Creating Metastable Schrödinger Cat States

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(Received 9 November 1994)

We propose a scheme using feedback to generate a macroscopic quantum superposition of coherent states in an optical cavity mode which experiences very little decoherence (due to dissipation).

PACS numbers: 42.50.Dv, 03.65.Bz, 42.50.Lc, 42.65.Ky

Over the last few years there has been considerable interest in schemes which can generate macroscopically distinguishable quantum states, commonly known as Schrödinger cats [1]. Within the field of optics several proposals for the generation of such superposition states in various nonlinear process [2–5] and in quantum nondemolition measurements [6] have been made. For example, it has been pointed out by Yurke and Stoler [3] that, in the presence of very low dissipation, a nonlinear system may convert a coherent state into a quantum superposition of macroscopically distinguishable states. Specifically, one imagines modeling a Kerr-like medium (e.g., optical fiber) as an anharmonic oscillator with the Hamiltonian ($\hbar = 1$)

$$H = \omega(a^\dagger a + \tfrac{1}{2}) + \kappa(a^\dagger a)^2, \qquad (1)$$

where $a^\dagger$ and a are the creation and annihilation operators of a photon of a single mode electromagnetic field with $[a, a^\dagger] = 1$. If the light field is in the coherent state $|\alpha_0\rangle$ initially ($t = 0$) then at the time $t = \pi/2\kappa$ (in a frame rotating at frequency ω) the state evolves into the coherent superposition state

$$|\alpha\rangle_{\rm YS} = (1/\sqrt{2})\,(e^{-i\pi/4}|\alpha_0\rangle + e^{i\pi/4}| - \alpha_0\rangle), \qquad (2)$$

where $|\alpha\rangle_{\rm YS}$ is called the Yurke-Stoler (YS) coherent state. At times such that $t = \pi/\kappa$ the system evolves to the state $| - \alpha_0\rangle$. This process repeats until the initial state is reconstructed at $t = 2\pi/\kappa$.

The photon number distribution and interferences in phase space of YS coherent states are known to be highly sensitive to even a small dissipative coupling [2,7,8]. This fact plus the smallness of the $\chi^{(3)}$ coupling constant κ makes the prospects of experimentally producing such states highly questionable.

When two modes interact via a Kerr medium, one mode (probe) can be used to perform a quantum nondemolition (QND) measurement of the photon number of the other mode (signal). To see this, consider the coupled signal-probe system described by the interaction Hamiltonian (in the interaction picture)

$$H_I = \hbar\kappa(a^\dagger a b^\dagger b), \qquad (3)$$

Where κ is proportional to the dispersive part of the third-order nonlinear susceptibility of a Kerr medium, and without loss of generality we assume that it is real and positive. Clearly, $a^\dagger a$ is a constant of the motion and is a QND variable for the signal. The Heisenberg equations of motion for a and b show that if one measures the phase shift of the probe (e.g., via a homodyne measurement of the probe quadrature) information on the signal photon number may be obtained.

We take advantage of these facts in the following system: Consider a cavity supporting two modes of different frequencies, where the annihilation operator a represents the system mode while the b mode is part of the measuring apparatus (Fig. 1). The two modes are coupled by the quadratic Hamiltonian (3). We assume that mode b is heavily damped through an output mirror at rate γ where the output field is mixed with a strong local oscillator field and enters a photodetector. The density operator for both modes obeys the master equation

$$\dot{W} = \mathcal{L}_0 W - iE[b + b^\dagger, W] - i\kappa[a^\dagger a b^\dagger b, W] + (\gamma/2)[2bWb^\dagger - b^\dagger bW - Wb^\dagger b], \qquad (4)$$

where E is the driving field amplitude and the internal dynamics of mode a is represented by the superoperator

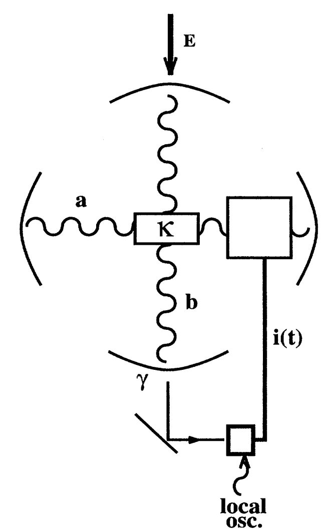

FIG. 1. Diagram of the QND-measurement scheme for the photon number $a^\dagger a$. The $\chi^{(3)}$ nonlinearity of the crystal couples the photon number of the a mode to that of the b mode. The b mode is damped at a rate γ and mixed with a local oscillator and together with the photodetector form the apparatus.

