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Effect of noise and modulation on the reflection of atoms from an evanescent wave

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We consider the reflection of cold atoms from a temporally modulated evanescent wave, with laser intensity noise, including stochastic surface adsorption. The stochastic surface adsorption is explicitly modeled by means of quantum trajectories while the effect of noise is modeled using the method of stochastic Hamiltonians. The results show that noise destroys quantum features such as interference and splitting, which is especially rapid for semiclassical states. For small noise, modulation can still produce a splitting of the atomic beam, with added dispersion resulting from heating of atoms. In order to distinguish the quantum features, a classical analysis is also presented. [S1050-2947(96)05907-0]

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I. INTRODUCTION

Considerable work has been done on the wavelike properties of particles of matter, showing that atoms are subject to phenomena such as interference and diffraction passing through slits and gratings [1,2]. These are examples of the spatial variation of the phase of the atomic wave function. Recently Steane *et al.* [3] published experimental results dealing with the temporal aspect of the wave nature of atomic particles. When the intensity of the evanescent field is temporally modulated, the system acts as a phase modulator, which is an important tool in studying phase space distribution of the cooled atom source or preparing diffraction limited beams and building new atomic interferometers. This turns out to be quite important for atomic interferometry since frequency and time intervals can be produced more accurately than distance intervals, thus promising higher precision methods. Owing to its significance in atom optics, we believe it is worthwhile to study the quantum and classical features of this special phase modulator theoretically.

Henkel *et al.* [4] have presented a theoretical analysis of the coherent reflection of atoms from a modulated evanescent wave using semiclassical (WKB) methods but noise was not taken into account. In order to properly assess the feasibility of interferometry in the time domain, it is necessary to include the loss mechanisms of this system. In this case the system becomes very complicated since, on top of the coherent Hamiltonian, laser intensity noise, time-dependent modulation and stochastic surface adsorption are also involved. The intensity noise is included by adding a stochastic term to the Hamiltonian and the evolution is described by a master equation while modulation is included by making the intensity-dependent coefficient time dependent. To include surface adsorption we treat the surface as an effective destructive coarse-grained measurement of atomic position, modeled as a Poisson jump stochastic process and the conditional evolution can be obtained from a deterministic Schrödinger equation. Most conveniently, the resulting master equation is solved by means of the method of quantum trajectories, combined with a second-order split-operator method for the Hamiltonian part of the dynamics. Furthermore, we also give a classical discussion of the system. The equivalent classical model is obtained by converting the master equation to a Wigner function equation and truncating to second-order derivatives. The resulting Fokker-Planck equation is then converted to Ito stochastic differential equations which are solved numerically, thus giving a more comprehensive analysis of the system.

II. QUANTUM ANALYSIS

The system consists of an atomic mirror resulting from a surface potential produced by the ac Stark shift of the atoms in an evanescent light field along the surface of a glass prism. The amplitude of the light field is modulated at a frequency of 950 kHz. Cesium atoms released from a magneto-optical trap (MOT) form a cold ''beam'' which is further velocity selected to have a narrow velocity distribution at the mirror surface. The cold atoms drop vertically, and are reflected. The reflected beam separates into distinct components, which may be observed by giving the atoms a velocity parallel to the surface.

A. Coherent Hamiltonian motion

In the following we only consider the dynamics in the direction normal to the evanescent field. In terms of dimensionless variables, the Hamiltonian of the system can be written as

$$H=\frac{y^2}{2}+\lambda x+\kappa e^{-x} \tag{2.1}$$

with the canonical commutation relations

$$[x,y]=iK. \tag{2.2}$$

We are using dimensionless position and momentum variables defined by $x=\alpha z$ where z is the displacement from the dielectric surface supporting the evanescent wave and α is the decay rate of the evanescent wave; $y=\alpha p_z/m\omega$ (where ω is a frequency scaling parameter, m is the mass of the atom, and p_z is the vertical momentum component along the z axis). While the scaled gravitational acceleration is $\lambda=\alpha g/\omega^2$, $\kappa=E\alpha^2/m\omega^2$, where $E=\hbar|\Omega_r|^2/\Delta$ is the amplitude of the evanescent potential in terms of the Rabi fre-

quency Ω_r and the detuning of the laser from the atomic transition Δ, and $K=\hbar\alpha^2/m\omega$ is the dimensionless Planck constant.

