



City Research Online

City, University of London Institutional Repository

Citation: Gambaro, A. M., Casalini, R., Fusai, G. and Ghilarducci, A. (2017). Quantitative assessment of common practice procedures in the fair evaluation of embedded options in insurance contracts. *Insurance: Mathematics and Economics*, doi: 10.1016/j.insmatheco.2017.10.005

This is the accepted version of the paper.

This version of the publication may differ from the final published version.

Permanent repository link: <http://openaccess.city.ac.uk/19128/>

Link to published version: <http://dx.doi.org/10.1016/j.insmatheco.2017.10.005>

Copyright and reuse: City Research Online aims to make research outputs of City, University of London available to a wider audience. Copyright and Moral Rights remain with the author(s) and/or copyright holders. URLs from City Research Online may be freely distributed and linked to.

City Research Online:

<http://openaccess.city.ac.uk/>

publications@city.ac.uk

Quantitative assessment of common practice procedures in the fair evaluation of embedded options in insurance contracts

Anna Maria Gambaro^{*}, Riccardo Casalini[†]
Gianluca Fusai[‡], Alessandro Ghilarducci[§]

Abstract

This work analyses the common industry practice used to evaluate financial options written on with-profit policies issued by European insurance companies. In the last years regulators introduced, with the Solvency II directive, a market consistent valuation framework for determining the fair value of asset and liabilities of insurance funds. A relevant aspect is how to deal with the estimation of sovereign credit and liquidity risk, that are important components in the valuation of the majority of insurance funds, which are usually heavily invested in treasury bonds. The common practice is the adoption of the certainty equivalent approach (CEQ) for the risk neutral evaluation of insurance liabilities, which results in a deterministic risk adjustment of the securities cash flows. In this paper, we propose an arbitrage free stochastic model for interest rate, credit and liquidity risks, that takes into account the dependences between different government bond issuers. We test the impact of the common practice against our proposed model, via Monte Carlo simulations. We conclude that in the estimation of options whose payoff is determined by statutory accounting rules, which is often the case for European traditional with-profit insurance products, the deterministic adjustment for risk of the securities cash flows is not appropriate, and that a more complete model such as the one described in this article is a viable and sensible alternative in the context of market consistent evaluations.

JEL classification codes: C63.

KEYWORDS: minimum guaranteed fund, embedded option, credit risk, liquidity risk, asset liability management.

^{*}Dipartimento di Studi per l'Economia e l'Impresa, Università del Piemonte Orientale "A. Avogadro", via E. Perrone 18 28100 Novara, Italy, Email: annamaria.gambaro@uniupo.it.

[†]Senior Manager, Deloitte Consulting Srl, Finance & Risk, Via Tortona 25, Milano, 20144, Italy, Email: rcasalini@deloitte.it.

[‡]Faculty of Finance, Cass Business School, City University, 106 Bunhill Row EC1Y 8TZ London, UK and Dipartimento di Studi per l'Economia e l'Impresa, Università del Piemonte Orientale "A. Avogadro", via E. Perrone 18 28100 Novara, Italy, Email: Gianluca.Fusai.1@city.ac.uk.

[§]Equity Partner, Deloitte Consulting Srl, Finance & Risk, Via Tortona 25, Milano, 20144, Italy.

1 Introduction

The most recent and widely adopted European Embedded Value (EEV) and Solvency II principles and standards require a market consistent approach for determining the fair value of asset and liabilities of insurance funds (see [11] and [12]).

According to the standard formula approved by the European Insurance and Occupational Pension Authority (EIOPA) and local regulators, government bonds issued by countries belonging to European Union all have the same risk¹, i.e. the credit and liquidity risk that they carry is not accounted in the valuation of insurance products. In order to cope with this assumption, it is a common practice by insurance companies to introduce a deterministic adjustment on assets cash flows, so that their present value, calculated discounting over the risk-free curve, and their market value, are equal. This approach in the context of market consistent evaluation, is called certainty equivalent [11, principle 13].

Hence, in the common model, credit and liquidity risk factors do not affect the volatility of the assets portfolio and the correlation between credit and liquidity spreads of different issuers is not considered at all. This has the further consequence that the tools generally adopted by insurance companies for Solvency II related valuations are not adequate for risk management, where these factors are usually included.

In this paper we propose a stochastic model for credit and liquidity risks, which allows for correlated movements across different issuers. Therefore, it is more suitable for risk management than the approach suggested by regulators.

In addition, we also disentangle the two sources of risk, credit and liquidity, in order to assess their relative importance. In fact, some econometric literature suggests that the liquidity effect is quite important in crisis period (see for instance [6]). An example of liquidity spread is reported qualitatively in Figure 1 for the German sovereign case. The historical series show that in several periods the Bund yield becomes smaller than the overnight rates in spite of a positive CDS premium. This behaviour can be interpreted as a *fly-to-liquidity* effect as explained in [6], i.e. there is a liquidity component in the bond spread and it turns out to be negative, hence the liquidity adjusting factor for zero coupon bonds (ZCBs) is greater than one as shown in Figure 2. This behaviour is also consistent when explained in terms of the re-denomination risk as suggested in a working paper of the European Central Bank ([33]). By viewing the liquidity risk as a *flight-to-quality* effect, we can avoid to deal with the financial market micro-structure, and focus on the *relative* importance of liquidity across European sovereign issuers.

In order to separate the effects of the two sources of risk, we consider firstly a model where the stochastic spread is driven by only one factor and we calibrate it on the Credit Default Swap (CDS) quotations; then we add a second stochastic factor to the spread and we calibrate it on the bonds yields. Assuming that CDS quotations are not affected by liquidity risk², we can isolate the contribution of the two stochastic components in the valuation of the portfolio.

For our numerical test, we focus on the case of segregated funds whose performance is determined by statutory accounting rules. This choice is due to the fact that the deterministic adjustment on cash flows, due to the application of the CEQ approach, is

¹For a more precise definition of bonds that are treated like government bonds under Solvency II standards see <https://eiopa.europa.eu/regulation-supervision/insurance/solvency-ii/solvency-ii-technical-specifications>

²This assumption is widely used in literature, see for instance [15] and [17].



Figure 1: The figure shows the historical series of the 5 year German Bund BVAL yields (red line), the Eur OIS 5Y rate (blue line) and the 5 year German CDS premium (green line).

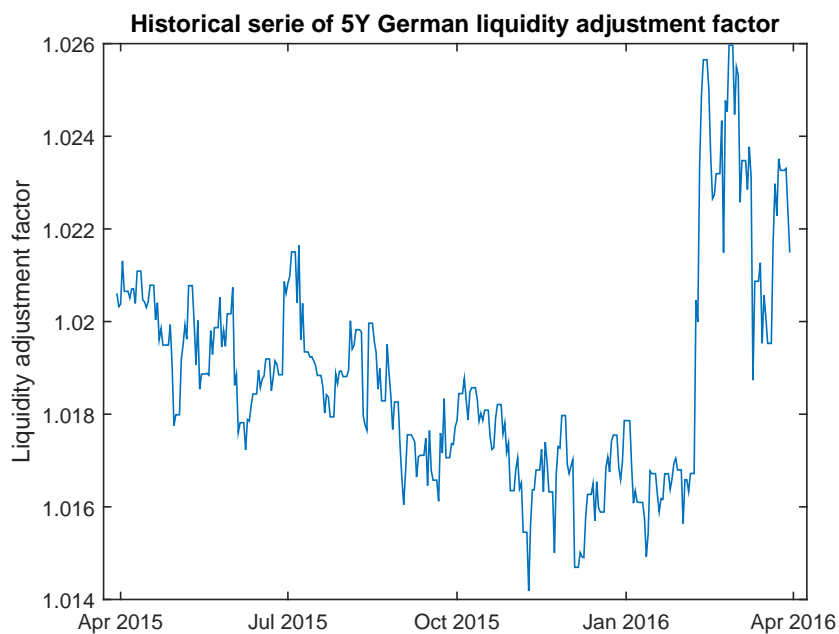


Figure 2: The figure shows the historical series of the 5 year German liquidity adjustment factor for ZCBs from April 2015 to April 2016.

particularly inappropriate in this case. Therefore, we test the impact of our new model versus the CEQ approach also with mark to market rules. Moreover, although we use Italian specific accounting rules for the segregated fund in order to produce our numerical results, very similar products are popular in other European countries (Germany and France are a good example), where they are traditionally used for saving or retirement³. The paper is organized as follows. Section 2 describes a with-profit segregated fund and explains the generally adopted (in market consistent evaluations) certainty equivalent approach used to evaluate the minimum guaranteed option. In Section 3 we describe our jointly stochastic model for interest rate, credit and liquidity risks and we perform the calibration of this model on market data. Section 4 presents the numerical evaluation of the embedded options. Results obtained with the common procedure are compared to the ones obtained with our model, inclusive of credit and liquidity risk. Conclusive remarks are presented in last section.

2 Quantitative assessment of the common practice

Fundamental aspects in the evaluation of insurance products and in particular of segregated funds are the statutory accounting rules which drive the profit sharing mechanism (between policyholder and shareholder) and ultimately, the shareholder obligations toward policyholders.

