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Robust Cooperative Guidance Law for Simultaneous Arrival

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***Abstract*—In the cooperative simultaneous arrival problem, a group of interceptors are guided to simultaneously engage a stationary target. However, some interceptors may not follow the prescribed guidance law during the engagement, which can lead to interception failures. This brief investigates a new robust cooperative simultaneous arrival problem in the presence of misbehaving interceptors. A robust cooperative guidance law (RCGL) integrated with a local filtering algorithm is designed. Without the knowledge of faulty interceptors (no fault diagnosis procedure is needed), the RCGL achieves a simultaneous arrival between normal interceptors if the misbehavior of faulty interceptors can be characterized by a threat model. By characterizing the contracting behavior of the maximum gap between impact time estimates of normal interceptors, sufficient conditions are established to guarantee the convergence of RCGL. Furthermore, regardless the network connections, the impact times of normal interceptors are upper bounded by the maximum initial time-to-go estimate of normal interceptors. Numerical comparison studies demonstrate the guidance performance of RCGL.**

***Index Terms*—Robust cooperative guidance law; cooperative control; simultaneous arrival.**

I. Introduction

The simultaneous arrival problem of multiple interceptors has become more interesting over the past few years (see, e.g., [1]–[8]). In general, the problem can be solved by two methods: 1) individual homing, e.g., [1]–[3]; 2) cooperative homing, e.g., [4]–[8]. Compared to the individual homing, the cooperative homing requires no predetermination of a common impact time. The group of interceptors synchronize the impact time by addressing the consensus problem of time-to-go estimates of interceptors.

The advancement of defense systems poses new challenges in homing guidance. It is important to increase the reliability of the cooperative guidance, especially, when some interceptors are destroyed or disturbed by the defense system of the target. However, studies on the robust simultaneous arrival problem in the presence of misbehaving interceptors are rare. The authors in [6] and [7] proposed two finite-time cooperative guidance laws (FTCGLs) based on classic graph theory and discussed the guidance performances of FTCGLs under communication faults and actuator faults. But the faulty interceptor must remain controllable and can not be the root of the communication structure. Since the defense systems of the target may destroy or disturb the interceptor and self-faults may happen during the engagement, the controllability of faulty interceptors is hard to preserve. It remains an open problem to design a robust guidance law such that 1) all normal interceptors can reach the target without identifying faulty interceptors; 2) all normal interceptors reach the target at the same time as much as possible.

In this brief, we consider a new robust cooperative simultaneous arrival problem when some interceptors may not follow the prescribed guidance law during the engagement. The unknown dynamics caused by faulty interceptors make the cooperative guidance design for the normal interceptors difficult. Inspired by the time-to-go approximate model in [4] and the notion of network robustness [9]–[11], we integrate a local filtering algorithm with other cooperative guidance law and present a useful robust cooperative guidance law (RCGL). If the misbehavior of faulty interceptors can be characterized by a threat model (each faulty interceptor sends the same value to all of its out-neighbors at each time-step), the RCGL can reduce the variance of impact times between normal interceptors without identifying faulty interceptors. Regardless the network connections, the impact times of normal interceptors are upper bounded by the maximum initial time-to-go estimate of normal interceptors, which can be seen as a safety condition. By discarding some extreme time-to-go estimates of in-neighbors at each time-step, the integrated local filtering algorithm of RCGL filters undesirable dynamics caused by faulty interceptors. Sufficient conditions are established to guarantee the consensus of time-to-go estimates of normal interceptors. The convergence analysis of RCGL is based on characterizing the contracting behavior of the maximum gap between impact time estimates of normal interceptors. Numerical comparison results demonstrate the effective guidance performance of RCGL.

The remainder of this brief is organized as follows. Section II formulates the robust simultaneous arrival problem with a single target and introduces the preliminaries. The main results of RCGL are given in Sections III and IV. In Section V, comparison simulation results of 5 to 1 robust engagement scenario are presented.

II. Problem Statement

Consider the scenario that a group of N interceptors attack a stationary target on a two-dimensional plane by assuming that the lateral and longitudinal planes are decoupled by means of roll control [12]. The planar engagement geometry is shown in Fig. 1.

In Fig. 1, for the ith interceptor, r_i is the rang-to-go; λ_i is the LOS angle; γ_i is the flight-path angle; σ_i is the

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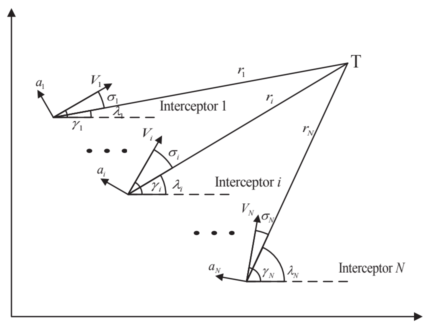

Fig. 1. Guidance geometry on N to 1 engagement scenario.

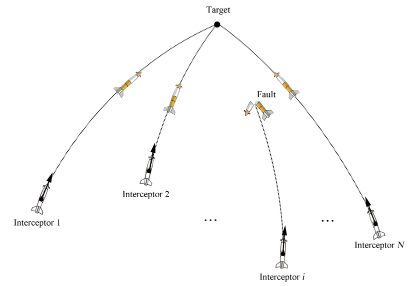

Fig. 2. N to 1 robust simultaneous arrival scenario.

heading error; V_i is the interceptor speed, which is assumed to be constant during the engagement; a_i is the acceleration, which is perpendicular to V_i. The planar interceptor-target engagement kinematics are given as

$$\begin{aligned}\dot{r}_i &= -V_i\cos(\sigma_i),\\ \dot{\lambda}_i &= -\frac{V_i\sin(\sigma_i)}{r_i},\\ \dot{\gamma}_i &= \frac{a_i}{V_i},\\ \sigma_i &= \gamma_i - \lambda_i, \quad i = 1, \cdots, N.\end{aligned} \tag{1}$$

where a_i is the control input for ith interceptor.

Suppose each interceptor uses well-known proportional navigation (PN) for homing as

$$a_i = N_s V_i \dot{\lambda}_i,$$

where N_s denotes the fixed navigation constant (in practice, N_s is usually chosen as $3 \le N_s \le 5$). When the heading error σ_i is small, the time-to-go of the ith interceptor can be approximated as [4]

$$\hat{t}_{go,i} = \frac{r_i}{V_i}\left(\frac{\sigma_i^2}{2N_s - 1} + 1\right), \quad i = 1, \cdots, N. \tag{2}$$

Note that σ_i is small in general cases.

Now consider a robust simultaneous arrival problem with N interceptors shown in Fig.2. The N interceptors are partitioned into a set of *normal interceptors* $\mathcal{N} = \{i \in 1, \cdots, N : i\text{th interceptors is normal}\}$, and a set of *faulty interceptors* $\mathcal{F} = \{i \in 1, \cdots, N : i\text{th interceptor is misbehaving}\}$, the number of faulty interceptor is upper bounded by F. The communications between interceptors happen at times $t_0, t_1, \cdots, t_k, \cdots$, and the communication period is τ, i.e., $t_k - t_{k-1} = \tau$.

