Morphological Region-Based Initial Contour Algorithm for Level Set Methods in Image Segmentation

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Abstract Initial Contour (IC) is the essential step in level set image segmentation methods due to start the efficient process. However, the main issue with IC is how to generate the automatic technique in order to reduce the human interaction and moreover, suitable IC to have accurate result. In this paper a new technique which we called Morphological Region-Based Initial Contour (MRBIC), is proposed to overcome this issue. The idea is to generate the most suitable IC since the manual initialization of the level set function surface is a well-known drawback for accurate segmentation which has dependency on selection of IC and wrong selection will affect the result. We have utilized the statistical and morphological information inside and outside the contour to establish a region-based map function. This function is able to find the suitable IC on images to perform by level set methods. Experiments on synthetic and real images demonstrate the robustness of segmentation process using MRBIC method even on noisy images and with weak boundary. Furthermore, computational cost of segmentation process will be reduced using MRBIC.

Keywords Initial contour · Level set · Image segmentation · Image processing

1 Introduction

Image segmentation is a significant and a challenging process in computer vision and image processing applications. Extensive researches, countless methods and

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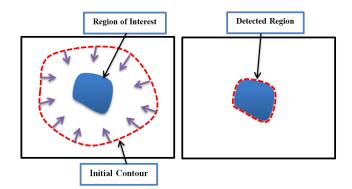


Fig. 1 Active contour and level set models concept.

numerous approaches have been proposed and applied for this matter [15,40]. Active contour models (ACMs, also called deformable models or snakes) [15,3,4, 19,36] are the most successful and accurate methods in image segmentation.

Active contour model, or snake model, was proposed by Kass et al. [15]. The models are on the basis of curve development hypothesis [16,4,19] and Level Set (LS) method [24]. The principal goal is to designate contours in as the zero level set of an implicit function determined in a higher dimension, generally described as the Level Set Function (LSF), and to develop the LSF under a Partial Differential Equation (PDE). Level set method was introduced by Osher and Sethian [25] for front propagation. The existing geometrics active contours or level set methods [3,19] are represented implicitly as level sets of a two-dimensional function that evolves in an Eulerian framework. Geometric active contours are independently introduced. However recently the level set method has been improved by researchers [9,17,10] to obtain suitable results. The concept of active contour and LS methods is demonstrated in Figure 1 which the specified initial guess as IC can move by image driven forces to the boundaries of the object of interest. The detail of this convergence is described in following sections.

The level set image segmentation has two main methods, namely: region-based [5] and edge-based method [4]. To extract the Region of Interest (ROI) in regionbased method the prior energies of an object are required. Whereas, in the edgebased method to specify the pixels on the edge, the gradient magnitude of each pixel of the image is essential [30]. Nevertheless, both the methods have their own drawbacks. The region-based method is sensitive to the variation of intensities in different objects and also the lack of edges in edge-based method which affects the results. Zhang et al. [41] proposed to combine the strengths of both methods together to remove their drawbacks. The active contours and level set methods require level zero or initial points (which is called Initial Contour (IC)) to initialize the evolution process. The IC is determined mostly manually depending on the images condition. IC replacement on object surface is illustrated in Figure 2 which generated at time zero.

In most image segmentations, using level set function, the contour level zero or IC has been chosen manually for all images [11,12,23,31,32], and segmentation process is done without selecting the proper IC which has an effect on segmenting the results. Images that are too noisy or need segmentation for the local regions,

will fail while using this method. As it is shown on Figure 2, the global segmentation with manual IC selection and performed Chan and Vese (C-V) level set method [5] fails to fulfill the segmentation of ROI. C-V Level set model is the most popular region-based image segmentation method which is based on Mumford-Shah segmentation approach [20]. C-V method claims that it does not depend on initial contour to start and it can automatically detect all of the contours. However, this paper demonstrates that the method is faster and more accurately by using a suitable algorithm to generate the IC. Comparatively, the C-V model can extract the object more accurately when the IC surrounds its boundary, therefore reducing the accuracy when it is not covering the proper areas of the image.