Time modulation of the evanescent field can be included by making κ time dependent, that is,

$$\kappa(\tau)=\kappa(1+\varepsilon\cos(\Omega\tau)), \tag{2.3}$$

where Ω is a scaled frequency variable, ε is the modulation strength, and $\tau=\omega t$ is the scaled time variable. The time evolution of the system is then governed by the time-dependent Schrödinger equation.

B. Surface adsorption

Any particle that reaches $x\leqslant 0$ sticks to the surface and is not reflected. In the quantum case even particles with insufficient initial energy can reach the surface by quantum tunneling. We call this stochastic surface adsorption and will model this as a ''destructive'' measurement of the particle's position. The measurement is very coarse grained, that is, it can give only two results: either a ''1'' if the particle is found at $x\leqslant 0$ or a ''0'' if the particle is found at $x>0$. If the result is 1, the particle is adsorbed and cannot be reflected. Thus for reflection we require only those states for which the result of all position measurements is 0. That is, we only need to keep track of the conditional states for which the measurement returns the result 0.

Let $\psi(x)$ be the position probability amplitude for the particle at some time. The probability to find the particle at $x\leqslant 0$ is

$$p_1=\int_{-\infty}^{0}dx|\psi(x)|^2, \tag{2.4}$$

while the probability to find the particle at $x>0$ is

$$p_0=1-p_1. \tag{2.5}$$

Now from the general theory of measurements, in terms of operations and effects [5], p_1 can be written in terms of a positive operator $\hat{\theta}$,

$$p_1=\langle\psi|\hat{\theta}|\psi\rangle=\mathrm{tr}(|\psi\rangle\langle\psi|\hat{\theta}), \tag{2.6}$$

where $\hat{\theta}$ is diagonal in the position basis because it is a coarse-grained position measurement, that is,

$$\hat{\theta}|x\rangle=\theta(-x)|x\rangle \tag{2.7}$$

where $\theta(x)$ is the unit step function defined as

$$\theta(x)=\begin{cases}1 & \text{if } x\geqslant 0\\ 0 & \text{if } x<0.\end{cases} \tag{2.8}$$

Equivalently,

$$\hat{\theta}\equiv\int_{-\infty}^{\infty}dx\,\theta(-x)|x\rangle\langle x|. \tag{2.9}$$

As the measurement only has two results, 1 or 0, $\hat{\theta}$ must have only two eigenvalues. Hence $\hat{\theta}$ is a projection operator which can be easily confirmed.

Equation (2.6) can be written as

$$p_1=\langle\tilde{\psi}^{(1)}|\tilde{\psi}^{(1)}\rangle, \tag{2.10}$$

where $|\tilde{\psi}^{(1)}\rangle=\hat{\theta}|\psi\rangle$ is the unnormalized conditional state corresponding to a 1 result.

The unnormalized conditional state corresponding to a 0 result (i.e., particle is found at $x>0$) is

$$|\tilde{\psi}^{(0)}\rangle=(\hat{I}-\hat{\theta})|\psi\rangle. \tag{2.11}$$

We now assume that the adsorption process may be approximated by a conditional Poisson process with rate γ. This means that when the atoms get near $x=0$ there is some rate of adsorption γ, which is equivalent to assuming that our measurement process is a jump process $dN(\tau)$ where

$$E(dN(\tau))=\gamma d\tau p_1. \tag{2.12}$$

Following Wiseman's theory [5], the evolution of the state conditioned on a sequence of results is given by the stochastic Schrödinger equation