The common practice for the implementation of a market consistent framework consists in using a certainty equivalent approach (CEQ) to evaluate assets, which for risky securities boils down to applying a risk adjustment to their cash flows. Therefore, in practical valuations, it becomes critical to deem which assets are risk free (and therefore risk adjusted according to CEQ), and which are not. In the latter case, the certainty equivalent approach may not be applied, depending on the sophistication of the calculators implemented. Unfortunately, according to Solvency II standard formula, all government bonds issued by sovereign countries belonging to the European Monetary Union are risk free⁴. This contrasts with the view of capital markets, which quote very different government bonds spreads (e.g. over the Euro overnight interest rate swap) on EMU sovereign issuers. The consequence is that insurance companies, in order to treat homogeneously government bonds under the certainty equivalent approach, heavily risk-adjust bonds cash flows. They have to do so in order to recover the assets market price as the present value of cash flows discounted over the risk free curve provided by EIOPA.

Unfortunately, the value of financial options embedded in insurance contracts (minimum guaranteed options) is not invariant to risk adjustment on cash flows because their pay-off is determined by statutory accounting rules, as explained in the next section.

2.1 Description of segregated fund characteristics

A segregated fund is a type of investment fund administered by insurance companies in the form of life insurance contracts offering certain guarantees to the policyholder, as a minimum rate of return (minimum guaranteed). Segregated funds are owned by the

³See Appendix A for details on Italian accounting rules. A good description of German traditional with-profit insurance, similar to the case treated in this paper, can be found in [2]. Another example covering the French case can be found in [18].

⁴<https://eiopa.europa.eu/regulation-supervision/insurance/solvency-ii/solvency-ii-technical-specifications>.

life insurance company, not the individual investors, and must be kept separate from the company's other assets.

These funds consist of a pool of investments in securities such as bonds and stocks but their value does not fluctuate according to the market value of the underlying securities. In fact, for the purpose of determining the rate of return of the fund, assets are evaluated at their amortized (average) cost, or book value, and income is computed according to the dividends, coupons and amortization payments accrued over the year, plus any realized gain or loss derived from the sales of assets with respect to their amortized cost. Segregated funds accounting rules are explained further in Appendix A. Every year at a specific date not necessarily coincident with the end of the fiscal year, this rate is published and shared with the policyholder for the part exceeding the minimum guaranteed (bonus rate), according to predefined contractual rules. The amount passed as a bonus to the policyholder is accrued in the statutory reserve representing the insurance company obligations to policyholders⁵.

To make the matter even more complicated, the determination of the return of the segregated fund (the credited or bonus rate) is subject to discretionary rules (or management actions) applied by the insurance company. These can be: the investment policy, the investment limits, and the crediting strategy; all these determine the gains and losses realization relative to the accounting measure.

Let $p(t)$ be the payoff of the annual profit earned by an insurer holding a minimum guaranteed investment fund, $N(t)$ be the reserve (or the insurance obligation) at time t , $F(t)$ be the annual rate of return of the segregated fund, \bar{r} be the minimum guaranteed rate, β be the policyholder participation coefficient and f be the fee charged by the insurer to the policyholder. The payoff $p(t)$ is given by

$$(2.1) \quad \begin{aligned} p(t) &= N(t) (F(t) - \max(\beta (F(t) - f), \bar{r})) \\ &= N(t) [(1 - \beta) F(t) + \beta f - (\bar{r} - \beta (F(t) - f))^+]. \end{aligned}$$

By the previous decomposition, it is evident that a guaranteed investment fund contains an embedded option which is similar to a set of annual floorlet sold by the insurance company to the policyholder. Each strip floorlet has payoff

$$(2.2) \quad c(t) = N(t) (\bar{r} - \beta (F(t) - f))^+.$$

Hence, the value of guarantees (VOG) is the sum of the floorlet discounted prices

$$(2.3) \quad VOG(0) = \sum_{t=1}^n \mathbb{E} \left[\frac{N(t) c(t)}{M(t)} \right],$$

where n is the number of years and $M(t)$ is the stochastic money market account⁶. The expectation is computed under the risk neutral measure, assuming that no-arbitrage holds and markets are complete.

However, the embedded option can not be evaluated as a "classical" strip of put options written on the segregated fund. In fact, the underlying of the option, i.e. the annual return $F(t)$, depends on both accounting and market value of assets, and the discretionary

⁵This type of financial guarantees where the bonus rate component is periodically consolidated in the statutory reserve is called "cliquet" and is typical of traditional endowment or saving insurance policies. See [31] for an overview of guaranteed products.

⁶The money market account $M(t)$ is defined later in equation (3.2)

management actions applied to the fund. Moreover, the notional amount of the floorlet $N(t)$ depends on the history of accrued rates. This makes the option path dependent. In fact, if the rate of return granted to the policyholder is greater than the minimum guaranteed, then the following year the notional amount, which corresponds to the value of the statutory reserve, will increase by a corresponding percentage.

For these reasons, in order to estimate the value of financial options embedded in a generic insurance product backed by a segregated fund (value of guarantees or VOG), we have to proceed simulating the fund applying a Monte Carlo approach, inclusive of an appropriate asset and liabilities management (ALM) model.

2.2 Description of the simulation apparatus

The first step of our ALM model is asset calibration. By this term, we mean a procedure apt to match the value of securities, calculated as a present value of contractual cash flows, to their observed market value at the valuation date. This procedure should not be confused with the calibration described in Section 3.1 of the interest, credit and liquidity risk models. In fact, while the purpose of the latter is to determine the values of a set of parameters so that the mathematical model describing the dynamic of some stochastic processes is in agreement with observed data, the former is an ad-hoc procedure used to correct any discrepancy in securities pricing which is not explained by the modelled risk factors. Generally speaking, the more complete the pricing model, the closer the price of a security should be to its observed market value. Since we want to compare results calculated simulating alternative models, we need to guarantee that the asset portfolio has the same initial value regardless the discounting factor's specification that is used to recover the present value of the securities. This means that all the residual value in assets pricing, not explained by the model, is captured by a constant factor. In practice, if the model used for the assets valuation includes all the relevant risk drivers, then the constant factor is negligible.

This asset calibration is performed basically in two ways: adjusting the cash flows (this is used under the CEQ approach) or adjusting the discount rate using a flat z-spread.

In our analysis both approaches have been utilised. The first one has been applied to test the standard approach where the discount rate is the risk free rate alone. In this case the asset calibration consists in correcting the cash flows of risky securities by a deterministic and constant probability of default and then discounting them by using a risk free curve. In formula, the price at valuation date (time zero) of the i -th coupon bond $CB_i(0, T)$ is

$$CB_i(0, T) = \sum_{s=0}^T \frac{C_{s,i}(1 - p_i)^s}{(1 + R(0, s))^s} \quad \text{with } 0 < p_i < 1,$$

where p_i is the calibration parameter specific for the i -th coupon bond, representing the i -th coupon bond's default rate, $R(0, s)$ is the risk free spot rate at time zero with tenor s , $C_{s,i}$ is the cash flow paid by the i -th coupon bond at time s , and T is the i -th coupon bond's maturity.

In mathematical terms, the z-spread approach consists in pricing each coupon bonds via

$$CB_i(0, T) = \sum_{s=0}^T \frac{C_{s,i}}{(1 + \tilde{R}(0, s) + z_i)^s}$$

where $\tilde{R}(s, 0)$ is the market (risky) spot rate at time zero with tenor s , and z_i is the z-spread specific for the i -th security. In more details, $\tilde{R}(0, s)$ is a stochastic interest rate whose dynamic is described in Section 3 (see equation (3.11)) and includes credit and liquidity spreads. This second approach is used to test our model against the CEQ standard procedure. The z-spread is calibrated initially for each security, added to the market curve of spot interest rate, and then kept constant during the simulation.

There are some remarkable differences between the two approaches. Using the z-spread approach, cash flows are not affected by asset calibration; in fact the z-spread affects only the discounting curve. The z-spread is greatly reduced by modelling appropriately a security's market discounting curve. That is the case of our sovereign bond model, which includes interest, credit and liquidity risk. Furthermore, the CEQ calibration factor (default probability) depends arbitrarily on the choice of the risk free curve, which is the only possible discounting curve in the CEQ model. Moreover, in the calibration may happen that the probability of default of the CEQ approach p_i does not stay between 0 and 1 (trying to capture all the residual risky components as credit and liquidity). This is the case for example of the German Bunds, when the risk free spot rate curve $(R(0, t))_{t>0}$ is extracted from the European swap curve.

After having calibrated the assets portfolio, using the risk free or the market spot rate curve at the valuation date, $R(0, t)$ or $\tilde{R}(0, t)$ respectively, then we use a Monte Carlo algorithm to simulate the term structure of interest rates over time. For each Monte Carlo scenario and for each projection year, we perform all the calculation steps of the Asset and Liability Management process that a real insurance company would do. Firstly, cash flows stemming from assets (such as coupons, dividends and capital reimbursements) and recurrent, or new business, premiums are collected and used to pay for contractual obligations which are come due. In case available cash is not enough to pay for them, assets are sold and gains or losses are accounted.

