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Complete List of Authors:	Das, Satyajit; University of Cambridge Department of Engineering, Engineering Kandan, Karthick; University of Cambridge Department of Engineering, Engineering Kazemahvazi, Sohrab; University of Cambridge Department of Engineering, Engineering Wadley, Haydn; University of Virginia, Materials Science Deshpande, Vikram; University of Cambridge Department of Engineering, Engineering
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Indentation response of a 3D non-woven carbon-fibre composite

S. Das^a, K. Kandan^{a,b}, S. Kazemahvazi^a, H.N.G. Wadley^c and V.S. Deshpande^{a*}

^a *Department of Engineering, University of Cambridge,
Trumpington Street, Cambridge CB2 1PZ, UK.*

^b *School of Engineering, De Montfort University,
The Gateway, Leicester LE1 9BH, UK.*

^c *Department of Material Science & Engineering, School of Engineering and Applied Science,
University of Virginia, Charlottesville, VA 22904, USA.*

Abstract

The indentation response of a three-dimensional (3D) non-interlaced composite comprising three sets of orthogonal carbon fibre tows in an epoxy matrix is investigated and the deformation/failure modes analysed by a combination of X-ray tomography and optical microscopy. The 3D composites have a near isotropic and ductile indentation response. The deformation mode includes the formation of multiple kink bands in the tows aligned with the indentation direction and shearing of the orthogonally oriented tows. This contrasts with the highly anisotropic response of traditional cross-ply composites where indentation results in either brittle tensile fracture of the plies or delamination failure depending on the indentation direction. Finite element (FE) calculations are also reported wherein tows in only one direction are explicitly modelled with the other two sets of orthogonal tows and the matrix pockets treated as an effective homogenous medium. The calculations are shown to capture the indentation response in the direction of the explicitly modelled tows with excellent fidelity but they under-predict the indentation strength in the other directions. This study has highlighted that in contrast to anisotropic and brittle laminated composites, 3D non-interlaced composites have a near isotropic and ductile indentation response. This makes them strong candidates for application as materials to resist impact loading.

*Corresponding author. E-mail address: vsd@eng.cam.ac.uk.

1. Introduction

Composite materials such as carbon fibre reinforced polymer (CFRP) composites made by laminating unidirectionally reinforced plies are extensively used in civil and aircraft structures due to their high specific strength and stiffness, superior corrosion resistance and improved fatigue resistance compared to conventional engineering materials such as aluminium and steel¹⁻³. However, such materials are susceptible to interply delamination^{4,5}, which traditionally has limited their use in situations where impact loading can occur. Nevertheless, lightweighting programs for transportation structures are driving renewed interest in composite materials for impact protection systems. For example, military vehicles are required to resist projectile impacts while in civilian aircraft applications the engines and airframes must be able to resist impacts from hail, bird strikes and impact by other foreign objects. In most cases, the structure needs to be able to retain its structural integrity after impact loading, which presents a serious challenge for traditional laminated composites.

Impact loads induce a range of damage modes in composites that seriously degrade their mechanical performance. These damage modes include matrix cracks, delamination between plies and fibre fracture. Delamination under impact loading is a particularly critical damage mode as it results in a significant reduction in the mechanical performance of the composite in spite of the fact that the fibres remain intact. A number of approaches have been proposed to tailor the fibre/tow architectures to enhance the delamination resistance. These include Z-pinning^{6,7}, stitching^{8,9} and knitting¹⁰ but these methods have had only marginal success as the reinforcements are unable to withstand the large out-of-plane stresses that are generated under impact loading. More recently, a range of methods have been developed to manufacture three-dimensional (3D) fibre preforms wherein tows are present in at-least three orthogonal directions; see Khokar¹¹ for a detailed review of these techniques. These 3D fibre preforms can be broadly classified into three categories: (i) 2D woven 3D fabrics produced by usual 2D weaving methods with mono-directional shedding; (ii) 3D woven 3D fabrics produced by a dual-direction shedding system and (iii) non-woven 3D fabrics without interlacing or interweaving produced by a technique known as “noobing” that is described in Section 2. This emerging ability to independently manipulate the volume fractions of fibre in three directions not only allows tailoring of the multi-axial properties of composites¹² but it also greatly reduces their susceptibility to delamination, and improves the impact resistance of CFRPs¹³⁻¹⁵. However, a key drawback of 3D composites compared to laminated counterparts is their reduced elastic modulus due to significant fibre waviness. This waviness also reduces compressive strength. For example, Kuo and Ko¹⁶ modified a conventional weaving machine to produce 3D composites with orthogonal non-woven yarns. Although the composite showed high ductility in compression, the inherent fibre waviness resulting from their modified weaving process, significantly reduced the compressive strength.

Impacts by hail particles or bird strikes against aircraft structures are typically categorised as relatively low velocity impacts. Nevertheless, visualisation of their damage modes is hindered by the difficulty of observing internal failure modes via methods like in-situ X-ray tomography (XCT) during impact events. Even interrupting the test to characterize damage modes can change the deformation/failure modes. Thus, damage visualisation in low velocity impact tests is typically restricted to post-mortem inspections. However, since stress waves travel and reflect multiple times from the edges of a target during the time over which the projectile is in contact

with the target^{17,18}, low velocity impacts can be well approximated to be quasi-static. Hence, quasi-static indentation tests are often employed to develop an initial understanding of the low velocity impact response of composites¹⁹ and a number of studies^{20,21} have demonstrated the equivalence between low velocity impact and quasi-static indentation tests.

This study reports the quasi-static indentation response of a 3D non-woven carbon fibre/epoxy composites manufactured by the noobing process. The uniaxial compressive response of this composite was recently studied by Das et al.²² and here we extend the understanding of the mechanical properties of this composite by investigating the indentation response. Both measurements and finite element simulations are reported and the indentation mechanisms of this 3D composite are contrasted with those of traditional cross-ply laminates in order to assess the suitability of 3D noobed composites for impact protection applications.

2. Materials and manufacture

The study primarily focuses on 3D noobed¹ composites in which three linear sets of orthogonal yarns are bound/tied together to produce a layerless 3D fabric. In order to put the behavior of such noobed composites in context, we also report the equivalent responses of a two-dimensional (2D) cross-ply composite. The details of these two materials including their manufacture are described in this section.

2.1 *Manufacture and structure of the noobed composites*

Dry 3D fabrics were produced in block form using the noobing method developed by Khokar²⁴ where readers can find details of the device used for its automated manufacture. With (X , Y , Z) forming a Cartesian co-ordinate system, the composite comprises an array of Z -yarns bound together by X and Y -yarns that traverse the rows and columns of the grid formed by the Z -yarns. These yarns loop as shown in Fig. 1a and externally bind the preform together (for clarity we do not show the looping of the Z yarns in Fig. 1a). This preform is therefore produced by a process that does not involve shedding as in a weaving process and results in structure consisting of three orthogonal non-interlaced X , Y and Z -yarns. Infusion of a polymer matrix was performed via a resin transfer molding process (RTM) to produce the 3D composite material.

The 3D noobed composites made for this study used Toray T700S 12k carbon fibre tows (non-twisted carbon fibre yarns are usually referred to as tows) in a NM FW3070 epoxy matrix². The carbon fibres were approximately $d = 7.2 \mu\text{m}$ in diameter and the 3D composite was anisotropic with 20% of the total number of tows in the Z -direction and 40% each in the X and Y -directions. Blocks of the 3D noobed composites of size 175 (X)mm \times 103 (Y)mm \times 45 (Z)mm were manufactured and specimens of required dimensions were cut from these blocks using a diamond band saw. X-ray computed tomography (XCT) images of the interior of the specimens on three orthogonal planes are included in Fig. 1b. These images clearly show that while the Z -direction tows have an approximately square cross-section the X and Y -direction tows are flattened in the Z -direction during the RTM process. Moreover, due to the orthogonal arrangement of the tows, pockets of pure matrix (in addition to the

¹ The acronym NOOB stands for Non-interlacing, Orientating Orthogonally and Binding²³

² Nils Malmgren AB, P.O.Box 2039 S-442 02 Ytterby Sweden.

matrix that exists between fibres within each tow) are regularly interspersed in the composite. The periodic unit cell as inferred from these XCT images is sketched in Fig. 1c (an average unit cell based on measurements at 30 different locations in the XCT images) and includes all the relevant dimensions of the tows and matrix pockets.

The composite comprises four principal phases: the X , Y and Z -direction tows and the matrix pockets. Based on the unit cell with dimensions sketched in Fig. 1c, the X and Y -direction tows comprise a volume fraction $v_X = v_Y \approx 29\%$ of the composite while the Z -direction tows occupy a volume fraction $v_Z \approx 17\%$ of the composite. The remainder $v_M = 25\%$ of the volume is occupied by the matrix pockets. To specify the overall carbon fibre volume fraction within the composite, recall that each tow comprises 12k fibres of diameter $7.2 \mu\text{m}$. Based on the tow cross-sectional areas from Fig. 1c the fibre volume fractions in the X and Y -direction tows are $f_X = f_Y \approx 68\%$ while the Z -direction tow comprises $f_Z \approx 30\%$ fibres. The overall fibre volume fraction in the composite then follows as $f = 2v_X f_X + v_Z f_Z \approx 45\%$.