$$d|\psi_c(\tau)\rangle=dN(\tau)\left(\frac{\hat{\theta}}{\sqrt{\langle\hat{\theta}\rangle_c}}-1\right)|\psi_c(\tau)\rangle + d\tau\left(-iH+\frac{\gamma}{2}(\langle\hat{\theta}\rangle_c-\hat{\theta})\right)|\psi_c(\tau)\rangle \tag{2.13}$$

or written in terms of the conditional evolution of the density operator

$$d\rho_c(\tau)=dN(\tau)\left(\frac{\hat{\theta}\rho\hat{\theta}}{\langle\hat{\theta}\rangle_c}-\rho\right) -d\tau\left[i[H,\rho]-\gamma\left(\frac{\hat{\theta}}{2}\rho-\rho\frac{\hat{\theta}}{2}-\langle\hat{\theta}\rangle_c\rho\right)\right]. \tag{2.14}$$

We only want to consider quantum trajectories for which the measurement result gives 0 (i.e., particle is never adsorbed). In this case $dN_c(\tau)$ is always zero and the stochastic Schrödinger equation reduces to

$$d|\psi_c^{(0)}(\tau)\rangle=\frac{\gamma d\tau}{2}(\langle\hat{\theta}\rangle_c-\hat{\theta})|\psi_c^{(0)}(\tau)\rangle-iHd\tau|\psi_c^{(0)}(\tau)\rangle. \tag{2.15}$$

If we do not require normalization, this is equivalent to

$$\frac{d}{d\tau}|\tilde{\psi}^{(0)}(\tau)\rangle=-i\left(H-i\frac{\gamma}{2}\hat{\theta}\right)|\tilde{\psi}^{(0)}(\tau)\rangle. \tag{2.16}$$

Then the total probability that no particle has been found at $x\leqslant 0$ for all times up to τ is $P^{(0)}(\tau)=\langle\tilde{\psi}^{(0)}|\tilde{\psi}^{(0)}\rangle$. This evolution is equivalent to a particle moving in the complex potential

$$v(x)\equiv-i\frac{\gamma}{2}\theta(-x), \tag{2.17}$$

where the value of γ is quite arbitrary provided it is big enough to account for the surface adsorption. We expect γ^{-1} to be short compared to the time during which the atom is interacting strongly with the evanescent wave.

C. Effect of laser intensity noise

To achieve reflection of a ground-state atom from an evanescent wave, the difference between the laser and the atomic transition frequency $\Delta=\omega_L-\omega_0$ is large compared to the Rabi frequency and the natural lifetime of the atomic transition. In this case, the effect of spontaneous emission is small and can be neglected. In what follows only the effect of intensity noise is considered. We consider two ways in which this noise can arise: (a) as explicit intensity noise on the classical driving laser and (b) due to a quantum intensity noise if the evanescent field is coupled to a cavity field, upon which phase measurements are made to monitor the atomic bounces. We discuss these cases separately although formally they have the same effect on the atomic system.

When modulation and noise are included, we have a time-dependent Hamiltonian with a random component, the corresponding Stratonovich evolution equation takes the form [5]

$$\dot{\rho}=-\frac{i}{K}[H_0(\tau),\rho]-\frac{i}{K}X(\tau)[F,\rho], \tag{2.18}$$

where $H_0(\tau)$ is the deterministic part of the total Hamiltonian H including modulation. The function F is a function of position. The fluctuations in the laser intensity is given by

$$X(\tau)=\sqrt{\eta}\,\frac{dW(\tau)}{d\tau}, \tag{2.19}$$

where η is the diffusion rate and $W(\tau)$ is the Wiener process.