Thereafter, crediting strategy is performed, i.e. assets are sold to meet the level of income targeted by the insurance company; usually this target is often calculated as the sum of a minimum guaranteed and a fixed management fee. This is typical of traditional insurance with-profit products because the performance of assets backing reserves, is commonly measured at book value rather than at market value⁷. Therefore, the insurance company has the ability of smoothing the return shared with policyholders (the credited rate) by selling assets whose market value is below (above) their book value when the accounting performance provided by the assets before any sale is above (below) the desired level. Ideally, the fund statutory performance (i.e. at book value) should be high enough to cover the fees due to shareholders and leave a return to policyholder above minimum guaranteed, in line with a stable long-term return. In practice, commercial considerations often affect the determination of the credited rate that is declared and shared with policyholder. After the crediting strategy step is completed, assets and liabilities are aligned and capital is injected or withdrawn depending on whether assets book value is lower or higher, respectively, than liabilities statutory value⁸. Finally, the portfolio is rebalanced according to investment limits.

⁷The way assets book value is calculated vary across countries and generally depends on the accounting category to which the asset is allocated (e.g. available for sales or immobilized). Coupon bonds are very often booked at historical amortised cost.

⁸The assets book value is calculated under the Local Generally Adopted Accounting Principles (LGAAP). Although this is similar to the old, Solvency I, coverage ratio, we wanted to keep our approach adherent to [10]. Indeed, aligning the value of the assets backing reserves at their statutory value provides to be useful in decomposing the embedded value of the in-force business.

Because the purpose of our analysis is to evaluate the impact of introducing a richer financial evaluation framework, which includes the stochastic credit and liquidity risk component of sovereign bonds, all the parameters affecting the ALM policy and the management actions⁹ are kept fixed, so that they do not depend either on the simulated economic scenario, or on time. For example, the target rate used in the crediting strategy is set constant and kept at its initial level during the simulations.

In order to minimize the turnover of the assets portfolio induced by allocation rebalancing, we firstly enforce a “Buy and Hold” strategy by setting lower and upper bounds on investments to 0% and 100% respectively. This is a sensible assumption on liability driven products for three reasons. First, because investment managers tend to match the maturity and duration of their assets and liabilities portfolio in order to offset interest rate risk on net asset value. Second, because insurance portfolios very often consist of long term investments, which cannot be disposed at an arms length (e.g. real estate or infrastructural investments). Third, because unnecessary turnover may deplete the reserve of unrealized gains or losses, which are fundamental to steer the bonus rate. Nevertheless, for completeness, we test also a “Constant Mix” approach where rebalancing occurs every year at constant allocation, although it is never applied as such in practice on this type of insurance products. See results in section 4.2.

Reinvestment strategy deserves a particular comment, since in our model new investments occur on constant maturity strategy¹⁰. This gives mainly two advantages: the exposure on key rate maturities can be controlled easily during the simulation, and no accounting option emerges to bias the results. In fact, modelling on an accounting base new bonds would introduce another option in the evaluation since bonds can be booked in more than one way, e.g. immobilized or available for sales.

Another important hypothesis is relative to the actuarial factors which are assumed to be deterministic (constant across scenarios) and independent from financial variables. For instance, the death rate of policyholders is deterministic and estimated from life tables. In the context of this study, assuming deterministic actuarial factors over time serves the purpose of having a greater focus on financial aspects. However, our model can be easily extended adding an affine stochastic mortality rate process (see for instance [24], [21], [34] and [37]).

Finally, VOG is calculated under the hypothesis that the insurance company is always solvent, i.e. it can not default, and it is always able to provide enough capital to cover statutory liabilities.

Without these assumptions, comparability of results would be greatly impaired.

Although the ALM apparatus just described is similar to the packages available on many commercial software widely used by actuaries, we implemented our on MATLAB[®], which allows for easier but sound implementation of financial calculation libraries.

⁹Management actions are the set of discretionary actions that a company can decide on its own, which may affect the annual return of the fund $F(t)$ in formula (2.2). In the context of this paper these are the crediting and the investment strategy. More in general, they can include also the type and extent of re-insurance, the type and extent of future underwriting business.

¹⁰A constant maturity strategy is an investment strategy where every year a bond with a specific maturity is purchased to be sold the following year. The proceeds are then used to finance the purchase of a new bond with the same maturity of the bond initially purchased.

3 The model

The model for sovereign bonds discussed in this section allows for three different sources of risk: interest rates, credit and liquidity. Credit and liquidity risks are modelled considering a specific term structure of spreads for each issuer.

At first, we model the risk free interest rate curve using the classical Vasicek model (see [36]). The dynamics of the short rate is described by the following stochastic differential equation

$$(3.1) \quad \begin{aligned} dr(t) &= a(\bar{r} - r(t))dt + \sigma dW(t), \\ r(0) &= r_0, \end{aligned}$$

where \bar{r} and a are the so called long run mean and speed of reversion coefficient (larger a faster r will move towards its mean and converge to \bar{r}), σ is the volatility parameter and $W(t)$ is a standard Brownian motion. The adoption of a Gaussian interest rate model is consistent with the recent experience of negative rates. Without loss of generality, the above dynamic is assumed to hold under the risk neutral measure.

The time t price of a risk free Zero Coupon Bond (ZCB) with maturity T is obtained by computing the following expectation under the risk neutral measure

$$P(t, T) = \mathbb{E} \left[\frac{M(0, t)}{M(0, T)} \right] = \mathbb{E} \left[e^{-\int_t^T r(s) ds} \right],$$

where the money market account, $M(0, t)$ is given by

$$(3.2) \quad M(0, t) = e^{\int_0^t r(s) ds}$$

It can be easily shown (see [8] page 59) that this expectation can be written as

$$(3.3) \quad P(t, T) = A(t, T)e^{-B(t, T)r(t)},$$

where

$$\begin{aligned} B(t, T) &= \frac{1 - e^{-a(T-t)}}{a}, \\ A(t, T) &= \exp \left[\left(\bar{r} - \frac{\sigma^2}{2a^2} \right) (B(t, T) - T + t) - \frac{\sigma^2}{4a} B(t, T)^2 \right]. \end{aligned}$$

In order to model the price of a bond issued by a defaultable issuer, we adopt an intensity model with zero recovery. This is equivalent, see for example [23], to add to the short rate a spread related to the creditworthiness of the issuer I . Therefore, the price of a defaultable ZCB is obtained as

$$(3.4) \quad P^I(t, T) = \mathbb{E} \left[e^{-\int_t^T (r(u) + s^I(u)) du} \right].$$

Assuming independence between spread and risk free short rate model¹¹, the price of the risky ZCB can be split into the product of two components, the risk free ZCB and a adjustment factor

$$P^I(t, T) = P(t, T) Adj_c^I(t, T),$$

¹¹A common assumption in literature, see for instance [7] and [9].

where

$$(3.5) \quad Adj_c^I(t, T) = \mathbb{E} \left[e^{-\int_t^T s^I(u) du} \right].$$

The adjustment factor can be interpreted as a survival probability, i.e. the probability that the issuer does not default in the time interval $[t, T]$.

The credit spread $s^I(t)$ is modelled as a positive stochastic process, e.g. the square-root Cox-Ingersoll-Ross (CIR, see [14]) process. Therefore, we write

$$(3.6) \quad \begin{aligned} ds^I(t) &= b_I (\bar{s}_I - s^I(t)) dt + \eta_I \sqrt{s^I(t)} dZ_I(t), \\ s^I(0) &= s_0^I, \end{aligned}$$

where Z_I is a standard Brownian motion, assumed to be independent from the Brownian motion driving the dynamics of the risk-free rate.

Given (3.6), then the adjustment factor can be expressed in closed form (see [8] page 66)

$$(3.7) \quad \begin{aligned} Adj_c^I(t, T) &= A_c^I(t, T) e^{-B_c^I(t, T) s^I(t)}, \\ A_c^I(t, T) &= \left[\frac{2h e^{(b+h)(T-t)/2}}{2h + (b+h)(e^{(T-t)h} - 1)} \right]^{2b\bar{s}_I/\eta^2}, \\ B_c^I(t, T) &= \frac{2(e^{(T-t)h} - 1)}{2h + (b+h)(e^{(T-t)h} - 1)}, \\ h &= \sqrt{b^2 + 2\eta^2}. \end{aligned}$$

A similar financial model is proposed in [20] in the so called multiple-curve Libor market, i.e. a Libor rate model with different tenors.

We can include in our model a liquidity risk factor, as well, by introducing a liquidity spread, $l^I(t)$, in the ZCB formula

$$(3.8) \quad P^I(t, T) = \mathbb{E} \left[e^{-\int_t^T (r(u) + s^I(u) - l^I(u)) du} \right].$$

We model it again using the Vasicek model

$$(3.9) \quad \begin{aligned} dl^I(t) &= k_I (\bar{l}_I - l^I(t)) dt + \phi_I dY^I(t), \\ l^I(0) &= l_0^I, \end{aligned}$$

where Y^I is a standard Brownian motion, assumed to be independent from the Brownian motions driving the dynamics of the risk-free rate and the credit spread.