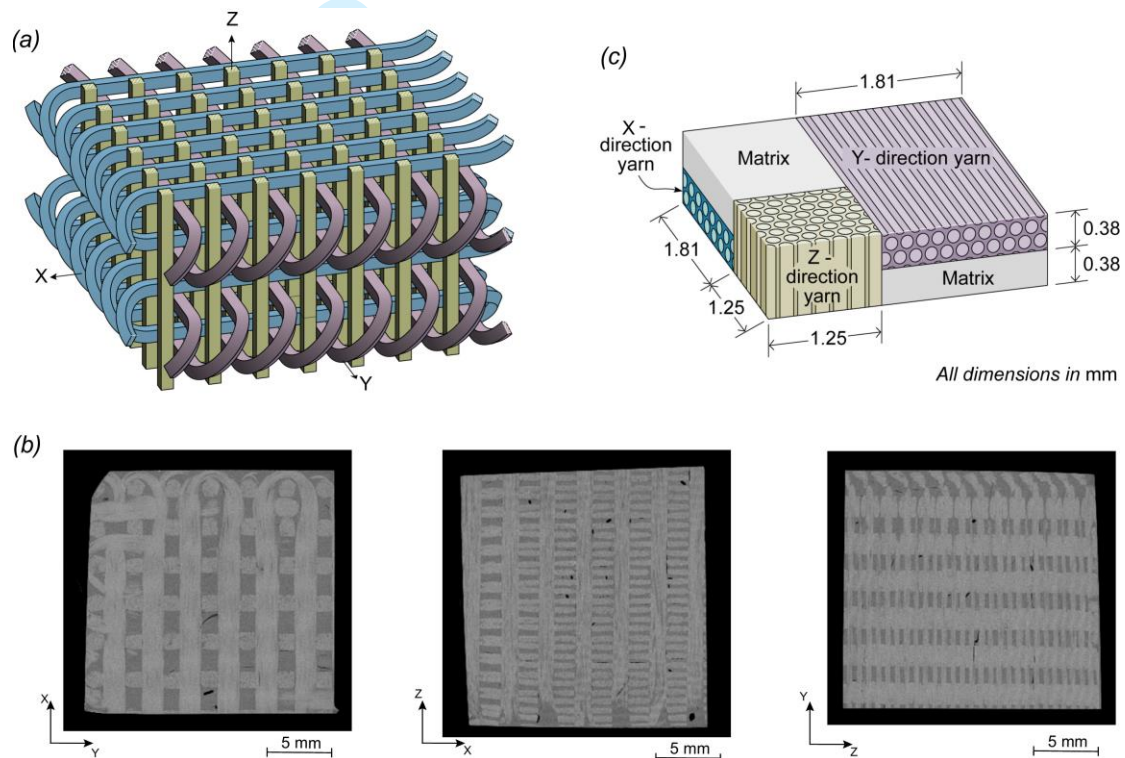


Fig. 1: (a) Sketch of the orthogonal non-woven yarns in the 3D noobed fabric that is infused with a polymer matrix to give the composite. (b) X-ray tomographic (XCT) scans of the 3D noobed composite showing sections on three orthogonal planes. The (X, Y, Z) co-ordinate system is indicated in (a). (c) Sketch of the unit cell of the 3D noobed composite employed in this study. The unit cell is inferred from the XCT images in (b).

2.2 The cross-ply 2D composite

The performance of the 3D noobed composite was compared with a standard laminated (2D) carbon fibre composite with constituent properties as close to those of the 3D noobed composite as possible. A cross-ply composite comprising 66% by volume IM7 fibres in a Hexply 8552 matrix was manufactured from prepregs supplied by Hexcel composites (ply thickness $125 \mu\text{m}$). Unlike the 3D noobed composite, 45 mm thick laminated composites typically are not manufactured from prepregs but in order to mimic the 3D composite as closely as possible, we constructed a 25 mm thick composite sheet with lay-up $(0/90)_{100}$. The mechanical

properties of this standard laminate are well established; see for example the Hexcel datasheet²⁵ and Russell et al.²⁶. We use a Cartesian co-ordinate system for the laminated system such that the Z -direction is perpendicular to the plane of the plies and the X and Y -directions are aligned with the fibre directions in the cross-ply composite.

3. Experimental protocol and measurements

The aims of the experimental study are (i) to measure the indentation response of the 3D noobed composites; (ii) investigate the deformation/failure mechanisms; and (iii) contrast the performance of the 3D composite with cross-ply laminated composite. We first describe the measurement protocols and then proceed to discuss the observations.

3.1 Measurement protocol

Back-supported indentation of the noobed composites in the X and Z -directions was carried out on cuboidal specimens of dimension $\sim 45 \text{ mm} \times 45 \text{ mm} \times 20 \text{ mm}$ (the blocks were 20 mm in the X and Z -directions for indentations carried out in those respective directions). Since the Y -direction is identical to the X -direction, results in only one of these directions are presented. The specimens were cut from the infused noobed composite blocks first with a diamond band-saw and then milled to the final dimensions to ensure parallel sides. Indentation was carried out in a screw-driven test machine using a hardened steel, flat-bottomed circular indenter of diameter $D = 8 \text{ mm}$. The indenter diameter was chosen so that at-least two Z -direction tows were aligned along the diameter of the indenter for indentation in the Z -direction (since the cross-section of the X -direction tows is smaller, at-least nine X -direction tows lay along this indenter diameter for indentation in the X -direction). The indentation was performed at the centre of the $45 \text{ mm} \times 45 \text{ mm}$ cross-sectional area of the specimen with the indentation load P measured from the load cell of the test machine and the indentation displacement δ measured via a laser extensometer. The indentation response is quantified in terms of the indentation stress $\sigma_I \equiv P/A$ versus the measure of indentation strain²⁷ $\varepsilon_I \equiv \delta/\sqrt{A}$, where $A = \pi D^2/4$ is the cross-sectional area of the indenter. All indentations were carried out at an applied displacement rate $\dot{\delta} = 0.2 \text{ mm min}^{-1}$.

The indentation tests were periodically interrupted to enable XCT visualisation of the deformation/damage evolution under the indenter. At high indentation displacements, the indenter was embedded in the specimen and could not be removed without causing additional damage. In these cases, the XCT was performed with the steel indenter still within the specimen and this resulted in relatively poor quality XCT images. In addition to these XCT images, high-resolution optical imaging of the interior of the specimens immediately under the indenter was also conducted at the end of the indentation experiment. The optical imaging involved polishing of the specimen to expose the specimen interior as follows. The unloaded specimen was polished with SiC abrasive paper first using a coarse-grit (P220-P400) until approximately the mid-section of the specimen was exposed. Then, another 1 mm or so of the specimen was further abraded using a fine-grit (P800-P4000) paper in order to obtain a clean and smooth surface for imaging. In order to maximise the resolution of the images while still imaging a large enough area to clearly expose the deformation/failure modes, the imaged area was divided into a grid comprising

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3 approximately 200 squares. Each of these squares was imaged separately and the
4 entire imaged section was then reconstructed by stitching together these sub-images.
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7 Indentation for the cross-ply composites in the Z -direction was conducted using the
8 same protocol. However, recall that cross-ply composite plates were only 25 mm
9 thick in the Z -direction. Thus, indentation in the X -direction was carried out using a
10 half-scale geometrically similar setup compared to all the other tests, i.e. the specimen
11 and indenter dimensions as well as the indentation rate were decreased by a factor of
12 two. The deformation/failure of the cross-ply specimens was visualised only using
13 XCT.
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16 3.2 Indentation of the noobed composite in the Z -direction

