Adaptive Behaviors Can Improve the System Consilience of a Network System

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Abstract

As a recently reported network property, consilience degree (CSD) indicates how well a network system integrates its topology and node activities together to serve a specific systemic goal. As is well-known, many natural and man-made systems are complex networks where, besides network topology, node activity states also play an important role in determining system performance. For example, a collaborative project involving friends is more likely to succeed than one involving enemies, even though the topology of organization network is the same. The concept of CSD can quantitatively distinguish the difference between the involvement of friends and the involvement of enemies. This paper reports a simulation study on the adaptive behaviors of nodes based on the selfish rule and the following-others rule, and the simulation results show that, based on such adaptive behavior of nodes, a network system will automatically evolve to a high level of system consilience. The simulation study also demonstrates that a high level of system consilience resulting from adaptive behavior will contribute to increased system resistance to external disturbances. The generality of adaptive behaviors in reality implies that CSD is an inherent attribute of real-world network systems, and therefore, the concept of CSD has significant application potential in the study of adaptive behavior in network systems.

Keywords: *Adaptive Behavior; Complex Network; Consilience; Disturbance.*

1. Introduction

Human society is a complex network system (Ostrom, 2009; Ball, 2012). To study various phenomena within it, the concept of "degree of connectedness" (CND) is often used, which indicates how many nodes are connected to a given node in a network system (Albert and Barabási, 2002; Boccaletti et al., 2006). For example, the method of CND distribution is applied to study the co-authorship between scholars (Newman, 2001), the sexual contacts in a community (Liljeros et al., 2001), and the collaborations between movie actors (Watts and Strogatz, 1998), and it proves that all of these human network systems share a feature of scale-free topology, where a few hub nodes have many links, while most other nodes have very few links. Another famous finding of CND-based study on human society is that our world is a small one of 6-degree of separation in terms of friendship (Kochen, 1989). Despite of the success of CND-based studies on human society, many complicated societal phenomena still remain unexplained and there is thus still a demand for new theories and methods for the study of complex network systems (OECD, 2011; Ball, 2012; Helbing, 2013).

Fig.1 illustrates an example where CND-based theories and methods become of little use. In spite of exactly the same network topology and human resources, Network System 1, by considering the similarity in expertise, is more likely to deliver a successful project than Network System 2 according to our daily common sense. Clearly, CND-based network theories and methods largely fail to quantify or distinguish the system capability to serve the purpose of delivering a successful project in Fig.1. This failure of CND is largely because it only focuses on network topology and ignores the nature of the interconnections between kinds of expertise. Actually, the performance of many real-world network systems is determined not only by network topology but is also largely dependent on whether the heterogeneity in nodes activities has been taken into account during the network topology design. In reality, it is often observed that connecting two nodes with conflictive activities will only degrade system performance. Therefore, to study real-world social network systems, various node activities, such as individual fitness (Caldarelli et al., 2002), friendship paradox (Eom and Jo, 2014), social norms and collaborative expectations (Peyton, 1998), epidemic dynamics (Pastor-Satorras and Vespignani 2001), and co-evolutionary activities (Nardini et al., 2008; Aoki and Aoyagi, 2012) have been introduced. In fact, the consideration of both topology and node activities simultaneously is also widely practiced in many domains of engineering and technology, for example, neural networks (Daido and Nakanishi 2004), power grids (Blaabjerg et al. 2006), protein networks (Maslov and Sneppen, 2002) and data mining (Hric et al. 2016; Peel et al. 2017). These case studies clearly show that focusing only on CND and network topology is not enough to understand real-world network systems. Besides, some general methods used for investigating assortativity mixing (Newman, 2003; Newman, 2010; Noldus and Van Mieghem, 2015), dyadic effect (Cinelli et al., 2017; Park and Barabási, 2007) and metadata (Eom and Jo, 2014; Peel et al. 2017) have also proved highly useful to study the importance of integrating network topology and node activities, and these methods have often been described as parallel concepts to the concept of CND.

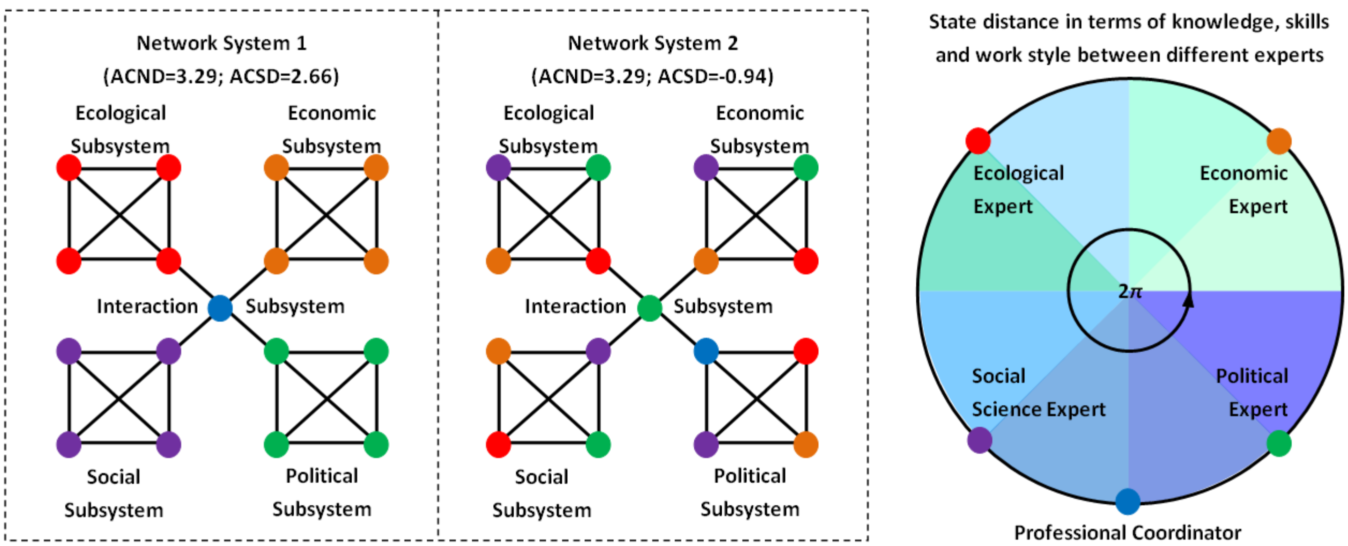

Fig.1. Networks of collaborating people (for the definition and calculation of ACND and ACSD, see Section 2).

We have recently proposed a new, fundamental network property, which we termed the *consilience degree* (CSD), which aims to quantitatively measure how well a network system integrates both the network topology and the node activity states together for a given systemic goal. In essence, CSD was proposed and positioned as an extension or generalization of CND (CND is just a special case of CSD) (Hu et al., 2014; Hu et al., 2017). In short, the CSD of a node is not only determined by how many other nodes it connects to, but also largely dependent of what kind of node activity states those connected nodes have. If a node is connected to many other nodes which have rather conflictive activity states to its own, then, although the node will have a large CND value, it will possess a very small CSD value. In other words, the concept of CSD can tell that connecting two conflictive nodes is not beneficial for system performance, while connecting two supportive nodes is desirable in terms of a given systemic goal. For example, in Fig.1, the nodes in those sub-systems of Network System 1 have much larger CSD values than those of Network System 2. Basically, system consilience, measured by the average CSD values of all nodes, may describe phenomena in human society, such as consensus of wills, convergence of opinions, and coordination of activities, which are all beyond the capability of the CND concept (Hu, et al., 2014; Hu, et al., 2017).

This paper, by investigating the relationship between adaptive behaviors and system consilience, aims to further demonstrate the usefulness of the CSD concept in studying complex network systems. It is well known that adaptive behaviors of individuals are common in complex network systems (Grefenstette, 1992; Ostrom, 2009; Ball, 2012; Shaukat and Chitre, 2016). The hypothesis of this study is that, if system consilience is related to adaptive behaviors in some sense, then the concept of CSD can be safely viewed as an inherent attribute of real-world systems, rather than an artificial network property in theory. The remainder of this paper is organized as follows. Section 2 briefly reviews the mathematical description of the CSD concept. Section 3 develops a model of adaptive behaviors based on the selfish rule and the following-others rule. Section 4 conducts a simulation study on system resistance to external disturbances, followed by some main conclusions to end the paper in Section 5.