In order to perform an average over the ensemble Wiener paths we transform Eq. (2.18) into the Ito form [6]

$$d\rho(\tau)=-\frac{i}{K}[H_0(\tau),\rho]d\tau-\frac{i}{K}[F,\rho]dW(\tau)$$
$$-\frac{\eta}{2K^2}[F,[F,\rho]]d\tau. \tag{2.20}$$

In our case $F=e^{-x}$, and after averaging over the noise, the corresponding master equation is

$$\frac{d\rho}{d\tau}=-\frac{i}{K}[H_0(\tau),\rho]-\frac{\eta}{2K^2}[e^{-x},[e^{-x},\rho]]. \tag{2.21}$$

In terms of quantum trajectories, solving the master equation (2.21) numerically is equivalent to solving the following Schrödinger equation between jumps,

$$\frac{d}{d\tau}|\tilde{\psi}\rangle=-\frac{i}{K}K_\sigma|\tilde{\psi}\rangle, \tag{2.22}$$

where the complex Hamiltonian K_σ is $K_\sigma=H_0(\tau)-i(\eta/2K)e^{-2x}$ and $H_0=(y^2/2)+\lambda x+\kappa(1+\varepsilon\cos\Omega\tau)e^{-x}$. As this equation is solved we compute the probability

$$p(\tau)=\langle\tilde{\psi}(\tau)|\tilde{\psi}(\tau)\rangle, \tag{2.23}$$

which is the probability for no jump up to time τ. This waiting time distribution can then be sampled to determine the time at which a jump occurs. If a jump does occur, the state after a jump $|\tilde{\psi}_{after}\rangle$ is defined in terms of the state before $|\tilde{\psi}_{before}\rangle$ in the following way:

$$|\tilde{\psi}_{after}\rangle=\frac{e^{-x}|\tilde{\psi}_{before}\rangle}{\sqrt{\langle\tilde{\psi}_{before}|e^{-2x}|\tilde{\psi}_{before}\rangle}}. \tag{2.24}$$

There has recently been a suggestion [7–9] that if the evanescent wave forming the atom mirror is coupled to an optical cavity, information about the atomic motion can be extracted by monitoring the phase of the cavity. In that case the cavity field would need to have a well defined phase, and thus there will be an irreducible level of intensity noise in the evanescent field. We now show that formally the effect of this noise on the atomic system is of the same form as Eq. (2.21), although the noise coupling term is quite different.

We now take the Hamiltonian to be [7]

$$H_c=\frac{y^2}{2}+\lambda x+\mu a^\dagger a e^{-x}, \tag{2.25}$$

where a is the annhilation operator for the cavity field sustaining the evanescent wave. In addition, we assume this cavity field is driven with a coherent laser field and undergoes damping in the usual way to acheive a steady state on a much shorter time scale than all time scales connected with the atomic motion. The total master equation is now [10]

$$\frac{dW}{d\tau}=-\frac{i}{K}[H_c,W]-iE[a+a^\dagger,W]$$
$$+\frac{\gamma_m}{2}(2aWa^\dagger-a^\dagger aW-Wa^\dagger a), \tag{2.26}$$

where W is the total density operator for the atom and field. The second term in this equation describes the coherent driving of the cavity while the last term describes the damping of the cavity at a rate γ_m. Following Ref. [10], we now adiabatically eliminate the cavity field to obtain a master equation for the atomic system alone. The result is

$$\frac{d\rho}{d\tau}=-\frac{i}{K}[H,\rho]-D[e^{-x},[e^{-x},\rho]], \tag{2.27}$$

where H is given in Eq. (2.1) but now the coupling constant κ is related to the coupling constant μ in Eq. (2.25) by $\kappa=\mu|\alpha_0|^2$ where $\alpha_0=2E/\gamma_m$. The last term in Eq. (2.27) represents the effect of quantum intensity fluctuations in the cavity field. The decoherence rate D is given by

$$D=\frac{8\mu^2E^2}{\gamma_m^3}. \tag{2.28}$$

In order to be able to make a phase determination of the cavity field it must have a well defined phase to begin with. For coherent driving this means we need E large and thus the decoherence term is large. Therefore a good measurement

leads to a large decoherence decay rate, as expected, from quantum back-action noise. Our quantum trajectory simulation can also account for this case simply by the replacement $\eta/2K^2 \rightarrow D$.