The communication topology among interceptors is described by the directed graph $\mathcal{G}$. The set of interceptors is defined as $\mathcal{V} = \{1, \cdots, N\}$. The adjacency matrix is defined as $\mathcal{A} = [\alpha_{ij}] \in \mathbb{R}^{N\times N}$, where $\alpha_{ii} = 0$ and $\alpha_{ij} = 1$ if the ith interceptor can get the information from the jth interceptor, otherwise $\alpha_{ij} = 0$. The jth interceptor is called an *in-neighbor* of ith interceptor if $\alpha_{ij} = 1$. The *in-neighbors* of $i$the interceptor are defined as a set $\mathcal{V}_i = \{j \in \mathcal{V} : \alpha_{ij} = 1\}$. The jth interceptor is called an *out-neighbor* of ith interceptor if $\alpha_{ji} = 1$ (i.e., the jth interceptor can get the information from the ith interceptor).

We make following assumptions throughout this brief.

Assumption 1: The speed of each interceptor is constant but may not be the same as that of other interceptors.

Assumption 2: Each faulty interceptor sends the same value to all of its out-neighbors at each time-step (e.g., for a faulty interceptor i, all the out-neighbors of i receive the same value from i at t_k).

Remark 1: In practice, Assumption 2 is nonrestrictive and easy to satisfy. For example, if the communication is realized through wireless broadcast, the faulty interceptor i naturally sends the same value to all of its out-neighbors.

The objective of this brief is to design a PN based RCGL to meet following demands:

1) all normal interceptors can reach the target without the knowledge of fault interceptors.
2) all normal interceptors reach the target at the same time as much as possible.

Remark 2: Although the threat model in Assumption 2 is defined according to the communication behavior, this threat model covers a wide range of faults in practice; not only the communication faults, but also actuator faults which cause undesirable changes in $\hat{t}_{go,i}$ are considered. It is plausible that some simple misbehaviors can be detected via an appropriate mechanism. However, for some complicate faults, especially in the short range guidance, it is hard to detect the faulty interceptors and reorganize the communications between normal interceptors. Moreover, the cooperative guidance performance will be degraded by the increasing time of fault diagnosis.

III. RCGL with a Local Filtering Algorithm

In this section, a novel cooperative guidance law with a local filtering algorithm is designed to solve the simultaneous arrival problem. By virtue of the integrated local filtering algorithm, the proposed cooperative guidance law is robust to the misbehaviors of faulty interceptors. With exchanging the time-to-go estimates $\hat{t}_{go,i}$ at discrete-time, the RCGL is designed as

$$a_i(t) = N_s\Bigg(1 + k_i \sum_{j\in\mathcal{R}_i(t_k)} w_{ij}(t_k)\left(\hat{t}_{go,i}(t_k) - \hat{t}_{go,j}(t_k)\right)\Bigg)V_i\dot{\lambda}_i, \quad \forall t \in [t_k, t_{k+1}), \tag{3}$$

where $i = 1, \cdots, N$, $k_i > 0$ are constants, $\mathcal{R}_i(t_k)$ is the set of retained in-neighbors of ith interceptor at t_k, $\hat{t}_{go,j}(t_k)$ and $w_{ij}(t_k)$ are defined in the local filtering algorithm running at times $\{t_k\}$.

Algorithm 1: Local Filtering for ith interceptor

Input: Fixed navigation constant N_s, upper bound of the number of faulty interceptors F, range-to-go $r_i(t_k)$, interceptor speed V_i, heading error $\sigma_i(t_k)$.

Output: In-neighbors after filtering $\mathcal{R}_i(t_k)$, time-to-go estimates of filtered in-neighbors $\{\hat{t}_{go,j}(t_k), j \in \mathcal{R}_i(t_k)\}$, time-varing comminication weights $w_{ij}(t_k)$.

1 ith interceptor estimates its time-to-go $\hat{t}_{go,i}(t_k) = \frac{r_i(t_k)}{V_i(t_k)}(\frac{\sigma_i(t_k)^2}{2N_s-1} + 1)$;
2 ith interceptor receives time-to-go estimates $\{\hat{t}_{go,j}(t_k), j \in \mathcal{V}_i\}$ from its in-neighbors and sends $\hat{t}_{go,i}(t_k)$ to its out-neighbors;
3 ith interceptor sorts the gathered estimates $\{\hat{t}_{go,j}(t_k), j \in \mathcal{V}_i\}$, and forms a sorted list;
4 **if** *there are less than F estimates larger (resp. smaller) than its own estimate $\hat{t}_{go,i}(t_k)$,* **then**
5 ith interceptor removes all estimates that are larger (resp. smaller) than its own estimate;
6 **else**
7 ith interceptor removes F largest (resp. F smallest) estimates;
8 **end**
9 Define $\mathcal{R}_i(t_k)$ as the set of in-neighbors of ith interceptor whose time-to-go estimate is retained at time-step t_k;
10 The time-varying communication weights are defined as

$$w_{ij}(t_k) = \begin{cases} \dfrac{\alpha_{ij}}{\sum_{j \in \mathcal{R}_i(t_k)} \alpha_{ij}}, & \forall j \in \mathcal{R}_i(t_k), \\ 0, & \text{otherwise.} \end{cases}$$

The RCGL has a simple structure which is composed of a traditional PN feedback loop, a cooperative time-to-go feedback loop and a novel local filtering algorithm. Note that RCGL is a continuous-time guidance law; however, the communications between interceptors are in discrete-time structures. The retained in-neighbors of ith interceptor are switching due to the local filtering algorithm. The RCGL uses the relative time-to-go errors $\sum_{j \in \mathcal{R}_i(t_k)} w_{ij}(t_k)\left(\hat{t}_{go,i}(t_k) - \hat{t}_{go,j}(t_k)\right)$ to adjust the curvature of the interceptors's trajectories; interceptors with smaller time-to-go estimates take detours, and interceptors with larger time-to-go estimates take shortcuts. When $\sum_{j \in \mathcal{R}_i(t_k)} w_{ij}(t_k)\left(\hat{t}_{go,i}(t_k) - \hat{t}_{go,j}(t_k)\right) = 0$ at any t_k, the simultaneous arrival is achieved, and RCGL becomes PN with fixed navigation constant N_s. In Algorithm 1, no additional procedure (e.g., fault detection) is needed, and the information used in the algorithm is the same as that of existing PN based cooperative guidance laws. The data flow structure of ith interceptor in the cooperative guidance is shown in Fig. 3. In the figure, at each time-step, ith interceptor communicates with its neighbors, removes some time-to-go estimates of in-neighbors according to the rules in Algorithm 1 and recalculates the communication weights $w_{ij}(\cdot)$. The controller calculates the acceleration command by using continuously measurements and sampled time-to-go estimates. The rigorous convergence analysis of RCGL is performed in the following section.

