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FREE-SURFACE FLUCTUATIONS AND TURBULENCE IN HYDRAULIC JUMPS

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Abstract: A hydraulic jump is the highly turbulent transition between a high-velocity impinging flow and a turbulent roller. The jump flow is characterised by some substantial air bubble entrainment, spray and splashing. In the present study, the free surface fluctuations and air-water properties of the hydraulic jump roller were investigated physically for relatively small Froude numbers ($2.4 < Fr_1 < 5.1$) and relatively large Reynolds numbers ($6.6\times10^4 < Re < 1.3\times10^5$). The shape of the mean free surface profile was well defined, and the time-averaged free-surface elevation corresponded to the upper free-surface, with the quantitative values being close to the equivalent clear-water depth. The turbulent fluctuation profiles exhibited a maximum in the first part of the hydraulic jump roller. The free surface fluctuations presented some characteristic frequencies between 1.4 and 4 Hz,. Some simultaneous free-surface measurements at a series of two closely located points yielded the free surface length and time scales of free-surface fluctuations in terms of both longitudinal and transverse directions. The length scale data seemed to depend upon the inflow Froude number, while the time scale data showed no definite trend. Some simultaneous measurements of instantaneous void fraction and free surface fluctuations exhibited different features depending upon the phase-detection probe sensor location in the different regions of the roller.

Keywords: Hydraulic jumps, Free-surface fluctuations, Turbulence properties, Time and length scales.

1. INTRODUCTION

A hydraulic jump is the highly turbulent transition from a high-velocity flow to a slower flow motion. The jump toe is a discontinuity between the impinging flow and the roller (Fig. 1). The hydraulic jump flow is characterised by some substantial air bubble entrainment, spray and splashing. In the turbulent flow region called the roller, two distinctive air-water regions are seen: the air-water mixing layer and the upper free-surface layer. In the mixing layer, there is a transfer of momentum from the high-velocity jet flow to the recirculation region above, as well as by an advective transport of the entrained air bubbles. Considering a hydraulic jump in a horizontal rectangular channel, the application of the equations of conservation of mass and momentum in their integral form yields a series of relationship between the flow properties downstream of and upstream of the jump:

$$\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1+8 Fr_1^2} - 1\right) \tag{1}$$

$$\frac{Fr_2}{Fr_1} = \frac{2^{3/2}}{\left(\sqrt{1+8 Fr_1^2} - 1\right)^{3/2}} \tag{2}$$

where d_1 and d_2 are respectively the upstream and downstream flow depths, Fr_1 and Fr_2 are the upstream and downstream Froude numbers respectively, the Froude number is defined as $Fr = V/\sqrt{gd}$, V is the flow velocity and g is the gravity acceleration. Equation (1) is called sometimes the Bélanger equation, first developed by Jean-Baptiste BÉLANGER in 1838 (BÉLANGER 1841, CHANSON 2009).

The hydraulic jumps are commonly encountered in hydraulic structures and stilling basins, storm waterways, water treatment plants and chemical processing plants. A classical example is the circular hydraulic jump in a sink. Figure 1A shows a hydraulic jump in an irrigation channel. The discharge per unit width was about 0.5 m^3/s corresponding to a Reynolds number of 1×10^5. Figure 1B presents a hydraulic jump in the inlet structure and mixer of a water treatment plant.

The turbulent free-surface properties above a hydraulic jump were rarely investigated, but for MOUAZE et al. (2005) and MURZYN and CHANSON (2009). The former study was based upon wire gage sensors commonly used for non breaking periodic waves. The latter was conducted with non-intrusive acoustic displacement sensors with a faster dynamic response.

In the present study, the free-surface fluctuations and air-water flow properties were investigated in hydraulic jumps with relatively small Froude numbers and large Reynolds numbers. Both non-intrusive acoustic displacement sensors and intrusive phase-detection probes were used. It is the aim of this work to characterise the unsteady free surface motion and the air-water flow properties in hydraulic jumps for a broad range of relatively small Froude numbers ($2.4 < Fr_1 < 5.1$).

