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Complex imaging with ray-rotating windows

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ABSTRACT

We study the imaging properties of windows that rotate the direction of transmitted light rays by a fixed angle around the window normal [A. C. Hamilton *et al.*, J. Opt. A: Pure Appl. Opt. **11**,085705 (2009)]. We previously found that such windows image between object and image positions with suitably defined complex longitudinal coordinates [J. Courtial *et al.*, Opt. Lett. **37**, 701 (2012)]. Here we extend this work to object and image positions in which any coordinate can be complex. This is possible by generalising our definition of what it means for a light ray to pass through a complex position: the vector from the real part of the position to the point on the ray that is closest to that real part of the position must equal the cross product of the imaginary part of the image position and the normalised light-ray-direction vector. In the paraxial limit, we derive the equivalent of the lens equation for planar and spherical ray-rotating windows. These results allow us to describe complex imaging in more general situations, involving combinations of lenses and inclined ray-rotating windows. We illustrate our results with ray-tracing simulations.

Keywords: ray rotation, generalised refraction, micro-optics, imaging

1. INTRODUCTION

One of the themes in our research is ray optics that is not restricted by wave optics. The justification is that it is possible to create light-ray fields which *look like* fields for which no corresponding phase fronts exist.[1] We achieve this with micro-structured, pixellated, windows. An example for such a window is an array of micro-telescopes,[2,3] each of which changes the direction of a small piece of the transmitted light wave according to some generalised law of refraction, which then propagates like a small bundle of light rays. The resulting light-ray field can then look as if the corresponding phase front is staircase-shaped *at every point*, which is not possible. The generalised laws of refraction for which this can happen are found in Ref.[4]

Imaging lies at the heart of optics. As part of our unrestricted-ray-optics research theme, we have investigated ray-optical imaging not constrained by wave optics.[7,8] We have also started to investigate generalised imaging for one of our favourite generalised laws of refraction, namely rotation of the light-ray direction by an arbitrary (but fixed) angle α around the local window normal.[9] In an earlier paper, we found that a planar, ray-rotating, window can be said to image between *complex* object and image distances.[10] In the following we choose our coordinate system such that the z axis coincides with the optical axis, which implies that complex object and image distances correspond to object and image positions with a complex z coordinate. The rays forming a complex object or image position lie on hyperboloids such as the one shown on the right of Fig. 1. The axes of these ray hyperboloids are parallel to the optical axis; the real part of the complex image position is located in the centre of the hyperboloid waist, the imaginary part of the z coordinate relates the angle of the rays to their distance from the hyperboloid waist.

Our previous work on imaging with ray-rotating windows cannot describe many situations, for example imaging of a ray hyperboloid whose axis is not perpendicular to the ray-rotating window, which happens in imaging with planar, but inclined, ray-rotating windows. Here we investigate such imaging. We also begin to study imaging with a spherical, ray-rotating, window, and find a generalisation of the equation describing spherical surface refraction.[11]

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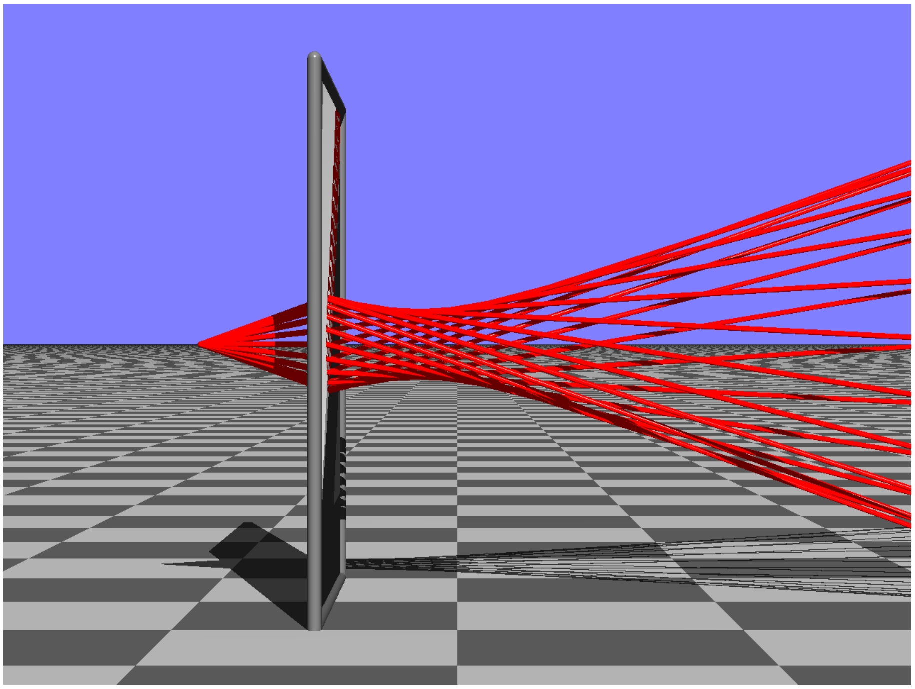