2. Mathematical definition of consilience degree (CSD)

Here, based on the original concept of CSD proposed in Hu et al. (2014 and 2017), we give a brief review of the mathematical description of CSD. Suppose there is a networked system, whose topology is given by $G(V,E)$, composed of node set V and link set E, where V has N_N nodes and E has N_E links. Let the adjacency matrix record all links, i.e., $M_A(i,j) = 1$ means that there is a link between nodes i and j, and otherwise $M_A(i,j) = 0$. It is known that the degree of connectedness (CND) of node i, indicating how many other nodes are connected to node i, is mathematically defined as (Albert and Barabási, 2002; Boccaletti et al., 2006):

$$k_{CN,i} = \sum_{j=1}^{N_N} M_A(i,j). \tag{1}$$

The consilience degree (CSD) of node i is defined as:

$$k_{CS,i} = \sum_{j=1}^{N_N} M_A(i,j) \cdot f_{CS}(\theta_i,\theta_j), \tag{2}$$

where $\theta_i = [\theta_{i,1},...,\theta_{i,N_{ASD}}]$ represents the activity state of node i, and $N_{ASD} \geq 1$ the dimension of that activity state (in many natural, engineering, or social-ecological systems, nodes have multi-dimensional activity state); $\underline{f_{CS}} \leq f_{CS}(\theta_i,\theta_j) \leq \overline{f}_{CS}$ is called the "consilience function", determining how the states of nodes i and j will affect the overall performance if the nodes are connected, and $\underline{f_{CS}}$ and $\overline{f}_{CS}$ are the lower and upper bounds, respectively. In Eq. (2), $M_A(i,j)$ represents the network topology, and $f_{CS}(\theta_i,\theta_j)$ introduces the node activities, which are the focus of this study.

In the real world, individual nodes may act differently, but their activities need to serve the same systemic goal. Through the network topology, nodes interact with each other. When a specific systemic goal is concerned, the differences in the activities will mean that if some nodes are connected, they may interact well, whilst some others will conflict with each other if connected. In general, the node activity state and the consilience function in Eq.(2) can effectively describe such real world situations. For example, if the similarity in node activities helps performance, then we can define $f_{CS}(\theta_i,\theta_j) = \overline{f}_{CS}$ when $\theta_i = \theta_j$, while if complementarity between node activities is desirable, then we may have $f_{CS}(\theta_i,\theta_j) = \overline{f}_{CS}$ when $|\theta_i - \theta_j| \geq \theta_T$, where θ_T is a problem-specific threshold.

The definition of node activity state is often highly problem-dependent. Many factors, such as signal synchronization, compatibility of facilities, complementarity or similarity of expertise, willingness of collaboration, social opinion, personal attitude and cultural (dis)similarity usually may be used in the definition. The consilience function $f_{CS}(\theta_i,\theta_j)$ may also be of any form depending on the nature of the system concerned. The definition of node activity state and the design of $f_{CS}(\theta_i,\theta_j)$ will play a crucial role in applying CSD to study real-world network systems, and some metadata methods (e.g., Eom and Jo, 2014; Peel et al. 2017) may provide inspiration.

In this study, for the sake of simplicity, we assume $f_{CS}(\theta_i,\theta_j) = \cos(\theta_i - \theta_j)$, which gives $-1 \leq f_{CS}(\theta_i,\theta_j) \leq 1$ and it then follows that $-k_{CN,i} \leq k_{CS,i} \leq k_{CN,i}$. In the case where $f_{CS}(\theta_i,\theta_j) = 1$ for any pair of connected nodes in the system, CSD becomes exactly CND, i.e., $k_{CS,i} = k_{CN,i}$. From Eq. (1) and Eq. (2), one may conclude that CSD is an extension of CND, while CND is just a special case of CSD. Therefore, CSD is a more general, more fundamental network property than CND. Basically, if a node connects to other nodes that have states more compatible to its own, then the node has a higher CSD, which may indicate that it has a better capability to integrate available resources in the system. Such a capability is fundamentally important to the system if it is to achieve a certain systemic goal, but traditional network properties, such as CND, synchronization, clustering coefficient and robustness, can barely capture or measure it (Hu et al., 2014; Hu et al., 2017). In real-world network systems, there is often a "being together – but better not" situation (e.g., the Network System 2 in Fig.1). CND studies only the first part, the "being together", while CSD completes the picture by disclosing the second part "but better not".

In the example of Fig.1, despite the fact that the two systems have exactly the same network topology and human resources, Network System 1, by organizing itself according to the similarity in expertise of its members, is likely to achieve a better performance than Network System 2. CND based network theories largely fail to quantify or distinguish the capability of the two network systems to serve a specific systemic goal, because the CND based network properties of the two systems are exactly the same, e.g., they have the same average connection degree (ACND). However, if one brings the functional expertise of the members of the network system into play, which can be measured in terms of differences in knowledge, skills and style between different experts (see the subplot on the right-hand side in Fig.1), the average consilience degree (ACSD) calculated based on differences in expertise can capture and describe the overall performance difference of the two network systems. Thanks to the concept of CSD, one can quantitatively tell that Network System 1 in Fig.1 will perform better because it has a larger ACSD. The calculations of ACND and ACSD are as following.

$$\bar{k}_{CN} = \frac{1}{N_N} \sum_{i=1}^{N_N} k_{CN,i} \, , \tag{3}$$

$$\bar{k}_{CS} = \frac{1}{N_N} \sum_{i=1}^{N_N} k_{CS,i} \, . \tag{4}$$

When compared with other concepts specifically proposed to study the integration of network topology and node activity states, for example, assortativity mixing, dyadic effect, metadata, and link weight, the CSD concept also exhibits some potential.

Basically, dyadic effect and assortativity mixing both describe to what extent the setup of a link is influenced by the similarity in certain attributes of those two nodes that are connected by the link. In the study of dyadic effect, the node attribute in question is assumed to be binary-valued (Cinelli et al., 2017; Park and Barabási, 2007). In the study of assortativity mixing, the value of node attribute could be a more complicated discrete or scalar quantity. For example, if we define an assortativity coefficient based on CND (i.e., a node is more likely to connect to another node which has a similar CND to its own), then the node attribute may have any value within a discrete set $\{0,1,2,\ldots,N_N\text{-}1\}$ (Newman, 2003; Newman, 2010; Noldus and Van Mieghem, 2015).

In the study of CSD, a node attribute is called a node activity state, which may have a multi-dimensional, continuous value. The simulation results of Hu et al. (2017) clearly show that even though two network systems may have the similar assortativity coefficients, they may have rather different system consilience levels, which means that CSD may bring extra information. The form of node activity state in CSD is similar to the vector data used in metadata studies. However, the concept of metadata itself focuses on node attributes, and it needs to be combined with some topology-oriented methods, in order to (like dyadic effect and assortativity mixing) study the correspondence, relationship, dependency and/or correlation between node attributes and network structure (Eom and Jo, 2014; Peel et al. 2017). In contrast, CSD is a concept directly integrating network topology and node attributes, and its main aim is to assess overall system performance, which is a level above studying the relationship between node attributes and network structure.

In effect, the concept of CSD may be viewed as similar to link weight. For example, if we replace the consilience function $f_{CS}(\theta_i,\theta_j)$ with an equivalent link weight $w_{i,j}$, we will get exactly the same computational result. However, the form of $w_{i,j}$ could intuitively make people think link that weight is an attribute of link, having nothing to do with node activity states (Albert and Barabási, 2002; Boccaletti et al., 2006). In other words, link weight is largely a topology-oriented concept. Actually, the connecting effect of a link often results from both link attribute and node activity state. For example, a link may itself have a transmitting efficiency (such as when the link is a road, a railway or a flightpath), and the states of two nodes may be supportive or conflictive. Once a link is set up between the two nodes, the actual connecting effect (measured by link weight) is a combinational result of link transmission efficiency and similarity in node activity states, but the form of a single $w_{i,j}$ cannot convey such a combination. Fortunately, an extended form of CSD in Hu et al (2017) can clearly tell which factors may contribute to the actual connecting effect, and to what extent.

As reported in Hu et al., 2014 and 2017, the concept of CSD, by developing many new CSD-based network properties and models, can facilitate a new theoretical framework for the study of complex network systems. As demonstrated by the simulation results in Hu et al., 2014 and 2017, such network properties and models are rather different from traditional CND-based network properties and models, and they open a new window to deepen our understanding of many real-world complex systems, such as social-ecological systems. For instance, a society that has a consensus of wills and practices a coordination of activities between individuals for the sake of disaster prevention, mitigation and relief is often observed to be less vulnerable to disasters, and a CSD-based simulation study quantitatively echoes such a real-life observation (Shi, et al. 2014).

Before progressing to the next sections, we present the major variables and coefficients used in this study in Table 1 so that readers may review any of them easily whenever they wish.

Table 1. A list of major variables and coefficients used in this study.

Variable/Coefficient	Description	How to be used in experiments
$c_{\mathrm{CC},i}$	Clustering coefficient of node *i*.	To be calculated during the evolutionary process of adaptive behaviors.
$c_{\mathrm{NF},i}(t)$	Functional capacity of node *i* at time *t*.	Initialized as $c_{\mathrm{NF},i}(0)=1$, and then to be calculated during the simulation of an attack history.
d_{DinNAS}	Diversity in node activity states in a network system.	To be calculated during the evolutionary process of adaptive behaviors.
d_{DinNNAS}	Difference in neighborhood node activity states of node *i*.	To be calculated during the evolutionary process of adaptive behaviors.
$f_{\mathrm{CS}}(\theta_i,\theta_j)$	Consilience function	Set as $f_{\mathrm{CS}}(\theta_i,\theta_j)=\cos(\theta_i-\theta_j)$.
$k_{CN,i}$, $k_{CS,i}$	Connectedness degree (CND) and consilience degree (CSD) of node *i*.	To be calculated after network topology and node activity state distribution are given.
$\bar{k}_{CN}$, $\bar{k}_{CS}$	Average CND (ACND) and average CSD (ACSD) of a network system.	To be calculated after network topology and node activity state distribution are given.
$M_{\mathrm{A}}(i,j)$	Adjacency matrix, $M_{\mathrm{A}}(i,j)$ = 1/0 means there is/isn't a link between nodes *i* and *j*.	Record network topology, and under adaptive behaviors.