(A, Left) Hydraulic jump in an irrigation channel on 10 May 2010 in Hualien County (Taiwan) - Flow from top right to bottom left (Shutter speed: 1/100 s)

(B, right) Hydraulic jump in the Molendinar water processing plant (Gold Coast, Australia) on 4 September 2002

Fig. 1 - Photographs of prototype hydraulic jumps

2. EXPERIMENTAL FACILITY AND INSTRUMENTATION

2.1 Experimental set-up and instrumentation

The experiments were performed in a 3.2 m long, 0.5 m wide horizontal rectangular channel (Fig. 2). The glass sidewall height was 0.45 m and the channel bed was made of PVC. The inflow conditions were controlled by a vertical gate with a semi-circular shape (∅ = 0.3 m), and the upstream gate opening was fixed during all experiments at h = 0.036 m. This channel was previously used with different flow conditions by CHANSON (2007), KUCUKALI and CHANSON (2008) and MURZYN and CHANSON (2009).

The water discharge was measured with a Venturi meter located in the supply line which was calibrated on-site. The discharge measurement was accurate within ±2%. The clear-water flow depths were measured using rail mounted point gages with a 0.2 mm accuracy. The pressure and velocity measurements in steady supercritical flows were performed with a Prandtl-Pitot tube. Its performances were compared with a British Standards design within 1% in wind tunnel tests for Reynolds numbers ranging from 1×10^5 to 9×10^5. The Prandtl-Pitot tube had an external diameter ∅ = 3.02 mm, the total head was measured through a 1 mm hole at the tip, and the distance between the tip of the probe and the lateral pressure points (Ø = 0.5 mm) was 9 mm.

Further details on the experimental facility, instrumentation and data sets were reported in CHACHEREAU and CHANSON (2010).

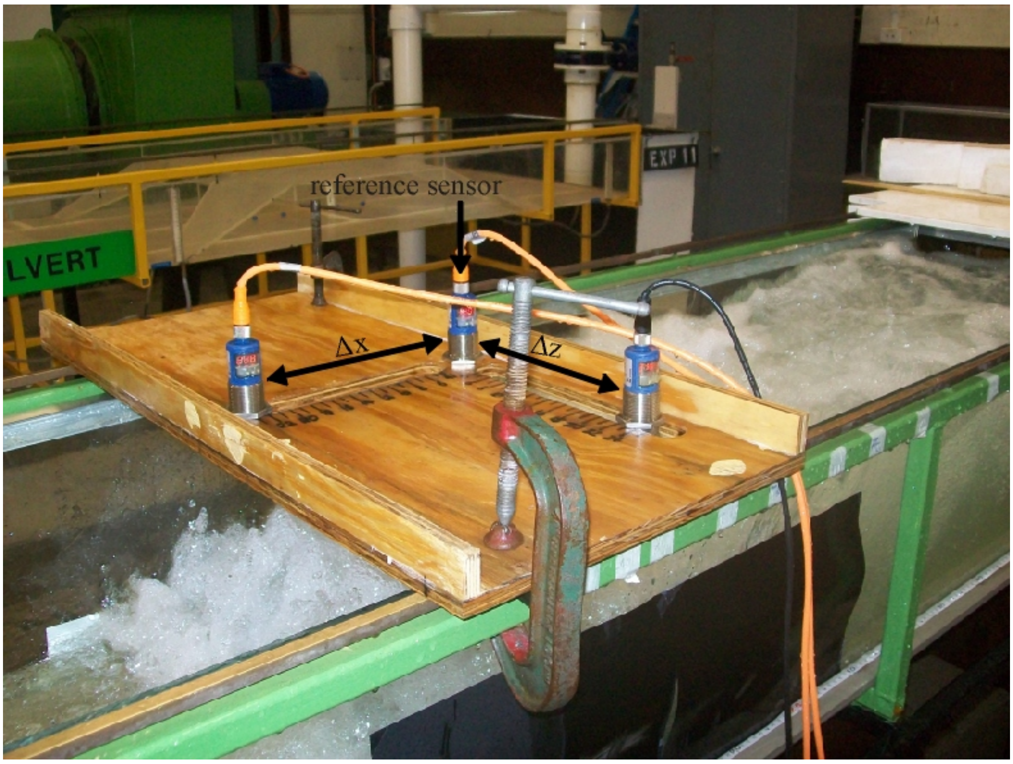


Fig. 2 - Experimental setup with three acoustic displacement meters and definition of the longitudinal and transverse separation distances Δx and Δz - Flow from left to right: d_1 = 39.5 mm, x_1 = 1.50 m, Fr_1 = 5.1, Re = 1.3×10^5

