A Model to account for the effects of Friction during Explosive Pinch

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1 Introduction

Safety is of paramount importance in the handling, processing and storage of explosives. Mechanical insults resulting from low-speed impact, that crush and pinch an explosive, have been identified as a possible ignition source. However, modelling such an ignition mechanism numerically with hydrocodes proves to offer some considerable challenges. Here we develop a model for the pinching of an explosive cylinder between two flat plates which accounts for the effects of friction at the contact between the plates and the explosive. An *ad hoc* analytical method of the axial pinching of an explosive cylinder by two flat plates moving at constant speed is developed and discussed in [1]. In this formulation it is assumed that as the material is compressed it is in perfect plastic flow under adiabatic conditions. The explosive reaction is modelled using a simple Arrhenius Law. The heating of the explosive due to mechanical heating and self heating due to the reaction are calculated. In the analysis presented there is no treatment of friction at the contact region between the plate and explosive. As a result of this simplification the dissipation calculated is constant throughout the sample. This is contrast with experiments conducted at AWE in which non-uniform heating is observed [2]. Sherwood and Durban [3] investigated the squeezing of a non-reactive viscoplastic solid in the presence of friction. It is suggested that their paper may form a strong basis to explore frictional effects in the configuration posed in [1]. Here we adopt the approach taken in [3] to describe the mechanical behaviour of an explosive sample subject to axial compression, and then introduce a simple Arrhenius Law, as in [1], to model the reaction. The work presented allows us to investigate the effects of frictional heating during compression and arrive at an improved model of the so called Pinch Test [1].

2 Mathematical Model

We study the Pinch Test, depicted in Figure 1. A cylinder of explosive material with radius $R(z,t)$ is placed between two parallel plates. The plates at $z = \pm h/2$, approach one another with constant velocities $\mp V$.

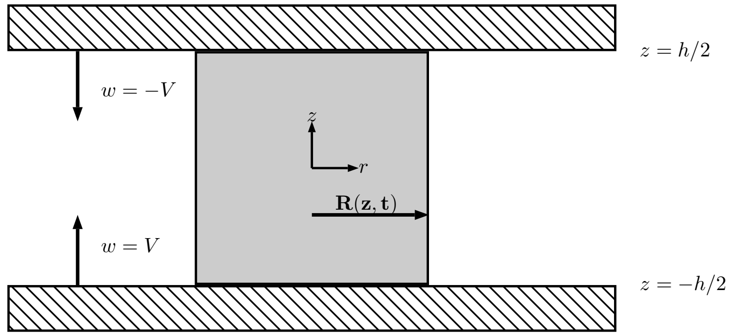

Figure 1: Configuration for the axial pinching of an explosive cylinder between two parallel plates.

The axis of the cylinder coincides with the the z-axis and the full height of the sample at time t is $h(t)$. The radial and axial velocity components are $u(r, z, t)$ and $w(r, z, t)$ respectively. We consider the explosive sample to behave as a rigid plastic, and impose a boundary condition on the shear stress at the plates which models the effects of friction [3]. In practice the sample height is typically much smaller than the sample radius, which provides a small parameter suitable for asymptotic analysis. A solution is obtained in the form of an expansion in h/r, where r is the radial coordinate.

We assume that the rigid plastic solid has yield stress in shear $k = 3^{-\frac{1}{2}}Y$, where Y is the compressive yield strength of the material. It is further assumed that the shear stress σ_{rz} at the plates is a fixed fraction m of this yield stress, i.e. $\sigma_{rz} = \mp mk$ on $z = \pm h/2$. The analysis is similar to Prandtl's cycloid solution for the plane two-dimensional compression of a rigid-plastic block between rough plates (see [4] p. 232). We impose the same friction coefficient m at both plates, so that the problem is symmetric about the centre plane $z = 0$. However, it is noted that the analysis can be extended in a straightforward manner to plates with differing friction coefficients. It is assumed that radial symmetry is maintained throughout.

3 Model Equations

Note that in the following we work in non-dimensional variables. The scalings for lengths, velocity components and stress components are the initial sample height h_0, plate velocity V, and compressive yield stress Y, respectively. A typical time scale for the problem is given by h_0/V. The temperature is scaled by a typical temperature difference ΔT.