17 The measured σ_I versus ε_I response of the noobed composite in the Z -direction is
18 plotted in Fig. 2a. After an initial elastic response, the composite displays an
19 approximately linearly hardening response. Unloading was performed from the three
20 instants A, B and C marked on the σ_I - ε_I curve in Fig. 2a (unloading followed
21 approximately the initial elastic slope and thus the unloading curves are omitted from
22 Fig. 2a for the sake of clarity). XCT images of a section of the specimen after
23 unloading from A, B and C are shown in Fig. 2b. These images are of sections lying
24 in the diametrical plane of the indenter and show views of the $X-Z$ plane. At the
25 onset of nonlinearity (A) nearly no permanent deformation or failure is observed.
26 Further along the hardening curves, shear bands orientated at approximately $\pm 45^\circ$
27 to the axis of loading, emanate from the edge of the indenter as seen in the images B and
28 C in Fig. 2b. These bands are associated with shearing of the X -direction tows and
29 kinking within the compressed Z -direction tows, although the kinking is not clearly
30 visible within the resolution of the XCT images. The optical micrograph
31 corresponding to the XCT image C in Fig. 2b is included in Fig. 2c. Multiple and
32 reflected kinks are now clearly visible in the Z -direction tows, and the deformation is
33 reminiscent of that observed during uniaxial compression of the same noobed
34 composite in the Z -direction²². It is thus instructive to compare the uniaxial
35 compression and indentation responses.
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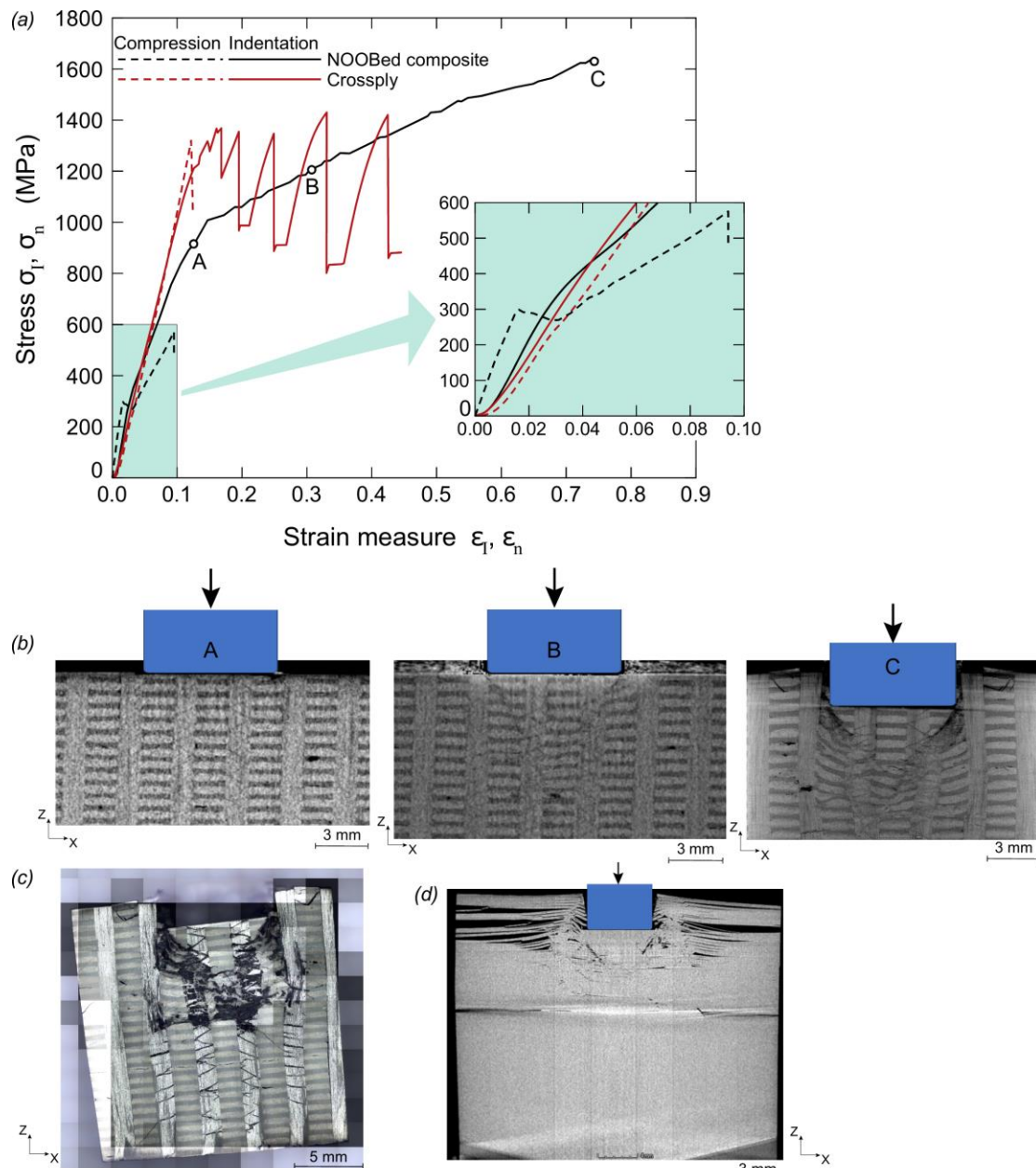


Fig. 2: (a) The measured Z-direction indentation ($\sigma_I - \epsilon_I$) and uniaxial compression ($\sigma_n - \epsilon_n$) responses of the 3D noobed and cross-ply composites. The inset shows a zoom-in of the early part of the loading history. (b) XCT images of the noobed composite showing the deformation under the indenter after unloading from locations A, B, and C marked in (a). (c) An optical micrograph of the deformation under the indenter of the noobed composite after completion of loading. (d) XCT image of the deformation/failure of the cross-ply specimen after completion of the indentation loading.

The Z-direction uniaxial nominal compressive stress σ_n versus nominal strain ϵ_n response of the noobed composite from Das et al.²² is included in Fig. 2a: the compressive ductility of the noobed composite in the Z-direction was about 10% and hence for the sake of clarity it is included in an inset that shows an enlarged view of the early part of the indentation response. We briefly summarise the compression results of Das et al.²² so as to enable us to contrast them with the indentation response measured here. The noobed composite has an elastic response until the onset of kinking, and then displays a hardening response when further kinks (some of which zig-zag) form within the Z-direction tows. Final collapse of the composite occurs by

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3 tensile failure of the X and Y - direction tows via the indirect tension²⁸ mechanism. In
4 comparing the indentation and uniaxial compression responses we emphasize that
5 while it is meaningful to compare the stress levels σ_I and σ_n , the indentation strain ε_I
6 as defined by Sargent and Ashby²⁷ is only an approximate measure of the strain under
7 the indenter and thus may not be directly comparable to ε_n . Keeping this in mind we
8 observe that the two main differences between the indentation and uniaxial
9 compressive response are: (i) the onset of non-linearity associated with the formation
10 of the first kink-band in the Z -direction tow occurs at a lower stress level under
11 uniaxial compression and (ii) the specimen under uniaxial compression fails at $\sigma_n \approx$
12 580 MPa while a continued hardening indentation response is observed even at an
13 indentation stress $\sigma_I \approx 1600$ MPa. These differences are best understood by recalling
14 that under indentation loading the stress state under the indenter has a significant
15 hydrostatic component that stabilises the formation of kinks and increases the stress
16 level required for the formation of the first kink band. Subsequently, continued
17 deformation occurs by the formation of multiple kinks within the Z -direction tow and
18 the shear deformation of the X and Y -direction tows. The continuing build-up of
19 hydrostatic stress results in the high hardening observed under indentation loading
20 while the overall hydrostatic stress remains low under uniaxial compression.
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26 3.3 Indentation of the noobed composite in the X -direction