 0031-9007/95/75(3)/418(4)$06.00

$\mathcal{L}_0$. We transform (4) to a displacement picture, where mode b is displaced to a coherent state near the vacuum. Specifically, we let $b \rightarrow b + \delta$ and $b^\dagger \rightarrow b^\dagger + \delta^*$. When $\delta = -2iE/\gamma$, the driving term in (4) is canceled. Assuming

$$\left|\frac{\kappa\langle a^\dagger a\rangle}{\gamma}\right| \sim \left|\frac{\langle \mathcal{L}_0\rangle}{\gamma}\right| = \epsilon \ll 1\,, \qquad (5)$$

we can adiabatically eliminate mode b as in Refs. [9,10]. This gives an equation of motion for the density operator ρ for mode a alone,

$$\dot{\rho} = \mathcal{L}_0\rho - i\kappa|\delta|^2[a^\dagger a,\rho] - \frac{2\kappa^2|\delta|^2}{\gamma}[a^\dagger a,[a^\dagger a,\rho]]. \qquad (6)$$

The second term on the right-hand side of Eq. (6) represents a detuning proportional to the displacement squared and can be eliminated by suitably tuning the cavity. The third term on the right-hand side corresponds to a diffusion in phase and is not small when $|\delta|$ is large. This term represents the backaction noise of the QND measurement.

Now we consider the homodyne measurement of the b mode where the resulting signal photocurrent has been shown by Wiseman and Milburn [9] to be

$$i(t) = \beta[\eta\gamma\langle b + b^\dagger\rangle + \sqrt{\eta\gamma}\xi(t)], \qquad (7)$$

where η is the photodetector efficiency, ξ represents Gaussian white noise, and β is proportional to the local oscillator amplitude and is assumed real and large. The expectation value of the normalized photocurrent $E[\bar{i}(t)] = \langle i(t)/\beta\sqrt{\eta\gamma}\rangle$ can be evaluated by expressing the two-mode density operator W in terms of ρ to second order in ϵ [9]. This gives

$$E[\bar{i}(t)] = -\frac{4\kappa E}{\gamma^{3/2}}\langle\rho a^\dagger a + a^\dagger a\rho\rangle, \qquad (8)$$

where we have assumed perfect detector efficiency $\eta = 1$.

We now want to consider feeding back the signal photocurrent to adjust the cavity detuning. A comprehensive quantum theory of feedback has been given recently by Wiseman and Milburn [11]. Using this theory, we can include the effects on mode a of feeding back the signal photocurrent onto the cavity detuning simply by adding the terms $i\chi[a^\dagger a, \rho a^\dagger a + a^\dagger a\rho] - (\chi^2/2)[a^\dagger a,[a^\dagger a,\rho]]$ to the master equation (6), where χ is the feedback gain. Thus,

$$\dot{\rho} = -i\chi[(a^\dagger a)^2,\rho] - \chi^2[a^\dagger a,[a^\dagger a,\rho]], \qquad (9)$$

where we have ignored the internal dynamics of mode a and defined $\chi = 4\kappa E/\gamma^{3/2}$. The above master equation is the main result of this paper. This equation is expected to be accurate provided the time delay in the feedback loop is small compared with the characteristic evolution time of the system. Notice that mode a behaves exactly like a single mode self-interacting with a Kerr medium with an additional phase diffusion term. The difference here is that the coupling χ is not necessarily small, since we are assuming that the driving E is quite large (so that δ is also large).

We note that the master equation (9) describes a mode in which amplitude, but not energy damping, takes place (unlike an oscillator coupled to a zero-temperature bath, for instance). Thus, the number operator $a^\dagger a$ is an exact pointer and quantum nondemolition observable. Since the number states $|n\rangle$ are the pointer basis, we expect the density operator to become diagonal in this basis. This is easily seen by expressing (9) in the number state basis and ignoring the first term on the right-hand side, we find the solution