N_N, N_E	Numbers of nodes and links in a network system.	Set as N_N=100, N_E=400.
$N_{SN,i}(t)$, $N_{DN,i}(t)$	Numbers of supportive, disturbing neighboring nodes of node i at time t.	To be updated during the evolutionary process of adaptive behaviors.
P_{CAS}, P_{RWL}	Percentages of nodes randomly chosen to change activity states, and to rewire links, respectively, at each simulated time instant.	P_{CAS}=2% & P_{RWL}=0%, P_{CAS}=0% & P_{RWL}=2%, and P_{CAS}=1% & P_{RWL}=1% in three different experiments.
$P_{SR,i}(t)$, $P_{FO,i}(t)$	Probabilities of applying the selfish rule, the following-others rule to node i at time t, given node i is chosen to conduct an adaptive behavior.	To be calculated during the evolutionary process of adaptive behaviors.
S_θ	The speed of adjusting state.	S_θ=0.2.
T_{SP}	How many simulation time units in a simulation period of evolutionary process.	T_{SP}=50000, except for Fig.6, where T_{SP}=1000.
$\alpha(i)$	Selfish coefficient of node i.	$\alpha(i)$=0.3 except for Fig.6, where $\alpha(i)$=0, 0.05,…,1.
α_R	Recovering ratio.	α_R=0.1.
θ_i	Activity state of node i.	Within [0 2π], change under adaptive behaviors.
$\Omega_{SN,i}(t)$, $\Omega_{DN,i}(t)$	Sets of supportive, disturbing neighboring nodes of node i at time t (simulated time in this study).	To be updated during the evolutionary process of adaptive behaviors.

3. Adaptive behaviors and system consilience

As discussed in Section 1, there is a fundamental question about the concept of consilience degree (CSD): Is CSD necessary or useful to study real-world network systems? To shed a little more light on this question, in this section, we will investigate the relationship between the concept of CSD and two adaptive behaviors which are well acknowledged in real-world systems.

3.1 Modeling adaptive behaviors

In many natural and social-ecological systems, individuals, i.e., network system nodes, usually keep changing their activity states and links according to two common rules: the selfish rule and the following-others rule (Ball, 2012). It is well known that individuals in a society usually pursue the maximization of their own benefits/profits is the result of the selfish rule (Axelrod, 1997; Wexler, 2006), and the herd effect widely observed in both human and animal systems can be explained with the following-others rule (Granovetter, 1978; Hirshleifer and Teoh, 2003). Under the selfish rule, a node is more likely to change its activity state according to the states of supportive neighboring nodes, and it is also more likely to rewire a link from a disturbing neighboring node to a supportive node. Under the following-others rule, all neighboring nodes are classified into two sets, the supportive set and the disturbing set. The node is more likely to change its activity state according to the set which has more nodes, and the node is also more likely to rewire a link from the smaller set to a node which is connected to the larger set but currently not connected to the node.

Suppose at time instant t=0, we have an initial network system without consilience design, and therefore the system consilience level is low. First, we give a mathematic description of node adaptive behavior under the selfish rule. Assume at simulation time instant $t \geq 0$, node i has $N_{SN,i}(t)>0$ supportive neighboring nodes, and node i is changing its activity state $\theta_i(t)$ under the selfish rule. Then, at the next time instant t+1, the activity state of node i will become

$$\theta_i(t+1) = \theta_i(t) + s_\theta \times \left(\frac{\sum_{j \in \Omega_{SN,i}(t)} \theta_j(t)}{N_{SN,i}(t)} - \theta_i(t)\right), \tag{5}$$

where S_θ is the speed of changing activity state, and $\Omega_{SN,i}(t)$ represents the set of all supportive neighboring nodes of node i at time t. Eq.(5) means that node i will adjust its activity state towards the mean value of all activity states of set $\Omega_{SN,i}(t)$.

Fig.2(a) gives an example of changing node activity state under the selfish rule. In Fig.2, the similarity in node colors represents the similarity in node activity states, red (resp. blue) links represent positive (rep. negative) effects between nodes because of their similar (resp. different) activity states, and we assume node 1 currently needs to adjust its activity state or links. In Fig.2(a), since neighboring nodes 2 and 3 are supportive to node 1 (they all have similar warm colors), under the selfish rule, node 1 changes its activity state to become even more similar to those of nodes 2 and 3, and as the result, the CSD of node 1 is increased.

Assume at time t, node i has $N_{DN,i}(t)$ disturbing neighboring nodes, which compose a set $\Omega_{DN,i}(t)$. If $N_{SN,i}(t)>0$, $N_{DN,i}(t)>0$ and node i needs to rewire its links under the selfish rule at time instant t, then it will randomly disconnect from a node in set $\Omega_{DN,i}(t)$ (assume node j is chosen), and then rewires the link to a supportive node which is linked to set $\Omega_{SN,i}(t)$ but not to node i at time instant t (assume node k is chosen). At next time t+1 after rewiring the link,

$$\Omega_{SN,i}(t+1) = \Omega_{SN,i}(t) + \{k\}, \quad N_{SN,i}(t+1) = N_{SN,i}(t) + 1, \tag{6}$$

$$\Omega_{DN,i}(t+1) = \Omega_{DN,i}(t) - \{j\}, \quad N_{DN,i}(t+1) = N_{DN,i}(t) - 1. \tag{7}$$

Fig.2(b) is an example of rewiring links under the selfish rule. Node 1 first disconnects a link from node 5, as they have conflictive activity states with each other. Then, node 1 rewires the link to node 4, which is a supportive neighbor of node 1's supportive neighboring nodes. This gives node 1 a good chance to improve its CSD.

Now, we mathematically describe how to change the activity state of node i under the following-others rule. Based on $\theta_i(t)$, the activity state of node i at time t+1 will become

$$\theta_i(t+1) = \begin{cases} \theta_i(t) + s_\theta \times (\frac{\sum_{j\in\Omega_{SN,i}(t)} \theta_j(t)}{N_{SN,i}(t)} - \theta_i(t)), & N_{SN,i}(t) > N_{DN,i}(t) \\ \theta_i(t) + s_\theta \times (\frac{\sum_{j\in\Omega_{DN,i}(t)} \theta_j(t)}{N_{DN,i}(t)} - \theta_i(t)), & N_{DN,i}(t) > N_{SN,i}(t) \end{cases}. \quad (8)$$

Eq.(8) shows that node i will change its activity state to follow the majority of its neighboring nodes, even if they currently have disturbing activity states to node i.

Fig.2(c) gives an example of changing node activity state under the following-others rule. Since most neighboring nodes of node 1 have cold colors, under the following-others rule, node 1 changes its activity state from warm color to cold color. This enables node 1 to get more supportive effects from its neighboring nodes, which causes the CSD of node 1 to increase.

Now, we explain how to rewire a link of node i under the following-others rule. If $N_{SN,i}(t) \geq N_{DN,i}(t) > 0$, then node i will rewire its links in the same way as under the selfish rule according to Eq.(6) and Eq.(7). If $0 < N_{SN,i}(t) < N_{DN,i}(t)$, then node i will randomly disconnect from a node in set $\Omega_{SN,i}(t)$ (assume node j is chosen), and then rewires the link to a node which is supportively linked to set $\Omega_{DN,i}(t)$ but not to node i at time t (assume node k is chosen). At next time t+1 after rewiring the link,

$$\Omega_{DN,i}(t+1) = \Omega_{DN,i}(t) + \{k\}, \quad N_{DN,i}(t+1) = N_{DN,i}(t) + 1, \quad (9)$$

$$\Omega_{SN,i}(t+1) = \Omega_{SN,i}(t) - \{j\}, \quad N_{SN,i}(t+1) = N_{SN,i}(t) - 1. \quad (10)$$

Fig.2(d) is an example of rewiring links under the following-others rule. In this case, node 1 has just 1 supportive neighbor, i.e. node 2, but 3 disturbing neighbors, i.e., nodes 3, 5 and 6. So, under the following-others rule, node 1 disconnects from node 2, and rewires the link to node 4, which is a supportive neighbor to the set of disturbing neighbors of node 1. This adjustment makes the CSD of node 1 temporarily decrease, but if it gets a chance to change its activity state under the selfish rule in future, its CSD will rise dramatically.

