find the following

$$\begin{aligned}
\dot{\alpha} &= \epsilon\beta^\dagger - \Gamma\alpha\beta\beta^\dagger - \gamma_a\alpha/2 \\
&\quad + \beta\beta^\dagger[\beta^\dagger \sinh\chi + \alpha(\cosh\chi - 1)] \, , \\
\dot{\alpha}^\dagger &= \epsilon\beta - \Gamma\alpha^\dagger\beta\beta^\dagger - \gamma_a\alpha^\dagger/2 \\
&\quad + \beta\beta^\dagger[\beta \sinh\chi + \alpha^\dagger(\cosh\chi - 1)] \, , \\
\dot{\beta} &= \epsilon\alpha^\dagger - \Gamma\beta\alpha\alpha^\dagger - \gamma_b\beta/2 \\
&\quad + \beta\beta^\dagger[\alpha^\dagger \sinh\chi + \beta(\cosh\chi - 1)] \, , \\
\dot{\beta}^\dagger &= \epsilon\alpha - \Gamma\beta^\dagger\alpha\alpha^\dagger - \gamma_b\beta^\dagger/2 \\
&\quad + \beta\beta^\dagger[\alpha \sinh\chi + \beta^\dagger(\cosh\chi - 1)] \, ,
\end{aligned} \tag{A3}$$

where the first three terms on the rhs represent the driving, the two-photon damping, and the mode damping, respectively. The final deterministic term is the feedback, where terms in the expansion of the exponential in (A1) with an odd number of commutators sum to the $\sinh\chi$ terms and the terms with even numbers of commutators add to the $(\cosh\chi - 1)$ terms.

To find the correlations in the noise terms ξ_i, we work indirectly by first calculating the equations of motion of all combinations of normally ordered products of four mode operators, $a, a^\dagger, b$, and $b^\dagger$, using the above amplitude equations. Thus, for instance, $d\alpha^2/dt = 2\alpha\dot{\alpha}$ and the first equation in Eqs. (A3) is substituted for $\dot{\alpha}$. There are ten such products, where two of them are the intensities of modes a and b. Since just the deterministic equations are used, the resulting equations contain no Ito correction terms. As these Ito correction terms are precisely the stochastic forces that is needed to calculate spectra, another set of equations is required that contain them.

The equations of motion for the ten bimode operators can also be calculated directly from their expectation values with the master equation (2.10), again using the full expansion of feedback term (A1). The resulting equations are the full equations of motion for these operators, including the Ito correction terms. Upon subtraction of these two sets of equations, only the stochastic force terms remain

$$\begin{aligned}
\langle\xi_{\alpha^\dagger}(t)\xi_\alpha(t')\rangle &= I_b[2\sqrt{I_aI_b}\cos\phi_+\sinh(\chi)(\cosh\chi-1)\\
&\quad+I_a(\sinh^2\chi-2\cosh\chi+2)\\
&\quad+(I_b+1)\sinh^2\chi]\delta(t-t')\ ,\\
\langle\xi_{\beta^\dagger}(t)\xi_\beta(t')\rangle &= I_b[2\sqrt{I_aI_b}\cos\phi_+\sinh(\chi)(\cosh\chi-1)\\
&\quad+I_b(\sinh^2\chi-2\cosh\chi+2)\\
&\quad+(I_a+1)\sinh^2\chi]\delta(t-t')\ ,\\
\langle\xi_\alpha(t)\xi_\alpha(t')\rangle &= I_b[2\alpha\beta^\dagger\sinh(\chi)(\cosh\chi-1)\\
&\quad+\alpha^2(\sinh^2\chi-2\cosh\chi+2)\\
&\quad+(\beta^\dagger)^2\sinh^2\chi]\delta(t-t')\ ,\\
\langle\xi_\beta(t)\xi_\beta(t')\rangle &= I_b[2\alpha^\dagger\beta\sinh(\chi)(\cosh\chi-1)\\
&\quad+\beta^2(\sinh^2\chi-2\cosh\chi+2)\\
&\quad+(\alpha^\dagger)^2\sinh^2\chi]\delta(t-t')\ ,\\
\langle\xi_\alpha(t)\xi_{\beta^\dagger}(t')\rangle &= I_b[(\alpha^2+(\beta^\dagger)^2)\sinh(\chi)(\cosh\chi-1)\\
&\quad+2\alpha\beta^\dagger(\sinh^2\chi-\cosh\chi+1)]\delta(t-t')\ ,\\
\langle\xi_\alpha(t)\xi_\beta(t')\rangle &= I_b[(I_a+I_b)\sinh(\chi)(\cosh\chi-1)\\
&\quad+\alpha\beta(\sinh^2\chi-2\cosh\chi+2)\\
&\quad+\sinh(\chi)(\cosh\chi+\alpha^\dagger\beta^\dagger\sinh\chi)\\
&\quad-\Gamma\alpha\beta+\epsilon]\delta(t-t')\ ,\\
\langle\xi_{\alpha^\dagger}(t)\xi_{\alpha^\dagger}(t')\rangle &= \langle\chi_\alpha(t)\chi_\alpha(t')\rangle^\dagger\ ,\\
\langle\xi_{\beta^\dagger}(t)\xi_{\beta^\dagger}(t')\rangle &= \langle\chi_\beta(t)\chi_\beta(t')\rangle^\dagger\ ,\\
\langle\xi_{\alpha^\dagger}(t)\xi_\beta(t')\rangle &= \langle\chi_\alpha(t)\chi_{\beta^\dagger}(t')\rangle^\dagger\ ,\\
\langle\xi_{\alpha^\dagger}(t)\xi_{\beta^\dagger}(t')\rangle &= \langle\chi_\alpha(t)\chi_\beta(t')\rangle^\dagger\ .
\end{aligned}\tag{A4}$$