$$\rho_{nm}(\tau) = \exp[-\chi^2(n - m)^2]\rho_{nm}(0)\,. \qquad (10)$$

We can solve Eq. (9) by expressing it in terms of the Q function $Q(\alpha,\alpha^*;\tau) = \langle\alpha|\rho(\tau)|\alpha\rangle$. This is a phase-space probability density for the simultaneous measurement of the operators $\hat{X}_1 = (a + a^\dagger)/2$ and $\hat{X}_2 = (a - a^\dagger)/2i$. The marginal distributions of the Q function do not equal the quantum mechanical distributions for $\hat{X}_1$ and $\hat{X}_2$. The Q-function marginals contain added noise arising from the quantum backaction when two canonical variables are measured simultaneously [12]. Using standard techniques [13], one finds the evolution equation for the Q function

$$\begin{aligned}\dot{Q}(\alpha,\alpha^*;\tau) = (\{[i\chi(2|\alpha|^2 + 1) - \chi^2]\alpha\partial_\alpha + \text{c.c.}\} \\ + [(i\chi - \chi^2)\alpha^2\partial^2_{\alpha\alpha} + \text{c.c.}] \\ + 2\chi^2|\alpha|^2\partial^2_{\alpha\alpha^*})Q(\alpha,\alpha^*;\tau)\,, \qquad (11)\end{aligned}$$

where $\partial_\alpha = \partial/\partial\alpha$.

The method of solution of Eq. (11) with initial condition $\rho(0) = |\alpha_0\rangle\langle\alpha_0|$ is similar to the methods used in Refs. [2,7]. Here we give the solution as

$$Q(\alpha,\alpha^*;\tau) = A_0\sum_{n,m=0}^{\infty}\frac{(\alpha\alpha_0^*)^n}{n!}\frac{(\alpha^*\alpha_0)^m}{m!}e^{|\alpha|^2 f(\tau)}e^{\mathcal{R}_{n,m}\tau}, \qquad (12)$$

where $A_0 = \exp[-(|\alpha|^2 + |\alpha_0|^2)]$, $f(\tau) = \exp[2i\chi \times (n - m)\tau] - 1$, and $\mathcal{R}_{n,m} = i\chi(n^2 - m^2) - \chi^2 \times (n - m)^2$. This solution is similar to the Q-function solution of an anharmonic oscillator coupled to a zero-temperature bath [2]. However, there is an important difference in the phase diffusion rate, which can be illustrated by calculating the moments. For instance, from the Q function (12) one can show that

$$\langle a\rangle = \alpha_0\frac{e^{-i\chi\tau}}{C(\chi\tau)^2}e^{-2|\alpha_0|^2[C(\chi\tau)-1]/C(\chi\tau)}e^{-\chi^2\tau}\,; \qquad (13)$$

whereas, for short times and small damping the damped anharmonic oscillator model of Milburn and Holmes [2] gives

$$\langle a\rangle = \alpha_0 e^{-i\chi\tau}e^{-|\alpha_0|^2[C(\chi\tau)-1]}e^{-|\alpha_0|^2[2-C(\chi\tau)]\bar{\gamma}\tau}\,, \qquad (14)$$

where $C(\chi\tau) = 2 - \exp[-2i\chi\tau]$. In the latter case the decay rate of the mode operator is proportional to