Figure 1. Complex image of a real point light source, created by a ray-rotating window (rotation angle $\alpha = 135°$). The real point light source lies at the apex of the cone of light rays (red cylinders) to the left of the ray-rotating window (framed with grey cylinders); the waist of the hyperboloid of light rays on the right of the window is interpreted as a complex image position. The image was calculated with scientific raytracer Dr TIM.[5,6]

2. LOCAL LIGHT-RAY ROTATION

Local light-ray rotation can be achieved with pairs of Dove-prism arrays,[12] or with two pairs of arrays of cylindrical lenses, such that the image-sided focal plane of one array of cylindrical lenses coincides with the object-sided focal plane of the other.[13] Each Dove-prism array, or pair of arrays of cylindrical lenses, that is positioned perpendicular to the optical axis, inverts one of the transverse components of the direction of transmitted light rays. Two such arrays, one immediately behind the other and rotated with respect to each other by an angle $\alpha/2$ around the optical axis, rotate the direction of transmitted light rays through an angle α around the z direction. (It is also known how to achieve local light-ray rotation around axes other than the window normal.[14])

Mathematically, the generalised law of refraction that is local light-ray rotation around the window normal can be described as refraction according to Snell's law at the interface between two media with a refractive-index ratio[15]

$$\frac{n}{n'} = \exp(\mathrm{i}\alpha). \tag{1}$$

It was this elegant mathematical description of light-ray rotation in terms of a complex refractive-index ratio that motivated us to describe the effect of light-ray-rotating, planar, windows, as imaging between complex object and image positions.

3. RAYS INTERSECTING COMPLEX POINTS

In order to investigate imaging between complex object and image points we need to define the meaning of a light ray passing through a complex position. We define a light ray with direction $\hat{\mathbf{a}}$ to pass through a complex position

$$\mathbf{P} = \mathbf{P}_r + \mathrm{i}\mathbf{P}_i \tag{2}$$

if and only if it passes through the position

$$\mathbf{C} = \mathbf{P}_r + \mathbf{P}_i \times \hat{\mathbf{a}}. \tag{3}$$

This definition, a generalisation of our earlier definition,[10] is the basis of the remainder of this paper.

In terms of ray hyperboloids, our definition can be understood as follows. The position $\mathbf{P}_r$ is the centre of the hyperboloids' waist plane. The vector $\mathbf{P}_i$ points in the direction of the hyperboloids' axis, and as our definition allows $\mathbf{P}_i$ to point in any direction, so does it allow the axis of the ray hyperboloids to be orientated arbitrarily. A ray with normalised direction $\hat{\mathbf{a}}$ passes the hyperboloid centre at a distance

$$d = |\mathbf{P}_i| \sin\theta, \tag{4}$$

where θ is the angle between $\hat{\mathbf{a}}$ and $\mathbf{P}_i$.