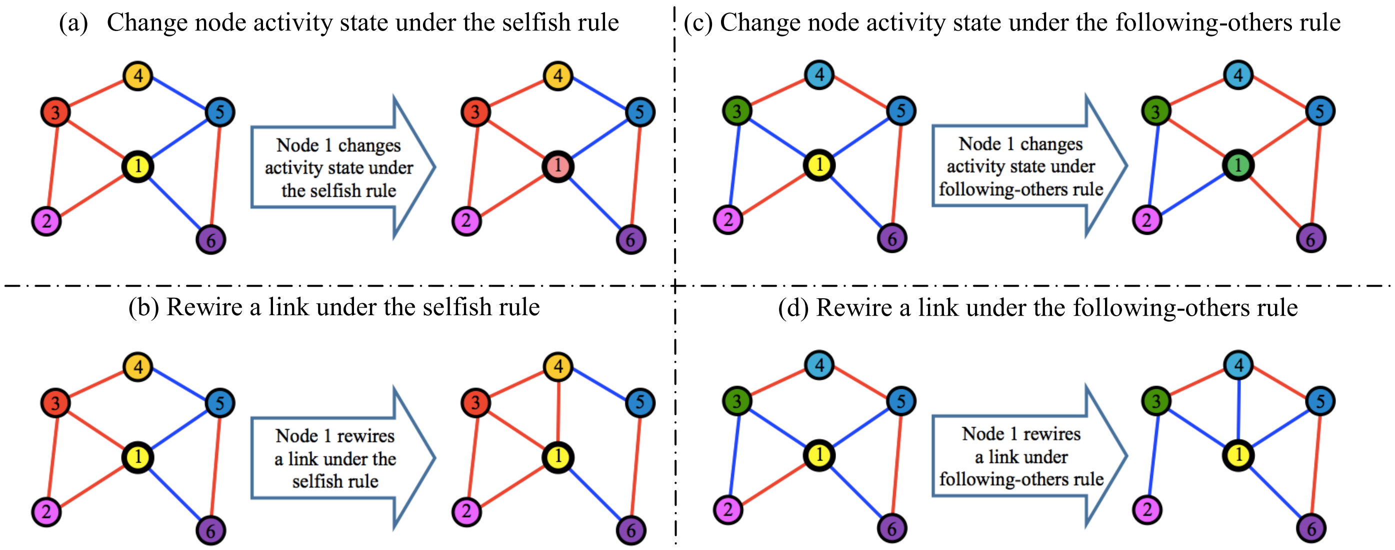

Fig.2. Examples of adaptive behaviors under the selfish rule and the following-others rule (Similarity in node colors represents the similarity in node activity states; red (resp. blue) links represents positive (resp. negative) effects between nodes because of their similar (resp. different) activity states).

Based on the above adaptive behaviors defined by Eq.(5) to Eq.(10), we can start an evolutionary process to evolve an initial network system. At each time instant of the evolutionary process, P_{CAS} percent of all N_{N} nodes will be randomly picked out to change their activity states, and P_{RWL} percent to rewire their links. Assuming node i is chosen to evolve at time t, then the probabilities of applying the selfish rule and the following-others rule are calculated as follows, respectively

$$P_{SR,i}(t) = \alpha(i) + (1-\alpha(i)) \times \frac{N_{SN,i}(t)}{N_{SN,i}(t) + N_{DN,i}(t)}, \tag{11}$$

$$P_{FO,i}(t) = 1 - P_{SR,i}(t), \tag{12}$$

where $0 \le \alpha(i) \le 1$ is a coefficient which determines how likely node i is to employ the selfish rule.

It should be noted that this paper aims to study the relationship between adaptive behaviors and system consilience, so the adaptive behaviors above are defined to be of a well-established form. In other words, we do not aim to introduce new adaptive behavior models, or to compare with any existing adaptive behavior models. Actually, in future extensions and applications of this study, a comparison between the adaptive behaviors defined above and some other relevant models is certainly worth investigating. For example, on the one hand, the selfish rule of Eq.(5) to Eq.(7) has something in common with the basic premise of Axelrod (1997) , which states that the more similar an actor is to a neighbor, the more likely it is that the actor will adopt one of the neighbor's traits. On the other hand, the culture evolution model of Axelrod (1997) is based on grid networks, while this study mainly focuses on small-world networks. In a grid network, the influence of an actor often needs to pass through many intermediate nodes to reach a spatially distant nodes, while in a small-world network, the influence of a node can spread much faster by passing through only a few intermediate nodes to reach any node in the system (Watts and Strogatz, 1998). Taking another example, the following-others rule of Eq.(8) to Eq.(10) is similar to the threshold model of Granovetter (1978), in particular, the selfish coefficient $\alpha(i)$ plays a role similar to a threshold in partially determining whether a node will employ the selfish rule or the following-others rule. However, the threshold model of Granovetter (1978) used a two-valued node attribute, while this study allows node activity state to vary continuously between 0 and 2π. It should be pointed out that the models of Axelrod (1997) and Granovetter (1978) have a closer focus on certain real-world applications. Therefore, comparing with and learning from such models may largely improve the application potential of this study, which will form the basis of extending this study in future research.

3.2 Simulation results

In this simulation, three trials were initially conducted. In the first of these, only node activity states changed according to the selfish and following-others rules. In the second one, only links between nodes changed according to the selfish and following-others rules. In the third test, both node activity states and links between nodes changed according to the selfish and following-others rules.

In each trial, the same initial network system with N_N=100 nodes and N_E=400 links was used. In the initial system, node activity states were distributed randomly within the range of [0 2π], and links were randomly set up between nodes according to the rule reported in Watts and Strogatz (1998). As a result, the ACSD of the initial system was almost 0. Then, the initial system started to evolve based on adaptive behaviors for a simulation period of T_{SP}=50000 simulated time units. In the evolutionary process, for the sake of simplicity, we set $\alpha(i)=0.3$ for all nodes, unless specified. At the final time instant *t*=50000, we inspected the evolved network system, and compared it with the initial network system. We also recorded how the ACSD value, the clustering coefficient (CC) value, the diversity in node activity states (DinNAS), and the difference in neighborhood node activity states (DinNNAS) changed during the evolutionary period; the ACSD value was calculated according to Eq.(4).

Clustering coefficient describes how tense a node and its neighbors are connected to each other by links (Albert and Barabási, 2002; Boccaletti, et al., 2006), and for node *i*, the CC value is usually calculated as

$$c_{CC,i}=\frac{2n_{E,i}}{k_{CN,i}(k_{CN,i}-1)}, \tag{13}$$

where $n_{E,i}$ is the number of all links existing in the cluster, which is composed of node *i* and all its $k_{\mathrm{CN},i}$ neighbors.

The diversity in node activity states (DinNAS) is assessed as following. The range of [0 2π] is evenly divided into N_{SS}=100 subsets. Then, we check the activity states of all the N_{N}=100 nodes in the network system at a given time instant, to see how many subsets have covered at least one node. Assume the activity states of all the N_{N}=100 nodes are distributed in $1 \leqslant N_{\mathrm{SS}} \leqslant 100$ subsets. Then, the DinNAS is measured by

$$d_{DinNAS}=\frac{n_{SS}}{N_{SS}}. \tag{14}$$

Based on Eq.(14), a larger value of d_{DinNAS} implies a better diversity in node activity states.

The difference in neighborhood node activity states (DinNNAS) is assessed as following. First, we measure the DinNNAS for each individual node, say, for node *i*, we have

$$d_{DinNNAS,i}=\sum_{j=1}^{k_{CN,i}}\left|\theta_i-\theta_{NS,i}(j)\right|, \tag{15}$$

where $\theta_{\mathrm{NS},i}(j)$ is the activity state of the *j*th node in the neighborhood set of node *i*, and the neighborhood set of node *i* has $k_{\mathrm{CN},i}$ neighbors connected to node *i*. Then, the average DinNNAS is calculated as

$$\bar{d}_{DinNNAS}=\frac{1}{N_N}\sum_{i=1}^{N_N}d_{DinNNAS,i}. \tag{16}$$

The results of three trials are given in Fig.3 to fig.5, respectively. In each figure, the left-top subplot (a) is the initial network system at time instant t=0, the right-top subplot (b) is the final evolved network system at time instant t=50000, the left-bottom subplot (c) gives the ACSD and DinNNAS curves, and the right-bottom subplot (d) gives the CC and DinNAS curves. When plotting a network system, we plot nodes with positive CSD values as triangles, while nodes with negative CSD values as circles. The color of a node is determined by the activity state of the node. Basically, the similarity in node colors indicates the similarity in node activity states. We also use different colors to indicate positive or negative effect of a link. If a link between two nodes has a negative (resp. positive) effect due to the dis/similarity in the activity states of these two nodes, we plot the link in blue (resp. red). The deeper the blue (resp. red) color the stronger the negative (resp. positive) effect of the link.