In the absence of body forces, the non-dimensional equations for axially symmetric equilibrium read

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \frac{\rho V^2}{Y}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right), \tag{1}$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \frac{\rho V^2}{Y}\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right), \tag{2}$$

where $(\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz})$ are the normal components of the stress tensor, σ_{rz} is the only active shear stress, ρ is the material density and u and w are the radial and axial velocity components. Note that (1) implies that $\sigma_{rr} = \sigma_{\theta\theta}$ on the axis $r = 0$. For the materials studied herein it is found that the ratio of inertial forces to yield stress is typically small for low-speed compression ($V < 50\ \mathrm{m\,s^{-1}}$). For example, for a sample of PETN with impact speed $V = 50\ \mathrm{m\,s^{-1}}$ we find $\rho V^2/Y \sim O(10^{-2})$.

The strain rate ε_{ij} is related to the deviatoric stress $s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk}$ by the coaxiality relation

$$\varepsilon_{ij} = \lambda s_{ij}, \tag{3}$$

and the von-Mises yield criterion for plastic flow holds

$$(\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{\theta\theta} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{rr})^2 + 6\sigma_{rz}^2 = 2. \tag{4}$$

The coaxiality relation (3), along with the yield criterion (4), gives

$$\lambda = \sqrt{\frac{3}{2}}(\varepsilon_{ij}\varepsilon_{ij})^{1/2}, \tag{5}$$

meaning that the factor λ will be a local function of the flow field between the plates.

To account for the effects of friction at the upper and lower plates we prescribe the shear stress to be a fraction of the yield stress, i.e.

$$\sigma_{rz} = \mp 3^{-1/2} m \quad \text{on} \quad z = \pm\frac{h}{2}, \tag{6}$$

where the friction factor m varies from $m = 0$ (perfectly smooth) to $m = 1$ (perfectly rough). Analyses of plastic deformation often use a boundary condition of this form, and the advantages of employing the friction factor m in plastic and viscoplastic flow problems have been demonstrated in several articles [5, 6]. It can been seen from (4) that m cannot be greater than 1.

As in [1], it is assumed that the heating of the explosive by mechanical dissipation and by self-heating as a result of chemical reaction takes place under adiabatic conditions. The initial temperature T_0 of the explosive is specified and the temperature $T(r, z, t)$ increases as a result of the heating from the reaction and by mechanical dissipation. The chemical reaction is modelled as a single step Arrhenius reaction, expressed in terms of the (dimensionless) mass fraction α of gaseous products by

$$\frac{\partial\alpha}{\partial t} = A(1-\alpha)\exp\left(-\frac{E}{T}\right), \tag{7}$$

where A is the pre-exponential factor and E is the activation energy. It is well known that multiple reactions, some endothermic, some exothermic are actually proceeding in parallel, but the one-step Arrhenius reaction serves as a preliminary model. The temperature growth is governed by the equation of conservation of energy, that is

$$\frac{\mathrm{D}T}{\mathrm{D}t} = \frac{V^2/(c_v\,\Delta T)}{\rho V^2/Y}\Phi + \Omega\frac{\partial\alpha}{\partial t}, \tag{8}$$

where c_v is the specific heat at constant volume, Ω is the non-dimensional specific heat of the reaction described by (7) and $\Phi = \varepsilon_{ij}\sigma_{ij}$ is the rate of mechanical dissipation. The effects of diffusion are neglected. This is justified by the small coefficient of the temperature diffusion term, which for the parameter values used in this study is found to be $O(10^{-8})$.

4 Results

In the absence of inertial stresses and friction at the plates, the equilibrium equations are satisfied by a uniform straining motion $(u, w) = (r/h, -2z/h)$, with $\sigma_{zz} = 1$ the only non-zero component of stress.

To satisfy the boundary condition (6) we seek a correction to the uniform straining motion. As in [3], we expand the velocity components as

$$u = -\frac{1}{r}\frac{\partial \Psi}{\partial z} = \frac{r}{h} - hw_1'(z) - w_2'(z)\frac{h^2}{r} + \cdots \,, \tag{9}$$

$$w = \frac{1}{r}\frac{\partial \Psi}{\partial r} = -\frac{2z}{h} + w_1(z)\frac{h}{r} + \cdots \,, \tag{10}$$

where the prime represents the derivative of a function of only one variable, and the corrections $w_i(z)$ are to be determined as part of the solution.