The understanding of our definition in terms of ray hyperboloids in turn allows the relationship with our earlier definition.[10] In the earlier definition, the axis of the ray hyperboloids was always perpendicular to the (planar) ray-rotating windows, in the z direction. In terms of our new definition, this corresponds to an imaginary part of the position pointing in the z direction, and so it was only the z (longitudinal) component, and therefore the distance from the window, which was complex. Our earlier definition was formulated in terms of object and image distances, and so these were complex. The equivalent of Eqn (4) in our earlier definition (see the imaginary parts in Eqn (3) in Ref.[10]) was

$$d = |\mathbf{P}_i| \tan\theta; \tag{5}$$

this agrees with the new definition for small angles θ.

4. IMAGING WITH PLANAR, RAY-ROTATING, SURFACES

In Ref.,[10] we used object and image distances, o and i, respectively the distance of the object *in front of* the window and of the image behind the window. Here we use z coordinates. For a window in the $z = 0$ plane, the image distance i is the same as the longitudinal coordinate of the image, z'; the relationship between the object distance, o, and the longitudinal coordinate of the object, z, is more complex:

$$z = -o^* \tag{6}$$

(see Eqn (7) in Ref.[10]). We found that a planar, ray-rotating, window (rotation angle α) imaged an object at object distance o to an image distance

$$i = -\exp(-\mathrm{i}\alpha) o^*. \tag{7}$$

In terms of the z coordinates we use in this paper, this equation becomes

$$z' = \exp(-\mathrm{i}\alpha) z, \tag{8}$$

or, expressing the complex exponential as a refractive-index ratio according to Eqn (1),

$$z' = \frac{n'}{n} z, \tag{9}$$

This equation still holds if the refractive-index ratio n'/n is not of the form $\exp(-\mathrm{i}\alpha)$ *, but only paraxially.[10] The transverse coordinates of the image were the same as those of the object, i.e.

$$x' = x, \quad y' = y; \tag{10}$$

these were required to be real for the results to hold.

*The outgoing light-ray direction can be obtained as follows. The modulus of the refractive-index ratio is the factor by which the transverse component of the normalised light-ray direction gets scaled before the longitudinal component gets adjusted such that the light-ray direction is normalised again.[15] This is equivalent to Snell's law.

We have now generalised these results to complex transverse object and image coordinates. For paraxial rays ($\theta \ll 1$) we obtain

$$x' = x_r + y_i + \mathrm{i}(x_i + \mathrm{i}y_i)\left(\frac{n'}{n}\right)^2, \tag{11}$$

$$y' = y_r - x_i + (x_i + \mathrm{i}y_i)\left(\frac{n'}{n}\right)^2, \tag{12}$$

$$z' = \frac{n'}{n}z, \tag{13}$$

where $x = x_r + \mathrm{i}x_i$ etc.

5. IMAGING WITH SPHERICAL, RAY-ROTATING, SURFACES

Lenses get their imaging properties due to Snell's-law refraction at spherical surfaces. Imaging at each individual spherical refractive-index interface of radius of curvature R can be described by the equation[11]

$$\frac{n'}{i} + \frac{n}{o} = \frac{n' - n}{R}, \tag{14}$$

where n and n' is the refractive index in front of and behind the interface, respectively, o is the distance of the object in front of the interface, i is the distance of the image behind the interface.

We have started to study imaging with spherical, ray-rotating, surfaces. In this initial calculation we have assumed cylindrical symmetry with respect to the z axis, i.e. $x = y = x' = y' = 0$. Paraxially (discarding terms with θ^2 or higher), we obtain the result

$$\frac{n'}{z'} - \frac{n}{z^*} = \frac{n' - n}{R}, \tag{15}$$

which is the same as Eqn (14), but using the longitudinal coordinates instead of object and image distances (see Eqn (6)).

Note that we do not use imaginary x and y components in this section, and so we could have obtained the results in this section with our previous definition.