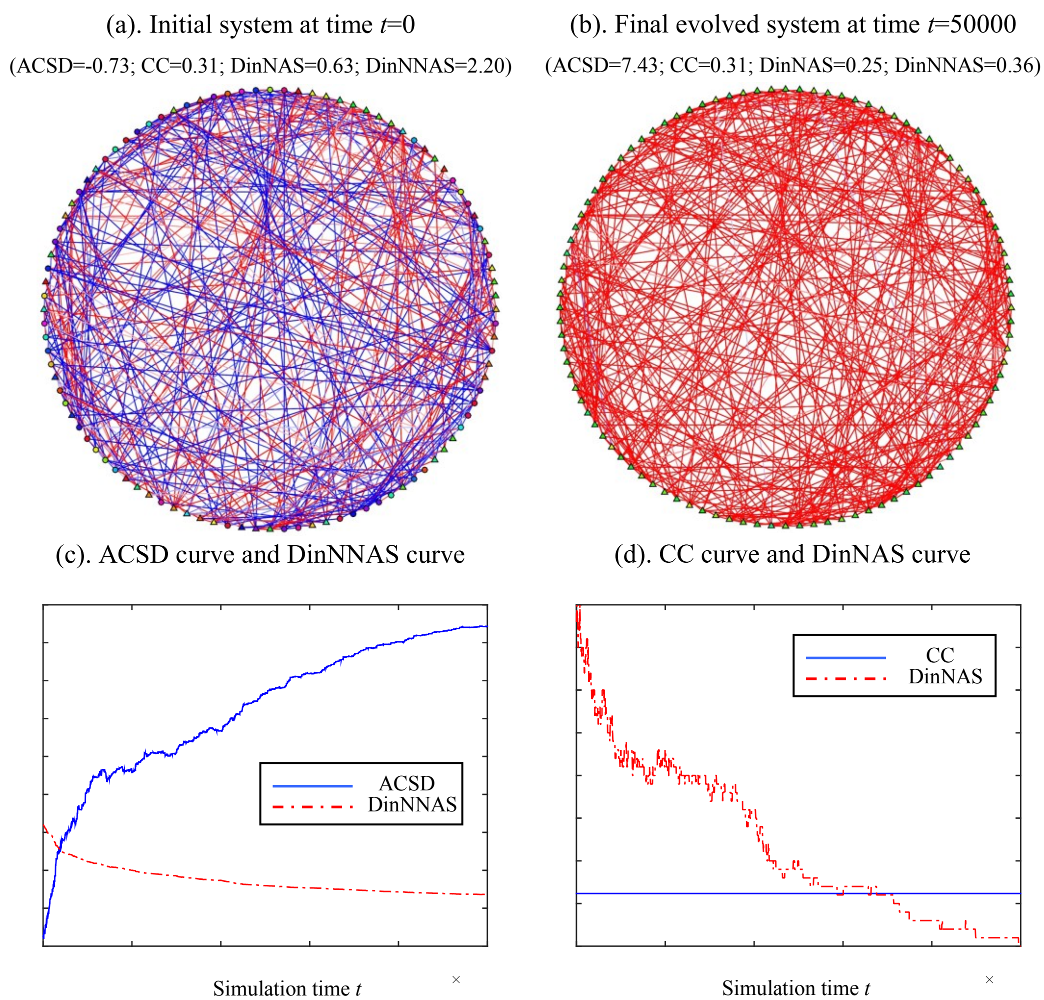

Fig.3. Only changing node activity states can improve the system consilience of a network system

(Trisl parameter setup: N_N=100, N_E=400, T_{SP}=50000, $\alpha(i)$=0.3, P_{CAS}=2% and P_{RWL}=0%).

From Fig.3, one can observe: (i) allowing node activity states to change according to adaptive behaviors can significantly improve the system consilience level (in this case, the ACSD increases from -0.73 to 7.43); (ii) because of adaptive behaviors, the diversity in node activity states (DinNAS) drops dramatically, which means at the end of evolutionary process, most nodes have similar activity states; (iii) during the co-evolutionary process, the difference in neighborhood node activity states (DinNNAS) decreases gradually, which is the reason why the ACSD increases; (iv), since no links are allowed to rewire, the network topology remains unchanged, and therefore, the CC remains the same.

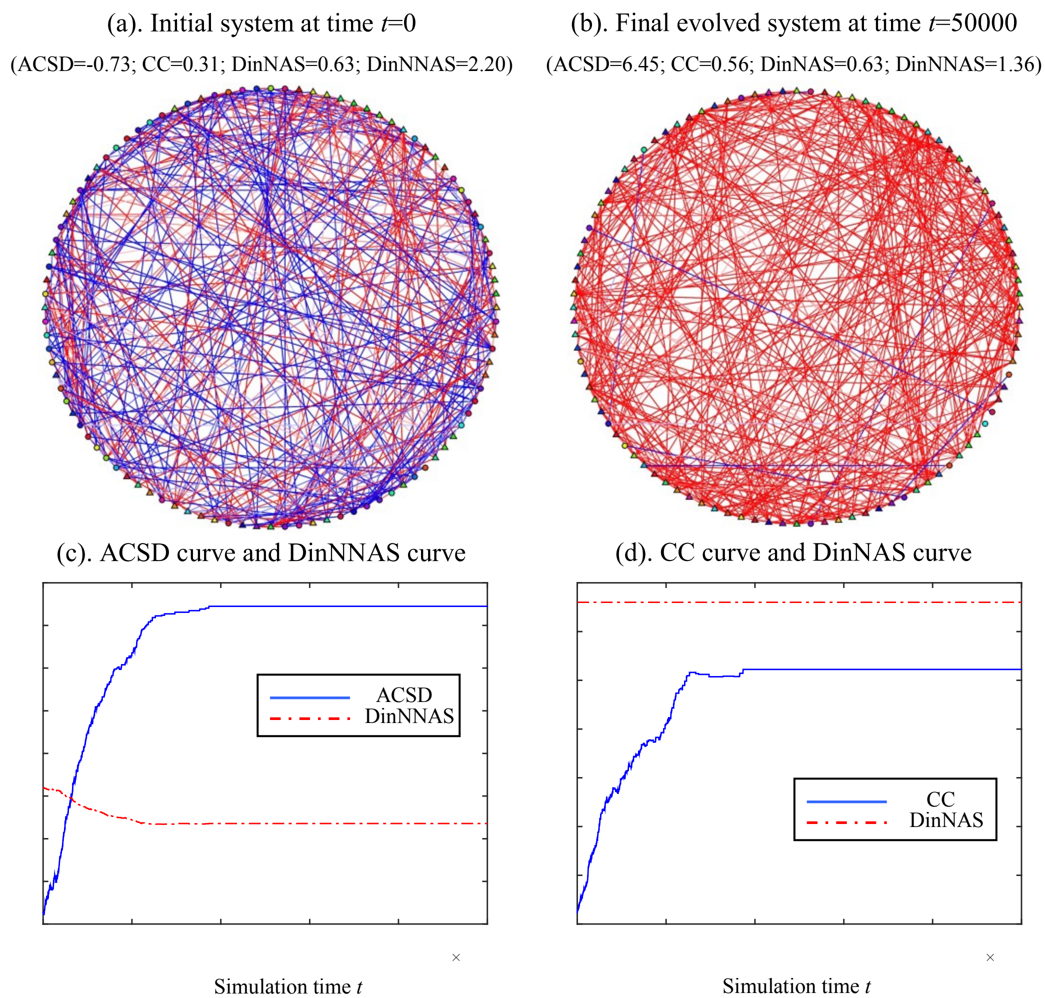

Fig.4. Only changing links between nodes can improve the system consilience of a network system (Trial parameter setup: N_N=100, N_E=400, T_{SP}=50000, $\alpha(i)$=0.3, P_{CAS}=0% and P_{RWL}=2%).

According to the definition of CSD in Eq.(2), when the consilience funtion is set as $f_{CS}(\theta_i,\theta_j)=\cos(\theta_i-\theta_j)$ as in this simulation, a system network with all nodes having the same activity state (i.e., no diversity at all) will have the potential maximal ACSD (here this value is eight). However, diversity is common and important to many natural and man-made systems. Then, can system consilience still be improved without sacrificing the diversity in node activity states?

Fortunately, this is possible if adaptive behaviors only change links between nodes. Fig.4 gives a test example where only links between nodes can change under the selfish rule and the following-others rule. From Fig.4, one can see: (i) only changing links between nodes under adaptive behaviors can still improve the system consilience level (in this case, the ACSD increases from -0.73 to 6.45), because the simulated adaptive behaviors re-group nodes according to the similarity in node activity states; (ii) as node activity states are not allowed to change, the DinNAS remains the same during the evolutionary process, which is a good news to a system pursuing diversity among individuals; (iii) since link rewiring under adaptive behaviors makes a node more likely to be connected to those nodes with similar activity states, the DinNNAS decreases over time; (iv) the CC goes up from 0.31 to 0.56, and this is mainly because the connections within a cluster with similar node activity states have been largely enhanced during the evolutionary process; (v) when compared with Fig.3, the ACSD, DinNNAS and CC curves soon reach certain stable levels, and this probably implies that, for a given, unchangeable diversity in node activity states, there might be an upper bound for the achievable ACSD.

