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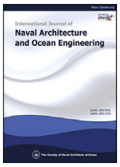

Parametric pitch instability investigation of Deep Draft Semi-submersible platform in irregular waves

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Abstract

Parametric pitch instability of a Deep Draft Semi-submersible platform (DDS) is investigated in irregular waves. Parametric pitch is a form of parametric instability, which occurs when parameters of a system vary with time and the variation satisfies a certain condition. In previous studies, analyzing of parametric instability is mainly limited to regular waves, whereas the realistic sea conditions are irregular waves. Besides, parametric instability also occurs in irregular waves in some experiments. This study predicts parametric pitch of a Deep Draft Semi-submersible platform in irregular waves. Heave motion of DDS is simulated by wave spectrum and response amplitude operator (RAO). Then Hill equation for DDS pitch motion in irregular waves is derived based on linear-wave theory. By using Bubnov-Galerkin approach to solve Hill equation, the corresponding stability chart is obtained. The differences between regular-waves stability chart and irregular-waves stability chart are compared. Then the sensitivity of wave parameters on DDS parametric pitch in irregular waves is discussed. Based on the discussion, some suggestions for the DDS design are proposed to avoid parametric pitch by choosing appropriate parameters. The results indicate that it's important and necessary to predict DDS parametric pitch in irregular waves during design process.

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Keywords: Deep Draft Semi-submersible platform; Parametric instability; Parametric pitch; Irregular waves; Mathieu

1. Introduction

The semi-submersible platform has been widely used for oil and gas exploration since it came out in 1960s because of its many advantages, such as less production cost, better hydrodynamic behavior and greater range of applicable water depth (Chen and Tan, 2008). But because of large heave motion, the dry-tree system cannot be utilized on the platform, which prohibits the development of semi-submersible platform greatly (Yang et al., 2009). Nowadays, many scholars put forward the conception of Deep Draft Semi-submersible platform (DDS) (Bindingsø and Bjørset, 2002, Halkyard et al., 2002), as shown in Fig. 1. They found by increasing the draft of semi-submersible platform from 25 m to 40 m, the heave motion can be improved tremendously. Although increasing draft can improve heave performance, it also brings new problems, such as parametric pitch and Vortex-Induced Motion (VIM) (Bai et al., 2014).

Parametric instability is a common phenomenon existing on offshore structures, such as ships, spar platforms, the tethers of tension-leg platforms (TLPs) and risers. When parametric instability occurs, motions in some degree of freedom will increase largely so that structures and crews will suffer harm. Over the past few years, several ships have suffered accidents due to parametric roll (France et al., 2003; Hua et al., 2006). Parametric instability of spar platforms and risers has also been observed in experiments (Hong et al., 2005; Huse et al., 1998).

In previous studies, researches about parametric instability are mainly focused on regular waves. Many scholars have

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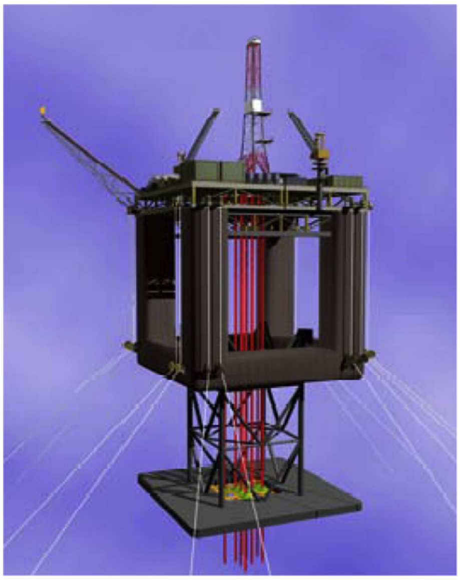


Fig. 1. Deep Draft Semi-submersible.

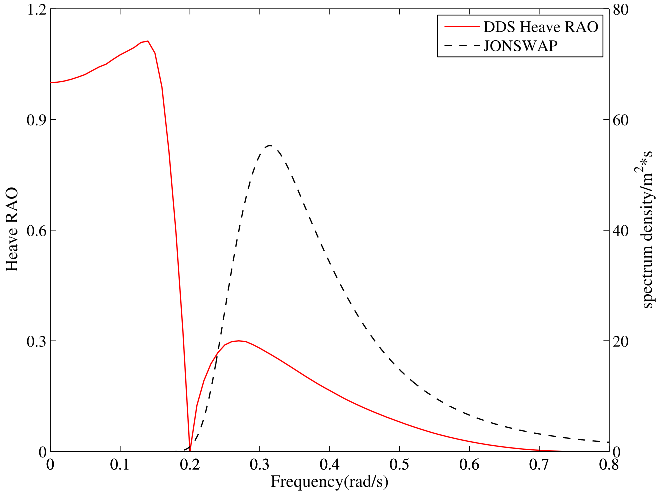


Fig. 2. JONSWAP spectrum and Heave RAO of Deep Draft Semi-submersible platform.

investigated parametric roll of ship in regular waves (Spyrou, 2000; Neves and Rodríguez, 2007; Kim and Kim, 2011). Most studies of parametric pitch of spar platforms were also limited to regular waves (Haslum and Faltinsen, 1999; Rho et al., 2002). However, the realistic sea conditions are irregular waves, which is more complex and random than regular waves. Nowadays, several scholars began to concentrate on the study of parametric instability in irregular waves. Lu et al. (2011) and Maki et al. (2011) gave their predictions of parametric roll of ships in irregular waves. Yang and Xu (2015) investigated parametric pitch of spar platforms in irregular waves. Yang and Xiao (2014) analyzed parametric instability of top-tensioned riser under combined vortex and irregular-waves excitation. The results of their researches indicate that parametric instability does occur in irregular waves. Therefore, it is necessary to investigate parametric instability in irregular waves.