IV. Convergence Analysis of RCGL

In this section, sufficient conditions are established to guarantee the convergence of RCGL. Before introducing the main results of this section, an important definition is given as follow

Definition 1 ((r,s)-robust graphs): For two positive integers r and s, a graph $\mathcal{G}$ is said to be (r,s)-robust if for any two disjoint nonempty subsets $\mathcal{S}_1$, $\mathcal{S}_2 \in \mathcal{V}$, at least one of the following holds:

1) Every interceptor in $\mathcal{S}_1$ has at least r in-neighbors outside $\mathcal{S}_1$.
2) Every interceptor in $\mathcal{S}_2$ has at least r in-neighbors outside $\mathcal{S}_2$.
3) There are at least s interceptors in $\mathcal{S}_1 \cup \mathcal{S}_2$ that each interceptor has at least r in-neighbors outside its respective sets.

From lines $4-7$ in Algorithm 1, we know that each interceptor periodically discards some of its in-neighbors. Definition 1 aims to capture the idea that for any two disjoint nonempty subsets, there are some interceptors within those sets that each of them has enough in-neighbors outside its respective sets. This definition plays a key role in our convergence analysis.

Define $\hat{T}_i(t_k) = \hat{t}_{go,i}(t_k) + k\tau$ as the impact time estimates of ith interceptors at time t_k. Note that $\hat{T}_i(t_0) = \hat{t}_{go,i}(t_0)$, and we have $\hat{t}_{go,1}(t_k) = \cdots = \hat{t}_{go,N}(t_k)$ if $\hat{T}_1(t_k) = \cdots = \hat{T}_N(t_k)$. Define $m(t_k) = \min\{\hat{T}_i(t_k), \forall i \in \mathcal{N}\}$ and $M(t_k) = \max\{\hat{T}_i(t_k), \forall i \in \mathcal{N}\}$; $m(t_k)$ and $M(t_k)$ are the lower and upper bounds of the impact time estimates of normal interceptors at t_k, respectively.

Theorem 1: Suppose that there are at most F faulty interceptors within a group of N interceptors. Under Assumptions 1, 2 and RCGL, the impact time estimates of normal interceptors $\hat{T}_i(t_k), \forall i \in \mathcal{N}$ are bounded within $[m(t_0), M(t_0)]$, regardless of the network connections of interceptors and the misbehaviors of faulty interceptors, if $\max\{k_i, \forall i \in \mathcal{V}\} < \frac{2N_s-1}{N_s \pi^2 \tau}$.

Proof: Since σ_i are small angles in general cases [4]; thus, we have $\sin(\sigma_i) = \sigma_i$ and $\cos(\sigma_i) = 1 - \frac{\sigma_i^2}{2}$. Substituting (3) into (1), we have

$$\begin{aligned} \dot{r}_i =& - V_i(1 - \frac{\sigma_i^2}{2}), \\ \dot{\sigma}_i =& - \frac{V_i \sigma_i}{r_i}\Bigg(N_s - 1 + N_s k_i \sum_{j \in \mathcal{R}_i(t_k)} w_{ij}(t_k) \\ & \cdot \left(\hat{t}_{go,i}(t_k) - \hat{t}_{go,j}(t_k)\right)\Bigg), \quad \forall t \in [t_k, t_{k+1}) \end{aligned} \tag{4}$$

where $i = 1, \cdots, N$.

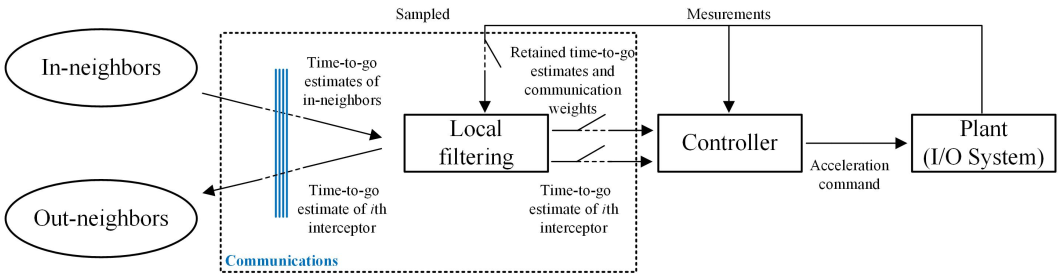

Fig. 3. Data flow structure of ith interceptor under RCGL

We can get the difference equation from the above results and (2)

$$\begin{aligned}\hat{t}_{go,i}(t_{k+1}) =& \hat{t}_{go,i}(t_k) + \int_0^\tau \dot{\hat{t}}_{go,i}(t) \\ =& \int_0^\tau \frac{\dot{r}_i}{V_i}(1+\frac{\sigma_i^2}{2(2N_s-1)}) + \frac{r_i\sigma_i\dot{\sigma}_i}{V_i(2N_s-1)}dt \\ &+ \hat{t}_{go,i}(t_k) \\ =& -K_i \sum_{j\in\mathcal{R}_i(t_k)} w_{ij}(t_k)\left(\hat{t}_{go,i}(t_k) - \hat{t}_{go,j}(t_k)\right) \\ &+ \hat{t}_{go,i}(t_k) - \tau, \end{aligned} \tag{5}$$

where $K_i = \frac{N_s\sigma_i^2 k_i\tau}{2N_s-1}$. Since $\max\{k_i, \forall i\in\mathcal{V}\} < \frac{2N_s-1}{N_s\pi^2\tau}$ and $\sigma_i^2 \le \pi^2$, we have $0 < K_i < 1$, $\forall i = 1,\cdots,N$.

The difference equations of impact time estimates are given as

$$\begin{aligned}\hat{T}_i(t_{k+1}) =& \left(1 - K_i\sum_{j\in\mathcal{R}_i(t_k)} w_{ij}(t_k)\right)\hat{T}_i(t_k) \\ &+ K_i \sum_{j\in\mathcal{R}_i(t_k)} w_{ij}(t_k)\hat{T}_j(t_k). \end{aligned} \tag{6}$$

Note that $\sum_{j\in\mathcal{R}_i(t_k)} w_{ij}(t_k) = 0$ if $\mathcal{R}_i(t_k) = \emptyset$. Otherwise we have that $\sum_{j\in\mathcal{R}_i(t_k)} w_{ij}(t_k) = 1$.

For a normal interceptor $i \in \mathcal{N}$, consider following situations.

1) $\mathcal{R}_i(t_k) = \emptyset$.
2) $\mathcal{R}_i(t_k) \subseteq \mathcal{N}$.
3) $\mathcal{R}_i(t_k) \cap \mathcal{F} \ne \emptyset$, where $\emptyset$ denotes the empty set.

For the first situation, all in-neighbors of ith interceptor are removed. Then the RCGL becomes a PN guidance law and $\hat{T}_i(t_{k+1}) = \hat{T}_i(t_k) \in [m(t_k), M(t_k)]$.

For the second situation, all the remaining interceptors are normal. Since $0 < K_i < 1$, together with (6), we can get that $T_i(t_{k+1})$ is a convex combination of itself and $\hat{T}_j(t_k), \forall j \in \mathcal{R}_i(t_k)$. Then, $\hat{T}_i(t_{k+1}) \in [m(t_k), M(t_k)]$.