Notice that the liquidity spread $l^I(t)$ can take negative values. This is relevant for example in the German sovereign case as previously shown. In practice, a negative liquidity spread allows us to capture the so called fly-to-liquidity effects; in other words a negative liquidity spread is an implicit convenience yield that arises to the owner of a liquid bond. If we assume that the liquidity spread is independent from both the risk free rate and the credit spread, then, simply, the risky ZCB formula P^I contains a second multiplicative adjustment factor that has the following closed form

$$(3.10) \quad Adj_l^I(t, T) = A_l^I(t, T) e^{+B_l^I(t, T) l^I(t)},$$

where

$$B_t^I(t, T) = \frac{1 - e^{-k(T-t)}}{k},$$

$$A_t^I(t, T) = \exp \left[\left(-\bar{l} - \frac{\phi^2}{2k^2} \right) (B(t, T) - T + t) - \frac{\phi^2}{4k} B(t, T)^2 \right].$$

The liquidity spread is calibrated using the bond yields quoted in the market. The independence between the credit and the liquidity spread is an assumption used in literature, see for instance [17], and it is desirable for the analytical tractability and the parsimony of the model. Due to the complex dependence structure between the credit and the liquidity risks highlighted by different empirical works, e.g. [4] and [19], we think that a linear correlation between the Brownian motions driving the credit and the liquidity spread dynamics is not appropriate. However, a more sophisticated model that takes this dependence into account can be introduced in our framework, following ideas in [28]. Finally, it should be clear to the reader that we are not attempting here to provide an absolute measure of liquidity similar to what required by Basel III¹², by engaging in complex econometric models, which would require, probably, more factors than what we have presently adopted. Rather, our aim is to complete our model in a parsimonious way by including an additional factor able to capture a further risk component, which is not necessarily explained by the credit spreads, and which may contribute to the relative fluctuation of prices of European sovereign bonds denominated in Euro. Finally, credit and liquidity spreads of different issuers can be correlated through the Brownian motions of the CIR or Vasicek processes, i.e. for $I \neq J$

$$dZ_I(t) dZ_J(t) = \rho_c^{IJ} dt,$$

$$dY_I(t) dY_J(t) = \rho_l^{IJ} dt.$$

3.1 Model calibration

In this section we describe a possible calibration procedure of the model. The implementation of the model requires firstly to identify the risk-free curve. EIOPA proposes a risk-free discounting curve based on Euribor 6 months par Interest Rate Swap (IRS) rates¹³. However, as highlighted in several papers, [3], [22], [27] and [30] among others, the Euribor par swap rate is affected by credit and liquidity risk of the interbank market, which is not negligible. To fix this problem the EIOPA curve contains a Credit Risk Adjustment (CRA). An additional Volatility Adjustment (VA) is applied to the curve (see [16]). These adjustments lead to a market consistency issue of the curve as highlighted in [25]. Therefore, according to the recent financial literature (for instance [26], [30] and [29]), we identify the overnight rate (in particular the Eonia rate for Euro currency) to be the best proxy for the risk free interest rate. In particular, given that there are no liquid options written on the Eonia rate, we calibrate the parameters of the stochastic model for $r(t)$ to the Euro overnight indexed swap (OIS) curve.

¹²See [5]. Here the liquidity of assets is measured as the ability of a security to be sold at an arm's length at little or no cost.

¹³More details about the construction of the EIOPA curve are given in Appendix B.

In the model previously presented in Section 3, ZCB price can be written as

$$\begin{aligned}
P^I(t, T) &= P(t, T) \text{Adj}_c^I(t, T) \text{Adj}_l^I(t, T) \\
&= e^{(-R(t, T) - S^I(t, T) + L^I(t, T))(T-t)} \\
(3.11) \quad &= e^{-\tilde{R}^I(t, T)(T-t)}
\end{aligned}$$

where $R(t, T)$ is the zero risk free rate, $S^I(t, T)$ and $L^I(t, T)$ are the credit and liquidity spreads, respectively, and $\tilde{R}^I(t, T)$ is the market risky spot rate, calculated between t and T for the issuer I . In order to estimate the credit component, we bootstrap the term structure of survival probabilities (i.e. the no-default probabilities of the issuer) from the quotations of credit default swap (CDS) spreads for each specific issuer. However, there are no liquid quotations of CDS for maturities longer than 10 years. Hence, we extract the long term survival probabilities from sovereign ZCB curves under the hypothesis that the long term liquidity spread remains constant and equal to the 10 years spread, i.e.

$$L^I(0, T) = L^I(0, T^*), \text{ for } T \geq T^* = 10.$$

Without this assumption, we should have too large values of long maturity liquidity spread¹⁴. Hence, we obtain the following formula for the issuer survival probabilities

$$PS^I(0, T) = \frac{P^I(0, T)}{P(0, T)} \left(\frac{P(0, T^*) PS^I(0, T^*)}{P^I(0, T^*)} \right)^{\frac{T}{T^*}}, \text{ if } T \geq T^*.$$

By the previous formula the issuer survival probabilities $PS^I(0, T)$ for $T > T^*$ are computed using (a) CDS quotations up to time T^* , i.e. $PS^I(0, T^*)$, (b) using the ZCB prices of the issuer I , $P^I(0, T)$ and $P^I(0, T^*)$, obtained from quoted sovereign spot curves and (c) using the risk free ZCB prices, $P(0, T)$ and $P(0, T^*)$, bootstrapped from the OIS curve.

By this procedure, we have now market implied term structures of $P(0, T)$, $PS^I(0, T)$ and $P^I(0, T)$ up to 30 years and we use them to calibrate the parameters of the processes $r(t)$, $s^I(t)$ and $l^I(t)$, i.e. the risk free short rate and the stochastic credit and liquidity spreads of the issuers. In particular, we consider CDS quotations and ZCB curves of Italian and German governments on March 30th, 2016. Calibration results on market quotations on March 30th, 2016 of the Vasicek model for $r(t)$, of the CIR model for $s^I(t)$ and of the Vasicek model for $l^I(t)$ are given in Table 1, 2 and 3 and Figure 3, 4 and 5, respectively. The calibrations are performed through the minimization of Root Mean Square Deviation (RMSD) between market data (ZCB prices and probabilities of default) and the corresponding model quantity. In Tables 1-3, we report the RMSD and the maximum relative error (MRE) between market and model quantities. Market data are reported in Appendix C. More details about the calibration procedure are given in Appendix D.

Finally, we estimate the historical correlations between the credit spreads of the two issuers, i.e. German and Italian governments, and we do the same for the liquidity spreads of the two issuers. The data set covers the period from March, 30th 2015 to March, 30th

¹⁴ This quantity is given by the following form (as in [36])

$$L^I(0, \infty) = -\bar{l}^I - \frac{1}{2} \frac{\phi_I^2}{k_I^2}.$$

2016 and it is composed by Euro OIS rates with maturity from 1 month to 10 years, CDS spread quotations of the two issuers with maturity from 6 months to 10 years and bond yield curves of the two issuers with maturity from 3 months to 10 years. For each date in the sample, we bootstrap the ZCB prices from OIS rates, then we choose the most liquid maturity and we invert formula (3.3) to extract $r(t)$. In this way, we obtain the historical time series of the risk free short rate $r(t)$. In order to extract the historical series of the stochastic credit spreads for each issuer, $s^I(t)$, we bootstrap in each date the issuer survival probability curve from CDS quotations, using OIS as discounting curve, and then we choose the most liquid maturity and we invert formula (3.7) to compute $s^I(t)$. The correlation calculated between the series of daily returns of the two credit spreads is assumed to be a proxy of the correlation between the two Brownian motions driving the CIR processes. Finally, we subtract the risk free and the credit components from the bond yields and the remaining part is identified with the natural logarithm of formula (3.10), so that we obtain also the series of the liquidity spreads $l^I(t)$. The correlation calculated between the series of daily returns is assumed to be a proxy of the correlation between the two Brownian motions driving the Vasicek processes of the two liquidity spreads. More details are given in Appendix D. The historical values of the correlation between Italian and German credit and liquidity spreads turn out to be $\rho_c = 0.2463$ and $\rho_l = 0.3341$, respectively. From a risk management point of view, it is very important to assess the impact of the correlation on the volatility of portfolio. For this reason, we simulate the portfolio using not only the historical based estimates of correlations but also considering extreme correlation scenarios, i.e. $\rho_c = \rho_l = \pm 1$, as well as the zero correlation case.

r_0	a	\bar{r}	σ	RMSD	MRE (%)
-0.0107	0.1993	0.0135	0.0006	4.9×10^{-3}	1.5

Table 1: Vasicek model: calibrated parameters on the Eonia curve on March 30th 2016. The result of the calibration is shown in Figures 3. The root mean square deviation (RMSD) and the maximum relative error (MRE) of the calibration are also reported.