Nguyen et al. [21] proposed the interactive continuous-domain convex active contour model for image segmentation. They used the segmentation result of the Geodesic method [2] for contour initialization. The result shows the robustness of this method but still the higher performance is depends on selection of IC. Sun et al. [35] proposed active contour model using local morphology fitting for automatic vascular segmentation on 2-D angiogram. The minimization of the energy associated with the active contour model is implemented within a level set framework. However, in this paper the manual interaction is involved to generate the IC. Sachdeva et al. [29] proposed content-based active contour (CBAC) method which uses both intensity and texture information present within the active contour to overcome above-stated problems capturing large range in an image. It also proposes a novel use of Gray-Level Co-occurrence Matrix to define texture space for tumor segmentation. The IC in this research is produced by radiologist and human infarction and accurate result relies on suitable selection of IC. Xie [37] introduced the IC independency method for LS image segmentation. Xie [38] utilized the magnetostatic active contour, persuading the contour to move through the constant points of vector flow which is the drawback of Generalized Gradient Vector Flow [39]. However, the method [37] to place the new contour, depends on the boundary of object and edge detection processes from initiation. Moreover, the segmentation of images still relies on the position of suitable initial contour (IC) on the image. Furthermore, the manual initialization is time consuming as well as not accurate in segmentation process [37, 38, 33, 6, 1]. Song et al. [33] proposed a technique to generate the IC by amplifying the difference of gradient value between edge and non-edge. Dizdaroglu et al. [8] proposed Structure-Based Level Set method (SBLS) method for retinal vasculature segmentation. The modified phase map is employed in order to obtain the skeletonization and segmentation process. Level set initialized automatically and initial contour is determined around vessels on the images using morphological dilation operator. Bai et al. [1] presented a novel region-based level set (NRBLS) method with using local and global information of image. The mean shift clustering is used to extract the global image information. The results are appropriate with generating suitable initial contours by regulating the clustering results.

In this paper, we propose a new local initial contour selection for level set method which shares the advantages of active contour methods and morphological operations on images. We utilize the statistical and morphological information inside and outside the contour to establish a region-based map function which is able to find the suitable IC on images to perform by level set method. Furthermore, the proposed method has no limitation on image conditions which perfectly provides the IC and object boundary in segmentation process.

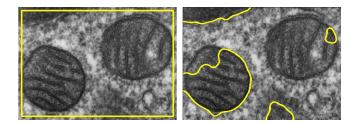


Fig. 2 Predefined IC selection with corresponding segmentation result.

This paper is organized as follows: In Section 2, we review the classic level set method and describes the formulation of our method and how to generate the initial contour. The numerical method of the proposed method is also summarized in the aforementioned section. Additionally, we provide description for implementing our method to segment various objects with dissimilar intensities. The advantages of our method over the manual IC selection are also discussed. Section 3 evaluates our method by extensive experiments on images. Section 4 concludes the paper.

2 Method

2.1 Level Set Segmentation

The level set method was proposed by Osher and Sethian [25] for front propagation, being applied to models of ocean waves and burning flames. Malladi [19] applied Osher [25] method for medical imaging purposes. Level set methods have attracted more and more researchers from different areas [7,14,22]. The concept of the level set method is to enclose a curve within a surface. Because of robust detection in image characteristics such as corners and topological changes, the level set method has been used extensively. The segmentation boundary can be defined as a part of the surface where the contour level is 0, i.e., the zero level set. Let φ represent the implicit surface such that

$$\varphi(x,t) = \pm d \tag{1}$$

Where x is a position in the domain (the image), t is time, and d is the distance between position x and the zero level set. The sign in front of d is positive if x is inside the zero level set. Otherwise, the sign is negative.