D. Numerical algorithm

The algorithm in our numerical simulation to solve the dynamics between jumps is the second order split-operator method [11]. The formal equation (2.22) is the same but we have to be very cautious in using it since the Hamiltonian will take different expressions in different cases. For example, when noise is not included, the Hamiltonian is described by $K_\sigma = H = y^2/2 + \lambda x + \kappa(1+\varepsilon\cos(\Omega\tau))e^{-x}$ while the effective Hamiltonian with intensity noise and surface adsorption is given by $K_\sigma = y^2/2 + \lambda x + \kappa(1+\varepsilon\cos\Omega\tau)e^{-x} - i(\eta/2K)e^{-2x} - i(\gamma/2)\theta(-x)$.

The second-order split-operator method solves the time-dependent Schrödinger equation by separating the Hamiltonian into two parts, that is, the kinetic energy part and the potential part. In our case we have

$$H = T(y) + V(x), \tag{2.29}$$

where $T(y) = y^2/2$, $V(x) = \lambda x + \kappa e^{-x}$ (without modulation and noise) or $\lambda x + \kappa(1+\varepsilon\cos\Omega\tau)e^{-x}$ (with modulation only), or $\lambda x + \kappa(1+\varepsilon\cos\Omega\tau)e^{-x} - i(\eta/2K)e^{-2x} - i(\gamma/2)\theta(-x)$ (when modulation, surface adsorption, and noise are all included), depending on whether the modulation, adsorption, and noise are included or not. Then the wave function $\phi(\tau)$ evolved over one time step is given by

$$\begin{aligned}\phi(\tau+\delta\tau) &= e^{-iH\Delta\tau/K}\phi(\tau) \\ &\simeq e^{-iT\Delta\tau/2K} e^{-iV\Delta\tau/K} e^{-iT\Delta\tau/2K}\phi(\tau),\end{aligned} \tag{2.30}$$

which is accurate up to second order in $\Delta\tau$. We have to work in both position and momentum spaces, the alteration between which is carried out by means of the fast Fourier tranform [12].

III. CLASSICAL ANALYSIS

It is instructive to compare our numerical results with the classical theory of atomic reflection. By means of the techniques described in [13], we can derive the Wigner function evolution equation from the master equation (2.21). Truncating the derived equation to second order in the spatial derivation, we obtain the corresponding classical model in terms of a Fokker-Planck equation [14],

$$\frac{\partial W}{\partial \tau}(x,y,\tau) = -\dot{y}\frac{\partial W}{\partial x} - \dot{x}\frac{\partial W}{\partial y} + \frac{1}{2}D(x)\frac{\partial^2 W}{\partial y^2}, \tag{3.1}$$

where $\dot{x} = y$ and $\dot{y} = \kappa e^{-x}$, and $D(x) \equiv D_0 e^{-2x}$ describes momentum diffusion due to laser intensity fluctuations and its correspondence with the quantum counterpart is $D_0 = \eta$. Since the initial energy is smaller than the potential height, the surface adsorption is negligible and is not included.

The equivalent Ito stochastic differential equations [14] are

$$dx = y\,d\tau \tag{3.2}$$

$$dy = \kappa e^{-x}d\tau + \sqrt{D_0}e^{-x}dW(\tau), \tag{3.3}$$

where $dW(\tau)$ is the usual Wiener increment. This defines a highly nonlinear momentum diffusion.

We solve this classical model by numerically integrating the Ito equations (3.2) and (3.3) for a cloud of initial phase-space point. We create an initial cloud of points in phase space which is Gaussian, use NAG routine G05DDF to produce the random Wiener increment, and solve the whole equations numerically by means of the fourth-order symplectic integrator [15].