2.1.1 Free surface measurements using acoustic displacement meters

The instantaneous free surface elevations were measured using several ultrasonic displacement meters

Microsonic™ located along and above the flume centreline. The sensors included six Mic+25/IU/TC with 0.18 mm accuracy and 50 ms response time, and one Mic+35/IU/TC sensor with 0.18 mm accuracy and 70 ms response time. The locations of the sensors were fixed for all experiments: they are listed in Table 1 where x is the longitudinal distance from the channel upstream end and x_1 is the hydraulic jump toe location. Each probe signal output was scanned at 50 Hz per sensor for 10 minutes. The sensors were calibrated on site before each day of experiments.

The displacement meter outputs included a few erroneous measurements when the angle between the free surface and horizontal was important and the reflected beam did not return to the acoustic displacement meter head. Another situation was when some air-water splashing was detected by the sensor instead of the free surface. This translated into spikes in the signals which were removed by a threshold technique.

Table 1 - Longitudinal positions of the acoustic displacement sensors (Jump toe location: x_1 = 1.50 m)

Sensor name	S_0	S_1	S_2	S_3	S_4	S_5	S_6
x (m) =	1.35	1.595	1.74	1.94	2.19	2.43	2.67
$x-x_1$ (m) =	-0.15	+0.095	+0.24	+0.44	+0.69	+0.93	+1.17
Sensor type	Mic+35	Mic+25	Mic+25	Mic+25	Mic+25	Mic+25	Mic+25

2.1.2 Two-phase flow measurements

The air-water flow properties were measured with a double-tip conductivity probe. The dual-tip probe was equipped with two identical sensors with an inner diameter of 0.25 mm. The distance between probe tips was Δx_{tip} = 7.12 mm. The probe was manufactured at the University of Queensland. The dual-tip probe was excited by an electronic system (Ref. UQ82.518) designed with a response time of less than 10 µs. During the experiments, each probe sensor was sampled at 20 kHz for 45 s. The displacement and the position of the probe in the vertical direction were controlled by a fine adjustment system connected to a Mitutoyo™ digimatic scale unit with a vertical accuracy of less than 0.1 mm.

The analysis of the probe voltage output was based upon a single threshold technique, with a threshold set at 50% of the air-water voltage range. The error on the void fraction was expected to be less than 1% using this technique. The single-threshold technique is a robust method that is well-suited to free-surface flows (CHANSON and CAROSI 2007). A number of air-water flow properties were derived from the probe signal analysis. These included the void fraction C defined as the volume of air per unit volume of air and water, the bubble count rate or bubble frequency F defined as the number of bubbles impacting the probe tip per second, and the air chord time distribution where the chord time is defined as the time spent by the bubble on the probe tip.

2.2 Inflow conditions

In the upstream supercritical flow, a turbulent boundary layer developed and its properties were investigated. For several discharges, the vertical distributions of velocity and pressure were measured with the Prandtl-Pitot tube for $0 < x < x_1 = 1.50$ m. All the experiments were carried out with the same upstream rounded gate opening h = 0.036 m, for which the flow depth immediately upstream of the roller toe differed depending on the discharges, ranging from $d_1 = 0.0395$ m to 0.0440 m (Table 2, 3rd column).