Country	s_0	b	\bar{s}	η	RMSD	MRE (%)
GER	0.0001	0.3659	0.0075	0.0742	4.1×10^{-3}	1.1
ITA	0.0001	0.4761	0.0389	0.1925	1.1×10^{-2}	4.8

Table 2: CIR model: calibrated parameters on CDS and sovereign ZCB curves on March 30th 2016. The results of the calibrations are shown in Figure 4. The root mean square deviation (RMSD) and the maximum relative error (MRE) of the calibrations are also reported.

Country	l_0	k	\bar{l}	ϕ	RMSD	MRE (%)
GER	-0.0078	0.9990	0.0090	0.0021	3.8×10^{-3}	1.0
ITA	-0.0001	0.4806	0.0261	0.0011	8.0×10^{-3}	3.4

Table 3: Vasicek model: calibrated parameters on the sovereign ZCB curves on March 30th 2016. The results of the calibrations are shown in Figure 5.

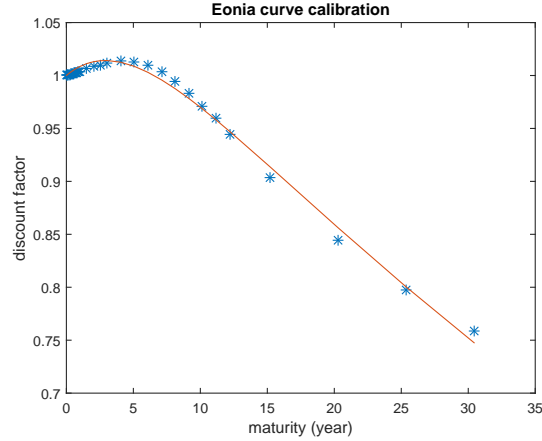


Figure 3: Calibration of the Eonia curve on March 30th, 2016 with the Vasicek model. Blue markers are the market quotations.

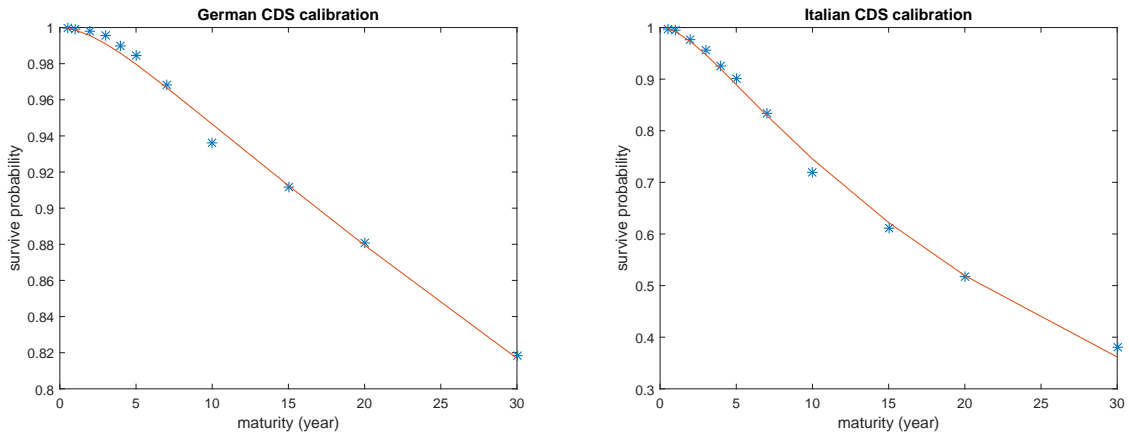


Figure 4: Calibration of the CDS and ZCB sovereign curves on March 30th, 2016 with the CIR model. Blue markers are the market quotations. The last four market quotations (15, 20, 25 and 30 years) are extracted from ZCB sovereign curves with the assumption of long term constant liquidity spread as explained in Section 3.1.

3.2 Risk neutral evaluation and martingale test

In order to prove that the Economic Scenario Generator (ESG) built upon the model presented in previous section is risk neutral and market consistent, as required by the regulator under Solvency II directive, martingale tests on sovereign coupon bonds with different maturities are performed. The martingale process is built by dividing the total return performance of an asset by a the total return performance of the cash account, i.e. the numeraire of the risk neutral measure. The results are shown in Figures 6-8, using the calibrated parameters presented in Tables 1-3.

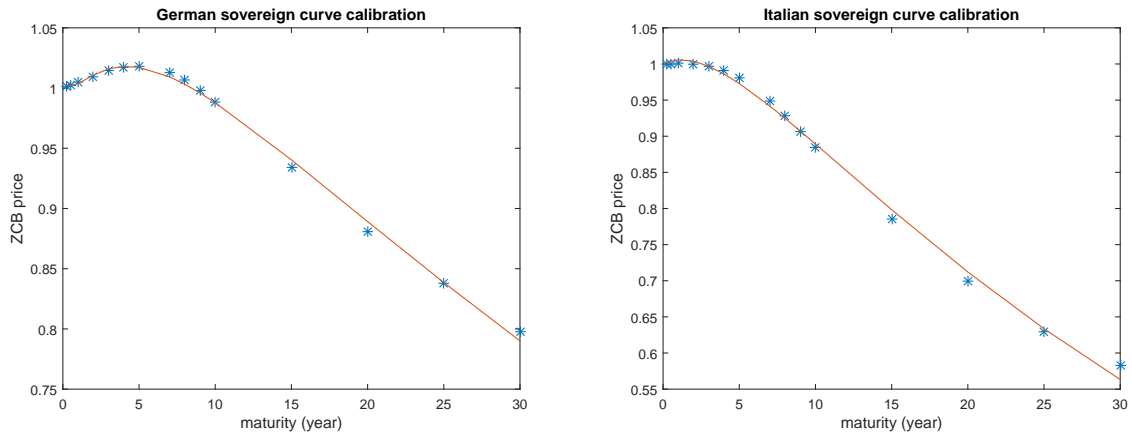


Figure 5: Calibration of the sovereign BVAL curves on March 30th, 2016 with Vasicek model for the risk free and the liquidity component and a CIR model for the credit factor. Blue markers are the market quotations.

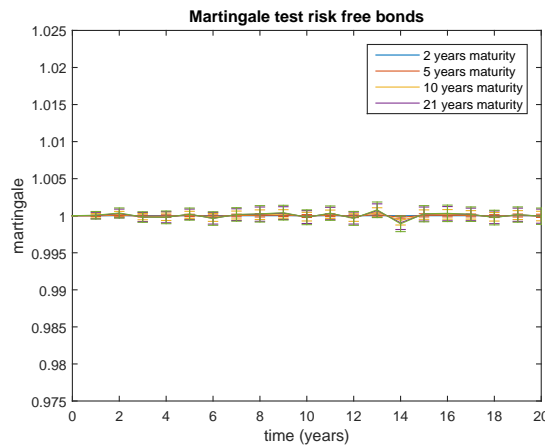


Figure 6: Martingale test performed on risk free coupon bonds with 100 Monte Carlo simulations and parameter reported in Table 1. The error bars are the 97.5% confidence intervals.

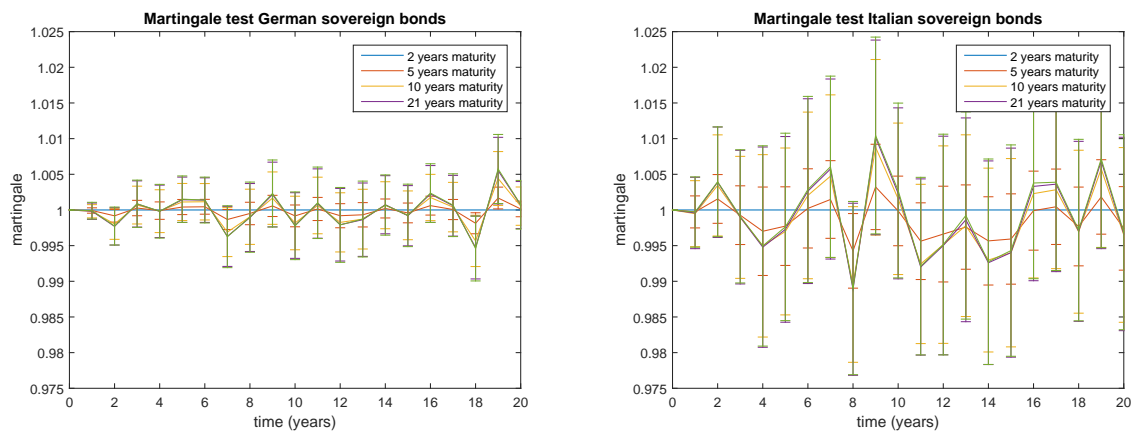


Figure 7: Martingale tests performed on sovereign German and Italian coupon bonds with 100 Monte Carlo simulations and parameter reported in Tables 1 and 2. The error bars are the 97.5% confidence intervals.