$$\varphi(x, y, 0) = \begin{cases} d(x, y, t) \, if(x, y) & inside \ the \ front \\ 0 & \\ -d(x, y, t) \, if(x, y) & outside \ the \ front \end{cases}$$
(2)

The concept of LS method is to enclose a curve within a surface. Figure 3(a) shows how the contour is enclosed within the surface of a cone and Figure 3(b) shows how two ICs are enclosed within the surfaces of the two adjacent cones. By using this ability of level set method, we are able to achieve our specific contour level by morphing the object's surface. The Figure 4 represents the level set progress from IC level zero. In this special case the contour is expanding and it is guaranteed to cross each grid point at most once. The T(x, y) demonstrates time

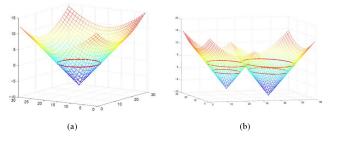


Fig. 3 (a) Circle contour enclosed within the cone. (b) Two circle contour enclosed within the surface of two cones.

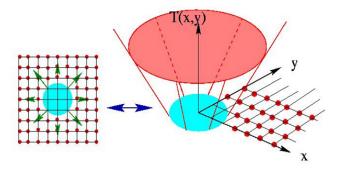


Fig. 4 Progress of level set method from initial contour level.

at which the contour crosses grid point (x, y). At any hight, t, the surface gives the set of points reached at time t.

To move the LS surface, let us define velocity field F which specifies how contour points move in time. Based on application-specifics; physical units such as time, position, normal, curvature, image gradient and magnitude will be specified. Then the initial value for the level set function, $\varphi(x, y, t)$, based on IC will be computed. The value of φ will be adjusted over different times and the current contour defined by $\varphi(x(t), y(t)) = 0$. Then the iteration will be repeated until the convergence on the boundary of the curve. The last obtained curve is the segmented area and final level in LS calculation. The algorithm of LS segmentation method is described as below:

The γ refers to the contour or front in LS. The position of front changes during each evolution.

Ma and Manjunath [18] describes that; in implementation under such a set up, a front might stop evolving at a position of two equidistant edges. In this

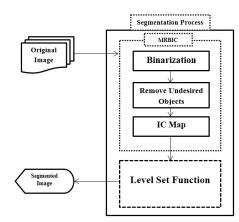


Fig. 5 Image segmentation process using MRIBC technique.

condition, even edge becomes like a realistic option that can evolve or stop moving. Moreover, if the zero level set is affected only by vectors of just one edge, situation will become worse and the entire set will dip into that edge. This might happen if the zero level set, is very close to only one side of the object in the image. In other words, generating the full vector field which covers the entire area and using it is the only viable choice [28].

However, effective level set methods cannot be used directly in all images due to several reasons: (1) complex computation; (2) complexity of parameter settings; and (3) finding the initial contours which are very sensitive where: (a) the speed of the level set method highly depends on the size and the position of initial curves as well as the complexity of objects, and also (b) in some conditions, coupled level set functions cannot converge the same placements of the initial contours.

2.2 Morphological Region-Based Initial Contour

Zero level is used to represent the initial contour or the start point for expansion or shrinkage in LSF. As mentioned in [27], the same IC even in different embedded LS methods will produce the same final result. Therefore, producing the suitable zero level, the result will be more accurate on image segmentation. In implementation of traditional LS methods [4,19,5,27], the upwind techniques are used to provide the numerical stability. The Figure 5 demonstrates the flow diagram of image segmentation process using proposed technique. In MRBIC technique the binarization step is used to simplify the image and remove background area from foreground. Moreover, unwanted objects eliminated in next step to reduce the computation time. The last step is to generate the IC which is most suitable IC for image.