Fig.4 clearly demonstrates that adaptive behaviors can improve system consilience without changing the diversity in node activity states. From Fig.3 and Fig.4, it seems that allowing diversity in node activity states to decrease may lead to a better system consilience. Suppose that both system consilience and the diversity in node activity states are desirable. Then, can adaptive behaviors achieve a good balance between system consilience and the diversity in node activity states? Fig.5 gives a test example where the initial network system is exactly the same as that in Fig.3 and Fig.4, but both the diversity in node activity states and links between nodes can change under the selfish rule and the following-others rule during the evolutionary process.

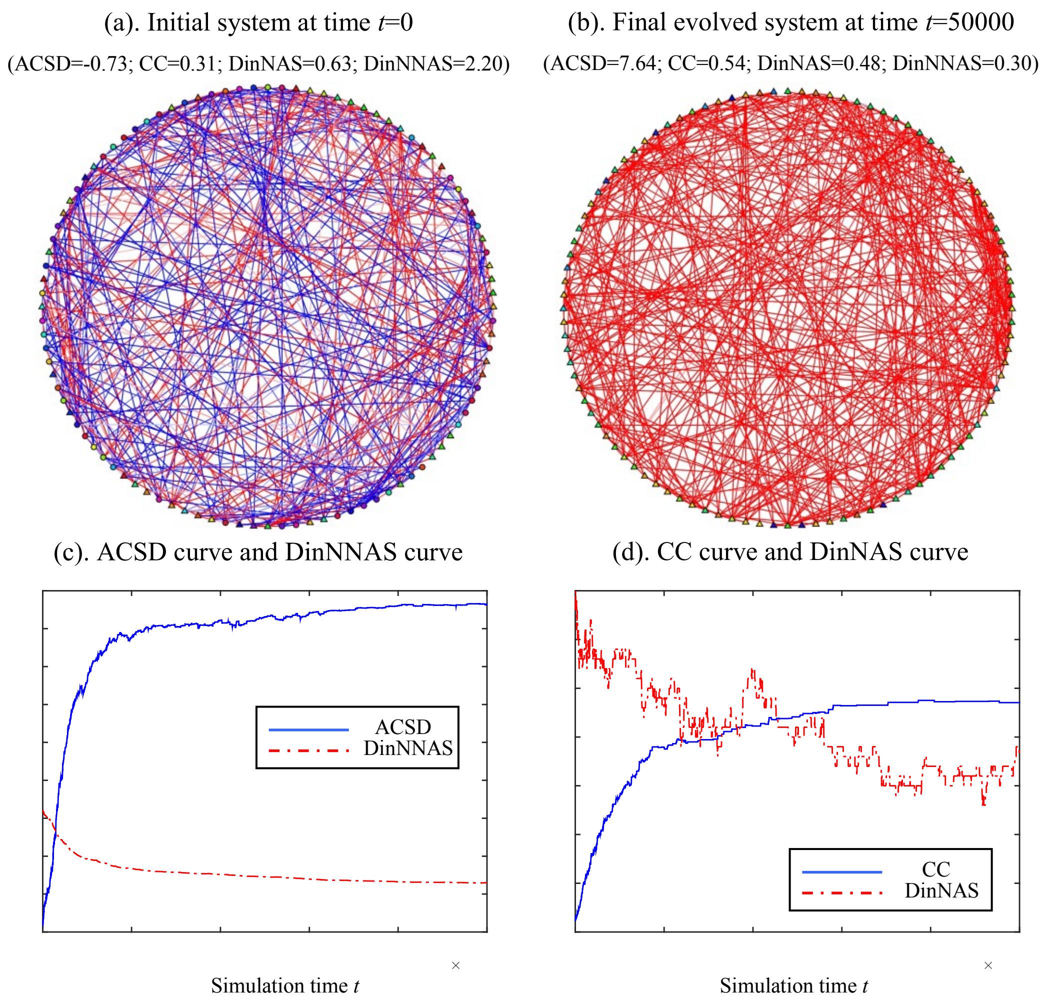

Fig.5. Allowing both the diversity in node activity states and links between nodes to change can improve the system consilience of a network system (Test parameter setup: N_{N}=100, N_{E}=400, T_{SP}=50000, $\alpha(i)$=0.3, P_{CAS}=1% and P_{RWL}=1%).

When compared with Fig.3, one can see that the diversity in node activity states is less changed in Fig.5, i.e., the DinNAS drops from 0.63 to 0.25 in Fig.3, while to just 0.48 in Fig.5. When compared with Fig.4, one can see that system consilience is largely improved in Fig.5, i.e., the ACSD increases from -0.73 to 6.45 in Fig.4, while this is 7.64 in Fig.5. Actually, by the end of evolutionary process, the ACSD in Fig.5 is even larger than that in Fig.3. This implies that allowing both the diversity in node activity states and links between nodes to change is more effective in a sense of Pareto optimality (Barr, 2004; Hu et al., 2013) than only changing the diversity in node activity states, because both system consilience and diversity levels are higher in Fig.5 than in Fig.3.

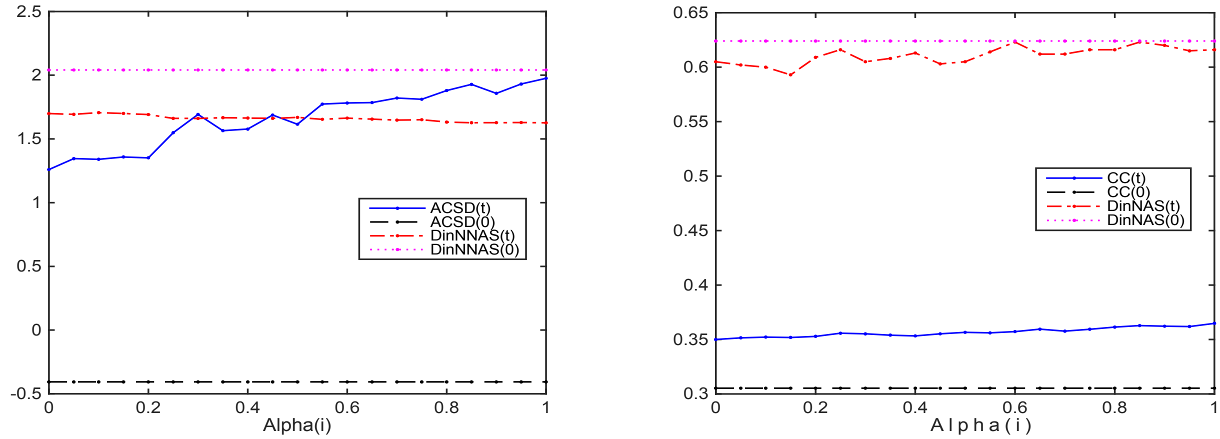

Fig.6. The influence of $\alpha(i)$ on ACSD, CC, DinNAS and DinNNAS of the final evolved network system (Test parameter setup: N_N=100, N_E=400, T_{SP}=1000, $\alpha(i)$=0,0.05,…,1, P_{CAS}=1% and P_{RWL}=1%).

We further conducted an experiment to test how the value of parameter $\alpha(i)$ influences ACSD, CC, DinNAS and DinNNAS of the final evolved network system during a period of T_{SP}=1000 simulated time units. It should be noted that in Fig.3 to Fig.5, the initial network system evolved for a period of T_{SP}=50000 simulated time units but here we only allowed 1000 simulated time units for system evolution. This is mainly because if the evolutionary period is too long, then a small value and a large value for $\alpha(i)$ can both eventually lead to similar ACSD, CC, DinNAS and DinNNAS. We set $\alpha(i)$=0,0.05,…,1. We applied each $\alpha(i)$ value to 100 randomly generated initial network systems and then obtained the average final ACSD, CC, DinNAS and DinNNAS of the 100 evolved systems at time t=1000. We plot how the average final ACSD, CC, DinNAS and DinNNAS change as $\alpha(i)$ increases from 0 to 1 in Fig.6, from which one can see that no matter what the value of $\alpha(i)$, the system consilience level will always be improved under adaptive behaviors when compared with the associated initial consilience level. Here we do not draw any conclusion such as which value of $\alpha(i)$ is the best, because (i) the main goal of this study is just to demonstrate that adaptive behaviors can improve system consilience level, and (ii) all experiments here are based on abstract network systems rather than real-world ones. A discussion about the best value of $\alpha(i)$ does not make much realistic sense here, and it would be better to optimize such a parameter value in future real-world case studies. For the same two reasons, here we do not go further with other experimental parameters, in order to avoid distracting readers' attention from the main goal of this study.

To summarize, from Fig.3 to Fig.6, one can see clearly that the simulated adaptive behaviors under the selfish rule and the following-others rule can improve the consilience level of a network system, no matter whether diversity in node activity states is allowed to change, or if links between nodes are allowed to rewire. Since the selfish rule and the following-others rule are very common in many real-world network systems, one can image that the consilience level of such real-world network systems should be reasonably high (at least significantly greater than zero) due to adaptive behaviors. Given the generality of such behaviors in reality, we therefore argue that CSD is an inherent attribute rather than an artificial concept, which underpins the fundamental importance of CSD to the study of real-world complex network systems such as social-ecological systems, that is to say that concept of CSD in Eq.(2) can provide an effective mathematical method to describe and study the consilience level of real-world network systems.