For the third situation, there is at least one faulty interceptor in $\mathcal{R}_i(t_k)$. For the reason that ith interceptor removes at most F in-neighbors which have larger (resp. smaller) time-to-go estimates than ith interceptor, and the number of faulty interceptor is upper bounded by F, there must be at least one normal interceptor (can be ith interceptor) in $\mathcal{N}$ that has a larger time-to-go estimate than the time-to-go estimates of all faulty interceptors in $\mathcal{R}_i(t_k) \cap \mathcal{F}$; furthermore, there must be at least one normal interceptor (can be ith interceptor) in $\mathcal{N}$ that has a smaller time-to-go estimate than the time-to-go estimates of all faulty interceptors in $\mathcal{R}_i(t_k)\cap\mathcal{F}$. Then we have $\hat{T}_i(t_{k+1}) \in [m(t_k), M(t_k)]$. Based on the above analysis, we can conclude that $\{M(t_k)\}$ and $\{m(t_k)\}$ are monotone and bounded sequences. Furthermore, $\hat{T}_i(t_{k+1}) \in [m(t_0), M(t_0)]$. ■

Remark 3: Theorem 1 shows that the impact time estimates of normal interceptors are always within $\left[\min\{\hat{t}_{go,i}(t_0), \forall i \in \mathcal{N}\},\ \max\{\hat{t}_{go,i}(t_0), \forall i\in\mathcal{N}\}\right]$. Note that $\hat{t}_{go,i} = 0$ only when $r_i(t) = 0$ (i.e., ith interceptor reaches the target). Together with Theorem 1, we can obtain $\hat{t}_{go,i}(t_k) + k\tau \le \max\{\hat{t}_{go,i}(t_0), \forall i \in \mathcal{N}\}$, which implies that all normal interceptors will reach the target no later than $\max\{\hat{t}_{go,i}(t_0), \forall i \in \mathcal{N}\}$, regardless misbehaviors of faulty interceptors.

Remark 4: When the communication period τ is sufficiently small, any positive constant k_i can guarantee the boundedness of $\hat{T}_i(t_k)$.

Theorem 2: Suppose that there are at most F faulty interceptors within a group of N interceptors and the communication graph $\mathcal{G}$ is $(F+1, F+1)$-robust. Under Assumptions 1 and 2, the difference between the upper and lower bounds of the impact time estimates of normal interceptors, i.e., $M(t_k)-m(t_k)$, monotonously converges to 0 if $\max\{k_i, \forall i\in\mathcal{V}\} < \frac{2N_s-1}{N_s\pi^2\tau}$.

Proof: The proof is stated in Appendix. ■

Corollary 1: Under assumptions in Theorem 2, $\bar{V}(t_k) = M(t_k) - m(t_k)$ exponentially converges to 0 as $k\to\infty$.

Proof: The proof follows similarly as in Theorem 2. ■

Remark 5: $\lim_{k\to\infty} M(t_k) - \lim_{k\to\infty} m(t_k) = 0$ implies that $\lim_{k\to\infty}\hat{t}_{go,i}$, $\forall i \in \mathcal{N}$ will reach an agreement and the simultaneous arrival will be achieved. Since the guidance time is finite in implementations, the difference $M(t_k)-m(t_k)$ will be nonzero. Nevertheless, $M(t_k) - m(t_k)$ is monotonously decreasing during the engagement and can be tuned by the parameters k_i and τ. Normally, under same faulty conditions and initial conditions, if τ is chosen to be sufficiently small, a system with larger gains $N_s k_i$ will have a smaller $M(t)-m(t)$ at the time instant t.

Remark 6: Under Assumption 2, the faulty interceptors are allowed to send any value as a time-to-go estimate to their out-neighbors. If a fault happens during the engagement, we do not assume the faulty interceptors know the time-to-go estimates of other interceptors.

Remark 7: In implementations, F is estimated according to the communication network reliability, the failure rate of interceptors and the robustness of RCGL that we want to

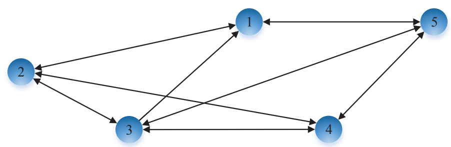

Fig. 4. Communication topology $\mathcal{G}_1$.

achieve. Note that we can get the maximum feasible value of F if the communication network has been set up. Furthermore, there is a tradeoff between the communication load and the robustness of RGCL. In general, the RGCL with larger F will have a higher communication cost. In this brief, we assume F is available. Specific steps for constructing a (r, s)-robust graph are given in [10].

Remark 8: In light of the existing cooperative guidance laws, there are only a few results cover the simultaneous arrival under fault conditions ([6], [7]). These results all based on the assumptions that faulty interceptors can still reach the target (i.e., the faulty interceptor is still controllable) and all the interceptors send correct time-to-go estimates to their out-neighbors. However, when faulty interceptors can not reach the target (e.g., actuator fault happens, $a_i = 0$ at all times) or faulty interceptors keep sending incorrect time-to-go estimates to their out-neighbors, the existing cooperative guidance laws can not guarantee the simultaneous arrival for normal interceptors. In fact, in the worst case, the normal interceptors which influenced by faulty interceptors can not even reach the target. Different from these existing guidance laws, by virtue of the embedded local filtering algorithm, the RCGL filters the undesirable effects of faulty interceptors and achieves the simultaneous arrival between normal interceptors.

V. Numerical Simulation

In this section, simulation studies are carried out to investigate the characteristics of RCGL. Consider an engagement scenario that 5 interceptors attack a single stationary target with initial conditions shown in Table I. The communication topology $\mathcal{G}_1$ between interceptors is shown in Fig. 4. The communication topology $\mathcal{G}_1$ is $(2, 2)$-robust according to Definition 1. During the engagement, Interceptor 3 is destroyed by the defense system of the target at 3 s.

A. Guidance Performance Analysis for RCGL

The parameters of robust guidance law are chosen as $\tau = 0.05$ s, $N_s = 3$ and $k_1 = \cdots = k_5 = 3.5$. The simulation results performed with RCGL are shown in Fig. 5. As it is shown, when Interceptor 3 is intercepted at 3 s, a_3 becomes $0\ m/s^2$ and the updating of $\hat{t}_{go,3}$ stops. From Figs. 5(b) and 5(c), we can see that time-to-go estimates of normal interceptors still reach an agreement after Interceptor 3 is intercepted; the RCGL achieves simultaneous arrivals by reshaping the trajectories of interceptors. In Fig. 5(d), we can see that there are some oscillations in the acceleration commands between 3 s and 6 s, which are caused by the local filtering actions of RCGL. The range of acceleration commands of normal interceptors are $-92.3\ m/s^2 \leq a_i \leq 54.3\ m/s^2$. In Fig. 6, we can observe that, before time-varying navigation gains achieve consensus, the normal interceptors with larger time-to-go estimates have larger time-varying navigation gains and the normal interceptors with smaller time-to-go estimates have smaller time-varying navigation gains. Then the time-varying navigation gains $N_s\Big(1 + k_i \sum_{j\in\mathcal{R}_i(t_k)} w_{ij}(t_k)\big(\hat{t}_{go,i}(t_k) - \hat{t}_{go,j}(t_k)\big)\Big)$ converge to a constant after 15 s, and the RCGL becomes PN with a navigation gain $N_s = 3$. The impact times T_i and initial time-to-go estimates $\hat{t}_{go,i}(t_0)$ are listed in the Table II. The impact times of normal interceptors are about 33.4 s and the dispersion of impact times is about 0.09 s. The RCGL reduces the impact time dispersion and achieves a simultaneous arrival. Note that the impact times T_i of normal interceptors are located within $\left[\min\{\hat{t}_{go,i}(t_0), \forall i \in \mathcal{N}\},\ \max\{\hat{t}_{go,i}(t_0), \forall i \in \mathcal{N}\}\right]$, which can be seen as a safety condition.