Though Deep Draft Semi-submersible platform is a conceptual platform, parametric pitch of the platform cannot be ignored. In this study, the parametric pitch property of DDS is analyzed in irregular waves. By using linear-wave theory and wave spectrum to describe irregular waves, combing with heave RAO, heave motion of DDS in irregular waves can be obtained. Then Hill equation for DDS pitch motion in irregular waves is derived through parameters replacing. By using Bubnov-Galerkin approach (Pedersen, 1980) to solve Hill equation, relevant stability chart can be obtained. Next, the differences of parametric pitch between regular-waves and irregular-waves excitation are compared. Lastly, the sensitivity of wave parameters on DDS parametric pitch in irregular waves is analyzed by stability charts.

2. Theory

2.1. Simulating heave motion of DDS in irregular waves

Linear-wave theory is widely used to describe irregular waves. According to linear-wave theory, irregular waves can be assumed as a superposition of a number of regular wave components. Usually, regular wave is described as cosine wave, so irregular waves can be simplified as follows,

$$\eta(t)=\sum_{i=1}^{n}\eta_i\cos(\omega_i t+\varepsilon_i) \tag{1}$$

where, η_i is the amplitude of each wave component and ω_i is the circular frequency of each wave component, ε_i is the stochastic initial phase.

Wave spectrum denotes wave energy spectrum density under each frequency. In this study, JONSWAP spectrum is chosen, whose expression is defined as follows,

$$S(\omega)=\frac{5H_s^2\omega_p^2}{16\omega^5}\cdot(1-0.287\lg\gamma)\cdot\exp\left(-\frac{5\omega_p^4}{4\omega^4}\right)\cdot\gamma^{\exp\left[-\frac{(\omega-\omega_p)}{2\sigma^2\omega_p^2}\right]} \tag{2}$$

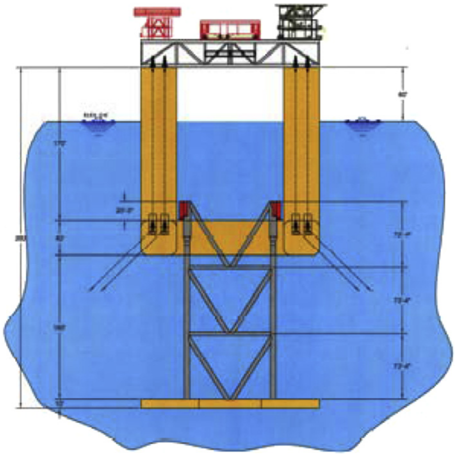


Fig. 3. Deep Draft Semi-submersible configuration.

Table 1
Dimensions of Deep Draft Semi-submersible.

Deck width (ft)	200	**Heave plate:**	
Column:		Heave plate truss height (ft)	200
No. of column	4	Heave plate width (ft)	200
Width (ft)	40	Heave plate depth (ft)	10
Length (ft)	40	Draft (bottom of heave plate) (ft)	320
Pontoon:		Heave natural period (s)	25.4
No. of Pontoon	4	Pitch natural period (s)	43
Height (ft)	40		
Width (ft)	40		

where, H_s is the significant wave height, ω_p is the peak wave frequency, ω is the wave frequency, γ is the peak enhancement factor, and σ is given as

$$\sigma = \begin{cases} 0.07 \ \text{if } \omega \leq \omega_p \\ 0.09 \ \text{if } \omega > \omega_p \end{cases} \tag{3}$$

Then the amplitude of each wave component η_i can be obtained,

$$\eta_i = \sqrt{2S(\omega_i)\cdot\Delta\omega} \tag{4}$$

where, $\Delta\omega$ is a frequency interval.

Response amplitude operator (RAO) is the ratio of amplitude of body in response to an incident wave of unit amplitude. Heave RAO represents heave motion amplitude of floating structure under the effect of unit simple harmonic wave, namely, $H(\omega_0) = (\xi_{heave}/\xi)\big|_{\omega=\omega_0}$. Fig. 2 shows JONSWAP spectrum and DDS heave RAO. Applying heave RAO to wave spectrum, heave motion can be obtained,

$$\xi_3(t) = \sum_{i=1}^{n} \xi_i \cos(\omega_i t + \varepsilon_i) \tag{5}$$

where, $\omega_i = i\omega_0$ and

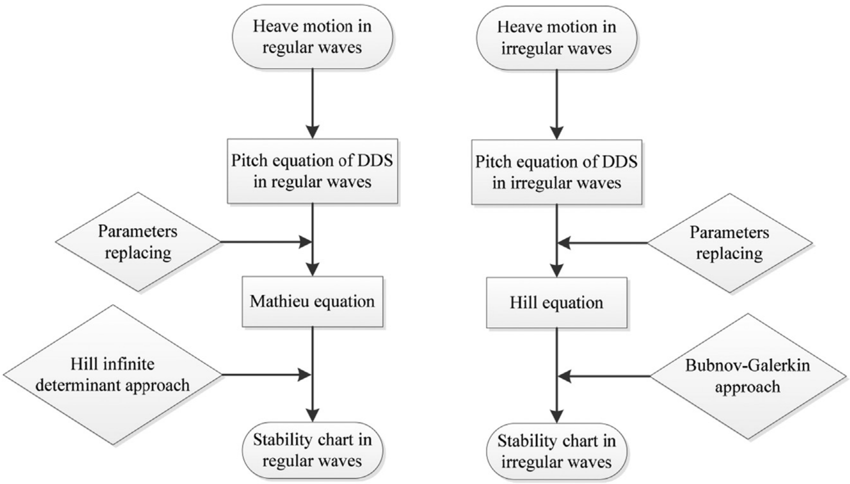


Fig. 4. Flow diagram of drawing stability charts in regular waves and irregular waves.