4. System consilience and system performance against disturbances

One may wonder: Why might a network system need a high consilience level? This is apparently a fundamentally important question for the study of adaptive behaviors and the concept of CSD. In this section, we attempt to explore a possible reason: system performance against external disturbances (e.g., natural disasters, terrorism attacks, and environmental changes). Actually, the concept of CSD was initially motivated by some practices and observations in disaster reduction and risk management, e.g., consensus of wills and coordination of activities often make a community more robust (or less vulnerable) to natural or man-made disasters (Shi et al. 2014).

4.1 Setup of anti-attack investigation

Here, we conducted a simulation trial to study the relationships between system performance against disturbances, system consilience and adaptive behaviors. In this test, we compared three network systems in terms of performance against disturbances. Each of these three network systems had N_{N}=100 nodes and N_{E}=400 links, and therefore, each network system had the same average CND (ACND) of 8.

The first network system, i.e., system (a) in Fig.7, was generated by the highly-acknowledged model of Watts and Strogatz (1999), which has a wide range of representations in the real world and can well explain the phenomenon of six degree of separation. Basically, the Watts and Strogatz (1999) model is purely based on CND, and the diversity in node activity states is not taken into account at all when setting up a link between nodes. Therefore, the resulting network systems usually have an average CSD (ACSD) around 0. For example, system (a) in Fig.7 had randomly diversified node activity states, which however were not considered when the network topology was developed. As a result, system (a) had an ACSD of -0.75.

The second network system, i.e., system (b) in Fig.7, was developed based on the adaptive behavior model as described in Section 3. System (a) in Fig.7 was used as the initial network system. Then, node activity states and links of system (a) changed, or evolved according to the selfish rule and the following-others rule as given in Section 3. This was actually a co-evolutionary process, because each node constantly changed its activity state and links by referring to its neighboring nodes. Such a co-evolutionary process went on for 1000 simulation time units. At the end of this evolutionary period, we obtained system (b). As has been proven in Section 3, the simulated adaptive behaviors can improve the level of system consilience, i.e., system (b) had an ACSD of 1.94, which was much higher than the ACSD of the initial system, i.e., system (a).

Given that the maximum theoretical ACND value is clearly 8 for a system with an ACND of 8, we wanted to develop a network system with an ACSD as close to the maximum possible to better understand the influence of system consilience on system performance against disturbances. To this end, we used the global CSD optimization model reported in Hu et al (2017). First, we removed all of the N_E links in system (a), and kept node activity states untouched. Then, we re-set N_E links one by one between nodes as follows. Each time when re-setting a link, we put the link between such a pair of nodes that would make the current ACSD maximal. In this way, we obtained system (c), which had an ACSD of 5.61, the closest to the maximal value of 8. As shown in Fig.7, system (c) had exactly the same diversity in node activity states as that of system (a), but different link distribution, or topology. This is likened to that organizing the same group of people in different way could result in rather different group performance. It should be noted that, compared with system (a), system (b) not only had a different link distribution but also different diversity in node activity states because of those adaptive behaviors in the model of Section 3.

Now, we exposed systems (a), (b) and (c) to the same series of external disturbances (random attack history) during a period of 10000 simulation time units. At the beginning of simulation, i.e., when the simulation time instant was t=0, all nodes had a functional capacity of 1, denoted as $c_{\mathrm{NF},i}(0)=1$ for node i at time t=0. Then, at each simulation time instant $0<t\leqslant 10000$, an attack event occurred, which randomly picked up 1% to 10% of the nodes and reduced their functional capacity to 0, i.e., if node i was under attack at time t, then, its functional capacity immediately became $c_{\mathrm{NF},i}(t)=0$. During the simulation process, if a node at time instant t, say node i, had functional capacity $c_{\mathrm{NF},i}(t)<1$, then, at the next time instant t+1, its functional capacity may recover to some extent as following

$$c_{NF,i}(t+1)=\min(1,\max(0,c_{NF,i}(t)+\alpha_R\cdot(c_{NF,i}(t) + \sum_{j=1}^{N_N} c_{NF,j}(t)\cdot M_A(i,j)\cdot f_{CS}(\theta_i,\theta_j))), \tag{17}$$

where $0<\alpha_R$ is recovering ratio. In this test, we set α_R=0.1, and the consilience function was again set as $f_{CS}(\theta_i,\theta_j)=\cos(\theta_i-\theta_j)$.

According to Eq.(17), we can see that the recovery of functional capacity at a node with $c_{\mathrm{NF},i}(t)<1$ largely depends on supports from its neighboring nodes. Even if all of its neighboring nodes have the maximal functional capacity, i.e., $c_{\mathrm{NF},j}(t)=1$, but if they are conflictive with node i in terms of node activity states, then they will not contribute to the recovery of node i, and instead, they will disrupt and even jeopardize the recovery that node. In other words, if without such conflictive neighboring nodes, node i might recover gradually based on its own remaining functional capacity $c_{\mathrm{NF},i}(t)$, given $0<c_{\mathrm{NF},i}(t)<1$, with conflictive neighboring nodes, the functional capacity of node i might never recover. Clearly, the CSD concept as defined in Section 2 can be used to well assess such supportive and conflictive situations. Basically, if a network system has a larger ACSD, then according to Eq.(17), its nodes will recover more quickly on average under attacks. The recovery behavior defined by Eq.(17) is highly consistent with common sense in daily life. For example, practices in disaster mitigation and relief often witness that whether people are supporting or looting each other largely determines community recovery performance after disasters (Shi et al., 2014).

We wish to point out that the recovery behavior defined by Eq.(17) is a dynamical process, because functional capacities of neighboring nodes may change from time to time. For example, suppose node i with $c_{\mathrm{NF},i}(t)<1$ has some supportive neighboring nodes as well as some conflictive neighboring nodes. At time t, if the term multiplied by α_R in Eq.(17) has a value of 0, then, the functional capacity of node i

will remain unchanged at time t+1. Then, assuming at time t+1, a few conflictive neighboring nodes are turned down by the random attack of time t+1, as a result, those supportive neighboring nodes will overwhelm the remaining conflictive neighboring nodes, and therefore, the functional capacity of node i will increase at time t+2. Then, assuming the random attack at time t+2 hits some supportive neighboring nodes and makes the balance shift to those conflictive neighboring nodes, the functional capacity of node i will decrease at time t+3.

During the random attack history of 10000 simulation time units, at each time instant $0 \leqslant t \leqslant 10000$, we checked the average node functional capacity (ANFC) in each of the 3 network systems, and we also counted the number of all non-functional nodes (ANFN) in each of the 3 network systems. In Fig.7, we plot ANFC and ANFN curves over the random attack history of 10000 simulation time units, in order to study how systems (a), (b) and (c) perform against attacks. Basically, if a network system can maintain a high level of ANFC and a low level of ANFN during the random attack history, then we can say the system has a good performance against attacks.

4.2 Results and discussions of anti-attack experiment

The results of the anti-attack investigation are given in Fig.7. From those ANFC and ANFN curves in Fig.7, one may have the following observations.

- System (a), which does not consider node activity states and therefore has no consilience design, has the poorest performance against attacks. Actually, in system (a), the number of nodes with a negative CSD value (such nodes are plotted as circles) is about 50, a half of N_N=100 nodes, and based on the recovering behavior defined in Eq.(17), most of such nodes, once hit by an attack, will never recover but simply remain with $c_{NF,i}(t)$=0. For the other half of N_N=100 nodes, they have positive CSD values, which means they can get effective supports from their neighboring nodes, and therefore, their functional capacities can usually recover over time more or less from attacks. For these nodes with positive CSD values, they have $c_{NF,i}(t)$>0 at most time. This explains why system (a) has the ANFN curve stabilized at a value about 48. Although the average number of functional nodes is about 52 over time, there are often some nodes in the recovering process and having $c_{NF,i}(t)$<1. Therefore, system (a) has the ANFC curve stabilized at a value about 42%, smaller than 52%.

- The co-evolutionary dynamics based on those adaptive behaviors of Section 2 gives system (b) a positive ACSD of 1.94. However, it should be noted that in system (b), there are still some nodes which have negative CSD values. For such nodes, they are very likely to stay with $c_{\mathrm{NF},i}(t)$=0 once they are hit by attacks. Apparently, as shown by the topology of system (b) in Fig.7, the number of nodes with negative CSD values is much smaller than that of system (a). Therefore, system (b) has the ANFN curve stabilized at a value about 28, 43% smaller than that of system (a). At the same time, system (b) has the ANFC curve stabilized at a value about 60%, 44% larger than that of system (a).