B. Comparison with other cooperative guidance law which has a fault diagnosis procedure

For the sake of guidance performance comparisons, a cooperative guidance law without local filtering algorithm is chosen as

$$a_i(t) = N_s\left(1 + k_i \sum_{j\in\mathcal{V}_i} \bar{w}_{ij}\left(\hat{t}_{go,i}(t_k) - \hat{t}_{go,j}(t_k)\right)\right)V_i\dot{\lambda}_i, \tag{7}$$

where,

$$\bar{w}_{ij}(t_k) = \begin{cases} \dfrac{\alpha_{ij}}{\sum_{j\in\mathcal{V}_i}\alpha_{ij}}, & \forall j \in \mathcal{V}_i, \\ 0, & \text{otherwise.} \end{cases}$$

Each interceptor is integrated with a fault diagnosis procedure which can detect the faulty interceptor and reorganize the communication between normal interceptors.

All parameters are chosen to be the same as that of Section V-A. Suppose that the fault diagnosis procedure needs 8 s to detect the fault and reorganize communications between normal interceptors (i.e., the fault diagnosis procedure accomplishes at 11 s). The simulation results in Fig. 7 show that guidance law (7) can achieve the simultaneous arrival by adding a fault diagnosis procedure. However, the trajectories of $\hat{t}_{go,i}$ and a_i are strong influenced by the misbehaviors of Interceptor 3 during 3 s to 11 s. In Fig. 7(b), the time-to-go estimate of Interceptor 3 becomes a leader, and all time-to-go estimates of normal interceptors try to follow the yellow line (during 3 s to 11 s). As depicted in Figs. 7(c) and 7(d), the accumulated errors in the time-to-go estimates of normal interceptors cause high acceleration commands a_i and hence distort engagement trajectories. The range of acceleration commands of normal interceptors are $-92.3\ m/s^2 \leq a_i \leq 69.87\ m/s^2$. The control costs $\int_{t_0}^{T_i} |a_i|\, dt$ of two guidance laws are shown in Fig. 8. The guidance law (7) use more control efforts than RCGL, since the normal interceptors take detours before fault diagnosis procedure accomplishes. The impact times T_i and initial time-to-go estimates $\hat{t}_{go,i}(t_0)$ are listed in the Table III. Note that the impact times of interceptors are about 38.4

TABLE I
ENGAGEMENT SCENARIO FOR 5 INTERCEPTORS

Parameters	Interceptor 1	Interceptor 2	Interceptor 3	Interceptor 4	Interceptor 5
Initial range-to-go(km)	7	7.5	7	10	8
Initial position(km)	$(-6.06, -3.5)$	$(-7.05, -2.57)$	$(-7, 0)$	$(-9.4, -3.42)$	$(-7.88, 1.39)$
Initial heading error(deg)	-20	10	-15	15	15
Velocity(m/s)	240	225	220	325	240
Target position	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$

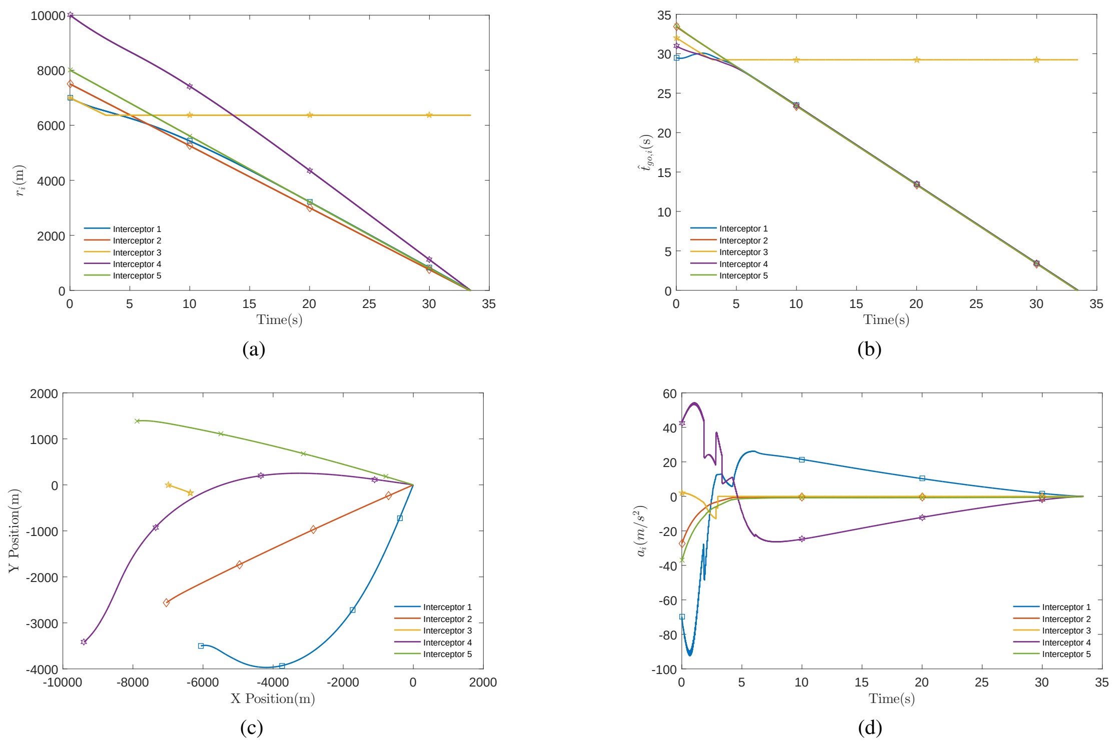

Fig. 5. Simulation results of RCGL. (a) Range-to-go r_i, $\forall i = 1, \cdots, 5$; (b) Time-to-go estimates $\hat{t}_{go,i}$, $\forall i = 1, \cdots, 5$; (c) Trajectories of interceptors; (d) Acceleration commands a_i, $\forall i = 1, \cdots, 5$.

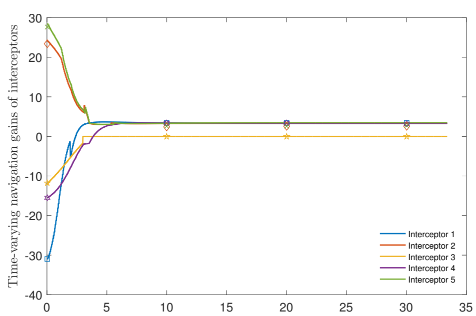

Fig. 6. Time-varying navigation gains $N_s\Big(1 + k_i \sum_{j\in\mathcal{R}_i(t_k)} w_{ij}(t_k)$ $(\hat{t}_{go,i}(t_k) - \hat{t}_{go,j}(t_k))\Big)$, $\forall i = 1, \cdots, 5$.