2.2.1 Binarization

The digitized images are generally manipulated and represented as gray scale images. In this regard, the binarization algorithm has been employed using variation of threshold values. The suitable threshold value will prevent the wrong selection of initial contours and furthermore the segmentation process. Therefore, with histogram based image thresholding and average of corresponding pixels in original gray-scale image, we can obtain the proper threshold value. Because of variety in image qualities and imaging difficulties using well-known thresholding methods such as Otsu's algorithm [26] is not efficient. Proposed algorithm contains four classes of pixel range to converts the gray scale image to a binary image. All the pixels in the input image are replaces with 1 where the luminance is greater than the level with the value 1 (white) and the other pixels are replaced with the value 0 (black). The four specified classes are in the range of [0,1]. This range is relative to the signal classes possible for the image's class. Equation 3 shows the threshold values and their pixel ranges.

Suppose f(i, j) is the gray-level value of pixel (i, j), and T is the threshold class value for image.

$$T_{i} = \begin{cases} T = 0.30 \text{ if } 20 \leq \mu f(i,j) < 65\\ T = 0.35 \text{ if } 65 \leq \mu f(i,j) < 90\\ T = 0.45 \text{ if } 90 \leq \mu f(i,j) < 140\\ T = 0.55 \text{ if } 140 < \mu f(i,j) < 220 \end{cases}$$
(3)

Where T has been selected by a specific gray-level average value (μ) and the thresholds are determined empirically.

After applying thresholding value to the image and obtaining the binarized image, the white sections will be the most probable Region of Interest (ROI) area and the black area is considered as a background.

2.2.2 Remove Undesired Objects

The binarized image may contain many objects which are not suitable for our computation. The aim of this stage is to analyse the connected components based on geometric properties (area and dimension) and then removing the undesired objects. The morphologically open binary image technique applied to removing all the connected components (objects) that have lesser than Predefined Pixels Value (PPV) and move to produce another binary image. To determine the PPV we used simple technique (Equation 4) with summarizing total numbers of column and raw and multiplying into 20%. Suppose to determine PPV for the image f(x, y) with x = 600, y = 400 the fallowing process is computed:

$$PPV = \sum (x, y) \times 20\% \tag{4}$$

$$PPV = 600 + 400 \times 20\% = 200$$

All the connected areas in image will be compared with PPV and if summation of each area is below than that, then the object is deemed as an undesired area. Whether It being the noises or unnecessary objects on images. The Figure 6 shows the refinement process, using morphologically open binary image technique on gray scale cell image by using disk structuring element and PPV pixels of neighboring size.

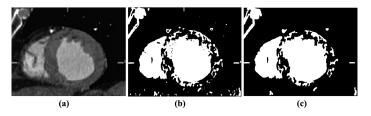


Fig. 6 Refinement process. (a) Original image. (b) Binarized image. (c) Removed undesired objects.

2.2.3 IC Map

We assumed the rectangular area to generate the contour since it makes the implementation faster and has a better result in segmentation process. Therefore, to select the IC for each image, image is divided into four areas (A) to represent the rectangular form of curve as IC and as well the necessity of level set for closed area of initial contour, 4 areas are selected. The reason to select the 4 areas is to cover the most of region of interest in image and reduce the computational time over selection more points. Let $f(x_n, y_m)$ be a subject image for segmentation, the divided four areas are defined as follows:

$$A = \begin{cases} A1: f(x_1:x_{n/2}, y_1: y_{m/2}) \\ A2: f(x_{(n/2)+1}:x_n, y_1: y_{m/2}) \\ A3: f(x_1:x_{n/2}, y_{(m/2)+1}: y_m) \\ A4: f(x_{(n/2)+1}: x_n, y_{(m/2)+1}: y_m) \end{cases}$$
(5)

The first connected pixels in A_i will be labeled by a number and scanning through each connected 8 neighboring pixels in the current area and labeling will be continued until all connected components have one. Suppose if f(i, j) is our binary image then the first connected pixels of ones (white pixels) in position (i, j)will be labeled by number one as first connected region.