27 The measured σ_I versus ε_I response for indentation of the noobed composite in the X -
28 direction is plotted in Fig. 3a with the corresponding uniaxial compressive response in
29 the X -direction from Das et al.²² again included in the inset. Now, unlike for
30 indentation in the Z -direction, there is negligible hardening with continued
31 indentation occurring at a constant stress $\sigma_I \approx 1200$ MPa. Further, there is no sign of
32 loss of load carrying capacity even at $\varepsilon_I \approx 0.8$ and this is in stark contrast with the X -
33 direction uniaxial compression response where the noobed composite underwent
34 catastrophic failure at $\sigma_n \approx 500$ MPa with a compressive ductility of about 2%; see
35 inset in Fig. 3a. XCT images of the deformation immediately under the indenter after
36 unloading from $\varepsilon_I \approx 0.8$ are included in Figs. 3b and 3c: these images show views of
37 the $X - Y$ and $X - Z$ planes for sections lying in the diametrical plane of the indenter.
38 The Z and Y -direction tows are seen to undergo large shear strains but the XCT
39 images do not have the resolution to clearly show the deformation/failure mechanisms
40 within the X -direction tows. Optical micrographs of the section in Fig. 3c are included
41 in Fig. 4 with multiple levels of magnification that clearly illustrate the kink bands
42 within the X -direction tows. Similar to the Z -direction tows in Fig. 2c we observe
43 multiple and reflected kink-bands in the X -direction tows. Thus, while the
44 deformation mechanisms for indentation in the X and Z -directions are similar, the
45 reduced hardening for indentation in the X -direction is presumably due to the lower
46 fibre volume fractions in the Z -direction tows, which makes them easier to shear. We
47 emphasize that under indentation loading these shear bands are confined under the
48 indenter and thus do not cause catastrophic failure.
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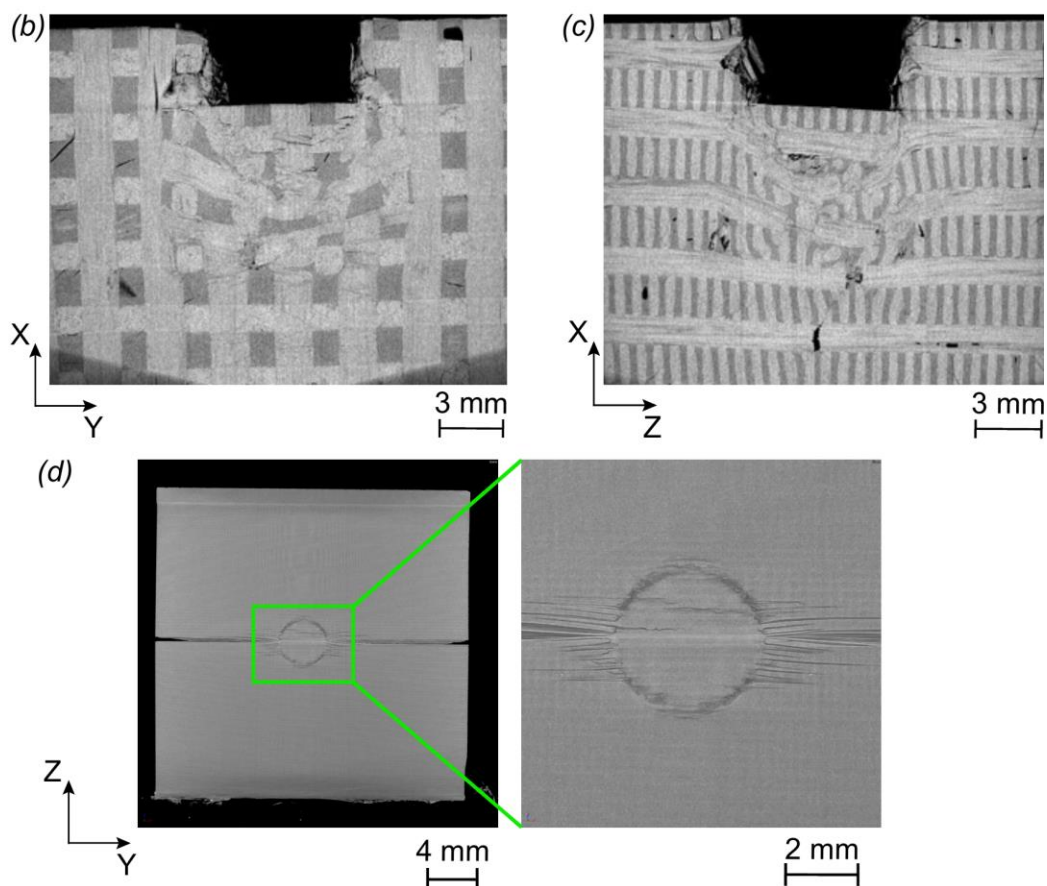
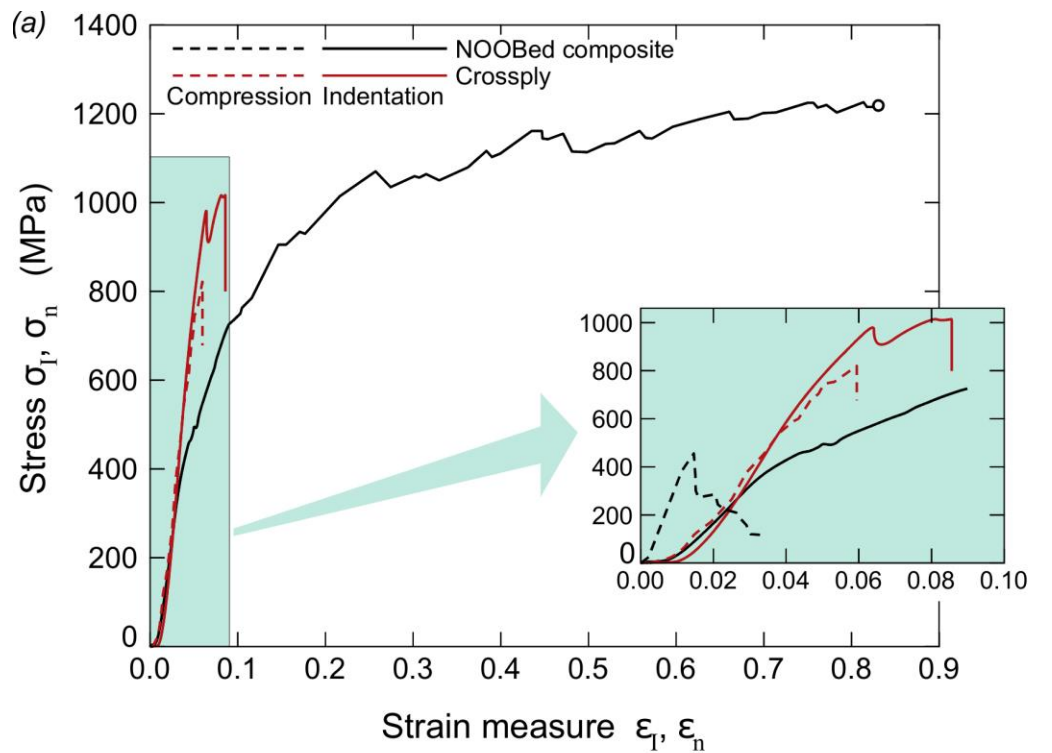


Fig. 3: (a) The measured X -direction indentation ($\sigma_l - \epsilon_l$) and uniaxial compression ($\sigma_n - \epsilon_n$) responses of the 3D noobed and cross-ply composites. The inset shows a zoom-in of the early part of the loading history. (b-c) XCT images of the noobed composite that show the deformation under the indenter after unloading from a strain level $\epsilon_l \approx 0.8$. (d) XCT image of the deformation/failure of the cross-ply specimen after completion of the indentation loading.

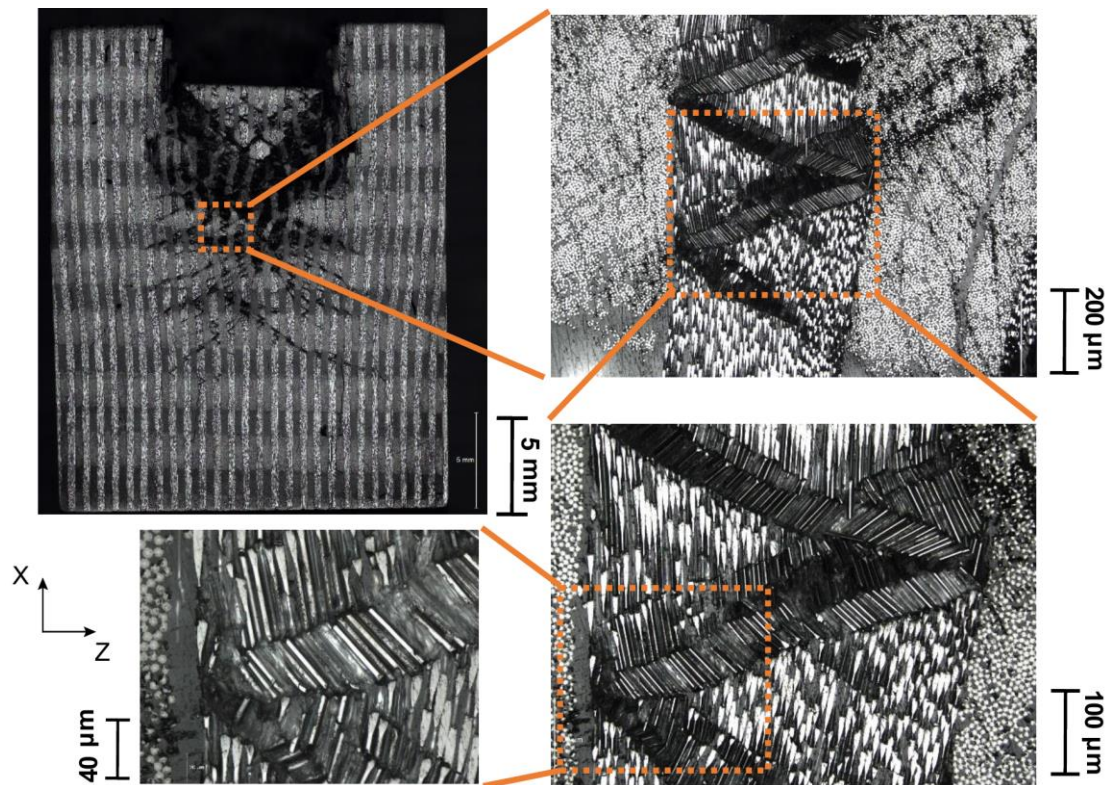


Fig. 4: Optical micrograph of the XCT section in Fig. 3c for the X -direction indentation of the noobed composite. Images at varying levels of magnification are included to illustrate the kink-bands within the X -direction tows.