Since the solution to the deterministic part can be found analytically [4]

$$x(\tau) = \ln\left\{\frac{\kappa}{E_i}\left[\cosh\left(\frac{\tau-\tau_b}{\tau_i}\right)\right]^2\right\}, \tag{3.4}$$

$$y(\tau) = \frac{2}{\tau}\tanh\left(\frac{\tau-\tau_b}{\tau_i}\right), \tag{3.5}$$

where $\tau_i = 2/y_0$ is the time the atom interacts with the evanescent wave and y_0 the initial momentum, $E_i = y_0^2/2 + \kappa e^{-x_0}$ is the initial energy with x_0 the initial position, while τ_b is the time at which the particles bounce, i.e., the time when the momentum is zero. We use the above analytical solution to check that our numerical calculation gives sufficient precision.

IV. NUMERICAL RESULTS

A. Parameters

We consider the cesium atom with $m = 2.21\times10^{-25}$ kg. The modulation frequency is 950 kHz, $\alpha = 10^7$ m^{-1}, $g = 9.81$ ms^{-2}. Let the scaling frequency $\omega = 10^6$ s^{-1} and we have the following scaled parameters: $\lambda = 9.8\times10^{-5}$, $\kappa = 5.0$, and $K = 0.05$.

The initial state is chosen to be a minimum uncertainty state which has the wave function

$$\phi(x,0) = (2\pi\sigma_x)^{-1/4}\exp\left(i\frac{y_0 x}{K} - (x-x_0)^2/4\sigma_x\right), \tag{4.1}$$

with initial mean position $x_0 = \langle x\rangle = 10$, initial mean momentum $y_0 = \langle y\rangle = -2.6$, and position variance $\sigma_x = 6.5$, while the momentum variance is $\sigma_y = K^2/4\sigma_x$. This corresponds to an atomic beam with a very large uncertainty in position compared with the length scale associated with the evanescent wave, while also having a well-defined momentum. The incoming atomic quantum state is thus close to a plane wave and very far from a classical particle interpretation.

B. Results

In all the calculations, scaled parameters are used and they do not have units. In the following figures, the adsorption has little influence since the atoms turn around before they get close enough to the surface.

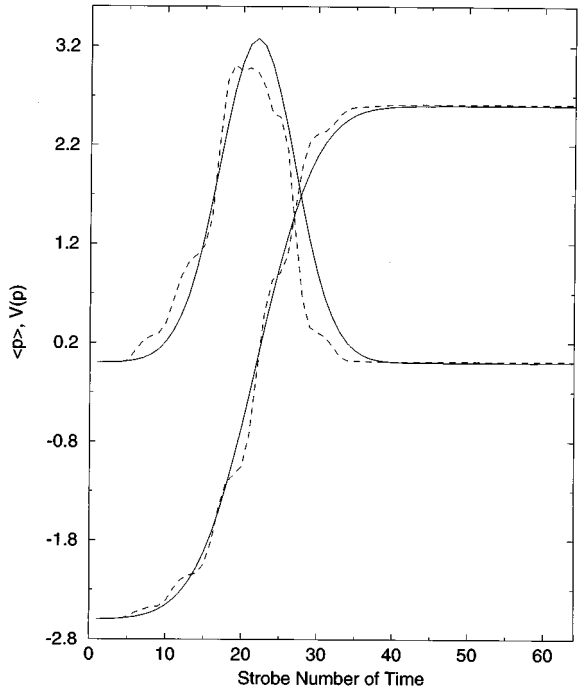

FIG. 1. The mean and variance of the momentum distribution (quantum): (a) solid lines, without modulation and noise; (b) dashed lines, with modulation at frequency 950 kHz and noise $\eta=0.006$; for both cases, the lower line represents the mean, and the upper line represents the variance.