6. LENSES WITH COMPLEX FOCAL LENGTHS

Eqn (1) suggests that a ray-rotating window is formally equivalent to the interface between media with complex refractive indices $n = 1$ and $n' = \exp(-\mathrm{i}\alpha)$. Substitution into the equation for paraxial imaging with spherical, ray-rotating, surfaces, expressed in terms of object and image distances, Eqn (14), gives

$$\frac{\exp(-\mathrm{i}\alpha)}{i} + \frac{1}{o} = \frac{\exp(-\mathrm{i}\alpha) - 1}{R}. \tag{16}$$

Comparison with the thin-lens equation

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f} \tag{17}$$

reveals that a spherical, ray-rotating, surface with a radius of curvature R behaves somewhat like a lens with complex focal length

$$f = \frac{R}{\exp(-\mathrm{i}\alpha) - 1}. \tag{18}$$

Fig. 2 shows an example of the effect of such a lens on a ray hyperboloid that corresponds to a point light source at the lens's complex focal point: after transmission through the lens, the rays are all parallel.

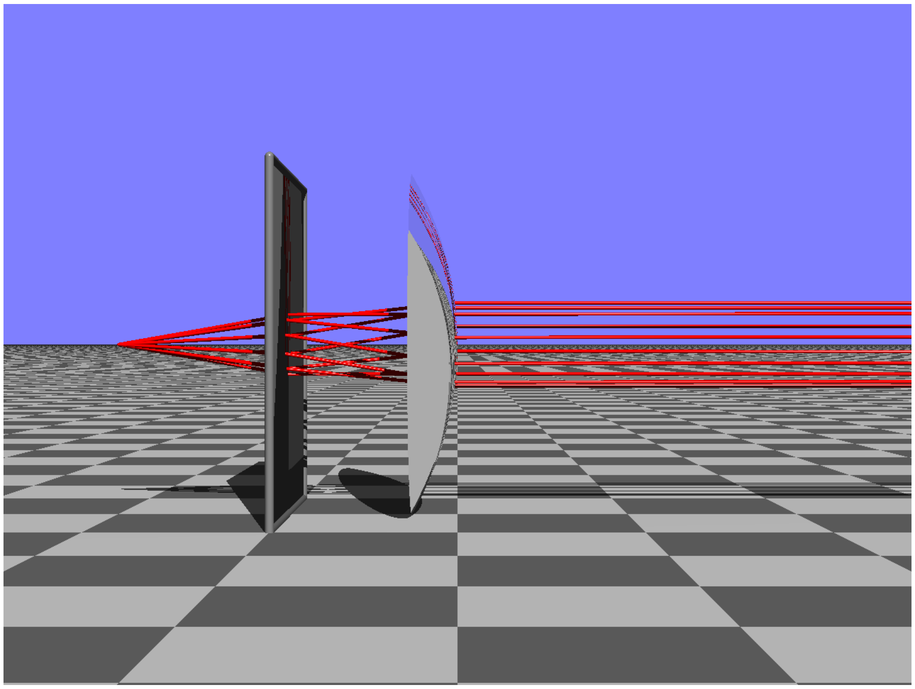

Figure 2. Imaging with a lens with a complex focal length. A ray cone travelling left to right is transmitted first through a planar, ray-rotating, window (distance from the cone apex 1, rotation angle 90°) and then a spherical, ray-rotating window (distance from the planar window 1, radius of curvature $R = -2$, rotation angle 90°, complex focal length $f = 1 - \mathrm{i}$). The planar, ray-rotating window images the cone apex at object distance $o = 1$ to an image distance $i = \mathrm{i}$. This corresponds to an object at the object-sided focal point of the spherical, ray-rotating, window, which transforms it to a parallel ray bundle. (More precisely, the image distance is given by the equation $-\mathrm{i}/i = \infty$, and so $i = \mathrm{i}\infty$.) The image was calculated with scientific raytracer Dr TIM.[5,6]

7. CONCLUSIONS

This work is part of a larger effort to develop mathematical frameworks that harness the new possibilities offered by ray optics not constrained by wave optics. Here we have progressed the formal description of imaging with ray-rotating surfaces in terms of complex object and image positions.

We intend to continue this work, by generalising it, for example dropping the cylindrical symmetry in the spherical-window case, and by searching for applications.

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