3.4 Comparison with the indentation response of cross-ply composites

Measurements of the Z -direction indentation response of the cross-ply laminates and the corresponding uniaxial compressive response are included in Fig. 2a. Under uniaxial compression in the Z -direction, the cross-ply laminates fail by indirect tension^{28,29} resulting in the elastic-brittle behaviour seen in Fig. 2a. The indentation behaviour is also dominated by indirect tension with the peak indentation and uniaxial compression strength being approximately equal (i.e. there is no enhancement of the indentation strength over the uniaxial compression strength as observed in the noobed composite). However, under indentation loading there is no catastrophic failure. Rather, the failure is progressive with significant load drops associated with the failure of plies immediately under the indenter. This is followed by a more gradual increase in the load, as the indenter compressively loads undamaged plies. Since the plies fail by the indirect tension mechanism, they recoil into the surrounding composite upon tensile failure, resulting in delamination as seen in the XCT image included in Fig. 2d (this image was taken at the end of the loading response shown in Fig. 2a). Similar behaviour has previously been reported for other cross-ply laminates including ultra high molecular weight polyethylene composites^{28,30}. The comparison in Fig. 2a indicates that the Z -direction indentation responses of the noobed and cross-ply composites are similar except that the cross-ply composite has a serrated response associated with ply failure while the noobed composite has a relatively smooth hardening curve.

While the cross-ply and noobed composites have reasonably similar Z -direction indentation responses, their X -direction indentation behaviours are markedly different

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3 as seen in Fig. 3a. Catastrophic failure of the cross-ply specimen occurs at an
4 indentation strain $\varepsilon_I \approx 0.09$ while the noobed composite exhibits a ductile indentation
5 response with no loss of load carrying capacity even at $\varepsilon_I \approx 0.8$. The failure of the
6 cross-ply specimen is a result of delamination that emanates from the indented region
7 as seen in the XCT image in Fig. 3d. This delamination is triggered by kink-band
8 formation in the plies with fibres aligned along the X -direction. This hypothesis is
9 consistent with the observation that the peak X -direction strength and corresponding
10 uniaxial compression strengths of the cross-ply composites are approximately equal;
11 see Fig. 3a.
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15 The comparisons between the indentation responses of the noobed and cross-ply
16 composites suggest that the noobed composites perform at-least on-par with cross-ply
17 composites in the Z -direction but have a greatly superior response in other directions.
18 The noobed composites have an indentation response reminiscent of a ductile metal,
19 albeit at significantly lower density. This combined with the absence of delamination
20 is expected to make 3D noobed composites attractive for impact and ballistic
21 protection applications where the direction of loading is uncertain and spall resistance
22 is an important requirement.
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26 27 **4. Finite element modelling of the indentation response**

28 In order to better understand the experimentally observed deformation and failure
29 modes of the noobed composites, FE calculations of the indentation responses in the
30 Z and X -directions were performed. The discrete microstructure comprising fibres and
31 matrix is not modelled explicitly. Rather, we use the homogenised model developed
32 by Das et al.²² which was shown to capture the uniaxial compressive response with
33 reasonable fidelity. In this approach the noobed composite was modelled as a two-
34 phase material comprising Z -direction tows within a *homogenised matrix* that
35 represents the smeared-out properties of the matrix pockets along with the X and Y -
36 direction tows.
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39 40 *4.1 Material model*

41 Two anisotropic materials are used to model the Z -direction tows and the
42 homogenised matrix. Here we detail the material properties used to describe both
43 these materials. Both materials are modelled as anisotropic elastic, perfectly plastic
44 materials with the anisotropic plasticity described by the Hill³¹ anisotropic plasticity
45 model. In the following, all the relevant anisotropic properties will be stated using the
46 global co-ordinate system. For example, E_Z^Z and E_X^Z denote the longitudinal and
47 transverse moduli, respectively of the Z -direction tow (the superscript specifies that
48 these properties relate to the Z -direction tow while the subscripts specify the direction
49 of the property). Similarly, E_X^X and E_Z^X are the longitudinal and transverse moduli,
50 respectively of the X -direction tow while E_X^h and E_Z^h are the moduli of the
51 homogenised matrix in the X and Z -directions, respectively.
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56 First consider the Z -direction tow. We model it as a transversely isotropic medium
57 with the Z or fibre direction being normal to the plane of isotropy. Then the elastic
58 strains ε_{ij}^e are related to the stresses σ_{ij} in the (X, Y, Z) co-ordinate system via 5
59 independent elastic constants as
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$$\begin{pmatrix} \varepsilon_{XX}^e \\ \varepsilon_{YY}^e \\ \varepsilon_{ZZ}^e \\ \varepsilon_{YZ}^e \\ \varepsilon_{XZ}^e \\ \varepsilon_{XY}^e \end{pmatrix} = \begin{pmatrix} 1/E_X^Z & -\nu_{YX}^Z/E_X^Z & -\nu_{ZX}^Z/E_Z^Z & 0 & 0 & 0 \\ -\nu_{XY}^Z/E_X^Z & 1/E_X^Z & -\nu_{ZX}^Z/E_Z^Z & 0 & 0 & 0 \\ -\nu_{XZ}^Z/E_X^Z & -\nu_{XZ}^Z/E_X^Z & 1/E_Z^Z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/(2G_{YZ}^Z) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/(2G_{YZ}^Z) & 0 \\ 0 & 0 & 0 & 0 & 0 & (1 + \nu_{XY}^Z)/E_X^Z \end{pmatrix} \begin{pmatrix} \sigma_{XX} \\ \sigma_{YY} \\ \sigma_{ZZ} \\ \sigma_{YZ} \\ \sigma_{XZ} \\ \sigma_{XY} \end{pmatrix}. \quad (4.1)$$

Here, the symbols G and ν denote the shear modulus and Poisson's ratio, respectively while the subscripts denote the directions of the property and the superscript Z denotes that these properties refer to the Z -direction tows (i.e. notation analogous to that for the Young's modulus E). The total strain rate is then written as the sum of the elastic and plastic strain rates such that

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^p, \quad (4.2)$$

with the plastic strain rate given by the associated flow rule

$$\dot{\varepsilon}_{ij}^p = \dot{\lambda} \frac{\partial \Phi}{\partial \sigma_{ij}}, \quad (4.3)$$

in terms of the plastic multiplier $\dot{\lambda}$ and the Hill yield potential Φ . This potential is specified in terms of the constants F, G, H, L, M and N as

$$2\Phi \equiv F(\sigma_{YY} - \sigma_{ZZ})^2 + G(\sigma_{ZZ} - \sigma_{XX})^2 + H(\sigma_{XX} - \sigma_{YY})^2 + 2L\sigma_{YZ}^2 + 2M\sigma_{ZX}^2 + 2N\sigma_{XY}^2, \quad (4.4)$$

such that continued plastic flow occurs with $\Phi = 1/2$. The six constants F, G, H, L, M and N then follow from six strengths with respect to the principal axes of anisotropy, i.e.

$$G + H = \frac{1}{(Y_X^Z)^2}, \quad F + H = \frac{1}{(Y_Y^Z)^2} \quad \text{and} \quad G + F = \frac{1}{(Y_Z^Z)^2}, \quad (4.5)$$

where Y_X^Z , Y_Y^Z and Y_Z^Z are the tensile strengths in the X , Y and Z -directions, respectively (note that the Hill model assumes equal compressive and tensile strengths). Similarly, the shear strengths Y_{YZ}^Z , Y_{ZX}^Z and Y_{XY}^Z give the remaining constants via

$$L = \frac{1}{2(Y_{YZ}^Z)^2}, \quad M = \frac{1}{2(Y_{ZX}^Z)^2} \quad \text{and} \quad N = \frac{1}{2(Y_{XY}^Z)^2}. \quad (4.6)$$

The material surrounding the Z -direction tows comprises the X and Y -direction tows as well as the matrix pockets. This material is modelled as a single effective medium labelled the homogenised matrix. Based on the symmetries of the micro-structure of this material, Das et al.²² modelled this homogenised matrix also as a transversely isotropic medium with the Z -direction being normal to the plane of isotropy. Thus, again we use an elastic law of the form Eq. (4.1) with plastic flow modelled via Hill's anisotropic plastic model. Derivations of the elastic and plastic properties of this effective medium are given in the Supplementary Information using the homogenisation procedure developed by Das et al.²². The elastic and plastic properties of the Z -direction tows and the homogenised matrix are summarised in Tables 1 and 2, respectively; these completely specify the elastic law and can also be used to determine the 6 constants of the Hill model.

Z-direction tow	$E_Z^Z = 65$	$E_X^Z = E_Y^Z = 4.2$	$\nu_{ZX}^Z = 0.25$	$\nu_{XY}^Z = 0.25$	$G_{XZ}^Z = G_{YZ}^Z = 1.7$
homogenised matrix	$E_Z^h = 7.1$	$E_X^h = E_Y^h = 54$	$\nu_{ZX}^h = 0.25$	$\nu_{XY}^h = 0.25$	$G_{XZ}^h = G_{YZ}^h = 2.8$

Table 1: The elastic properties of the transversely isotropic Z-direction tows and the homogenised matrix in the 3D noobed composite. All the moduli are given in GPa.