TABLE II
IMPACT TIME OF NORMAL INTERCEPTORS UNDER RCGL

	Interceptor 1	Interceptor 2	Interceptor 4	Interceptor 5
Initial time-to-go(s)	29.49	33.43	30.96	33.54
Impact time(s)	33.42	33.35	33.44	33.38
Impact time dispersion(s)	0.09			

s, which is larger than $\max\{\hat{t}_{go,i}(t_0), \forall i \in \mathcal{N}\}$. In fact, the impact time of the normal interceptor under guidance law (7) is related to the time of fault diagnosis; if the time of fault diagnosis is longer, the impact time will be delayed. If the fault diagnosis fails, the guidance law (7) can not even guarantee normal interceptors reaching the target.

The above simulation results demonstrate that RCGL has better performances when faults happen during the engagement. The simultaneous arrival can be achieved without any additional fault diagnosis procedure, which enhances the reliability of the cooperative guidance.

VI. CONCLUSIONS

This brief considers a new robust cooperative simultaneous arrival problem. A distributed cooperative guidance law RCGL is proposed based on discrete-time communications. By virtue of a novel local filtering algorithm, the RCGL can achieve a simultaneous arrival between normal interceptors without the

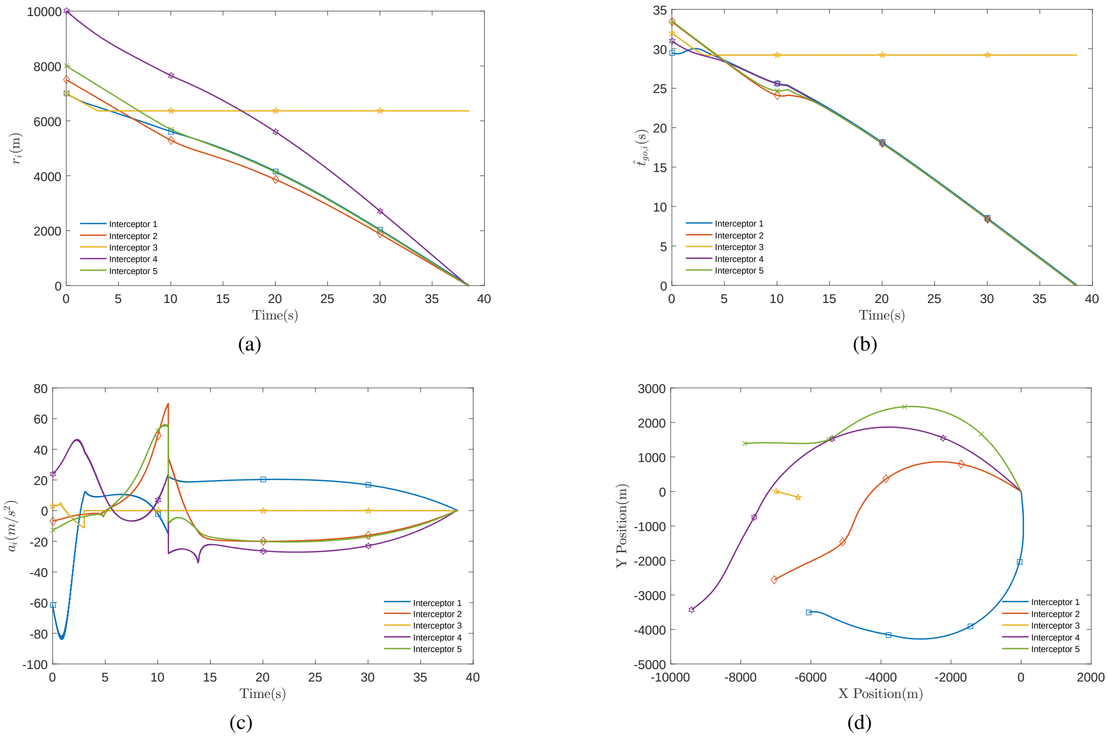

Fig. 7. Simulation results of cooperative guidance law (7) with a fault diagnosis procedure. (a) Range-to-go r_i, $\forall i = 1, \cdots, 5$; (b) Time-to-go estimates $\hat{t}_{go,i}$, $\forall i = 1, \cdots, 5$; (c) Acceleration commands a_i, $\forall i = 1, \cdots, 5$; (d) Trajectories of interceptors.

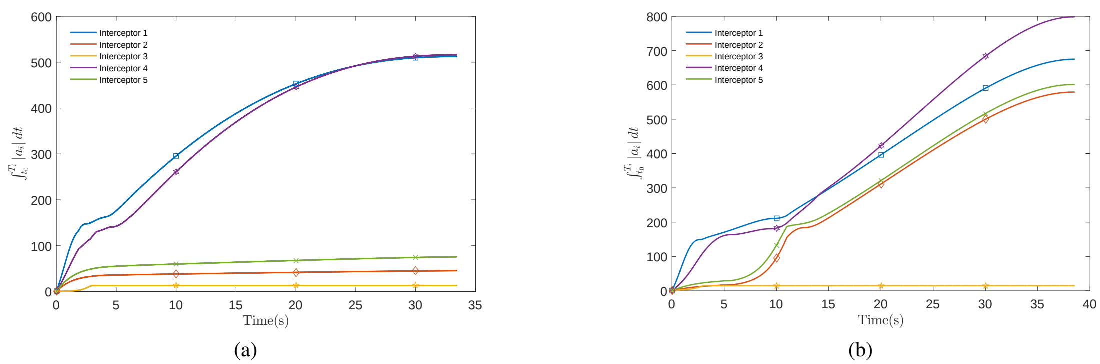

Fig. 8. Control cost of two guidance laws. (a) RCGL; (b) Cooperative guidance law (7) with fault detection procedure.

TABLE III
IMPACT TIME OF NORMAL INTERCEPTORS UNDER GUIDANCE LAW (7) WITH A FAULT DIAGNOSIS PROCEDURE

	Interceptor 1	Interceptor 2	Interceptor 4	Interceptor 5
Initial time-to-go(s)	29.49	33.43	30.96	33.54
Impact time(s)	38.54	38.41	38.42	38.42
Impact time dispersion(s)	0.13			

knowledge of faulty interceptors (or any fault diagnosis procedure). Furthermore, the impact times of normal interceptors are upper bounded by the maximum initial time-to-go estimate of normal interceptors, regardless the network connections. Compared to the existing cooperative guidance laws, RCGL is fully distributed and requires no additional information; thus it reduces the communication burden in practice implementations. The comparison of simulation results shows that the RCGL can enhance the reliability of the cooperative guidance. Future research may include extensions to the case with manoeuvrable targets.