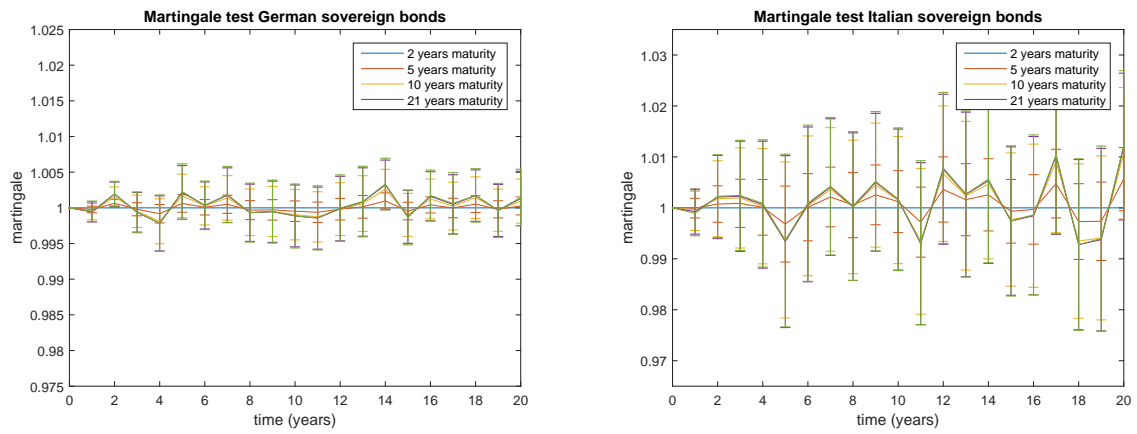


Figure 8: Martingale tests performed on sovereign German and Italian coupon bonds with 100 Monte Carlo simulations and parameter reported in Tables 1, 2 and 3. The error bars are the 97.5% confidence intervals.

4 Numerical results

In this section, we compare the value of contractual options embedded in European insurance (with-profit) traditional products, assuming that all government bonds are risk free, with the value provided by the more complete model described in previous sections. Overcoming the need of adopting a CEQ approach makes the evaluations more sensible and robust, and ultimately, more consistent with a risk management framework. In particular for the credit and liquidity model we study the role of the correlation among issuers. The possibility to stress parameters such as correlation is of paramount importance for a model to be of use as a risk management tool ¹⁵. This opportunity is not achievable under the commonly adopted framework. In order to test the consistency of our approach, we compare the full model including credit and liquidity spreads with a (less complete) model that includes only credit spread and with one (incomplete) including only the risk free interest rate as stochastic factor. In the latter case, we evaluate the embedded option using the (standard) CEQ approach and a z-spread adjustment calibrated at valuation date. Significantly, all the results agree with theory. In particular, results show that adjusting cash flows for risk undesirably affects the value of the option through the statutory accounting rules of the segregated fund.

4.1 Description of the tested portfolio

The ALM set-up described in the previous section is used to simulate, over a 20 years time-horizon a portfolio of endowments, i.e. life liabilities with death, surrender and maturity benefit (no annuities), which has a total duration (modified) of about nine years and which runs-off in approximately 20 years. Liabilities have an average minimum guaranteed of 3% (\bar{r} in formula 2.1), a total value (mathematical reserve) of one billion and a vintage year of 4 years. The average policyholder participation coefficient β is near to one and fixed fees are set very low at 25 basis points.

The life liabilities are backed by a portfolio of government bonds with fixed or floating rate, issued by Italy or Germany (90% Italian and 10% German) with a total duration (modified) of about nine years. The asset portfolio has an operating (current) accounting return higher than 3% for the next 5 years.

The initial portfolio and liabilities compositions are derived from a real insurance fund whose equity and corporate components are reallocated to sovereign bonds, keeping constant the issuers proportions.

4.2 Evaluation of the embedded option

We perform a Monte Carlo simulation using the apparatus described above and a set of stochastic scenarios consisting of the risk free rate, the Italian and German credit spreads and the Italian and German liquidity spreads. The stochastic model adopted is described in Section 3 and the calibration parameters are reported in Tables 1, 2 and 3 in Section 3.1.

The embedded option is evaluated under three alternative set-ups. In set-up 1, the stochastic scenarios used to evaluate the VOG are generated using all available risk

¹⁵The approach presented in this paper is also in agreement with the prudent person principle. For more details please check [1, Paragraph 6]

factors, which are interest rate, credit and liquidity; in the set-up 2 only interest rate and credit risk are used. In set-up 3 only interest rate risk is considered.

The asset calibration is carried out using a z-spread for set-up 1 and 2. The z-spread is very few basis points when the complete model is used (set-up 1). For set-up 3, we test both the z-spread method and the CEQ approach.

In the first two set-ups different dependence scenarios between the two issuers are generated and tested. Historical correlations for credit and liquidity spreads of the two issuers are estimated in calibration, see Section 3.1. Extreme values (-1 and 1) of the correlation between Brownian motions of credit and liquidity spreads are tested. Moreover, the portfolio is simulated also in the hypothesis of independent issuers. VOG results are reported in table 4.

Table 4 shows clearly that correlation affects sensibly the results and that adding the liquidity risk to the model increases the value of the option. This is consistent with the diversification effect which is expected when correlation turns negative. In fact, increasing the correlation increases the variance of the portfolio and makes the options more expensive. Although the standard error is still material with only 1000 simulations, also in the set-up 2 it is evident that a diversification effect is operating.

In Table 7, we compare the VOG obtained using only stochastic interest rate (set-up 3) with the VOG obtained with our complete model (set-up 1). The set-up 3 evaluation is done applying a risk adjustment to securities cash flows, according to the CEQ approach. The difference in results of Table 7 is striking, using both a buy & hold or a constant mix investment strategy.

The explanation is that the option is written on an underlying that depends on accounting rules that are not invariant to arbitrary risk adjustments of cash flows.

Compared to the model used to generate the results reported in Table 7, the results in Table 4, set-up 1, are obtained using five risk factors (encompassing all the assets classes in position which are German and Italian government bonds), instead of just one as in the set-up 3. Therefore, we would have expected a result of the CEQ model close to the one we have got using set-up 1 and perfect correlation. Because the magnitude of the discrepancy observed between these models cannot be explained by some missing risk factor or by a smaller volatility of the richer model compared to the CEQ, the only explanation must be the appropriateness of the constant and arbitrary adjustment derived from the application of the CEQ approach. From Appendix A, it will be clear that any risk adjustment applied to a security's cash flows would change the assessment of the statutory income through the gains or losses at maturity of bonds available for sales, the coupons received, the difference between accrued interest, and finally, the calculation of average book value, and that all these changes do not necessarily compensate. To confirm our interpretation of the numerical results, we have calculated the VOG evaluating the segregated fund return (which is the option's underlying) at market value, i.e. we have evaluated the fund applying the same principles as for the assets classified in the Fair Value Through Profit and Loss (FVTPL) category as defined by International Financial Reporting Standards (IFRS). In line with our expectation, the choice of the accounting rules has a large impact on the value of the embedded option. The most relevant aspect is that the results obtained with a CEQ approach and with our model in case of perfect correlation, are quite close if the fund is evaluated "mark to market" (see Table 8). VOG in Table 8 are higher than VOG in Tables 4 and 7, because accounting rules reduce the volatility of the underlying fund (e.g. immobilized assets don't show return volatility by definition if they can be held to maturity; see also Section 2.2). These results have an

interesting implication, with respect to a quite controversial debate on whether or not accounting rules have an impact on the economic value of assets.

Finally, we perform some sensitivity analysis in order to assess the importance of some features of the model as the introduction of a liquidity spread or the correlation between the sovereign issuers. The results are reported in Tables 5 and 6.

The reasons why the CEQ approach is not appropriate for the evaluation of this option can be summarised as follows:

- the return of the funds and hence the value of the option depends on accounting conventions;
- the value of the option also depends on the interaction of assets and liabilities which is not completely under control of the insurance company and which depends on unobservable (in the capital market) variables such as mortality and surrender rates;
- the value of the option depends on discretionary actions (management actions) defined by the company such as commercial targets on segregated fund statutory return, investment rules including the guidelines to classify newly purchased assets on the segregated fund balance sheet, the criteria used to sell assets for paying contractual obligations or for smoothing the fund return using unrealised gains or losses.

Correlation (Spread corr.)	VOG - set-up 1 (Std error)	VOG - set-up 2 (Std error)
-1 (-60.2%)	11,525,803 (501,307)	9,876,695 (467,384)
0 (-0.8%)	10,456,238 (504,367)	9,844,966 (498,935)
Hist (19.1%)	11,824,693 (579,445)	9,840,467 (542,230)
1 (99.0%)	11,643,768 (610,698)	10,365,180 (569,352)

Table 4: The table reports the Value of Options and Guarantees (VOG) calculated running 1000 stochastic simulations; in parenthesis is reported in the first column the average correlation observed on simulated spread and in the second and third columns the standard error of VOG. The option is evaluated using buy & hold ALM rebalancing strategy. In all cases, the segregated fund is evaluated using Italian traditional accounting rules (see Appendix A for details).

set-up 1 vs set-up 2		
Correlation	Abs. difference	Rel. variation (%)
-1	1,649,108	15%
0	611,271	6%
Hist	1,984,226	18%
1	1,278,588	12%

Table 5: This table reports the absolute difference and the relative variation between the values obtained with and without the liquidity spread in Table 4. The relative variation is calculated as the absolute difference divided by the average between the two prices.