In each area the biggest connected component must be kept and the other labeled connected components will be eliminated. In this case, if there are two or more large connected components of the same importance in area, the region which is closer to outer boundary due to covering more objects will be selected. The labeled region i in area A_i is defined as:

$$\mathbf{R}_i = region_i$$

j

And the actual number of pixels in each region or weight of region is defined as:

$$w_i = |R_i|$$

The centroid of biggest labeled connected component or region W_i in each area is considered a desired point for initial contour of area A_i . The region selection is shown as:

$$W = \{w_1, w_2, w_3, \dots, w_n\}$$
$$r_j = Max(W)$$

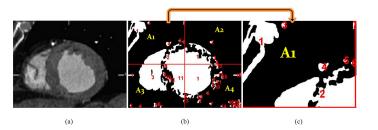


Fig. 7 Region selection. (a) The original gray scale cell image, (b) Four divided areas and labeled regions, (c) Enlarged area A_1 which shows the labeled regions.

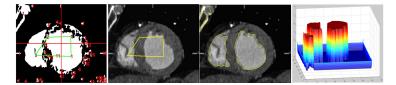


Fig. 8 Selected initial contours in four areas within threshold image and respectively generated initial contour on original image and segmentation result.

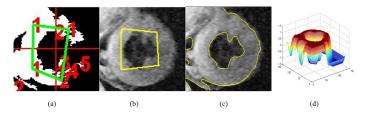


Fig. 9 IC generation process in medical image. (a) IC selected on threshold image, (b) IC on original image, (c) segmented image, (d) domain surface of segmented image.

Where W is set of all the regions in area A_i and r_j is the biggest area with position j.

The Figure 7 demonstrates the original gray scale cell image with four divided areas and labeled regions. As it is shown there are many regions in each area.

The centroid of region with biggest area is the initial contour point in current area. The Figure 8 shows the labeled regions and centroid of each selected region with biggest weight by subjecting them to generate the initial contour and segmentation result respectively.

Nevertheless, during the thresholding process, some divided areas of image may contain zero pixels and be without any region. To overcome this issue the pixel values of each part will be verified and if the value of area is equal to zero, then the thresholding value will be reduced by 0.1. The process will repeat until the pixel value is greater than zero which means there is at least one region to select. The Figure 9 is the gray scale medical image, Figure 9(a) and (b) demonstrate the IC generated using MRBIC method and Figure 9(c) and (d) represent the segmentation result using C-V level set method.

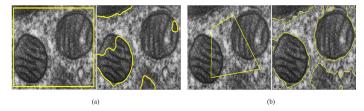


Fig. 10 Segmentation results. (a) Segmentation using predefined IC selection. (b) Segmentation process using MRBIC method.

3 Results and Discussion

Proposed method generates more suitable initial contour to employ in level set based function. The statistical and morphological information of each image has been extracted to establish a region-based map function. However, compared with traditional manual IC generation, the result demonstrates that the MRBIC method is faster and has more accuracy and is also easy to implement. The proposed method is applied on publicly available databases such as 20 images from STARE dataset [13] with 24 bit color plane (standard RGB) at 605×700 , 20 images from DRIVE dataset [34] with 8 bit per color plane at 768×584 and 32 images from our dataset in 8 bit gray level at 400×400 which contains medical and natural images.. The segmentation results compared with three different states of arts to demonstrate the performance of each method. Moreover, the segmentation results of each method is evaluated with publicly available ground truth of each dataset using several quality measures. MRBIC algorithm is implemented on Matlab 7.12 on a 2.0-GHz Intel Pentium IV PC. In each experiment we choose the level set (C-V) method parameters as $sigma = 5, \mu = 3, \Delta = 5$, and time step $\Delta t = 1.$

The Figure 10 shows the result of a grayscale cell image with the considered ROIs inside the image. The Figure 10(a) shows the corresponding segmentation result using traditional method on IC selection respectively. The Figure 10(b) demonstrates the original image with our proposed method for selection of initial contour which is inside the considered regions. By using MRBIC method, the result of level set function converges in 40 iterations takes only 0.20 min, while the segmentation converges in 300 iterations takes 2.45 min using manual and predefined IC.