Table 2 - Developing boundary layer properties upstream of the hydraulic jump (Present study)

Q (m^3/s)	h (m)	d_1 (m)	Fr_1	Re	x/d_1	δ/d_1	δ_1/d_1	δ_2/d_1	δ_3/d_1	V_{max} (m/s)
0.0446	0.036	0.0440	3.1	8.9×10^4	9.1	0.151	0.0096	0.0085	0.0162	2.60
					17.0	0.138	0.0058	0.0055	0.0106	2.59
					21.1	0.114	0.0059	0.0054	0.0103	2.57
					24.3	0.120	0.0076	0.0068	0.0129	2.55
					25.0	0.125	0.0088	0.0078	0.0147	2.55
					31.8	0.117	0.0077	0.0068	0.0129	2.53
0.0490	0.036	0.0405	3.8	9.8×10^4	9.9	0.129	0.0088	0.0078	0.0147	2.86
					18.5	0.173	0.0104	0.0094	0.0179	2.84
					27.2	0.158	0.0097	0.0088	0.0169	2.82
					34.6	0.135	0.0104	0.0091	0.0172	2.80
0.0545	0.036	0.0395	4.4	1.1×10^5	10.1	0.237	0.0215	0.0181	0.0335	3.18
					19.0	0.332	0.0333	0.0280	0.0518	3.15
					27.8	0.422	0.0380	0.0323	0.0602	3.07
					35.4	0.467	0.0354	0.0304	0.0567	3.01
0.0627	0.036	0.0395	5.1	1.3×10^5	10.1	0.177	0.0154	0.0131	0.0243	3.63
					19.0	0.250	0.0229	0.0195	0.0362	3.63
					27.8	0.334	0.0310	0.0262	0.0486	3.63
					35.4	0.394	0.0348	0.0297	0.0552	3.57

The measurements showed that the pressure distributions were hydrostatic. The velocity profile measurements indicated that the supercritical flow was partially developed, consisting of a developing boundary layer and an ideal fluid flow region above (Fig. 3). For each flow condition, the boundary layer thickness, displacement, momentum and energy thicknesses were calculated (Table 2). The complete data set is reported in Table 2 (7th to 10th columns) together with the free-stream velocity V_{max} (Table 2, 11th column). The boundary layer growth data were best correlated by:

$$\frac{\delta}{d_1} \propto \left(\frac{x - x_o}{d_1}\right)^{4/5} \qquad (3)$$

where δ is the boundary layer thickness, d_1 is the water depth at the position x = 1.40 m, corresponding to a location immediately upstream of the jump toe for all the experiments conducted in this study, x is the longitudinal distance from the upstream gate, and x_o is the virtual origin of the boundary layer that was function of the flow conditions. For a wall jet configuration, the virtual origin x_o of the boundary layer is usually not located at the opening but upstream of the gate (SCHWARZ and COSART 1964, CHANSON 1997).

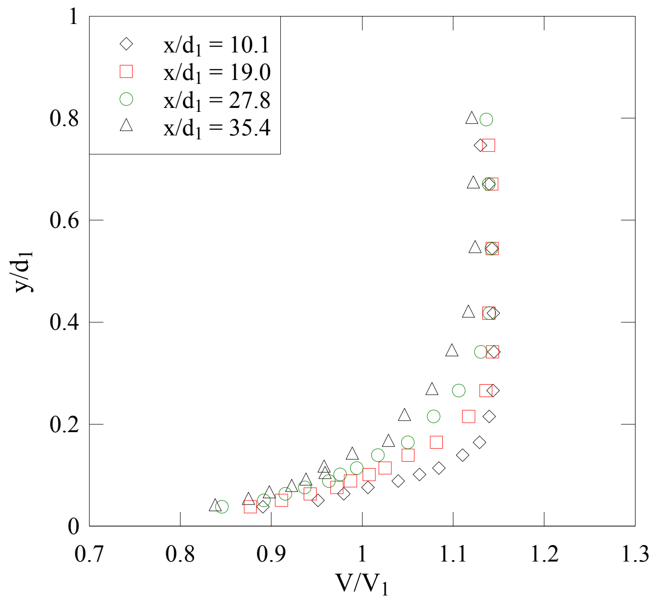


Fig. 3 - Dimensionless velocity profiles in the developing flow upstream of the hydraulic jump for Fr_1 = 5.1, d_1 = 0.0395 m, h = 0.036 m, Re = 1.3×10^5

Notes: Q: flow rate; h: gate opening; d_1: water depth immediately upstream of the jump (measured at x = 1.40 m); Fr_1: Froude number at x = 1.40 m; x: longitudinal position; Re: Reynolds number; δ: boundary layer thickness; δ_1: displacement thickness; δ_2: momentum thickness; δ_3: energy thickness; V_{max}: free-stream velocity.