Extreme correlations analysis	VOG - set-up 1	VOG - set-up 2
Abs. difference	1,368,455	524,713
Rel. variation (%)	13%	5%

Table 6: This table reports the absolute difference and the relative variation between the extreme values obtained stressing the correlation in Table 4. The relative variation is calculated as the absolute difference divided by the smaller of the two prices.

Buy & Hold		Constant Mix	
VOG set-up 3 (CEQ)	VOG set-up 1	VOG set-up 3 (CEQ)	VOG set-up 1
36,503,519 (97,921)	11,643,768 (610,698)	15,399,976 (254,428)	5,922,923 (1,415,055)
Abs. difference	24,859,752	Abs. difference	9,477,053
Rel. variation (%)	103%	Rel. variation (%)	89%

Table 7: The table reports the value of VOG calculated using the CEQ approach with only the interest rate as stochastic risk factor and our full model (set-up 1) with correlation equal to 1. The option is evaluated using two different ALM rebalancing strategies, buy & hold and constant mix. In all cases, the segregated fund is evaluated using Italian traditional accounting rules (see Appendix A for details).

Buy & Hold		Constant Mix	
VOG set-up 3 (CEQ)	VOG set-up 1	VOG set-up 3 (CEQ)	VOG set-up 1
187,566,084 (450,605)	181,630,924 (6,337,511)	187,003,992 (570,700)	203,839,753 (7,445,830)
Abs. difference	5,935,160	Abs. difference	-16,835,761
Rel. variation (%)	3.2%	Rel. variation (%)	-9%

Table 8: The table reports the value of VOG calculated using the CEQ approach with only the interest rate as stochastic risk factor and our full model (set-up 1) with correlation equal to 1. The option is evaluated using two different ALM rebalancing strategies, buy & hold and constant mix. In all cases, the segregated fund is evaluated using the FVTPL (fair value through profit or loss) accounting rule of IFRS (International Financial Reporting Standards).

Conclusion

The European directive also known as Solvency II has driven more focus on the need of sounder risk management practice by insurance companies. At the same time, it has introduced standards in the evaluation of assets and liabilities in the attempt of creating a fairer playing field in the insurance sector. Although the European regulator has succeeded in its attempt, this is not without critics. One of most debated issues is the assumption that all European government bonds has to be evaluated using a risk free discounting curve. This assumption together with the adoption, mandatory in the Solvency II framework, of market consistent evaluations, has pushed insurance companies to adopt the certainty equivalent approach to cope with the complexity of simulation apparatus needed to carry out all the necessary calculations, while being consistent with the assumptions of the (Solvency II) Standard Formula. In this paper we have analysed the consequences of oversimplified risk models, in particular where risk adjustment is applied to options whose underlying depends on accounting rules. Moreover, we have introduced a new model for European government bonds that is more consistent with prices observed in capital markets and at the same time, more flexible to be used for day-to-day risk management. Finally this work presents interesting possibilities for further extensions such as the introduction of a model for corporate bonds and the relaxation of simplifying assumptions concerning the insurance risks and the management actions.

References

- [1] AIFIRM, M. R. C. Prudent Valuation guidelines and sound practices. *available at <http://www.aifirm.it/wp-content/uploads/2016/06/Documento-AIFIRM-Mar2016-Prudent-Valuation-Guidelines-and-Sound-Practices.pdf>* (2016).
- [2] ALEXANDER KLING, ANDREAS RICHTER, J. R. The interaction of guarantees, surplus distribution, and asset allocation in with profit life insurance policies.
- [3] AMETRANO, F., AND BIANCHETTI, M. Bootstrapping the illiquidity: Multiple yield curves construction for market coherent forward rates estimation. *Risk Books, Incisive Media* (2009).
- [4] BAI, J., AND COLLIN-DUFRESNE, P. The cds-bond basis. *AFA 2013 San Diego Meetings Paper available at SSRN: <https://ssrn.com/abstract=2024531>* (2013).
- [5] BASEL COMMITTEE ON BANKING SUPERVISION. Basel iii: The liquidity coverage ratio and liquidity risk monitoring tools.
- [6] BEBER, A., BRANDT, M., AND KAVAJECZ, K. Flight-to-Quality or Flight-to-Liquidity? Evidence from the Euro-Area Bond Market. *Rev. Financ. Stud.* 22, 3 (2009), 925–957.
- [7] BRIGO, D., AND MASETTI, M. A Formula for Interest Rate Swaps Valuation under Counterparty Risk in presence of Netting Agreements. *available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=717344* (2005).
- [8] BRIGO, D., AND MERCURIO, F. *Interest Rate Models: Theory and Practice.*, 2nd ed. Springer, 2006.
- [9] BRIGO, D., AND MORINI, M. CDS Market Formulas and Models. *available at <http://www.imperial.ac.uk/~dbrigo/cdsmktfor.pdf>* (2005).
- [10] CASTELLANI, G., DE FELICE, M., MORICONI, F., AND PACATI, C. Embedded value in life insurance.
- [11] CFO-FORUM. Market Consistent Embedded Value Basis for Conclusions. *available at http://www.cfoforum.nl/downloads/CFO-Forum_MCEV_Basis_for_Conclusions_April_2016.pdf* (April 2016).
- [12] CFO-FORUM. Market Consistent Embedded Value Principles-Guidance. *available at http://www.cfoforum.nl/downloads/CFO-Forum_MCEV_Principles_and_Guidance_April_2016.pdf* (April 2016).
- [13] CORVINO, G. *I prodotti assicurativi vita rivalutabili. Caratteristiche, regolamentazione e gestione.* EGEA, 2003.
- [14] COX, J., INGERSOLL, J., AND ROSS, S. A Theory of the Term Structure of Interest Rates. *Econometrica*, 53 (1985), 385–407.
- [15] DUFFIE, D., AND SINGLETON, K. *Credit Risk: Pricing, Measurement, and Management.* Princeton University Press, 2003.

- [16] EIOPA. Technical documentation of the methodology to derive EIOPA's risk-free interest rate term structures. <https://eiopa.europa.eu/regulation-supervision/insurance/solvency-ii-technical-information/risk-free-interest-rate-term-structures> (May 2016).
- [17] F. LONGSTAFF, S. M., AND NEIS, E. Corporate yield spreads: Default risk or liquidity? new evidence from the credit default swap market. *Journal of Finance* 60 (2005), 2213–2253.
- [18] FABRICE BOREL-MATHURIN, PIERRE-EMMANUEL DARPEIX, Q. G. S. L. Main determinants of profit sharing policy in the french life insurance industry.
- [19] FONTANA, A., AND SCHEICHER, M. An analysis of euro area sovereign cds and their relation with government bonds. *Journal of Banking & Finance* 62 (2016), 126–140.
- [20] GRBAC, Z., AND RUNGALDIER, J. *Post-Crisis challenges and Approaches.*, 1st ed. Springer, 2015.
- [21] HARI, N., WAEGENAERE, A. D., AND NIJMAN, T. E. Longevity risk in portfolio of pension annuities. *Insurance: mathematics and economics* 42 (2008), 505–519.
- [22] HENRARD, M. The irony in the derivatives discounting part II: the crisis. *SSRN eLibrary* (2009).
- [23] JARROW, R., AND TURNBULL, S. Pricing Derivatives on Financial Securities Subject to Credit Risk. *The Journal of Finance* L, 1 (1995).
- [24] JEVTIC, P., AND REGIS, L. Assessing the solvency of insurance portfolios via a continuous-time cohort model. *Insurance: mathematics and economics* 61 (2015), 36–47.
- [25] KAROUI, N. E., LOISEL, S., PRIGENT, J., AND VEDANI, J. Market inconsistencies of the market-consistent European life insurance economic valuation: pitfalls and practical solutions. *working paper*, <https://hal.archives-ouvertes.fr/hal-01242023> (2015).
- [26] KENYON, Z. Short-rate pricing after the liquidity and credit shocks: Including the basis. *Risk Mag.* (2010), 83–87.
- [27] MERCURIO, F. Interest rates and the credit crunch: new formulas and market models. *Technical report, Bloomberg Portfolio Research Paper*, 2010–01-FRONTIERS (2010).
- [28] MONFORT, A., AND RENNE, J. Decomposing euro-area sovereign spreads: credit and liquidity risks. *Review of Finance* 18, 6 (2014), 2103–2151.
- [29] MORENI, N., AND PALLAVICINI, A. Parsimonious HJM modelling for multiple yield-curve dynamics. *Quant. Finance*, 14 (2014), 199–210.
- [30] MORINI, M. Solving the puzzle in the interest rate market. *SSRN eLibrary* (2009).