The Figure 11 demonstrates the segmentation results of images using different methods. The ROI in these images are the blood vessels in retinal images obtained from STARE [13] dataset. With using predefined technique the the level set has failed to segment the ROI and just the outer boundary of image is segmented. The third column is results obtained form [8]. The initial contour is generated using morphological dilation operator around the vessels. As it shows in focused images, their result are more promising compare to predefined technique but comparing to the proposed MRBIC segmentation some regions are failed to segment.

The Figure 12 shows the multi object global segmentation of a galaxy image having weak edges. Result show that the segmentation using our proposed method has an accurate result while segmentation of the predefined fails to segment it accurately. The iteration using proposed method in first row is 70 and it takes

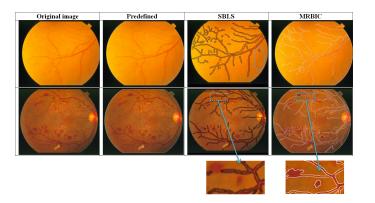


Fig. 11 Segmentation of STARE dataset images comparing with different approaches.

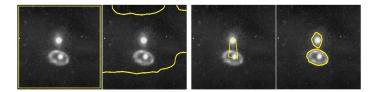


Fig. 12 Galaxy image segmentation with subject to multi object segmentation.



Fig. 13 Segmentation of noisy image using proposed method and predefined IC selection method.

 $0.42~{\rm min}$ to segment. The iteration for second row is 300 and takes about 2.5 min, however it does fail when it comes to segmentation.

Segmentation of particularly noisy images is a challenging process in which the level set method has an advantage. But our experiments show that segmentation of noisy images depends on accurate selection of IC. The Figure 13 demonstrates the IC selection using proposed method which covers every object inside a very noisy image. The result shows the segmentation process has accurately segmented the object. On second row the IC selection has been done pre-defined and results show the object is segmented but there are many unwanted segmented objects that cause the problems in segmentation.

The Figure 14 show some IC and their corresponding segmentation results using our proposed method and respectively segmentation. Moreover, the Figure 15 demonstrates the segmentation process of a hand x-ray image and corresponding domain surface on each iteration level. It shows that the completion of image segmentation within 40 iteration which is very fast and suitable for segmentation process.

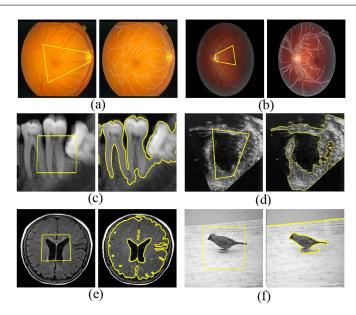


Fig. 14 Some segmentation results of different images, each two images represents IC and related segmented image using MRBIC technique. a) from STARE dataset. b) from DRIVE dataset. c)-f) from our dataset

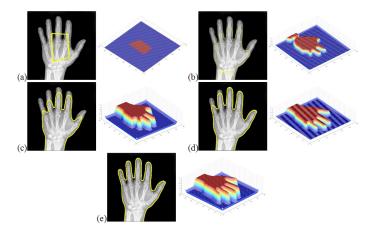


Fig. 15 Segmentation result on x-ray image. (a) IC selection on first step for x-ray image of hand. (b) Progress of level set function on iteration=10. (c) Iteration=20. (d) Iteration=30. (e) Iteration=40.