Z-direction tow	$Y_Z^Z = 1300$	$Y_X^Z = Y_Y^Z = 170$	$Y_{XZ}^Z = Y_{XZ}^Z = Y_{ZY}^Z = 85$
Homogenised matrix	$Y_Z^h = 1260$	$Y_X^h = Y_Y^h = 940$	$Y_{XZ}^h = Y_{XZ}^h = Y_{ZY}^h = 92$

Table 2: The plastic/failure strengths of the Z-direction tows and the homogenised matrix in the 3D noobed composite. In this table, all the strengths are in MPa.

4.2 Description of the boundary value problem

The finite element (FE) calculations were performed using the commercial FE package ABAQUS. In order to reduce the size of the computation, cubic specimens comprising a grid of 16×16 Z-direction tows were employed (Fig. 5a). In these specimens, the Z-direction tows had cross-sectional dimensions $1.25 \text{ mm} \times 1.25 \text{ mm}$ and were arranged in a cubic grid with a centre-to-center tow spacing of 3.06 mm consistent with the unit cell sketched in Fig. 1c. The same specimen was used for both the Z and X-direction indentation calculations with the specimen discretised using 8-noded linear brick elements (C3D8R in the ABAQUS notation) of size $\sim 0.05 \text{ mm}$. Perfect bonding was assumed between the homogenised matrix and the Z-direction tows but an imperfection to trigger the formation of a kink-band was introduced into the Z-directions tows (this imperfection influences the predictions of the Z-direction indentation response but has virtually no effect on the X-direction indentation behaviour). The imperfection is sketched in Fig. 5b and comprised a region within which fibres were assumed to be misaligned. This imperfect region was located at mid-height in all the Z-directions tows, had a width $w = 200 \mu\text{m}$ and was inclined at an angle $\beta = 20^\circ$ with respect to the X-direction. The misalignment was specified by rotating the principal axes of the material anisotropy such that the material Z-direction was at an angle ϕ with respect to the global Z-direction in the X – Z plane as shown in Fig. 5b. Such a prescription of the initial imperfection to initiate a kink-band is commonly employed^{32,33} and consistent with a range of experimental observations³⁴.

The boundary conditions for simulating indentation were as follows. Employing symmetry of the problem we analysed a quarter of the specimen and employed symmetry boundary conditions on two perpendicular planes that intersect along the axis of the cylindrical indenter; see sketch in Fig. 5a for the setup for simulating the Z-direction indentation response. Displacements in the direction of indentation were completely restrained on the bottom of the surface of the specimen with the specimen sides specified to be traction-free. The circular flat-bottomed indenter was modelled as a rigid body with the general contact option in ABAQUS used to model contact between the indenter and the specimen. Loading was specified by applying an indentation displacement to the indenter: the applied displacement and resulting

work-conjugate load were used to define ε_I and σ_I in a manner analogous to the experiments. The FE analysis was conducted using the large deformation/non-linear geometry option.

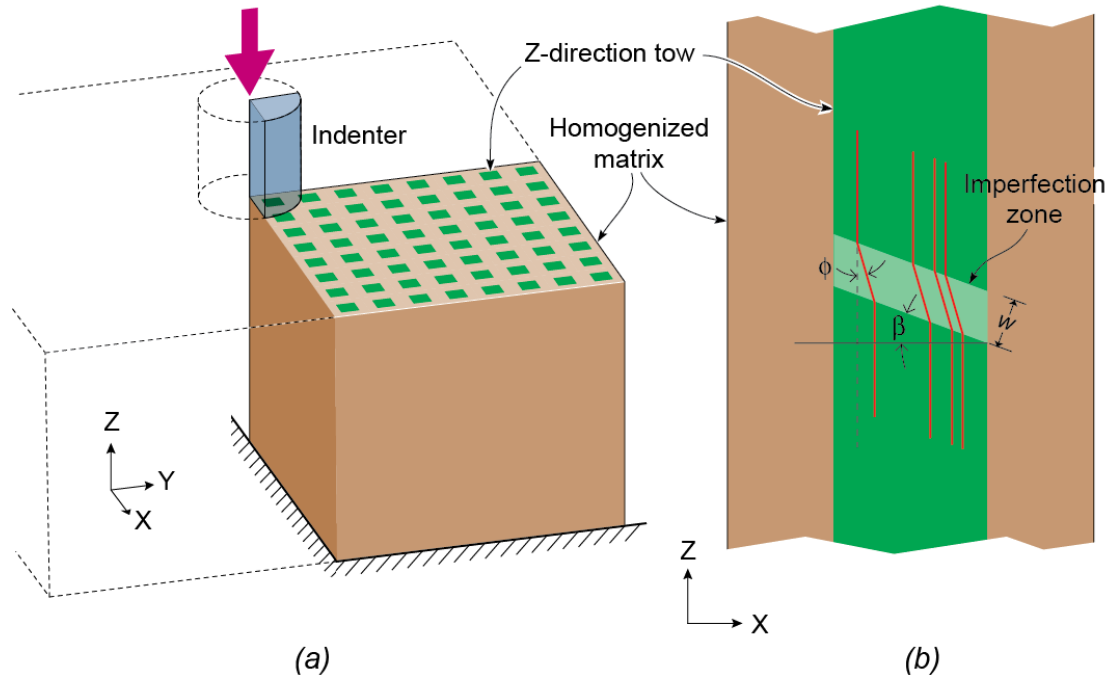


Fig. 5: (a) Sketch of the quarter specimen used in the FE analysis of the Z-direction indentation. (b) An illustration of the geometric imperfection included in the Z-direction tows.

4.3 Numerical predictions of the indentation response

Predictions of the Z and X-direction indentation responses of the noobed composites are included in Figs. 6a and 6b, respectively. First consider the case of indentation in the Z-direction. The maximum applied displacements in the FE calculations corresponded to an indentation strain $\varepsilon_I \approx 0.3$ as mesh distortion prevented the FE calculations from being carried out any further. Excellent agreement is observed between measurements and predictions over the range of displacements computed in the FE analysis including an accurate prediction of the onset of non-linearity and the subsequent hardening rate. Corresponding predictions of the deformation modes under the indenter at instants P, Q, and R marked in Fig. 6a are shown in Fig. 6c. In Fig. 6c we show contours of the plastic strain ε_{ZX}^p on a plane that sections the specimen along the X-Z diametrical plane of the indenter. Here we see that very early in the deformation history (P) a kink-band has formed in a centrally located Z-direction tow with the development of plasticity at the edge of the indenter. However, this plasticity causes no significant non-linearity in the overall indentation response. Multiple and reflected kink-bands developed within the Z-direction tows with increasing indentation displacement, and at the end of the calculation (R) we see extensive zig-zagged kink-band formation along as well as bands of plasticity, emanating from the edges of the indenter, reminiscent of the shear bands seen in Fig. 2c.

On the other hand, the results in Fig. 6b show that the FE calculations grossly under-predict the X-direction indentation strength. The associated predicted deformations at points L, M and N marked in Fig. 6b are shown in Fig. 6d. The deformations here are

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3 shown on a section through the specimen along the $X - Y$ diametrical plane of the
4 indenter with contours of the plastic strain ε_{XY}^p included in Fig. 6d. The FE
5 calculations correctly predict the development of shear bands emanating from the
6 edge of the indenter that shear the Z -directions tows. However, the X and Y -direction
7 tows are not explicitly modelled here and thus the approach does not include
8 predictions of kink-bands in the X -direction tows as seen in Fig. 4. We associate the
9 poor fidelity of the FE model of Das et al.²² in this case to the fact that the X -direction
10 tows are not explicitly modelled, i.e. this study suggests that a more comprehensive
11 model which explicitly includes tows in all directions is required to capture the full
12 multi-axial response of the noobed composites.
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18 5. Concluding remarks