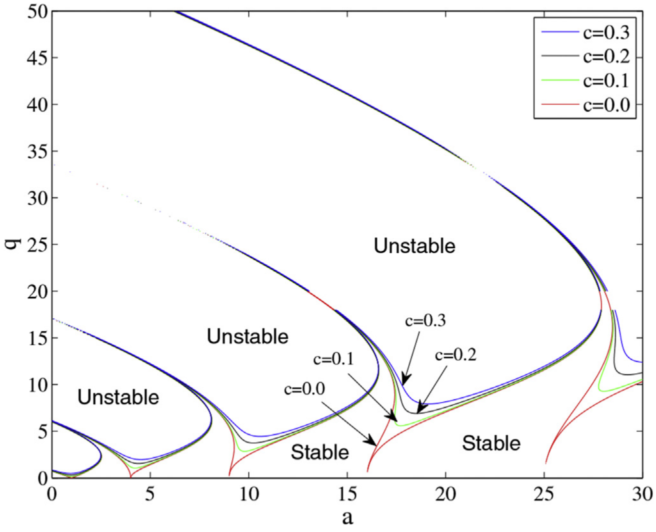


Fig. 5. Stability chart under regular-waves excitation.

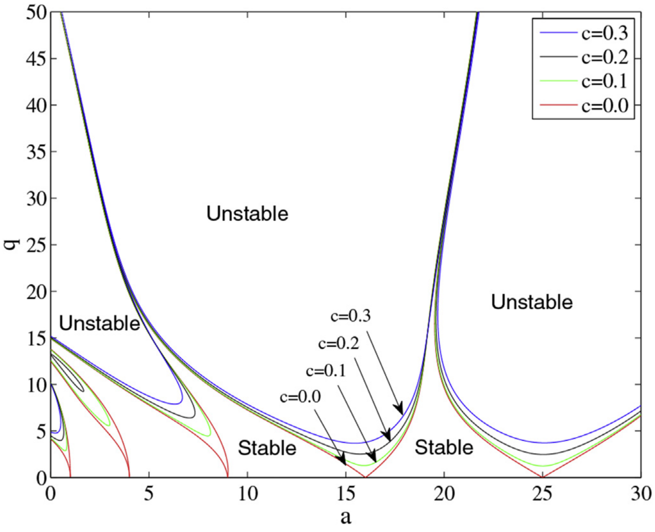


Fig. 6. Stability chart under irregular-waves excitation.

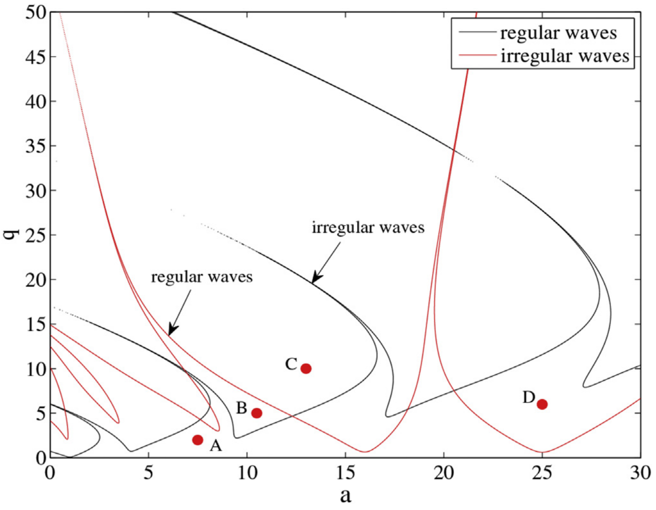


Fig. 7. Stability chart under regular-waves and irregular-waves excitation.

$$\xi_i = H(\omega_i)\cdot\eta_i = H(\omega_i)\cdot\sqrt{2S(\omega_i)\cdot\Delta\omega} \tag{6}$$

2.2. Mathieu equation and Hill equation derivation

Considering the couple of heave and pitch motion, the pitch motion equation of Deep Draft Semi-submersible platform (DDS) can be defined as follows,

$$(I_{55}+A_{55})\ddot{\xi}_5(t)+C_{55}\dot{\xi}_5(t)+\rho g\nabla\left(\overline{GM}-a\xi_3(t)\right)\xi_5(t)=0 \tag{7}$$

where, I_{55} is the pitch moment of inertia, A_{55} is the added pitch moment of inertia, C_{55} is linear damping, ρ is the density of sea water, ∇ is the displaced volume, $\overline{GM}$ is initial value of metacentric height, $\xi_5(t)$and $\xi_3(t)$ are pitch and heave motion, respectively, a is change of metacentric height caused by unit heave motion.

In regular waves, heave motion can be simplified as harmonic motion, namely, $\xi_3(t)=\xi_3\cos\omega_3 t$, where ξ_3 and ω_3 are amplitude and frequency of heave motion, respectively.

In irregular waves, the expression of heave motion has been given by Eqs. (5) and (6).

By replacing parameters, pitch motion equation of DDS in regular waves can be rewritten as a form of Mathieu equation as shown in Eq. (8),

$$\ddot{\xi}_5(\tau)+c\dot{\xi}_5(\tau)+(\alpha+qcos\tau)\xi_5(\tau)=0 \tag{8}$$

Table 2
Properties of design point A, B, C and D.