The experiment shows segmentation, using level set function is dependent on proper IC selection. Iteration of level set and time of segmentation process will change based on the selection of IC. Even though, using the proposed algorithm produces accurate results but still suffers from imaging difficulties such as extra noise or low contrast and etc. All this will affect the segmentation results. It is also noteworthy that even the size of images affect the segmentation time.

Method	Sensitivity (%)			Specificity (%)			Precision (%)		
	STARE	DRIVE	Ours	STARE	DRIVE	Ours	STARE	DRIVE	Ours
Predefined	53.14	47.01	32.25	68.08	62.28	73.51	57.31	69.44	68.40
SBLS [8]	69.26	77.04	66.31	97.26	96.13	78.06	76.33	74.60	79.43
MRBIC	73.42	69.65	74.21	96.08	97.22	98.13	78.51	68.02	82.46
Song [33]	68.24	71.32	59.09	94.71	93.56	81.12	68.28	72.08	65.03
NRBLS [1]	76.35	78.23	72.64	80.33	83.92	91.03	81.01	65.64	79.08

Table 1 Average statistical results of segmentation methods

Table 2 Average statistical results of segmentation methods

Method	Dice Coefficient (%)			Time (S)			Accuracy (%)		
	STARE	DRIVE	Ours	STARE	DRIVE	Ours	STARE	DRIVE	Ours
Predefined	43.24	50.02	47.30	164	178	220	62.41	71.02	69.31
SBLS [8]	70.68	68.26	62.08	106	121	145	94.41	93.65	89.03
MRBIC	68.23	73.19	75.70	96	103	112	92.89	96.08	97.44
Song $[33]$	63.05	64.38	59.88	173	148	178	87.07	90.37	72.60
NRBLS [1]	77.65	67.04	64.32	147	153	123	89.92	94.21	88.23

Sensitivity rate which shows the true positive rate, demonstrates the rate of correct segmentation results 6. Specificity rate demonstrates the correctly detection of negatives results 7. Precision rate quantifies the rate of identified TP with respect to all positive results 8. Accuracy demonstrates the overall performance of segmentation results 9. The higher rate in this measurement shows better performance of segmentation of images each dataset. The Table 1 and Table 2 show the evaluation and performance of image segmentation using predefined, SBLS [8], Song [33], NRBLS [1] and proposed MRBIC techniques. As can seen in Table 2 for the accuracy, for instance, when compared with other methods, our proposed method demonstrates better quantitatively on DRIVE and our database with 96.08% and 97.44%. Nevertheless, difference in image quality and size of image in each database causes the different result for the methods. SBLS method has shown better result on segmentation of STARE database but less than MRBIC for DRIVE and our image database.

$$Sensitivity = \frac{TP}{TP + FN} \tag{6}$$

$$Specificity = \frac{TP}{FP + TN} \tag{7}$$

$$Precision = \frac{TP}{TP + FP} \tag{8}$$

$$Accuracy = \frac{TP + TN}{TP + FP + FN + TN} \tag{9}$$

$$Dice coefficient = \frac{2TP}{(TP + FP) + (TP + FP)}$$
(10)

4 Conclusion

In this paper, we proposed a rapid region-based IC generation for image segmentation with utilizing level set function. Most level set methods are quite dependent selection of IC position on image to start the process of segmentation. Comparatively, the C-V model can extract the object more accurately when the IC is surrounded by its boundary and accuracy reduces when IC is not covering the proper area of the image.

Proposed Morphological Region-Based Initial Contour (MRBIC) method generates the most suitable IC for level set function. The method extracts the statistical and morphological information of image to produce the IC. Extensive experiments on various images compared with predefined IC selection method and results demonstrate superior accuracy and speed of the process, using MRBIC method for level set image segmentation. Furthermore, proposed method is easy to implement and robust, which can be used with any images as well as noisy and weak edges ones has no limitation on image conditions.

Nevertheless, for future work, the proposed method can be performed with other level set and active contour methods with extensive standard datasets and improved for segmentation of images with mentioned difficulties

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