2.3 Experimental flow conditions

Two series of experiments were conducted. The first series focused on some basic observations of hydraulic jump properties and some detailed free-surface characteristics. The experiments were performed with Froude numbers between 2.4 and 5.1 corresponding to Reynolds numbers between 6.6×10^4 and 1.3×10^5. During the second series of experiments, the free-surface fluctuations and two-phase flow properties were recorded simultaneously. The upstream Froude numbers ranged between 3.8 and 5.1 and the Reynolds numbers between 9.8×10^4 and 1.1×10^5.

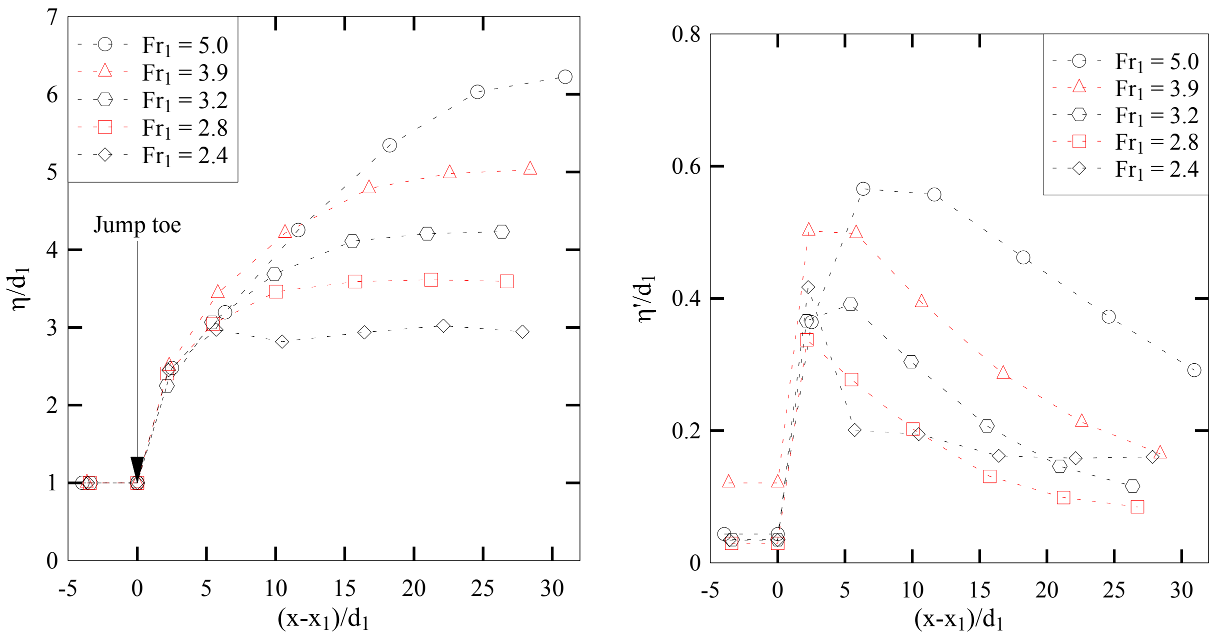
For both series of experiments, the upstream rounded gate opening was set at h = 0.036 m, and the jump toe was located at x_1 = 1.50 m. For these conditions, the inflow depth ranged from 0.042 down to 0.038 m depending upon the flow rate and the inflow conditions were partially-developed (Table 2).

3. FREE-SURFACE FLUCTUATIONS

3.1 Free-surface profiles

For Froude numbers Fr_1 less than 2.4, the hydraulic jump was undular. That is, the front was followed by a train of secondary waves or undulations. For $Fr_1 > 2.4$, the jump had a marked breaking roller, with some increasing air entrainment and air-water projections with increasing Froude number (Fig. 2). For the remaining sections, the inflow Froude number Fr_1 was larger than or equal to 2.4, and thus corresponded to breaking jumps without undulations.