Design point	Behavior (regular waves)	Behavior (irregular waves)
A	Stable	Stable
B	Unstable	Stable
C	Unstable	Unstable
D	Stable	Unstable

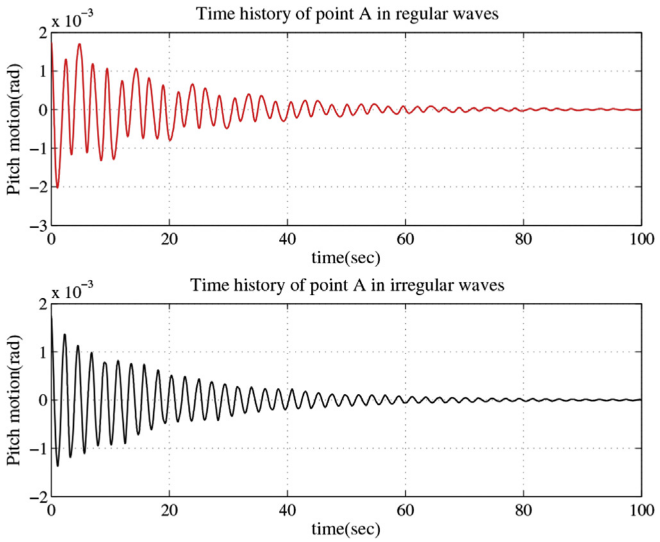


Fig. 8. Time history of point A in regular waves and irregular waves.

The substitutions are as follows,

$$\tau=\omega_3 t;\quad c=\frac{C_{55}}{(I_{55}+A_{55})\omega_3};\quad \alpha=\frac{\rho g\nabla\overline{GM}}{(I_{55}+A_{55})\omega_3^2}=\frac{\omega_5^2}{\omega_3^2};$$

$$q=-\frac{\rho g\nabla\alpha\xi_3}{(I_{55}+A_{55})\omega_3^2};$$

where ω_5 is pitch natural frequency and ω_3 is heave motion frequency.

Through parameters change, pitch motion equation of DDS in irregular waves can be rewritten as a form of Hill equation as shown in Eq. (9),

$$\ddot{\xi}_5(\tau)+2c\dot{\xi}_5(\tau)+(\alpha+2q\phi(\tau))\xi_5(\tau)=0 \tag{9}$$

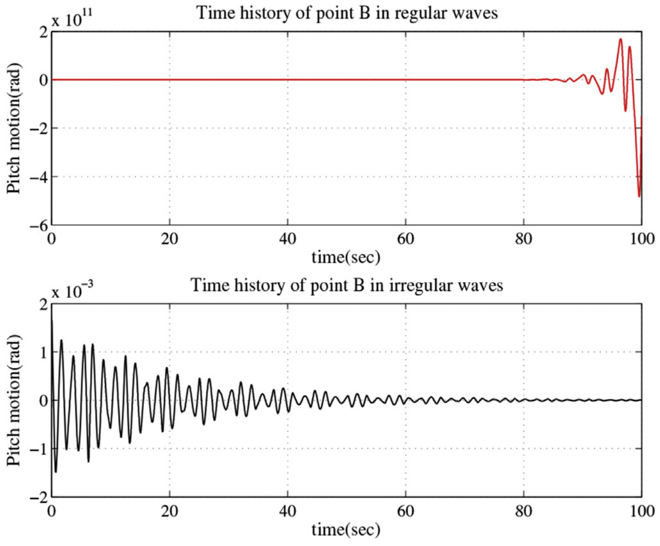


Fig. 9. Time history of point B in regular waves and irregular waves.

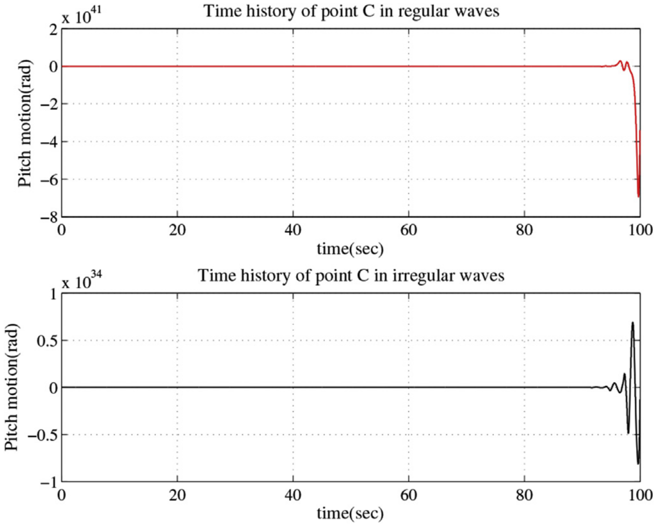


Fig. 10. Time history of point C in regular waves and irregular waves.

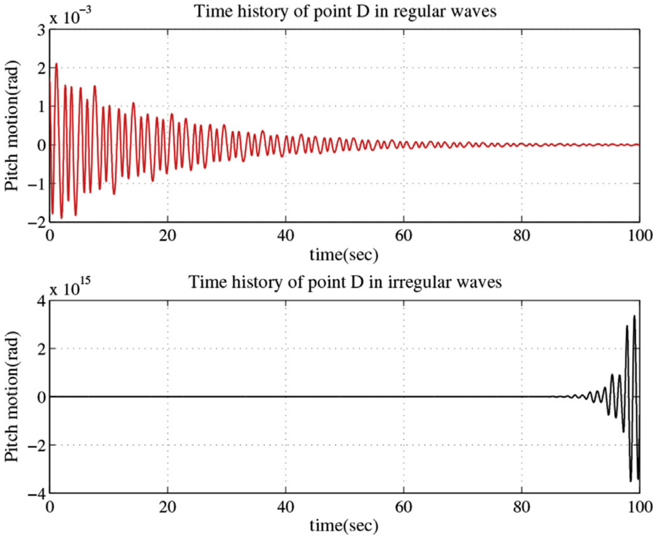


Fig. 11. Time history of point D in regular waves and irregular waves.