- [31] PITACCO, E. From benefits to guarantees looking at life insurance products in a new framework.
- [32] PWC. IFRS versus German GAAP. Summary of similarities and differences.
- [33] SANTIS, R. D. A measure of redenomination risk. *European Central Bank working paper series* (April 2015).
- [34] SCHRAGER, D. Affine stochastic mortality. *Insurance: mathematics and economics* 38, 1 (2006), 81–97.
- [35] THEROND, P. Provisions techniques relatives aux actifs, financiers des assureurs. *Resource Actuarielles* (2008).
- [36] VASICEK, O. An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5 (1977), 177–188.
- [37] VIGNA, E., AND LUCIANO, E. Mortality risk via affine stochastic intensities: calibration and empirical relevance. *Belgian Actuarial Bulletin*, 8 (2008), 5–16.

A Segregated fund accounting rules

In this section we describe briefly how the performance of fixed income securities (coupon bonds) is calculated in case of Italian segregated funds. More information can be found in the book [13]. Similar rules are applied to traditional saving and pension products also by other countries in continental Europe (e.g. France, Germany; see [35] or [32]).

The components of the accounting performance are the determination of the bond current (periodical) income and the average book value.

A bond’s current income is made of the following elements:

- coupons paid during the year or calculation period,
- difference between initial and final accrued interests, during the calculation period,
- amortisation,
- any realized gain or loss due to sales of part or all the quantity in position,
- any realized gain or loss at a bond’s maturity date.

The amortisation depends on the classification the bond receives when it is purchased. The same asset can have more than one classification in the same segregated fund. Admissible classifications are of two types: Immobilised, or Available for Sales. When a bond is classified as immobilised it cannot be sold before maturity and the difference between price paid when the asset was purchased and its value at maturity (reimbursement) can be amortised linearly every year. If instead a bond is classified as available for sales, the bond can be sold at any time, but only the difference between issue price and reimbursement price can be amortized. Therefore, a remarkable characteristic of assets classified as available for sale within a segregated fund, is that in case they are purchased above par, the difference between the face value and the price is accounted as a loss (negative income or a cost) when the bond matures. Obviously, the same applies with opposite sign when assets are purchased below par.

In order to calculate the statutory accounting return or performance, the income assessed during the calculation period has to be divided by the average book value. The average book value is the time weighted average of the book values of the assets in position during the calculation period. A numerical example may help understanding. Let us assume the calculation period is one year, that a bond with a notional of 1000 Euro is purchased at a price of 100 at the beginning of the year and then another bond of the same type with a notional of 1000 Euro is bought at a price of 110 after 6 months and hold until year's end. Then the average book value is $1000 \cdot 0.5 + 2100 \cdot 0.5 = 1550$.

For those that have some knowledge of financial assets performance measurement, this is a way to compute a money weighted performance.

B EIOPA curve construction

In this Appendix, for aim of completeness, we describe the procedure to obtain the regulatory-specific risk free curve. Our presentation is based on the EIOPA official technical documentation as of May, 30th 2016 ([16]).

EIOPA curve is based on the bootstrapping of the 6 months Euro swap rates from 1 year maturity onwards.

The credit risk adjustment (CRA) is applied through a parallel downward shift of the observed par swap rates. For the Euro curve, the CRA is the difference between the 3 months OIS rate and the 3 months Euro swap rate, in spite of the fact that in the technical documentation is said “*The maturity of the OIS rate used to derive the CRA is consistent with the tenor of the floating legs of the swap instruments used to derive the term structure.*”

After the CRA, a Smith-Wilson method (described in details in EIOPA technical documentation) is used to extrapolate forward rates between a maturity of 20 years (the “last liquid point”) and a maturity of 60 years (the “convergence point”). The one-year zero-coupon forward rate is assumed to converge towards a Ultimate Forward Rate (UFR), that for the Euro zone is set equal to 4.2%. As specified in the documentation: “*The control input parameters for the interpolation and extrapolation are the last liquid point, ultimate forward rate (UFR), the convergence point and the convergence tolerance.*”

Finally, a Volatility Adjustment (VA) treatment is applied on the ZCB curve. The VA is published by EIOPA at least on a quarterly basis for each relevant currency. The technical documentation defines the VA in the following way: “*The volatility adjustment (VA) is an adjustment to the relevant risk-free interest rate term structure. The VA is based on 65% of the risk-corrected spread between the interest rate that could be earned from bonds, loans and securitisations included in a reference portfolio and the basic risk-free interest rates.*”

As highlighted in [25], the two adjustments, CRA and VA, create a market consistency issue of the curve. In particular, it is not clear why the CRA for the Euro currency is based on a different tenor with respect to the swap curve. In addition, after the VA correction the curve is no more risk free, since this adjustment contains the credit and liquidity risk of bonds loans and securization.

C Market Data

We report market data on March 30th, 2016 used for calibrating the model. Market data are taken from the provider Bloomberg.

Maturity	Mid swap rate (%)	Maturity	Mid swap rate (%)
04-Apr-16	-0.3470	03-Apr-18	-0.4106
08-Apr-16	-0.3574	01-Oct-18	-0.3908
15-Apr-16	-0.3555	01-Apr-19	-0.3813
02-May-16	-0.3482	01-Apr-20	-0.3315
01-Jun-16	-0.3503	01-Apr-21	-0.2548
01-Jul-16	-0.3507	01-Apr-22	-0.1547
01-Aug-16	-0.3645	03-Apr-23	-0.0437
01-Sep-16	-0.3690	02-Apr-24	0.0724
03-Oct-16	-0.3696	01-Apr-25	0.1819
01-Nov-16	-0.3724	01-Apr-26	0.2855
01-Dec-16	-0.3760	01-Apr-27	0.3655
02-Jan-17	-0.3816	03-Apr-28	0.4624
01-Feb-17	-0.3860	01-Apr-31	0.6499
01-Mar-17	-0.3936	01-Apr-36	0.8062
03-Apr-17	-0.3982	01-Apr-41	0.8659
02-Oct-17	-0.4067	02-Apr-46	0.8818

Table 9: Term Structure of zero rates from EONIA swap market quotes on March 30th, 2016.

CDS Germany		CDS Italy	
tenor (year)	spread (bps)	tenor (year)	spread (bps)
0.5	5.29	0.5	28.45
1.0	5.44	1	36.00
2.0	6.71	2	67.06
3.0	8.94	3	86.62
4.0	14.86	4	114.10
5.0	18.38	5	122.36
7.0	27.01	7	150.81
10.0	38.54	10	186.00

Table 10: Term Structure of German and Italian CDS spreads on March 30th, 2016.

D Calibration procedure

The calibrations are performed through the minimization of the sum of squared differences between model and market data as follows:

$$\min_{\Theta} \frac{\sum_{n=1}^N (\text{Market}_n - \text{Model}_n(\Theta))^2}{N},$$

German sovereign curve		Italian sovereign curve	
tenor	yield(%)	tenor	yield (%)
0.25	-0.504	0.25	-0.166
0.5	-0.422	0.5	-0.053
1	-0.460	1	-0.058
2	-0.481	2	-0.002
3	-0.469	3	0.093
4	-0.426	4	0.230
5	-0.361	5	0.396
7	-0.183	7	0.772
8	-0.079	8	0.965
9	0.024	9	1.143
10	0.117	10	1.300
15	0.467	15	1.827
20	0.679	20	2.145
25	0.776	25	2.360
30	0.844	30	2.382

Table 11: Term Structure of zero rate of German and Italian BVAL sovereign curves on March 30th, 2016.

where N is the number of available market quotations, $Market_n$ and $Model_n$ are the market and the model quantities, respectively, and Θ is the vector of the model parameters.

Once we have calibrated the model, for each curve, we report the root mean square deviation (RMSD) and the maximum relative error (MRE), defined as follows

$$\begin{aligned}
RMSD &= \frac{1}{\sqrt{N}} \sqrt{\sum_{n=1}^N (Market_n - Model_n^*)^2} \\
MRE &= \max_{n=1, \dots, N} \frac{|Market_n - Model_n^*|}{Market_n}
\end{aligned}$$

where $Model_n^*$ is the model quantity calculated in correspondence of the optimal parameters vector, Θ^* .

The market quantities used for calibration purpose are ZCB prices for the Eonia and sovereign bonds curves, and CDS quotes.

The calibration is performed in three steps. Firstly a Vasicek model is calibrated on the Eonia ZCB curve. Secondly a CIR model is calibrated on the issuer survival probability curve. Finally the liquidity parameters (Vasicek) are calibrated on the sovereign or corporate ZCB curve, fixing the other parameters previously obtained.

We report the explicit form of the historical correlation estimated between the liquidity spreads through the procedure described in Section 3.1:

$$(D.1) \quad \rho_l^H = Corr(dl^1(t), dl^2(t)) = \rho_l \frac{1 - e^{-(k_1+k_2)dt}}{\sqrt{(1 - e^{-2k_1dt})(1 - e^{-2k_2dt})}} \frac{2\sqrt{k_1k_2}}{k_1 + k_2}$$

where ρ_l is the correlation between the two Brownian motions. Hence, the estimated correlation is not exactly equal to the model correlation. However, for small dt the

difference between the two correlations goes to zero, for this reason we choose historical series of daily returns.

We do not have an explicit formula for the correlation between credit spreads, but the estimation procedure follows similar reasoning.