The longitudinal free surface profiles were recorded for a series of Froude numbers ranging from 2.4 to 5.1 (Table 3). The instrumentation consisted in seven acoustic displacement meters located at different longitudinal positions (Table 1) and sampled simultaneously at 50 Hz for 10 minutes. Figure 4A presents some typical mean free surface profiles. In Figure 4A, η is the time-averaged free-surface elevation above the invert, x is the longitudinal position of the sensor and d_1 is the inflow depth immediately upstream of the hydraulic jump toe. The data showed some longitudinal profiles that were very close to the photographic observations through the glass sidewalls. In a hydraulic jump, the flow properties immediately upstream and downstream of the jump roller must satisfy the continuity and momentum principles (HENDERSON 1966, LIGGETT 1994). Equation (1) shows the classical result in a rectangular, horizontal, smooth channel, and it is compared the present experimental data in Figure 5 as well as with other data sets. The results showed a close agreement between the data and theory as expected.



(A, Left) Dimensionless time-averaged free surface profile η/d_1

(B, Right) Dimensionless free surface fluctuations η'/d_1

Fig. 4 - Free surface profile measurements in hydraulic jumps

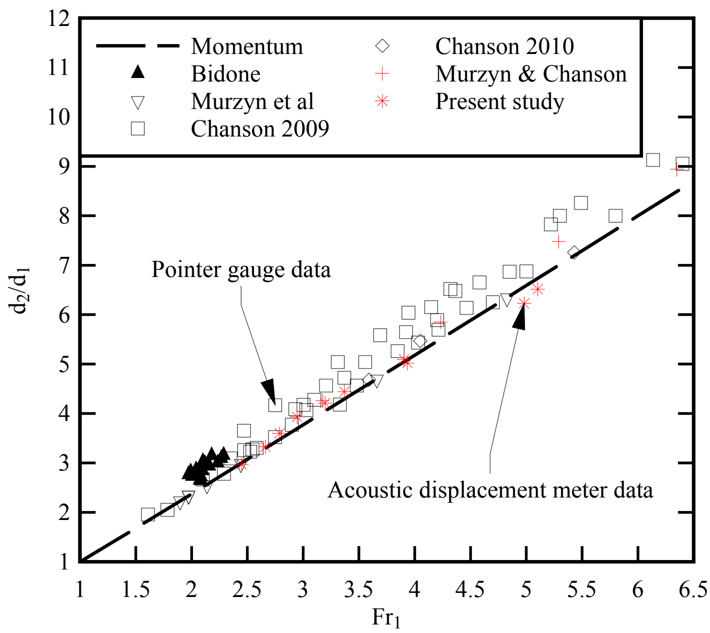


Fig. 5 - Ratio of conjugate depths d_2/d_1 in hydraulic jumps - Comparison between the momentum principle

(Eq. (1)), experimental data obtained using acoustic displacement meters (Red symbols: MURZYN and CHANSON 2009, Present study) and data based upon pointer gauges (Black & white symbols: BIDONE 1819, MURZYN et al. 2007, CHANSON 2009,2010)

Table 3 - Experimental conditions of the free surface profile experiments (Present study)

Q (m^3/s)	B (m)	h (m)	x_1 (m)	d_1 (m)	δ (m)	Fr_1	Re	d_2 (m)	F_{fs} (Hz)
0.033	0.50	0.036	1.50	0.0420	--	2.4	6.6×10^4	0.1247	2.1-5
0.0365	0.50	0.036	1.50	0.0425	--	2.7	7.3×10^4	0.1414	2.8-3.7
0.040	0.50	0.036	1.50	0.0438	--	2.8	8.0×10^4	0.1576	2.65-3
0.0446	0.50	0.036	1.50	0.0454	0.0051	2.9	8.9×10^4	0.1785	2.5-3
0.0468	0.50	0.036	1.50	0.0444	--	3.2	9.4×10^4	0.1870	1.6-3.8
0.049	0.50	0.036	1.50	0.0442	0.0055	3.4	9.8×10^4	0.1963	1.85-3.9
0.0515	0.50	0.036	1.50	0.0412	--	3.9	1.0×10^5	0.2068	1.7-2.9
0.0545	0.50	0.036	1.50	0.0430	0.0184	4.4	1.1×10^5	0.219	1.6-3.8
0.0573	0.50	0.036	1.50	0.0378	--	5.0	1.1×10^5	0.235	1.95-2.55
0.0627	0.50	0.036	1.50	0.0395	0.0156	5.1	1.3×10^5	0.257	1.8-2.4