APPENDIX
PROOF OF THEOREM 2

Theorem 1 shows that $\{M(t_k)\}$ is nonincreasing and $\{m(t_k)\}$ is nondecreasing, respectively. Since every bounded monotone sequence of real numbers has a limit, $\lim_{k\to\infty} M(t_k)$ and $\lim_{k\to\infty} m(t_k)$ exist. Define $\lim_{k\to\infty} M(t_k) = A_M$ and $\lim_{k\to\infty} m(t_k) = A_m$, we will show $A_M = A_m$ by seeking a contradiction.

Suppose that $A_M \neq A_m$, we can define $\epsilon_0 = \frac{A_M - A_m}{2}$. For any $\epsilon_i \in \mathcal{R}$, we define $\mathcal{T}_M(t_k, \epsilon_i) = \{i \in \mathcal{V} : \hat{T}_i(t_k) > A_M - \epsilon_i\}$ and $\mathcal{T}_m(t_k, \epsilon_i) = \{i \in \mathcal{V} : \hat{T}_i(t_k) < A_m + \epsilon_i\}$. Note that $\mathcal{T}_M(t_k, \epsilon_i)$ contains normal and faulty interceptors that have $\hat{T}_i(t_k)$ larger than $A_M - \epsilon_i$, $\mathcal{T}_m(t_k, \epsilon_i)$ contains normal and faulty interceptors that have $\hat{T}_i(t_k)$ smaller than $A_m + \epsilon_i$.

Define a positive constant $\epsilon < \min\{\epsilon_0, \frac{\alpha^N}{1-\alpha^N}\epsilon_0\}$, where $\alpha \in (0,1)$ is a constant to be defined latter. Define t_{k_ϵ} as the time-step such that $M(t_{k_\epsilon}) < A_M + \epsilon$ and $m(t_{k_\epsilon}) > A_m - \epsilon$, $\forall t_k > t_{k_\epsilon}$; the existence of t_{k_ϵ} is guaranteed by the convergence of $\{M(t_k)\}$ and $\{m(t_k)\}$. It is obvious that $\mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$ and $\mathcal{T}_m(t_{k_\epsilon}, \epsilon_0)$ are nonempty and $\mathcal{T}_M(t_{k_\epsilon}, \epsilon_0) \cap \mathcal{T}_m(t_{k_\epsilon}, \epsilon_0) = \emptyset$. Note that communication graph $\mathcal{G}$ is $(F+1, F+1)$ robust, and the number of faulty interceptors is upper bounded by F; if there are normal interceptors in $\mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$ and $\mathcal{T}_m(t_{k_\epsilon}, \epsilon_0)$ (i.e., $\mathcal{N} \cap \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0) \neq \emptyset$ and $\mathcal{N} \cap \mathcal{T}_m(t_{k_\epsilon}, \epsilon_0) \neq \emptyset$), there is at least one of these normal interceptors that has at least $F+1$ in-neighbors outside of its set. Note that $\mathcal{N} \cap \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0) \neq \emptyset$ and $\mathcal{N} \cap \mathcal{T}_m(t_{k_\epsilon}, \epsilon_0) \neq \emptyset$ are true. Otherwise, we have $\hat{t}_{go,i}(t_{k_\epsilon}) \geq \frac{A_M + A_m}{2}$, $\forall i \in \mathcal{N}$ or $\hat{t}_{go,i}(t_{k_\epsilon}) \leq \frac{A_M + A_m}{2}$, $\forall i \in \mathcal{N}$, which contradicts the fact that $\{M(t_k)\}$ and $\{m(t_k)\}$ monotonously converge to A_M and A_m respectively. Without loss of generality, suppose that the normal interceptor $i \in \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0) \cap \mathcal{N}$ has at least $F+1$ in-neighbors outside of $\mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$. Then, we have $\hat{T}_j(t_{k_\epsilon}) \leq A_M - \epsilon_0$, $\forall j \in \mathcal{V}_i \cap \complement_{\mathcal{V}} \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$, where $\complement_{\mathcal{V}} \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$ denotes the relative complement of $\mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$ with respect to $\mathcal{V}$. Since normal interceptor i removes at most F in-neighbors which are in $\complement_{\mathcal{V}} \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$, at least one interceptor belongs to $\mathcal{V}_i \cap \complement_{\mathcal{V}} \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$ will be retained in $\mathcal{R}_i(t_{k_\epsilon})$. Assume that none of σ_i, $\forall i \in \mathcal{N}$ reaches zero before normal interceptors reach the target (i.e, there exists a positive constant K_m, such that $K_i > K_m$). Together with (6), we have that the impact time estimate of the normal interceptor i at time $t_{k_\epsilon+1}$ has the following property

$$\begin{aligned}\hat{T}_i(t_{k_\epsilon+1}) &\leq (1 - \frac{K_m}{N}) M(t_{k_\epsilon+1}) + \frac{K_m}{N}(A_M - \epsilon_0) \\ &\leq A_M - \alpha\epsilon_0 + (1-\alpha)\epsilon \leq A_M - \epsilon_1, \end{aligned} \tag{8}$$

where $\epsilon_1 = \alpha\epsilon_0 - (1-\alpha)\epsilon$ and $\alpha = \frac{K_m}{N}$. Since $0 < K_m < K_i < 1$, we have $\alpha \in (0,1)$ and $\epsilon_1 < \epsilon_0$. To get (8), we have used the fact that $w_{ij}(\cdot) \geq \frac{1}{N}$. Note that for any normal interceptor $j \notin \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$, we still have $\hat{T}_j(t_{k_\epsilon+1}) \leq A_M - \epsilon_1$; since such a interceptor j will use its own impact time estimate $\hat{T}_j(t_{k_\epsilon})$ at $t_{k_\epsilon+1}$. Similarly, if a normal interceptor $p \in \mathcal{T}_m(t_{k_\epsilon}, \epsilon_0)$ has at least $F+1$ in-neighbors outside of $\mathcal{T}_m(t_{k_\epsilon}, \epsilon_0)$, we can obtain

$$\hat{T}_p(t_{k_\epsilon+1}) \geq A_m + \alpha\epsilon_0 - (1-\alpha)\epsilon \geq A_m + \epsilon_1.$$

Furthermore, for any normal interceptor $q \notin \mathcal{T}_m(t_{k_\epsilon}, \epsilon_0)$, we still have $\hat{T}_q(t_{k_\epsilon+1}) \geq A_m + \epsilon_1$.

Based on the above analysis, we know that at least one of following statement is true if both $\mathcal{N} \cap \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0) \neq \emptyset$ and $\mathcal{N} \cap \mathcal{T}_m(t_{k_\epsilon}, \epsilon_0) \neq \emptyset$:

1) At least there is one normal interceptor $i \in \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)$ whose $\hat{T}_i(t_{k_\epsilon})$ decreases to $A_M - \epsilon_1$ (or below) at $t_{k_\epsilon+1}$. Then, we have $\|\mathcal{T}_M(t_{k_\epsilon+1}, \epsilon_1) \cap \mathcal{N}\| < \|\mathcal{T}_M(t_{k_\epsilon}, \epsilon_0) \cap \mathcal{N}\|$, where $\|\cdot\|$ denotes the cardinality of a set.
2) At least there is one normal interceptor $j \in \mathcal{T}_m(t_{k_\epsilon}, \epsilon_0)$ whose $\hat{T}_j(t_{k_\epsilon})$ increases to $A_m + \epsilon_1$ (or above) at $t_{k_\epsilon+1}$. Then, we have $\|\mathcal{T}_m(t_{k_\epsilon+1}, \epsilon_1) \cap \mathcal{N}\| < \|\mathcal{T}_m(t_{k_\epsilon}, \epsilon_0) \cap \mathcal{N}\|$.