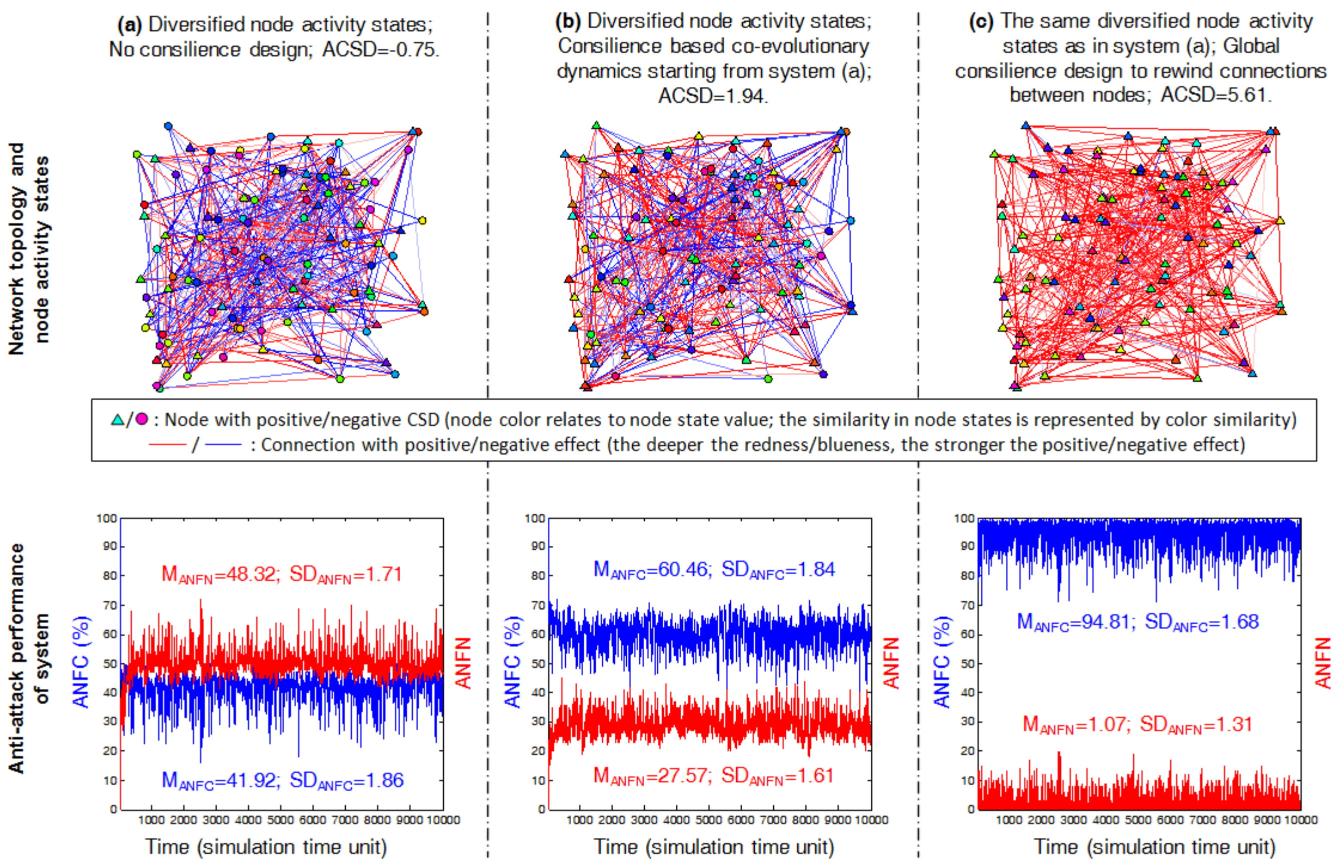

Fig.7. Comparison between anti-attack performances of three network systems with different ACSD values (Investigation parameter setup: N_N=100, N_E=400, and α_R=0.1).

- In system (c), based on the global optimization model of Hu et al (2017), every link is carefully set up in order to maximize ACSD. As the result, all nodes in system (c) have positive CSD values, and the ACSD of system (c) is 5.61, the largest one in all of the 3 network systems, and much larger than the ACSD of system (b), let alone the ACSD of system (a). Under the same attack history, system (c) has the ANFN curve maintained at a value about 1, and the ANFC curve stabilized at a value around 95%. In other words, those attacks have almost no impact on the performance of system (c), because almost all nodes are functioning well with $c_{\mathrm{NF},i}(t) \approx 1$ on average all time.

- Now, we come back to the fundamental question about CSD: Why might a network system need a high consilience level? In other words, the fact that adaptive behaviors can naturally/automatically improve the consilience level of a network system must be for certain purposes. The results from systems (a), (b) and (c) in Fig.7 might give us a clue about one of such purposes, i.e., a higher level of system consilience implies a better system performance against attacks. As is well known, system robustness against attacks is often a top systemic goal, which makes some sense of CSD.
- One might argue that adaptive behaviors, although they widely exist in many natural and man-made network systems, do not lead to the highest consilience level in Fig.7. Does this mean consilience is not so really important in real world? The highest consilience level in Fig.7 is achieved by the global optimization model of Hu et al (2017), which basically demands a central governor having all necessary global information and absolute power to decide on every local detail of network design. However, having such a central governor is often an ideal situation, too good to be true. Unfortunately, in most natural and man-made network systems of real world, there is no such a central governor. Actually, in the real world, such a central governor is often not desirable, because many natural and man-made network systems are of multi-agent or multi-stakeholder, which means each node has self-governing power and usually does not want to be governed by others. In many natural systems, having such a central governor is often not practicable, either, because it is very difficult, if not impossible, for a natural system to collect all necessary global information. In fact, most natural systems, if not all, are driven and run by decentralized, local-information-based, multi-agent-based self-organizing mechanisms, such as those adaptive behaviors defined in Section 3. Surprisingly but quite reasonably, although without any powerful central governor, nature still finds its way to achieve a high level of system consilience, i.e., through a co-evolutionary process based on multi-agent adaptive behaviors, in

order to, for example, deal with external disturbances effectively. In other words, based on the multi-agent-based, self-organizing feature, nature has evolved adaptive behaviors to achieve a high level of system consilience. When compared with a central governor strategy, such adaptive behaviors might not be ideal from the viewpoint of achieving the highest level of system consilience, but they are the most suitable to the multi-agent-based, self-organizing feature of natural systems.

- It should be emphasized that another reason for why adaptive behaviors do not give a very high level of system consilience in Fig.7 is because system (b) evolves from system (a) through just 1000 simulation time units. In Section 3, we allowed an initial system to evolve over 50000 simulation time units, which led to a very high level of system consilience. In the experiment of this section, we deliberately restrict system (a) to evolve for only 1000 simulation time units, for two reasons: (i) we want to show the influence of different levels of system consilience, and (ii) in reality, external environment usually changes constantly and rapidly, and there could not be much or enough time for a network system to evolve and become fully fit to the external environment of a specific time instant.
- We would also like to point out that, although system (c) has a better anti-attack performance than system (b) in Fig.7, it does not mean the adaptive behavior model is less useful than the global optimization model. There are many natural systems as well as many decentralized social-ecological systems (such as market-driven economic systems) where there lacks an explicit/strong central governing body, and individuals compete with each other largely based on local/limited information. The adaptive behavior model reported in Section 3 is much more useful to study these real-world network systems. Of course, advances in modern scientific technologies, such as information communication technologies, enable many social systems to be aware and then to make use of global information for making decisions/policies. Therefore, the results of Fig.7 might imply that, if a social system with a central governing body could make and implement proper top-down central policies/strategies for all individuals in the system, then it could make most of the diversity in individuals and achieve a better performance than in a natural, decentralized, self-organizing situation. How to modify and improve the adaptive behavior model is apparently worth further investigation in future, because many real-world

systems are driven by adaptive behaviors. How to make a proper top-down policy/strategy for a centralized system is also a great application area for consilience theory.

5. Conclusions

This paper reports a simulation study on the adaptive behaviors based on the selfish rule and the following-others rule. The simulation results show that, because of such adaptive behaviors, a network system will automatically evolve to a high level of average consilience degree. Since selfish and following-others behaviors are common in both natural and man-made systems, we conclude that system consilience is an inherent, fundamental property of real-world complex network systems. The simulation study also demonstrates that a high level of system consilience, as a result of adaptive behaviors, will contribute to a better system performance against external disturbances. Therefore, the concept of consilience degree may help with the understanding of complex phenomena in many real-world network systems.

This paper takes a specific and tightly defined goal to just demonstrate the relationship between adaptive behaviors and system consilience, and does not pursue many other issues in depth. For example, regarding why a network system might need a high consilience level, besides anti-attack performance as discussed in the paper, there could be some other reasons, such as cost-efficiency and sustainability of system, which are all worth further investigation. More efforts can also be made to conduct comprehensive investigations, in order to test the influence and learn the role of those model parameters by changing their values over a wide range. It will be interesting to design trials to compare system consilience with some other relevant concepts, e.g., link weight, metadata and assortativity coefficient, so that by comparing how they will change/evolve under adaptive behaviors, we may gain a deeper understanding concerning the merits and demerits of system consilience. Besides the selfish rule and the following-others rule, attention may be paid to include more other relevant adaptive behaviors, such as the majority rules in culture evolution models (Axelrod, 1997) and threshold models (Granovetter, 1978). In particular, future research needs to put a focus on applications of theoretical consilience concept in various real-world, natural and man-made network systems, and in such applications, more knowledge is expected to be obtained regarding how to define node activity state and how to design consilience function in a more realistic way.

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