Substitution of the expansions (9) and (10) into the governing equations and application of the appropriate boundary conditions and von-Mises yield criterion gives radial and axial velocity components

$$u = \frac{r}{h} + \frac{3^{1/2}}{m}\left(1 - 4m^2z^2/h^2\right)^{1/2} - \frac{3^{1/2}}{2m}[(1-m^2)^{1/2} + m^{-1}\sin^{-1} m] + O(h/r), \tag{11}$$

$$\begin{aligned} w = &-\frac{2z}{h} - \frac{3^{1/2}h}{4mr}\left[m^{-1}\sin^{-1}(2mz/h) + \frac{2z}{h}(1 - 4m^2z^2/h^2)^{1/2}\right] \\ &+ \frac{3^{1/2}z}{2mr}[(1-m^2)^{1/2} + m^{-1}\sin^{-1} m] + O((h/r)^2). \end{aligned} \tag{12}$$

This allows for computation of the mechanical dissipation in the sample:

$$\Phi = (1 - 4m^2z^2/h^2)^{1/2}\left(\frac{2}{h} - w_1'\frac{h}{r}\right) + \frac{8}{3^{1/2}}\frac{m^2z^2}{h^2}\left[\frac{3}{h^2} + \frac{(hw_1'')^2}{4}\right]^{1/2}, \tag{13}$$

where $w_1(z)$ is known, but omitted here for brevity. We observe that for $m = 0$ (free-slip boundary) the mechanical dissipation is uniform throughout the sample. However, for non-zero m the mechanical heating is spatially dependent. Figure 2 shows the first order correction to the radial velocity component, and the mechanical dissipation in the sample, both as a function of z for several values of m. We see that as m increases, i.e. as we approach a no-slip condition at the plates, the heating due to mechanical dissipation increases near the boundary. The prediction of increased heating near boundaries is consistent with drop weight experiments, and with numerical numerical simulations of pinch [7].

The asymptotic results for the mechanics of the problem are substituted into the energy equation (8), which is then integrated in time using a Crank-Nicolson scheme. Figure 3 shows the rise of the maximum temperature of (a) HMX and (b) PETN with time for varying values of friction factor: $m = 0$; $m = 0.1$; $m = 0.5$; and $m = 0.9$. It is clear by inspection to determine the time at which thermal runaway commences. We observe that an increase in friction factor m (i.e. more friction between the plates and the explosive sample) causes thermal runaway to commence at an earlier time. This is due to the spatially dependent mechanical dissipation (13), which causes the temperature in localised regions to increase at a faster rate than the bulk temperature in the sample.

The relative sensitivities of the two materials studied is well captured by the model. For PETN we find that runaway commences at around 390 microseconds with $m = 0.1$ compared with around 220 microseconds with $m = 0.9$. Clearly we see a remarkable decrease in the time to runaway for PETN when m is large. This is contrasted with HMX, where we find that thermal runaway has not yet fully commenced when the sample becomes fully pinched at $t = 509$ microseconds, which is consistent with experimental results [1].