19 The quasi-static indentation response of a 3D noobed (the acronym NOOB stands for
20 Non-interlacing, Orientating Orthogonally and Binding) composite comprising carbon
21 fibre tows in an epoxy matrix has been investigated. The measurements indicate that
22 the indentation response is ductile with continued indentation occurring by a
23 combination of kink-band propagation in the tows aligned with the indentation
24 direction and shearing of the tows in the orthogonal directions. Importantly, the
25 response is nearly isotropic with the indentation strength being approximately equal in
26 three orthogonal directions. By contrast, the out-of-plane indentation of a traditional
27 cross-ply carbon fibre laminate occurs by progressive tensile ply failure resulting in a
28 serrated load versus displacement curve. Nevertheless, the average indentation
29 strength in this direction is approximately equal to that of the 3D noobed composite.
30 However, cross-ply composites delaminate when indented in the in-plane direction
31 resulting in an elastic-brittle response.
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35 A finite element (FE) analysis of the indentation response of the noobed composites is
36 also presented wherein only one set of tows is explicitly modelled while the other two
37 sets of orthogonal tows and the matrix pockets are homogenized into an effective
38 medium. This simplified approach accurately captures the indentation response in the
39 direction of the tows that are explicitly modelled but under-predicts the indentation
40 strength in the other directions. The modelling study suggests that all tows of the 3D
41 composite need to be explicitly considered to capture the multi-axial response of these
42 composites with adequate fidelity.
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46 Traditional cross-ply carbon fibre laminates and state-of-the-art ultra high molecular
47 weight polyethylene composites³⁵ have a saw-tooth indentation load versus
48 displacement curve for indentation perpendicular to the plane of the plies. Moreover,
49 these composites are extremely brittle with delamination limiting the energy
50 absorption for indentation along the ply directions. We have demonstrated here that
51 3D noobed composites not only have smooth load-displacement curves that are
52 reminiscent to those of ductile metals for indentation in all directions, but also
53 undergo no delamination. This is all achieved at densities significantly lower than
54 structural metals that display equivalent ductility. These noobed composites are thus
55 strong candidates for applications such as impact protection where the loading
56 direction is not known a-priori and spall resistance is an important requirement.
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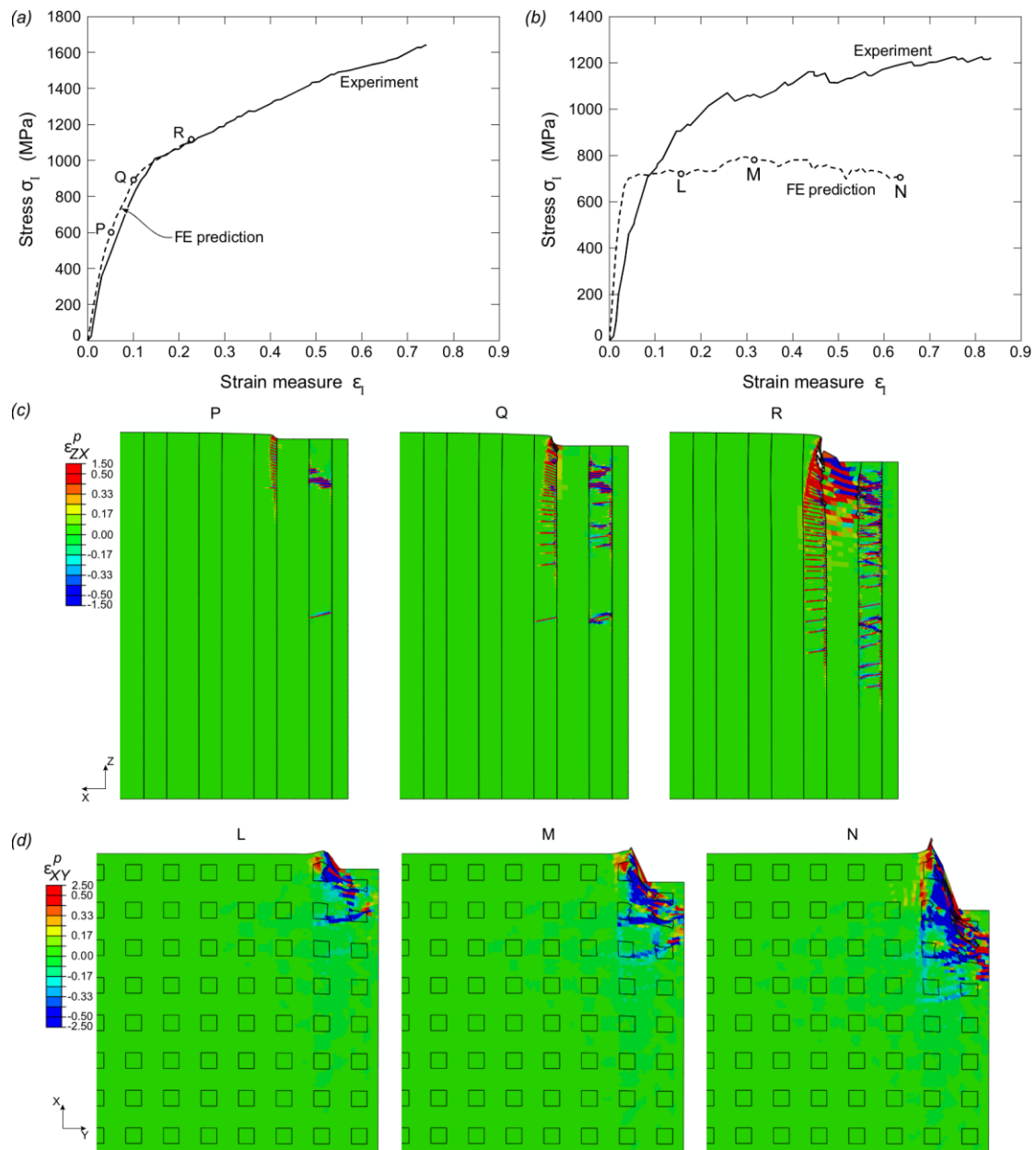


Fig. 6: Comparison between the measured and FE predictions of the (a) Z-direction and (b) X-direction indentation responses of the noobed composites. The corresponding FE predictions of the deformation along with distributions of the plastic strain are shown in (c) for Z-direction and in (d) X-direction indentation. The columns in (c) and squares in (d) indicate the Z-direction tows.

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Supplementary information

Estimates of the effective properties of the tows and homogenised matrix

The 3D noobed composite comprised Toray T700S 12k carbon fibre tows in a NM FW3070 epoxy matrix. The properties of these two constituents as given by the fibre and matrix manufacturers are:

(i) The fibre Young's modulus and Poisson's ratio are $E_f = 210$ GPa and $\nu_f = 0.25$, respectively while the tensile strength of the fibres, $\sigma_f = 4$ GPa.

(ii) The matrix Young's modulus and Poisson's ratio are $E_m = 3$ GPa and $\nu_m = 0.25$, respectively while the matrix tensile yield strength, $\sigma_m = 140$ MPa.

Here we summarise the procedure developed by Das et al.^{S1} to use these properties of the constituents to derive estimates of the effective properties of the different phases in the 3D noobed composite comprised of four phases. The Z -direction tows have a significantly larger cross-sectional area compared to the X and Y -direction tows. Thus, in the finite element (FE) calculations in Section 4 we explicitly considered the Z -direction tows but modelled the X and Y -direction tows and the matrix pockets that surround the Z -direction tows as a single effective medium referred to as the homogenised matrix. We shall thus first derive effective properties for the tows and then use them to estimate properties of the homogenised matrix. All the relevant anisotropic properties will be stated using the global co-ordinate system. For example, E_Z^Z and E_X^Z denote the longitudinal and transverse moduli, respectively of the Z -direction tow (the superscript specifies that these properties relate to the Z -direction tow while the subscripts specify the direction of the property). Similarly, E_X^X and E_Z^X are the longitudinal and transverse moduli, respectively of the X -direction tow while E_X^h and E_Z^h are the moduli of the homogenised matrix in the X and Z -directions, respectively.

S1. Elastic properties

The tows are assumed to be transversely isotropic with the fibre direction normal to the plane of isotropy. We first consider the Z -direction tows. The longitudinal modulus E_Z^Z is given by the Voigt bound as $E_Z^Z = f_Z E_f + (1 - f_Z) E_m$ while the transverse moduli $E_X^Z = E_Y^Z$ are given by the equivalent Reuss bound. Since the Poisson's ratios of the matrix and fibres are assumed equal, we take $\nu_{ZX}^Z = \nu_{ZY}^Z = \nu_m$ and the shear modulus G_{XZ}^Z is estimated from a Reuss bound such that

$$\frac{1}{G_{XZ}^Z} = \frac{2(1 + \nu_f)f_Z}{E_f} + \frac{2(1 + \nu_m)(1 - f_Z)}{E_m}. \quad (S1.1)$$

The five independent elastic constants required to describe the elastic properties of the transversely isotropic Z -direction tows are listed in Table S1. Equivalent estimates can be evaluated for the X (or Y)-direction tows with f_Z replaced by f_X . These properties are also listed in Table 1 for the X -direction tow. Note that the X -direction is normal to the plane of isotropy for the X -direction tow and hence the components of the elasticity tensor listed in Table 1 differ for the X and Z -direction tows.

We proceed to calculate the properties of the homogenised matrix that surrounds the Z -direction tows. From the unit cell sketched in Fig. 1c it is clear that this homogenised matrix is an orthotropic effective material with Young's moduli equal in the X and Y -directions. Thus, in order to simplify the constitutive description it is

reasonable to assume that this homogenised matrix is also transversely isotropic with the Z -direction being normal to the plane of isotropy. Again, since all the constituents have equal Poisson's ratios it is reasonable to take $\nu_{ZX}^h = \nu_{XY}^h = \nu_m$. The Voigt estimate for the moduli $E_X^h = E_Y^h$ is given as

$$E_X^h = \frac{\nu_X(E_X^X + E_Y^X) + \nu_m E_m}{2\nu_X + \nu_m}, \quad (\text{S1.2})$$

while that for modulus E_Z^h is

$$E_Z^h = \frac{2\nu_X E_Y^X + \nu_m E_m}{2\nu_X + \nu_m}. \quad (\text{S1.3})$$

Similarly, the shear modulus G_{XZ}^h is given by the Voigt bound as

$$G_{XZ}^h = \frac{\nu_X(G_{YZ}^X + G_{XZ}^X) + \nu_m \frac{E_m}{2(1 + \nu_m)}}{2\nu_X + \nu_m}, \quad (\text{S1.4})$$

where $G_{YZ}^X = 0.5E_Y^X/(1 + \nu_{ZY}^X)$. The five independent elastic constants for this effective medium are listed in Table S1.