The standard deviation of the water elevation η' was recorded and Figure 4B presents η'/d_1 as a function of the dimensionless distance from the jump toe $(x-x_1)/d_1$ for the same flow conditions as Figure 4A. Basically, some small free-surface fluctuations were observed upstream of the jump toe ($x-x_1 < 0$). A significant increase in free surface fluctuation was observed immediately downstream of the jump toe ($x-x_1 > 0$) for all Froude numbers, and the free-surface fluctuations reached a maximum value η'_{max} within the roller. This maximum value η'_{max} increased with increasing Froude numbers (Fig. 6). The large standard deviations in free-surface elevations were believed to be linked with a large number of air-water projections above the roller and jump toe. Further downstream, the free-surface fluctuations η' decreased with increasing distance from the jump toe. The results were consistent with the earlier studies of MOUAZE et al. (2005), KUCUKALI and CHANSON (2008) and MURZYN and CHANSON (2009).

The peak of turbulent fluctuations was observed for $(x-x_1)/d_1 < 7$ (Fig. 4B). That is, the peak in turbulent free-surface fluctuations was located in the first half of the roller as previously observed by MOUAZE et al. (2005) and MURZYN and CHANSON (2009). For example, the standard deviation of the free-surface elevation was nearly 0.6 times the inflow depth ($0.6d_1$) for Fr_1 = 5.0 (Fig. 4B). The free surface profile became more turbulent with increasing Froude number. Figure 6 summarises the dimensionless peak of turbulent fluctuations η'_{max}/d_1 as a function of the inflow Froude number. The present data are compared with the data fit proposed by MURZYN and CHANSON (2009):

$$\left(\frac{\eta'}{d_1}\right)_{max} = 0.116\left(Fr_1 - 1\right)^{1.235} \tag{4}$$

Equation (4) is shown in Figure 6 together with the data. There is a good agreement, but for the lowest Froude number. The present data were further in close agreement with the data of MADSEN (1981), MOUAZE et al. (2005), KUCUKALI and CHANSON (2008) and MURZYN and CHANSON (2009) (Fig. 6).

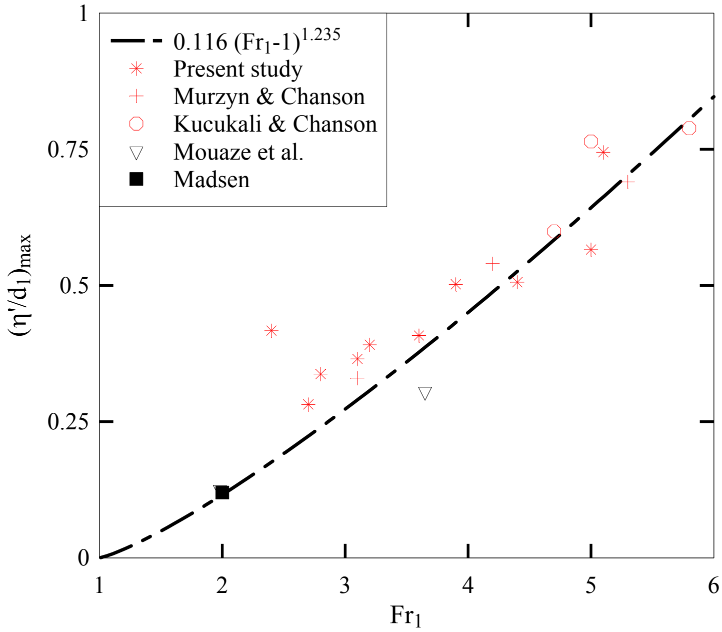


Fig. 6 - Maximum of turbulent fluctuations η'_{max}/d_1 in hydraulic jumps as a function of Froude number Fr_1 - Comparison between Equation (4) and experimental data (MADSEN 1981, MOUAZE et al. 2005, KUCUKALI and CHANSON 2008, MURZYN and CHANSON 2009, Present study) - Red symbols are acoustic displacement meter data