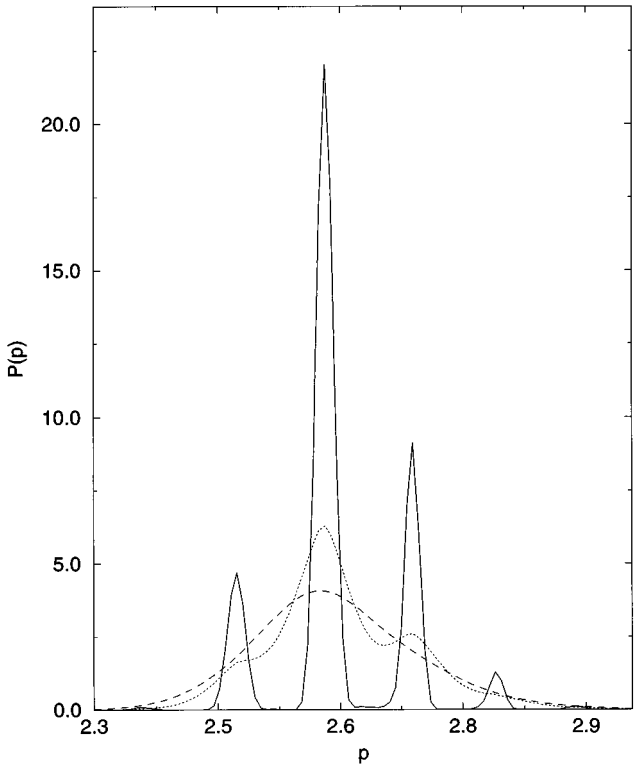

FIG. 2. Far-field momentum distributions with modulation and at different noise levels (quantum): (a) solid line, without modulation and noise; (b) dotted line, with modulation at frequency 950 kHz and noise $\eta=0.002$; (c) dashed line, with modulation at frequency 950 kHz and noise $\eta=0.006$.

1. Quantum case

The mean and variance of the momentum distribution are shown in Fig. 1, where the solid lines are for the case without modulation and noise, the dashed lines are for the case when both modulation and noise are involved, while for both cases the lower line represents the mean and the upper line represents the variance. When there is neither modulation nor noise, the mean and variance are very smooth and regular, and only change quickly around the turning point. But the case with modulation and noise is very different, both mean and variance are changed immensely. Apparently, the modulation and noise add features to the mean and variance; hence we will expect some interesting structures to appear in the position and momentum distributions as well.

The far-field momentum distributions with modulation and at different noise levels are given in Fig. 2, where the solid line is the case without noise, the dotted line is the case when the noise is $\eta=0.002$, and the dashed line is the case when the noise is $\eta=0.006$. Without noise, the modulation causes the momentum distribution to split into several well-defined peaks. The separation between these peaks is the same as that predicted by Henkel *et al.* [4]. But when noise is included, it is a very different story. As noise increases, the peaks are quickly washed away and the splitting disappears.

The far-field position distributions with modulation and at different noise levels are given in Fig. 3, where the solid line is the case without noise, the dotted line is the case when the noise is $\eta=0.006$, and the dashed line is the case when the noise is $\eta=0.06$ (note that the noise levels here are different from those of the momentum distributions). Similar to the momentum distribution, the modulation also causes splitting in the position distribution. But unlike the momentum distribution, the splitting is not easily destroyed by noise. Even at relatively high noise levels, the peaks are still resolvable.

2. Classical case

The mean and variance of the momentum distribution are shown in Fig. 4, where the solid lines are for the case without modulation and noise, the dashed lines are for the case when both modulation and noise are involved, while for both cases the lower line represents the mean and the upper line represents the variance. It shows clearly that modulation and noise do not influence the mean and variance much and we cannot see the complicated structures which occur in the quantum case. As a result, we may not be able to see any splitting at all, which is really what we have found out numerically.

V. CONCLUSIONS AND DISCUSSIONS

In summary, we have presented a complete analysis of the motion of atoms reflected from an evanescent wave, with intensity modulation including both intensity noise and surface adsorption. Modulation causes both the momentum and position distributions to split into multiple-peaked structures, which make it easy to use the system as a phase modulator. This phase modulator has an advantage of being able to rapidly change the modulation factor of the light wave forming the mirror, which gives a direct control over the phases and intensities of the reflected de Broglie waves and allows one

