Leveraging Retransmissions in Wireless Networked Control Systems with Packetized Predictive Control

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***Abstract*—In this work we propose a communication and control joint design strategy using packetized predictive control and halted Chase combining hybrid automatic repeat request to minimize the bandwidth required to provide ultra-reliable low latency communications towards wireless networked control systems. The joint design nature comes from tunning at the same time both the length of the control horizon, a control parameter, and the maximum number of transmission attempts, a communication variable. Numerical results show that by using this strategy and correctly choosing the parameters, significant savings in bandwidth can be achieved.**

I. Introduction

Factory automation constitutes one of the most relevant applications of ultra-reliable low latency communications (URLLC) [1]. Using a wireless network in networked control systems (NCS) provides great flexibility, quick deployment and increased modularity to a wide range of factory automation applications [2], however, wireless communications introduce random probabilities of packet dropouts, which pose a significant challenge to the stability and performance of the NCS [3]. Since the target error rates and latencies can be very stringent in NCS applications (*i.e.* 10^{-9} and 1 ms) [4], joint design techniques—wherein both communication and control parameters are considered at the same time—are crucial to provide URLLC performance whilst minimizing wireless resource consumption [5].

One way of dealing with random packet dropouts in the controller to actuator link is to use a model predictive controller (MPC) to generate K control signals and send them in one packet to the actuators, such that if the next packet fails, the following instruction is available for execution. This is known as packetized predictive control (PPC) [3]. In this work we propose a joint design strategy using PPC and halted Chase combining hybrid automatic repeat request (CC-HARQ), wherein any failed packet is retransmitted up to z times and the message is combined at the receiver with all previous failed attempts using maximum ratio combining before decoding.

II. System Model

A. Control Architecture

Consider a networked control architecture with a wireless link connecting the controller to the actuator—which is positioned at the plant—, as illustrated in Figure 1. PPC is performed such that a buffer stores the vector $u(t)$, which contains K control signals to be used if a packet is dropped.

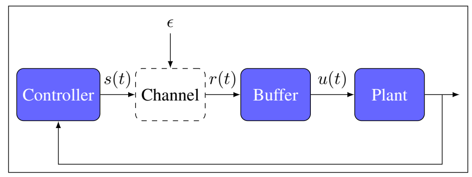

Fig. 1: Control architecture with a wireless link connecting controller and actuator considering ϵ packet drop probability.

In PPC, sending longer packets to include the additional control signals creates a trade-off, as it increases the message length (which imposes higher transmit rates for the same latency) resulting in a higher probability of a communication error, while at the same time it making the system more fault tolerant. The stability and performance of the NCS depend on the packet drop rate ϵ_{c} and on the communication latency λ.

B. Communication Model

At every instant t a packet containing K control signals is sent to the buffer, such that the received signal, $r(t)$, is expressed by $r(t) = h(t)s(t)+w(t)$. Here $s(t)$ is the generated signal at the controller, $w(t)$ is the additive white Gaussian noise (AWGN) and $h(t)$ is the wireless channel gain, which is random and follows a Nakagami-m distribution. Since in NCS the mobility is very low, the channel does not vary between symbols. Also, slow frequency hopping is considered, so subsequent transmissions are performed in an uncorrelated frequency channel and thus the channel model is block-fading.

The probability of dropping a packet ϵ depends on the condition of the wireless channel. As there is a strict maximum deadline, the capacity at finite block length should be considered in order to obtain achievable rates. However, since the channel is block-fading and the rates are relatively high, as is the SNR, the average error probability can be tightly approximated by the outage probability [6].

Considering the header length L_{H} and the control signal length L_{U}, both in bits, the total length of the forward packet L_{T} is equal to $L_{\mathrm{H}} + KL_{\mathrm{U}}$. Lastly, the minimum bit rate R_{b} to meet the target latency is

$$R_{\mathrm{b}} = \frac{L_{\mathrm{T}}}{\lambda}. \quad (1)$$

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We use the minimum bit rate because using a larger one would result in a higher error probability for the same bandwidth B.

III. Proposed Solution

The proposed solution works by the controller attempting to send a packet containing K control signals every λ seconds. If the packet fails, a retransmission is attempted using the CC-HARQ strategy and this is repeated up to z times. This is enabled by the fact that in PPC at any sampling instant t, K control signals are transmitted and thus if a message is decoded on instant $t+j$, the controller can obtain $K-j$ relevant control signals. The maximum value of z which guarantees the availability of every control signals is $\lceil K/2 \rceil$[1].

By carefully choosing the value for z and K, it is possible to optimize communication resources while still meeting the control design target. Since energy is not critical at the controller or the actuator, the focus of this work is to minimize the required bandwidth B for each transmission[2].