The intensity-phase Ito equations are now easily found

$$\begin{aligned}
\dot I_a &= 2\sqrt{I_aI_b}\epsilon\cos\phi_+-2I_aI_b\Gamma-\gamma I_a\\
&\quad+I_b[\sqrt{I_aI_b}\sinh2\chi\cos\phi_+\\
&\quad+(I_b+I_a+1)\sinh^2\chi]+\xi_{I_a}\ ,\\
\dot I_b &= 2\sqrt{I_aI_b}\epsilon\cos\phi_+-2I_aI_b\Gamma-\gamma I_b\\
&\quad+I_b[\sqrt{I_aI_b}\sinh2\chi\cos\phi_+\\
&\quad+(I_b+I_a+1)\sinh^2\chi]+\xi_{I_b}\ ,\\
\dot\phi_+ &= -(I_b+I_a)\sin(\phi_+)(\epsilon+I_b\sinh\chi\cosh\chi)/\sqrt{I_aI_b}\\
&\quad+\sin2\phi_+\sinh^2(\chi)(I_b^2+I_a^2)/I_a+\xi_{\phi_+}\ ,\\
\dot\phi_- &= -(I_b-I_a)\sin(\phi_+)(\epsilon+I_b\sinh\chi\cosh\chi)/\sqrt{I_aI_b}\\
&\quad+\sin2\phi_+\sinh^2(\chi)(I_b^2-I_a^2)/I_a+\xi_{\phi_-}\ .
\end{aligned}\tag{A5}$$

The noise correlations can be calculated using Eqs. (3.11). The semiclassical, steady-state intensities, stability regions, and spectra obtained from the above Ito equations are qualitatively similar, for both small and large $|\chi|$, to the results obtained in the main body of this paper. For example, the above-threshold steady-state intensity with $\cos\phi_+^0=1$ is

$$I=\frac{\gamma-2\epsilon-\sinh^2\chi}{2e^\chi\sinh\chi-2\Gamma}\ ,\tag{A6}$$

and the spectrum of fluctuations in the intensity difference is

$$\bar S_{I_-}(\omega)=\frac{\Gamma e^\chi+\sinh\chi(-1+2\epsilon-2\epsilon e^{2\chi}-\Gamma e^\chi\sinh\chi+\sinh^2\chi)}{e^\chi(e^\chi\sinh\chi-\Gamma)(1+\omega^2)}\ .\tag{A7}$$

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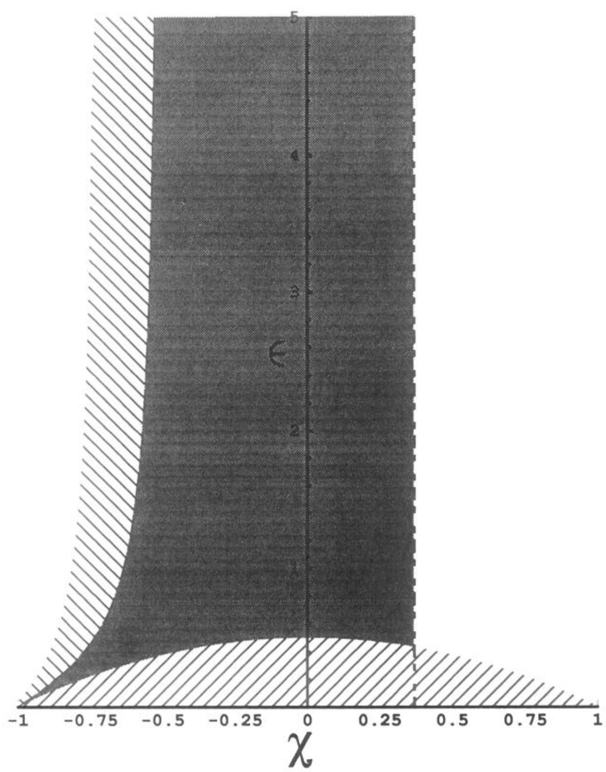

FIG. 2. Above- and below-threshold stability regions with $\Gamma = 0.5$ and $\gamma = 1.0$.