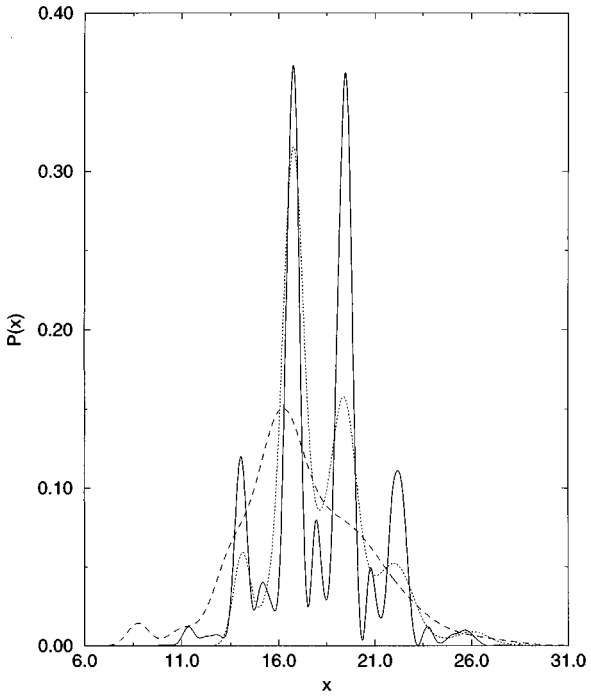

FIG. 3. Far-field position distributions with modulation and at different noise levels (quantum): (a) solid line, without modulation and noise; (b) dotted line, with modulation at frequency 950 kHz and noise $\eta = 0.006$; (c) dashed line, with modulation at frequency 950 kHz and noise $\eta = 0.06$.

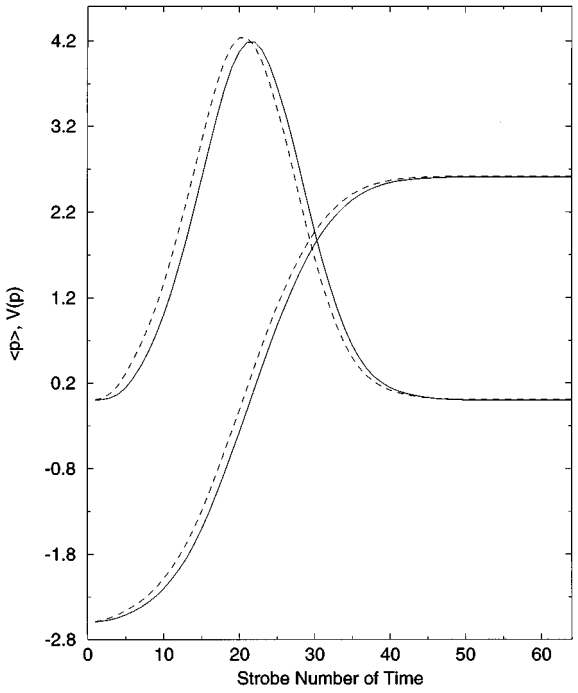

FIG. 4. The mean and variance of the momentum distribution (classical): (a) solid line, without modulation and noise; (b) dashed line, with modulation at frequency 950 kHz and noise $\eta = 0.006$; for both cases, the lower line represents the mean, and the upper line represents the variance.

to build simple and useful atomic interferometric devices. When noise is added, the multiple peaks broaden and are not well resolved, especially in momentum domain. But provided the noise is not too high, splitting in both position and momentum distributions can be easily observed. When noise is not low enough, it will be difficult to see splitting in momentum, but it is still possible to observe splitting in position. It is interesting that the splitting of the momentum distribution should be so sensitive to the noise, suggesting that the multiple-peak structure in the momentum distribution is ultimately a kind of quantum interference feature. The noise levels used in our calculation are very low, so it is important to experimentally reduce the noise as much as possible in order to see the multiple peaks discussed in this paper.

Finally, our results indicate that modulation cannot cause beam splitting in the classical case. Furthermore the noise level does not have much influence on the distributions. Therefore the beam splitting is a purely quantum phenomenon, and can be attributed to the very large position uncertainty and well defined momentum of the incoming atomic state.

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