The substitutions are as follows,

$$\tau=\frac{\omega_0}{2}t;\quad c=\frac{C_{55}}{(I_{55}+A_{55})\omega_0};\quad q=\frac{2\rho g\nabla\alpha}{(I_{55}+A_{55})\omega_0^2};$$

$$\alpha=\frac{4\rho g\nabla\overline{GM}}{(I_{55}+A_{55})\omega_0^2}=\left(\frac{2\omega_5}{\omega_3}\right)^2;$$

$$\phi(\tau)=-\xi_3(\tau)=-\sum_{i=1}^{n}\xi_i\cos(\omega_i t+\varepsilon_i);$$

3. Stability charts for DDS parametric pitch in regular waves and irregular waves

In this paper, a Deep Draft Semi-submersible which was put forward by Halkyard in 2002 (Halkyard et al., 2002) is chosen to investigate parametric pitch. The configuration of Deep Draft Semi-submersible is shown in Fig. 3 and the dimensions are displayed in Table 1.

3.1. Comparison of stability charts under regular-waves and irregular-waves excitation

The stability chart is used to analyze the properties of parametric instability. It includes transition curves below which are stable regions and above which are unstable regions. By observing which region the parameters of offshore structure fall into, it's convenient to estimate if the offshore structure is stable or not.

By applying the approach of Hill infinite determinant to solve Mathieu equation, the stability chart under regular-waves excitation can be easily obtained. By using the Bubnov-Galerkin approach, Hill equation can be effectively solved, then the corresponding stability chart under irregular-waves excitation can be gained. Fig. 4 provides the process of drawing stability charts in regular waves and irregular waves. Figs. 5 and 6 show the stability charts under regular-waves and irregular-waves excitation, respectively.

It can be easily observed from Figs. 5 and 6 that as damping coefficient increases, transition curves move upward and unstable regions become smaller. When damping coefficient is zero, unstable region becomes most. Therefore, it's effective to suppress parametric instability by increasing damping.

Figs. 5 and 6 also show several differences. As Fig. 6 shows, the unstable regions under irregular-waves excitation distribute much wider and more randomly than that under regular-waves excitation. The stable region under regular-waves excitation may be unstable under irregular-waves excitation, also waves are irregular in real sea condition, so it's of great importance to analyze parametric pitch of DDS in irregular waves.

3.2. Dynamic response analysis in time domain from stability chart

To be more obvious, the stability charts under regular-waves and irregular-waves excitation are drawn in one diagram, as shown in Fig. 7. The differences between regular-waves and irregular-waves stability charts are more apparent from the diagram. The first three unstable areas are much larger for regular-waves regions than that for irregular-waves regions, so it's conservative to predict parametric pitch from regular-waves excitation. For higher-order zones, many stable regular-waves zones fall into unstable irregular-waves zones, so it's dangerous to estimate parametric pitch in regular waves. Through the above analysis, it's of great necessity to predict parametric pitch of DDS in irregular waves.

In order to examine the accuracy of stability chart, four representative design points are chosen to analyze their time history. Table 2 shows the properties of point A, B, C and D.

Fig. 8 shows time history of point A under regular-waves excitation and irregular-waves excitation. In both conditions, the amplitude of pitch motion decreases to a small value within 100 s, so the system is stable. Fig. 9 shows time history of point B under regular-waves excitation and irregular-waves excitation. The amplitude of pitch motion in