Since $\epsilon_1 < \epsilon_0$, we have $\mathcal{T}_M(t_{k_\epsilon+1}, \epsilon_1) \cap \mathcal{T}_m(t_{k_\epsilon+1}, \epsilon_1) = \emptyset$. Define ϵ_j recursively as $\epsilon_j = \alpha\epsilon_{j-1} - (1-\alpha)\epsilon$, $\forall j \geq 1$, one can obtain $\epsilon_j < \epsilon_{j-1}$ and $\mathcal{T}_M(t_{k_\epsilon+j}, \epsilon_j) \cap \mathcal{T}_m(t_{k_\epsilon+j}, \epsilon_j) = \emptyset$. If there are still normal nodes in $\mathcal{T}_M(t_{k_\epsilon+j}, \epsilon_j) \cup \mathcal{T}_m(t_{k_\epsilon+j}, \epsilon_j)$, we can repeat the above analysis at $t_{k_\epsilon+j+1}$, then we have either $\|\mathcal{T}_M(t_{k_\epsilon+j+1}, \epsilon_{j+1}) \cap \mathcal{N}\| < \|\mathcal{T}_M(t_{k_\epsilon+j}, \epsilon_j) \cap \mathcal{N}\|$, or $\|\mathcal{T}_m(t_{k_\epsilon+j+1}, \epsilon_{j+1}) \cap \mathcal{N}\| < \|\mathcal{T}_m(t_{k_\epsilon+j}, \epsilon_j) \cap \mathcal{N}\|$, or both. Subsequently, there exists a time-step $t_{k_\epsilon+\bar{k}}$ such that $\mathcal{N} \cap \mathcal{T}_M(t_{k_\epsilon+\bar{k}}, \epsilon_{\bar{k}}) = \emptyset$ or $\mathcal{N} \cap \mathcal{T}_M(t_{k_\epsilon+\bar{k}}, \epsilon_{\bar{k}}) = \emptyset$. Since $\|\mathcal{N} \cap \mathcal{T}_M(t_{k_\epsilon}, \epsilon_0)\| + \|\mathcal{N} \cap \mathcal{T}_m(t_{k_\epsilon}, \epsilon_0)\| \leq N$, we obtain $\bar{K} < N$. For the case $\mathcal{N} \cap \mathcal{T}_M(t_{k_\epsilon+\bar{k}}, \epsilon_{\bar{k}}) = \emptyset$, we can obtain that the impact time estimates of all normal interceptors satisfy $\hat{T}_i(t_{k_\epsilon+\bar{k}}) \leq A_M - \epsilon_{\bar{k}}$, $\forall i \in \mathcal{N}$; for the case $\mathcal{N} \cap \mathcal{T}_m(t_{k_\epsilon+\bar{k}}, \epsilon_{\bar{k}}) = \emptyset$, we can obtain that the impact time estimates of all normal interceptors satisfy $\hat{T}_i(t_{k_\epsilon+\bar{k}}) \geq A_m + \epsilon_{\bar{k}}$, $\forall i \in \mathcal{N}$.

In the following analysis, we will show that $\epsilon_{\bar{k}} > 0$, which contradicts the fact that $\{M(t_k)\}$ monotonically converges to A_M (in the first case) or that $\{m(t_k)\}$ monotonically converges to A_m (in the second case). Note that

$$\begin{aligned}\epsilon_{\bar{k}} &= \alpha\epsilon_{\bar{k}-1} - (1-\alpha)\epsilon \\ &= \alpha^2\epsilon_{\bar{k}-2} - \alpha(1-\alpha)\epsilon - (1-\alpha)\epsilon \\ &= \alpha^{\bar{k}}\epsilon_0 - (1-\alpha^{\bar{k}})\epsilon \geq \alpha^N\epsilon_0 - (1-\alpha^N)\epsilon.\end{aligned}$$

Since $\epsilon < \frac{\alpha^N}{1-\alpha^N}\epsilon_0$, we have $\epsilon_{\bar{k}} > 0$, which provides the contradiction. Thus, we can conclude that $A_M = A_m$, the impact time estimates $\hat{T}_i$, $\forall i \in \mathcal{N}$ converge to a same value.

References

[1] I.-S. Jeon, J.-I. Lee, and M.-J. Tahk, "Impact-time-control guidance law for anti-ship missiles," *IEEE Trans. Control Syst. Technol.*, vol. 14, no. 2, pp. 260–266, March 2006.

[2] G. A. Harrison, "Hybrid guidance law for approach angle and time-of-arrival control," *J. Guidance, Control, Dyn.*, vol. 35, no. 4, pp. 1104–1114, 2012.

[3] J. I. Lee, I. S. Jeon, and M. J. Tahk, "Guidance law to control impact time and angle," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 43, no. 1, pp. 301–310, January 2007.

[4] I.-S. Jeon, J.-I. Lee, and M.-J. Tahk, "Homing guidance law for cooperative attack of multiple missiles," *J. Guidance, Control, Dyn.*, vol. 33, no. 1, pp. 275–280, 2010.

[5] Y. Zhang, D. Yu, Y. an, and Y. Wu, "An impact-time-control guidance law for multi-missiles," in *Proc. IEEE Int. Conf. Intell. Comput. Intell. Syst.*, vol. 2, Nov 2009, pp. 430–434.

[6] P. Zhang, H. H. Liu, X. Li, and Y. Yao, "Fault tolerance of cooperative interception using multiple flight vehicles," *J. Franklin Inst.*, vol. 350, no. 9, pp. 2373 – 2395, 2013.

[7] J. Zhou and J. Yang, "Distributed guidance law design for cooperative simultaneous attacks with multiple missiles," *J. Guidance, Control, Dyn.*, pp. 2439–2447, 2016.

[8] M. Snyder, C. Li, and Z. Qu, "A new parameterized guidance law for cooperative air defense," in *Proc. 36th AIAA Aerosp. Sci. Meeting New Horizons Forum Aerosp. Expo.*, 2012, p. 850.

[9] S. M. Dibaji and H. Ishii, "Resilient consensus of second-order agent networks: Asynchronous update rules with delays," *Automatica*, vol. 81, pp. 123 – 132, 2017.

[10] H. J. LeBlanc, H. Zhang, X. Koutsoukos, and S. Sundaram, "Resilient asymptotic consensus in robust networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 4, pp. 766–781, April 2013.

[11] H. Zhang, E. Fata, and S. Sundaram, "A notion of robustness in complex networks," *IEEE Trans. Control Netw. Syst.*, vol. 2, no. 3, pp. 310–320, Sept 2015.

[12] P. Gurfil, "Robust guidance for electro-optical missiles," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 39, no. 2, pp. 450–461, April 2003.