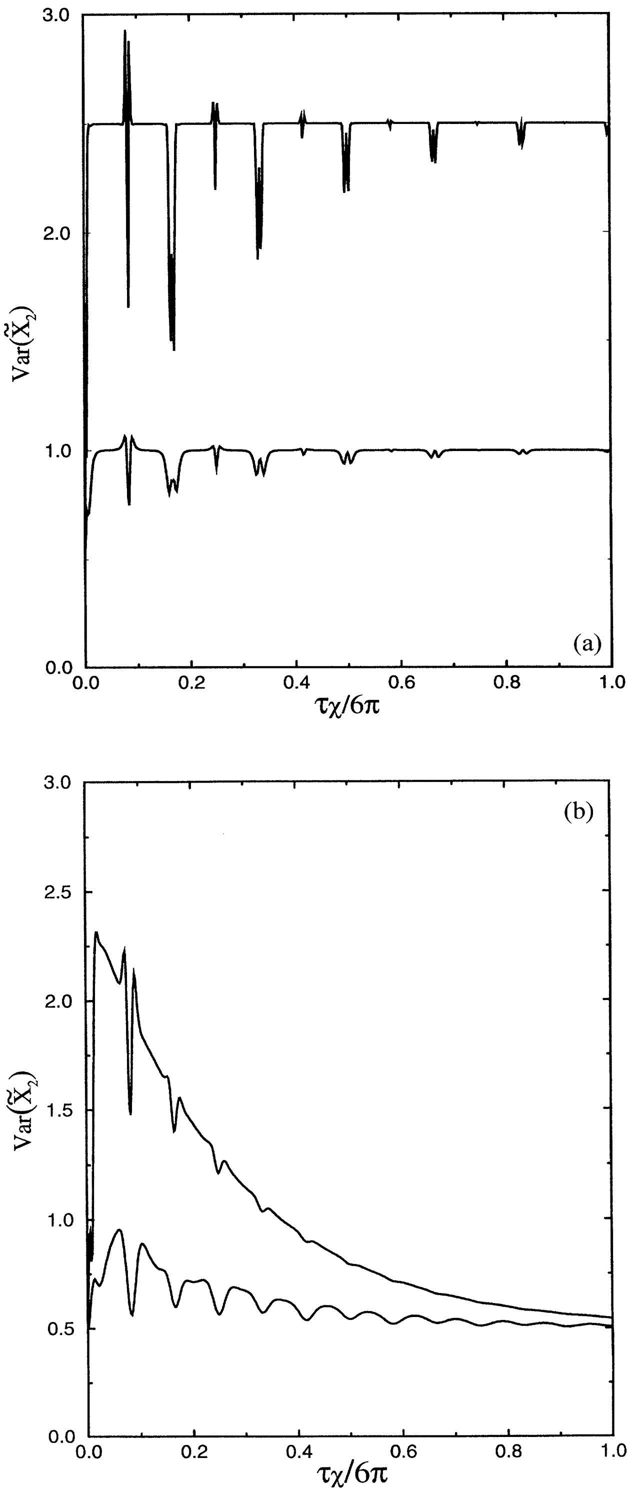

FIG. 2. Variance of the approximate quadrature $\tilde{X}_2$ versus time for (a) the model described in this paper with $\chi = 0.1$, and (b) an anharmonic oscillator coupled to a zero-temperature bath with $\chi = 0.1$ and $\bar{\gamma} = 0.01$. For both graphs, the top curve is for an initial coherent state with mean photon number $|\alpha_0|^2 = 4.0$ and the bottom curve with $|\alpha_0|^2 = 1.0$.

$\bar{\gamma}|\alpha_0|^2$, where $\bar{\gamma}$ represents the coupling to the zero-temperature bath. For a coherent state with a large initial photon number this corresponds to a very rapid decay. On the other hand, Eq. (13) shows a modest decay rate independent of the photon number.

This behavior is illustrated in Fig. 2, where we show the variance of the approximate quadrature $\tilde{X}_2$ versus time. Figure 2(a) is produced from the Q-function solution (12) with $\chi = 0.1$, whereas Fig. 2(b) is created from the solution given by Milburn and Holmes of the anharmonic oscillator coupled to a zero-temperature bath [2] with $\chi = 0.1$ and $\bar{\gamma} = 0.01$. The top curve in each graph is for an initial coherent state with mean photon number $|\alpha_0|^2 = 4.0$ and the bottom curve with $|\alpha_0|^2 = 1.0$. Each curve represents three complete recurrences of the initial state $|\alpha_0\rangle$. An approximate YS state is seen as a steep valley in the variance curve bordered on each side by a sharp peak.

The top curve in Fig. 2(b) is the most telling as it shows that no YS states are formed after the first recurrence even though the coupling to the external bath is small ($\bar{\gamma} = 0.01$). On the other hand, the top curve in Fig. 2(a), shows that near YS states continue to form for much longer interaction times. This behavior becomes more pronounced as $|\alpha_0|^2$ increases.

Of course, every cavity exhibits some damping; however, it is not difficult to imagine a cavity with a decay time much longer than the arbitrarily small feedback-induced nonlinear response time. Using the parameters of Fig. 2(a), one can estimate that the cavity decay constant for mode a must be $<10^{-4}$ if the curves are not to be noticeably altered.

Finally, it is important to remember that Fig. 2 was made assuming equal nonlinear couplings χ in order to illustrate the differences in time scales for the decoherence of the YS states. In reality the feedback-induced nonlinear coupling χ in Eq. (9) can be much greater than the nonlinear coupling of a normal $\chi^{(3)}$ crystal. For the experimentalists, this means that by using the feedback scheme outlined in this paper much shorter interaction times and interaction lengths will be required to produce the YS coherent superpositions.

This work was in part supported by an ARC grant.

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