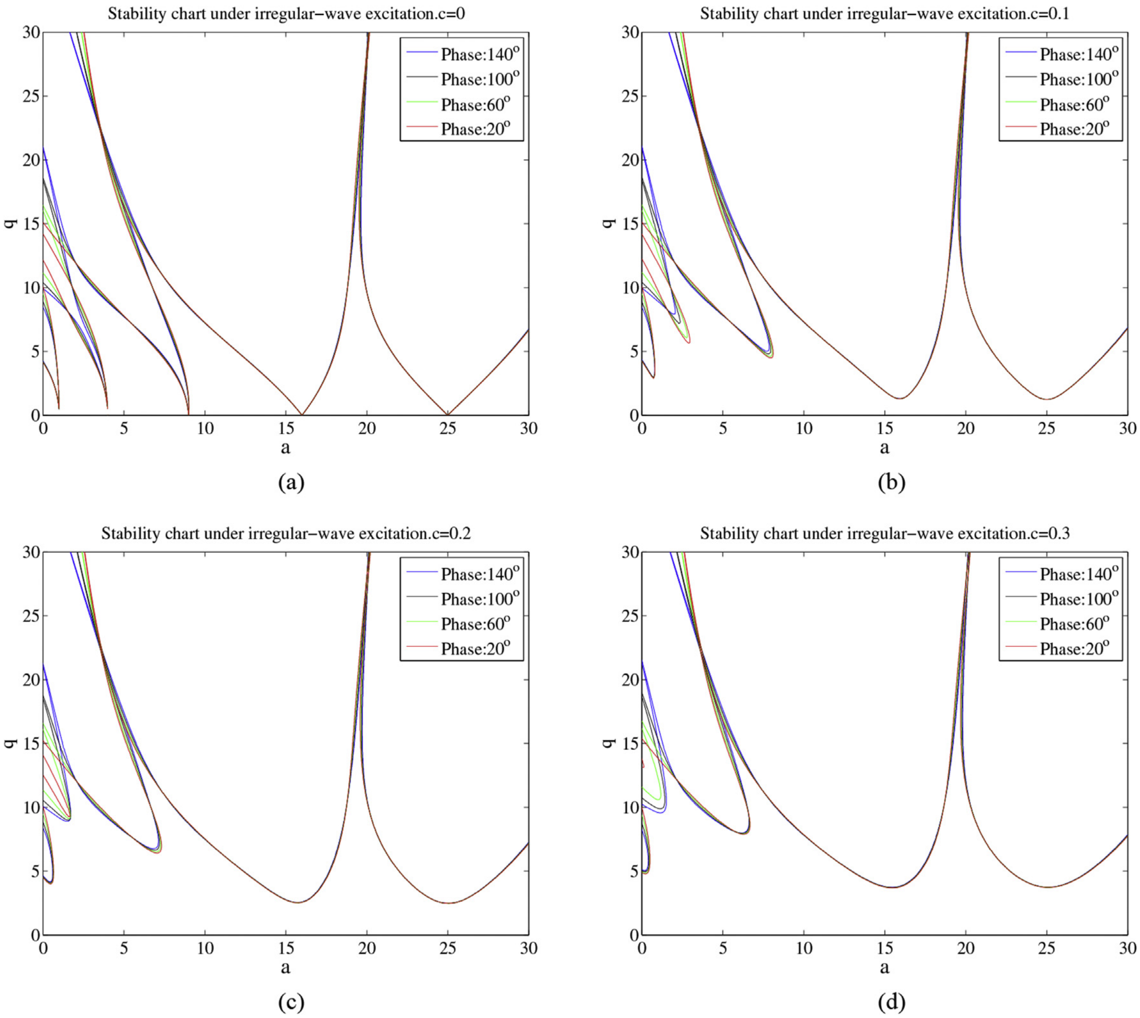


Fig. 12. Stability chart with different wave phases in irregular waves.

regular waves increases to very large within 100 s, but declines to a small value in irregular waves during the same period Therefore, pitch motion of DDS is unstable in regular waves and stable in irregular waves. Fig. 10 provides time history of point C in regular waves and irregular waves. It can be easily concluded that pitch motion of DDS is unstable in both conditions. Time history of point D in regular waves and irregular waves is plotted in Fig. 11. The amplitude of pitch motion decreases to a small value in regular waves but becomes very large in irregular waves within 100 s. Thus, pitch motion of DDS is stable in regular waves and unstable in irregular waves.

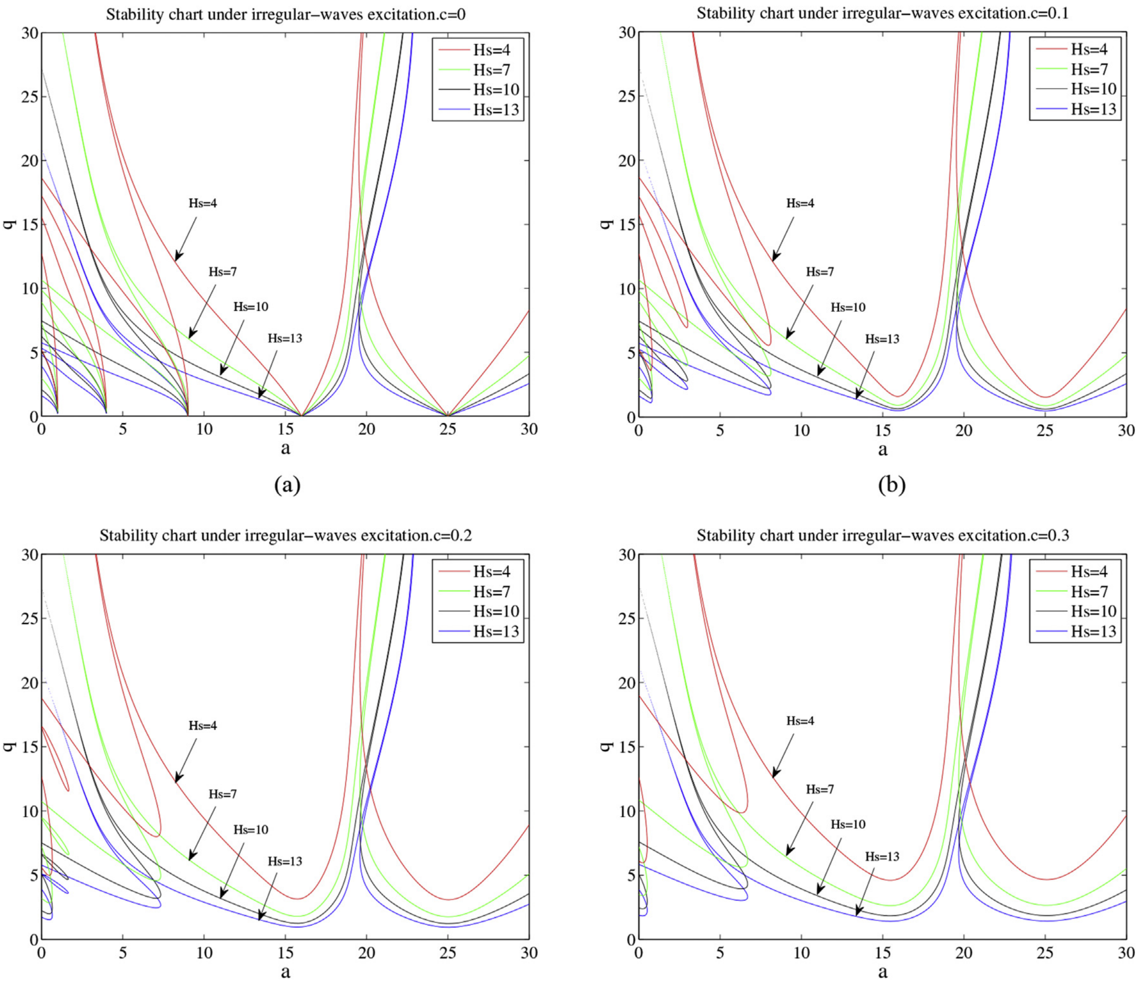
The stability properties of all points conform to the stable region they belong to in Fig. 7. Stability chart drawn through analytical method agrees well with the time history obtained by numerical simulation, so the stability chart can be used to predict parametric instability in initial design. Special attention should pay to point D, which is stable predicted in regular waves but unstable predicted in irregular waves. It's risky to predict parametric pitch in regular waves when the real sea conditions are irregular waves.