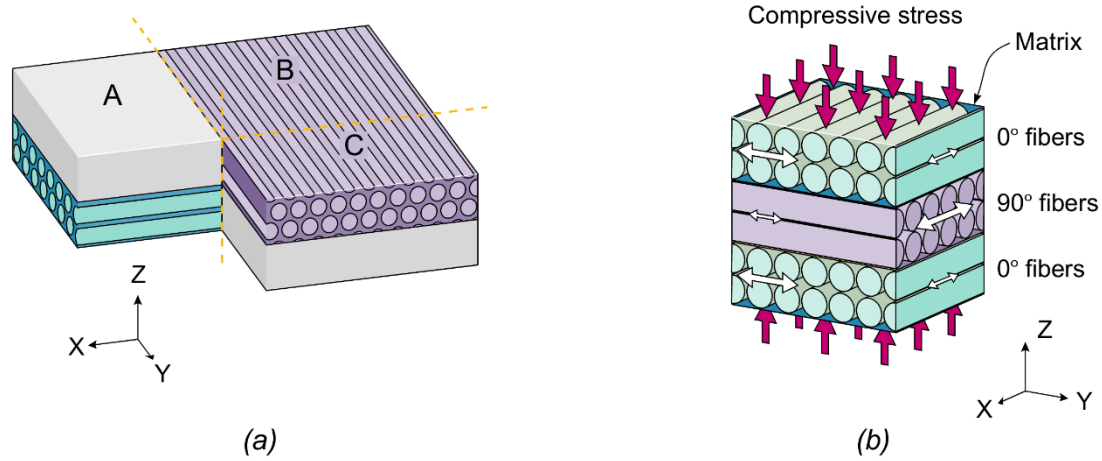


Figure S1: (a) Sketch of the homogenised matrix within the unit cell with the constituents of the homogenised matrix also indicated. The three regions A, B and C into which the homogenised matrix within the unit cell is divided for the analysis of the effective properties are also indicated. (b) Sketch of the indirect tension mechanism operative during the compression of region B in the Z -direction.

Z-direction tow	$E_Z^Z = 65$	$E_X^Z = E_Y^Z = 4.2$	$\nu_{ZX}^Z = 0.25$	$\nu_{XY}^Z = 0.25$	$G_{XZ}^Z = G_{YZ}^Z = 1.7$
X-direction tow	$E_Z^X = E_Y^X = 8.8$	$E_X^X = 142$	$\nu_{XZ}^X = 0.25$	$\nu_{ZY}^X = 0.25$	$G_{XZ}^X = G_{XY}^X = 3.6$
homogenised matrix	$E_Z^h = 7.1$	$E_X^h = E_Y^h = 54$	$\nu_{ZX}^h = 0.25$	$\nu_{XY}^h = 0.25$	$G_{XZ}^h = G_{YZ}^h = 2.8$

Table S1: The elastic properties of the transversely isotropic tows and the homogenised matrix in the 3D noobed composite. The X and Y -direction tows have identical properties with the super/subscript X replaced by Y . All the moduli are given in GPa.

S1.2 Plastic/failure strengths

In estimating the plastic/failure strengths of the different phases we note that the strength for tensile loading along the fibre direction is limited by the failure strength σ_f of the fibres while loading in other directions (e.g. transverse or shear loading) is limited by flow of the matrix around fibres. Since the fibre strength is significantly greater than the matrix strength, the fibres may be assumed to be rigid for the purposes of analysis of strength in matrix flow governed regimes. With this understanding we proceed to develop estimates for the anisotropic strengths of the tows and the homogenised matrix.

First consider the Z-direction tow. The longitudinal tensile strength is limited by fibre fracture and directly given by a Voigt estimate as $Y_Z^Z = f_Z \sigma_f + (1 - f_Z) \sigma_m$. The calculation of the transverse strength is more complex. A Reuss estimate assuming rigid fibres will specify that the transverse strength is equal to that of the matrix which is a poor estimate as the rigid fibres constrain the flow of the matrix and enhance the strength. Bele and Deshpande^{S2} provided a simple analytical estimate (verified via FE calculations) for the transverse strength of a composite comprising rigid cylinders dispersed in a plastic matrix. Here we use that prescription to estimate the transverse and shear strengths of the tow. The Hashin^{S3} lower bound for the Young's modulus E of a composite comprising a volume fraction f_Z of rigid inclusions in an incompressible matrix of modulus E_m is

$$\frac{E}{E_m} = 1 + \frac{5f_Z}{2(1 - f_Z)}. \quad (\text{S2.1})$$

This linear bound can be transformed to an estimate of the strength using the method proposed by Suquet^{S4} by using the bound (S2.1) as the properties of a fictitious linear comparison composite. The transverse strength is then given as

$$Y_X^Z = Y_Y^Z = \sigma_m \sqrt{\frac{E}{E_m} (1 - f_Z)}, \quad (\text{S2.2})$$

with E/E_m given by Eq. (S2.1). The shear strengths are assumed to be related to the transverse strength via a Tresca yield criterion such that $Y_{XY}^Z = Y_{XZ}^Z = Y_{ZY}^Z = Y_X^Z/2$. These properties of the Z-direction tow are listed in Table S2. The plastic/failure strengths for the X-direction tow can also be estimated in an analogous manner and these predictions are also listed in Table S2.

Next consider the homogenised matrix sketched in Fig. S1a. Uniaxial loading in the X-direction results in longitudinal and transverse loading of the X-direction and Y-direction tows, respectively as well as loading of the matrix pockets. The average stress sustained by this homogenised material at failure then follows as

$$Y_X^h = \bar{A}_{ZY}^X (Y_X^X + \sigma_m) + 0.5 \bar{A}_{ZY}^Z (\sigma_m + Y_Y^X), \quad (\text{S2.3})$$

with $Y_Y^h = Y_X^h$. In order to calculate the strength Y_Z^h it is convenient to divide the X – Y plane of the homogenised matrix into three regions A, B and C as shown in Fig. S1a. The uniaxial stress in the Z-direction over regions A and C is limited to the matrix yield strength σ_m while compression of region B is equivalent to the compression of a cross-ply laminate. The out-of-plane compression of a cross-ply laminate results in the development of tensile stresses in the fibres due to the anisotropic Poisson expansion of the cross-ply; see Fig. S1b. This so-called indirect

tension mechanism was analysed by Attwood et al.^{S5} who showed that the compressive strength of cross-ply laminates approximately equals the in-plane tensile strength Y_X^X of each lamina. The strength Y_Z^h then is given by the average over the three regions such that

$$Y_Z^h = \frac{2\bar{A}_{XY}^m \sigma_m + (1 - \bar{A}_{XY}^Z - 2\bar{A}_{XY}^m) Y_X^X}{1 - \bar{A}_{XY}^Z}, \quad (\text{S2.4})$$

where \bar{A}_{XY}^m is the area fraction that the matrix pockets occupy in the $X - Y$ plane on the surface of the unit cell (it is equal to the ratio of the area of region A to the area $(1.81 + 1.25)^2 \text{ mm}^2$ of the unit cell projected on the $X - Y$ plane). We assume all shear strengths to be equal ($Y_{ZX}^h = Y_{ZY}^h = Y_{XY}^h$) and given by a Voigt bound such that

$$Y_{ZX}^h = \frac{2v_x Y_{XY}^X + v_m \sigma_m / 2}{2v_x + v_m}, \quad (\text{S2.5})$$

where we have assumed that the matrix shear strength is $\sigma_m / 2$ (Tresca yield criterion). These plastic collapse and failure strengths are listed in Table S2.

Z-direction tow	$Y_Z^Z = 1300$	$Y_X^Z = Y_Y^Z = 170$	$Y_{XY}^Z = Y_{XZ}^Z = Y_{ZY}^Z = 85$
X-direction tow	$Y_X^X = 2800$	$Y_Z^X = Y_Y^X = 200$	$Y_{XY}^X = Y_{XZ}^X = Y_{ZY}^X = 100$
Homogenised matrix	$Y_Z^h = 1260$	$Y_X^h = Y_Y^h = 940$	$Y_{ZX}^h = Y_{ZY}^h = Y_{XY}^h = 92$

Table S2: The plastic/failure strengths of the tows and the homogenised matrix in the 3D noobed composite. In this table all the strengths are in MPa.

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