The outage probability $P_{\text{out},z}$ after z rounds in CC-HARQ is approximated at high signal to noise ratio (SNR) by [7]

$$P_{\text{out},z} \approx \frac{\left(\frac{m\gamma_0}{\bar{\gamma}}\right)^{mz}}{\Gamma(mz+1)}, \quad (2)$$

where m is the Nakagami-m parameter which relates to line of sight (LOS) conditions, $\gamma_0 = 2^{Rb/B} - 1$ is the outage threshold from the channel capacity, $\Gamma(.)$ is the gamma function and the average SNR $\bar{\gamma}$ is obtained via

$$\bar{\gamma} = \frac{P_{\text{t}}}{BN_0 M_{\text{l}} A_0 d^{\alpha}}. \quad (3)$$

In (3), P_{t} is the transmit power, N_0 is the noise spectral power density, A_0 is the loss at a reference distance, d is the distance between transmitter and receiver, α is the path loss exponent and M_{l} is a link margin which accounts for any unforeseen losses, including the noise figure. Since a minimum error probability must be guaranteed, $P_{\text{out},z} \leq \epsilon_{\text{c}}$ must hold. Thus, using (2) and (3) we arrive at

$$B\left(2^{\frac{R_{\text{b}}}{B}} - 1\right) \leq \beta, \quad (4)$$

where $\beta = \frac{(\epsilon_{\text{c}}\Gamma(mz+1))^{1/mz} P_{\text{t}}}{mN_0 M_{\text{l}} A_0 d^{\alpha}}$. Using W_{-1}, the lower part of the main Lambert-W function, (4) becomes

$$B \geq \frac{-R_{\text{b}}\ln(2)}{W_{-1}\left(\frac{-Rb\ln(2)}{\beta}\right)}. \quad (5)$$

Since W_{-1} can only yield real values for inputs greater than or equal to $-1/\mathrm{e}$ (where e is Euler's constant), using (1) and analyzing the argument of W_{-1} in (5) provides the bound

$$K \leq \frac{1}{L_{\text{U}}}\left(\frac{\beta\lambda}{\mathrm{e}\ln(2)} - L_{\text{H}}\right). \quad (6)$$

[1]Using $z > \lceil K/2 \rceil$ violates the target error probability because when a message is decoded after having failed $j > \lceil K/2 \rceil$ times, only $K-j$ control signals are not outdated and the next packet would have to succeed in $K-j$ attempts, which is less than $\lceil K/2 \rceil$ by definition.

[2]The excess bandwidth consumed due to slow frequency hopping between different transmission attempts is not an issue since in CC-HARQ the average number of retransmissions is typically close to one, even for large values of z, when the target error probability is very low, as in URLLC.

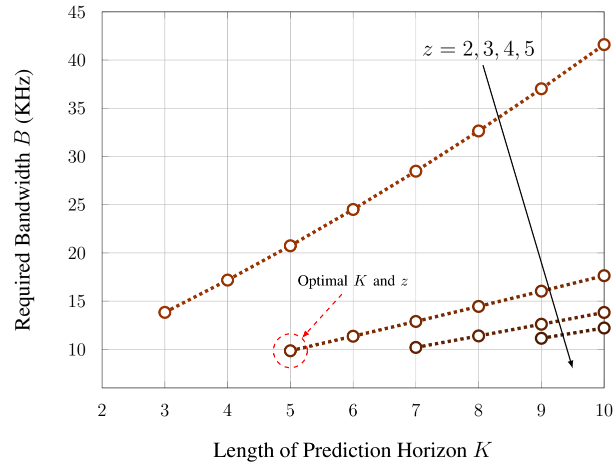

Fig. 2: Required bandwidth for different values of K and z.

Finally, we write the optimization problem as

$$\underset{K \in \mathbb{N}^*, z \in \mathbb{N}^*}{\text{minimize}} \quad B = \frac{-R_{\text{b}}\ln(2)}{W_{-1}\left(\frac{-Rb\ln(2)}{\beta}\right)} \quad (7a)$$

$$\text{subject to} \quad z \leq \lceil K/2 \rceil, \quad (7b)$$

which can be tackled numerically.

IV. Numerical Results

To illustrate the effect of K and z in the required bandwidth, we have performed numerical simulations computing B from (7) using $\epsilon_{\text{c}} = 10^{-9}$, $\lambda = 1$ ms, $P_{\text{t}} = -30$ dB, $d = 10$ m, $L_{\text{H}} = 40$ bits, $L_{\text{U}} = 16$ bits, $M_{\text{l}} = 10$ dB, $N_0 = -204$ dB, $A_0 = 30$ dB, $m = 1$ and $\alpha = 3$. Figure 2 shows the minimum required bandwidth for different values of K. Each curve represents a value of z, ranging from 2 to 5. Using the inequality in (6), we can observe that K would have to assume a negative value for $z = 1$, which is not possible. Here the best choice of parameters is $K = 5$ and $z = 3$.

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