4. Sensitivity of wave parameters on DDS parametric pitch in irregular waves

Irregular waves can be described by linear-wave theory and wave spectrum. According to Eq. (1) and Eq. (2), irregular waves contain three important wave parameters, namely, wave phase, significant wave height and wave peak period (peak wave frequency). It's important and worthy to investigate the sensitivity of the three wave parameters on DDS parametric pitch in irregular waves.

4.1. Sensitivity of wave phase on parametric pitch in irregular waves

Wave phase will influence the expression of irregular waves and DDS heave motion, thus affecting DDS parametric pitch. Four different wave phases are chosen to analyze their sensitivities on parametric pitch. Stability charts for different wave phases are drawn under four different damping coefficients which vary from 0 to 0.3, in 0.1 intervals. Significant wave height and wave peak


Stability chart under irregular–waves excitation.c=0

Stability chart under irregular–waves excitation.c=0.1

Stability chart under irregular–waves excitation.c=0.2

Stability chart under irregular–waves excitation.c=0.3

(a) (b)

(c) (d)

Fig. 13. Stability chart with different significant wave heights in irregular waves.

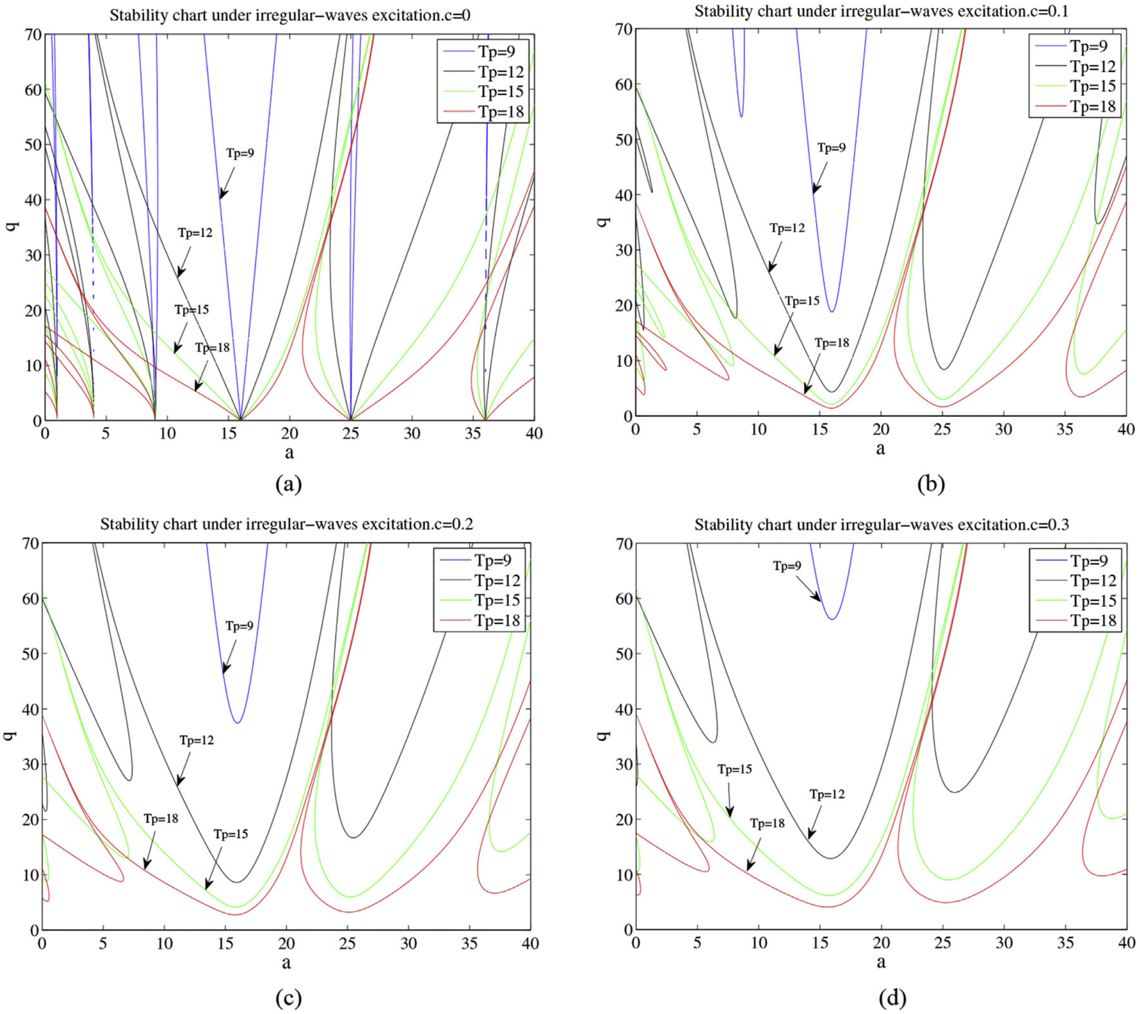
period remain the same while wave phase varies. Fig. 12 shows stability chart for different wave phases under different damping coefficients. For different damping coefficients, stability charts show different properties, but they also reveal several similarities:

(1) For all damping coefficients, the first three unstable regions are sensitive to wave phase, especially for the second unstable region. But for higher-order unstable regions, the influence of wave phase is slight.
(2) As wave phase increases, unstable region becomes smaller. Therefore, lager wave phase will result in smaller unstable region.