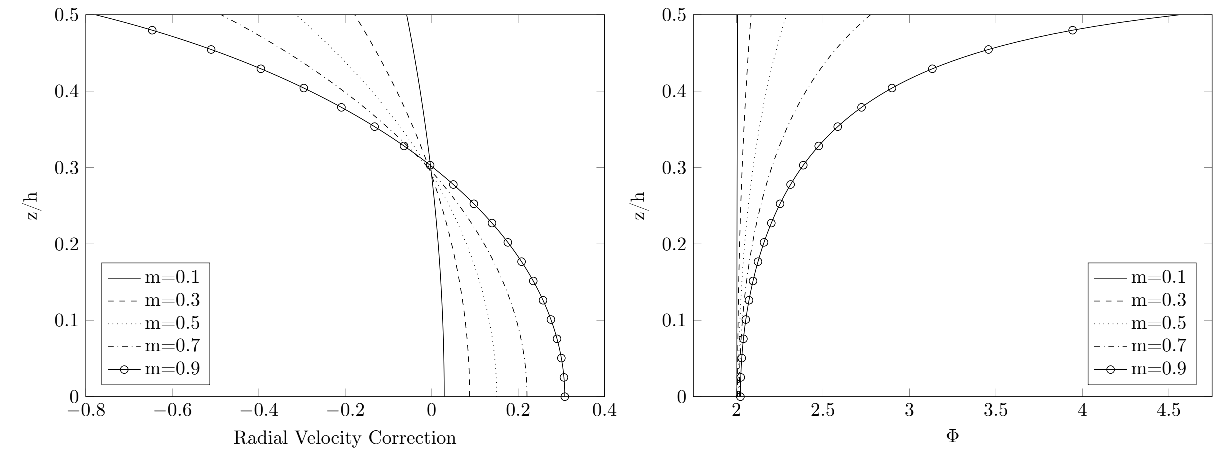

Figure 2: Left: Non-dimensional correction to the radial component of velocity for several values of m. Note that $m = 0$ gives zero correction, i.e. radial velocity independent of z. Right: Non-dimensional mechanical dissipation rate as a function of z for several values of m. For $m = 0$ the dissipation in uniform throughout the sample.

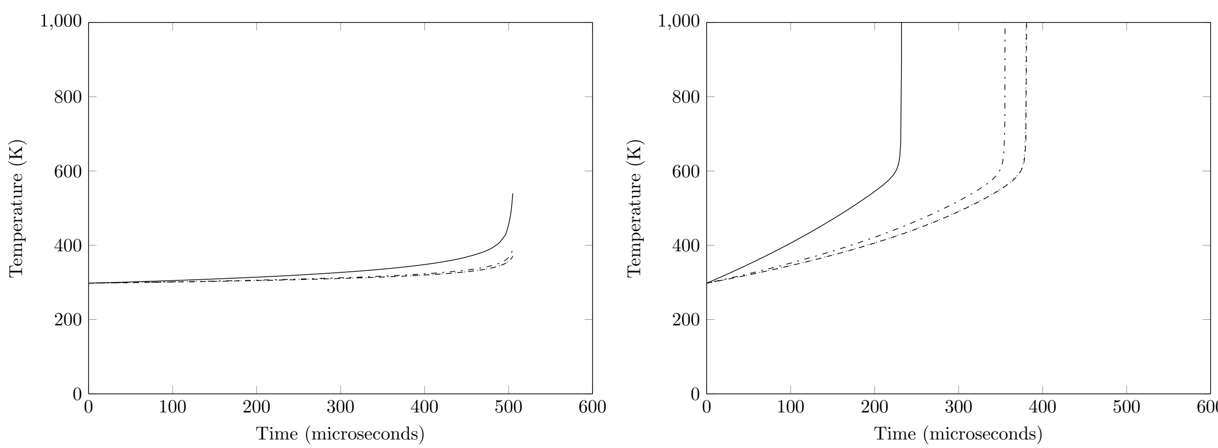

Figure 3: Rise of the maximum temperature of the sample of HMX (left) and PETN (right) during the Pinch Test with an impact speed $V = 50\ \mathrm{ms}^{-1}$ and an initial temperature $T_0 = 298$K. Here we have used the values $m = 0$ (- -); $m = 0.1$ (..); $m = 0.5$ (-.); $m = 0.9$ (–). The curves for $m = 0$ and $m = 0.1$ virtually coincide.

5 Conclusion

An extension to the *ad hoc* model developed by Curtis [1] has been presented which allows for the inclusion of friction at the walls. This permits a solution in the form of an expansion in the ratio of the sample height-to-radius, h/r. The results presented indicate that friction at the plates induces shear, which in turn gives rise to higher temperatures and ultimately leads to thermal runway commencing more quickly after the initial insult. The model predicts localised heating at the contact region between the plates and explosive sample, which is consistent with findings in experiments conducted at AWE [2]. It should be emphasised that the additional heating discussed here is due to changes to the deformation which result from the partial slip boundary condition, and that further heating due to friction at the plates is not included.

The application of asymptotic methods to solve for the velocity components, and thus mechanical dissipation, is novel to the explosives safety problem considered here, and demonstrates that such methods may be of use in other configurations. For given material properties and experimental parameters the model presented may be used to calculate the spatial dependence of heating due to mechanical dissipation, and make qualitative predictions about key outcomes such as time to runaway. Current work is continuing to exploit the simple geometry of the Pinch Test to study other hot spot mechanisms. For example, one could consider the effect of allowing small variations in the initial temperature or in various material properties. This may give rise to localised hot spots within the sample and cause thermal runaway to commence before the bulk temperature is high enough for reaction. Such a mechanism may be particularly important in the case where the constitutive behaviour of the material is greatly influenced by temperature.

Acknowledgments

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