4.2. Sensitivity of significant wave height on parametric pitch in irregular waves

Significant wave height will affect wave spectrum, and wave spectrum will influence the expression of irregular waves, thus affecting DDS heave motion. Therefore, significant wave height will have an effect on DDS parametric pitch. Four different significant wave heights are chosen to investigate their sensitivities on parametric pitch. For each damping coefficient, stability charts for different significant wave heights are drawn. Wave phase and wave peak period remain constant while significant wave height changes. Stability charts for different significant wave heights under different damping coefficients are plotted in Fig. 13. From Fig. 13, the following conclusions can be made:

(1) As significant wave height increases, unstable region becomes larger. The reason is that larger significant wave height means larger heave motion, hence, DDS parametric pitch is more likely to happen. Therefore, larger significant wave height will lead to larger unstable region. It is suggested that DDS should avoid working under severe sea conditions.
(2) When damping coefficient is zero, transition curves touch the horizontal axis, and transition curves for different significant wave heights meet at the same point, as illustrated in Fig. 13(a). With damping increasing, unstable region becomes smaller.



(a) (b)

(c) (d)

Fig. 14. Stability chart with different wave peak periods in irregular waves.

4.3. Sensitivity of wave peak period on parametric pitch in irregular waves

Wave peak period will influence the distribution of wave spectrum, thus affecting DDS heave motion so as to influence DDS parametric pitch. Four different wave peak periods are chosen to investigate their sensitivities on parametric pitch. Wave phase and significant wave height maintain constant while wave peak period varies. Fig. 14 provides stability charts for different wave peak periods under different damping coefficients.

It is clearly seen from the figure that unstable region gets smaller when wave peak period decreases. When damping coefficient is zero, the first two unstable regions for wave peak period 9 s can be negligible. As damping coefficient increases, lower-order unstable regions disappear gradually. The reason is that if wave peak period is small, it will be far away from heave motion natural period so that heave motion is moderate. Thus, parametric pitch is unlikely to occur. As wave peak period increases and if it is close to heave motion natural period, excessive heave motion will happen, so parametric pitch occurs. Because wave peak period changes within a certain range in realistic sea condition, so it is recommended that heave motion natural period of DDS should be designed to be out of the range of wave peak period so as to avoid parametric pitch.

4.4. Discussion of parametric pitch suppression for DDS design

Parametric pitch of DDS is influenced by various parameters. It is important and necessary to discuss how to control DDS parametric pitch in engineering. Based on the above discussion, the following measures are recommended to avoid DDS parametric pitch in design process:

(1) As is seen from the stability chart, instability regions become smaller when damping increases. Therefore, increasing DDS heave and pitch damping can help avoiding parametric pitch. Damping plates are recommended and different hull forms can also be taken into consideration.

(2) Wave parameters of irregular waves have an great influence on DDS parametric pitch. Large significant wave height will result in large instability regions, so DDS should avoid working in severe sea conditions as far as possible. If heave motion natural period of DDS is close to wave peak period, parametric pitch is more likely to happen. It is suggested that heave motion natural period of DDS should be designed to be away from the range of wave peak period.

(3) When the ratio of heave motion period and pitch motion natural period satisfies 1:2, parametric pitch is very likely to happen. Thus, large pitch motion natural period is preferable. Pitch motion natural frequency is provided by function $\omega_5 = \sqrt{\rho g \nabla \overline{GM}/(I_{55}+A_{55})}$. As can be seen from the above function, small metacentric height will result in small pitch natural frequency, namely, large pitch natural period. It is recommended to reduce metacentric height on the premise of satisfying the criteria.

5. Conclusions

Parametric pitch of Deep Draft Semi-submersible platform is investigated in condition of irregular waves. Hill equation for DDS pitch motion in irregular waves is derived and solved through Bubnov-Galerkin approach. Then the corresponding stability chart is obtained. By analyzing time history of representative points, the accuracy of stability chart is examined. Then the differences of parametric pitch under regular-waves and irregular-waves excitation are compared and the sensitivity of wave parameters on DDS parametric pitch in irregular waves is investigated by drawing stability charts. From the presented work, the following conclusions can be made:

(1) Parametric pitch of DDS will occur not only in regular waves but also in irregular waves. Parametric pitch may occur in irregular waves while it is predicted safe in regular waves. Therefore, it's important and necessary to predict DDS parametric pitch in irregular waves.

(2) Time history of representative points agrees well with the stability chart, so Bubnov-Galerkin approach can successfully analyze the stability of Hill equation. Besides, stability chart is a useful and convenient tool to predict parametric pitch during design process.

(3) Parametric pitch predicted from regular-waves and irregular-waves excitation shows different properties. For low-order zones, the unstable regions for regular-waves are much larger than that for irregular-waves. Thus it's conservative to predict parametric pitch in regular waves. For high-order zones, many stable regular-waves zones overlap with unstable irregular-waves zones. Hence it will be risky to predict parametric pitch in regular waves. Therefore, it's reasonable and necessary to predict parametric pitch in irregular waves.

(4) DDS parametric pitch will be greatly influenced by wave parameters of irregular waves. Wave phase will affect the low-order unstable regions but have little effect on high-order unstable regions. Significant wave height and wave peak period have a great effect on stability chart. By adjusting DDS working condition and choosing appropriate parameters, parametric pitch of DDS can be effectively avoided.

Finally, this study is only a qualitative analysis, so experiments should be conducted in the future research. In addition, analysis of stability charts combing with probability analysis will be investigated.

Acknowledgments

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