3.2 Free-surface frequencies

A spectral analysis of the acoustic displacement meter signal outputs was performed. The results showed a dominant characteristic frequency between 1.6 and 4 Hz. Figure 7A summarises the characteristic frequencies of the free surface fluctuations in hydraulic jumps for Froude numbers, between 3.1 and 5.1, as a function of the dimensionless distance to the jump toe $(x-x_1)/d_1$. For some positions, the results indicated two characteristic frequencies. In these cases, both frequencies are shown on Figure 7A. In other cases, the data presented a flat zone of maximum amplitude. For this situation, both ends of the range were recorded. These zones are represented on Figure 7A in the form of two points linked with a dashed line. Overall the dominant frequencies showed relatively little effect of the longitudinal distance $(x-x_1)/d_1$ (Fig. 7A).

Figure 7B shows the characteristic free-surface fluctuation frequency in the hydraulic jump roller as a function of the inflow Froude number. The data are shown with the range of data scatter and they are compared with the data of MURZYN and CHANSON (2009) obtained with the same instrumentation, sampling rate and sampling duration (50 Hz for 10 min.). Despite some scatter, the data were close and showed a slight decrease in dimensionless free-surface frequency with increasing Froude number. Both data sets were best correlated by:

$$\frac{F_{fs}\ d_1}{V_1} = 0.143\ \exp(-0.27\ Fr_1) \qquad 2.4 < Fr_1 < 6.5 \quad (5)$$

with a normalised correlation coefficient of 0.62. Equation (5) is compared with the data in Figure 7B.

The horizontal oscillations of the jump toe were recorded and the data are shown in Figure 7B. The results are plotted in terms of a Strouhal number defined as $F_{toe}d_1/V_1$ where F_{toe} is the jump toe oscillation frequency. They were compared with earlier jump toe oscillation data (MURZYN and CHANSON 2009, CHANSON 2010) and with the dimensionless free-surface fluctuation frequencies. Despite some scatter, the range of jump toe fluctuation frequency was lower than the free-surface fluctuation frequency observations particularly at small inflow Froude numbers (Fig. 7B). The finding differs from results in hydraulic jumps with large Froude numbers which showed similar jump toe fluctuation and free-surface fluctuation frequencies for $Fr_1 > 7$.

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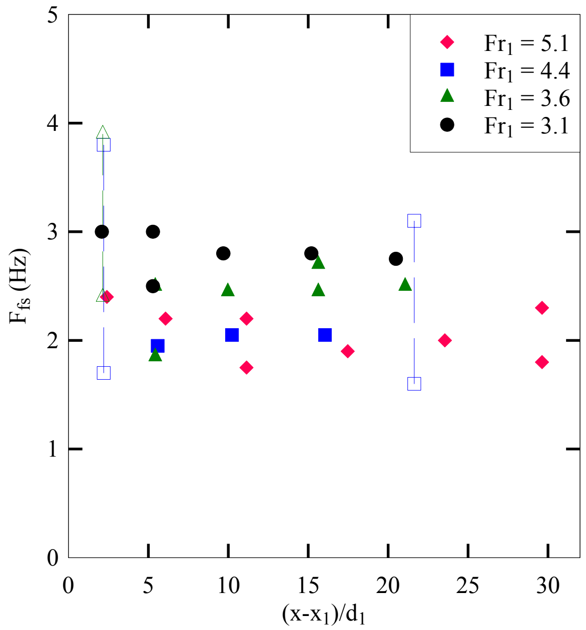
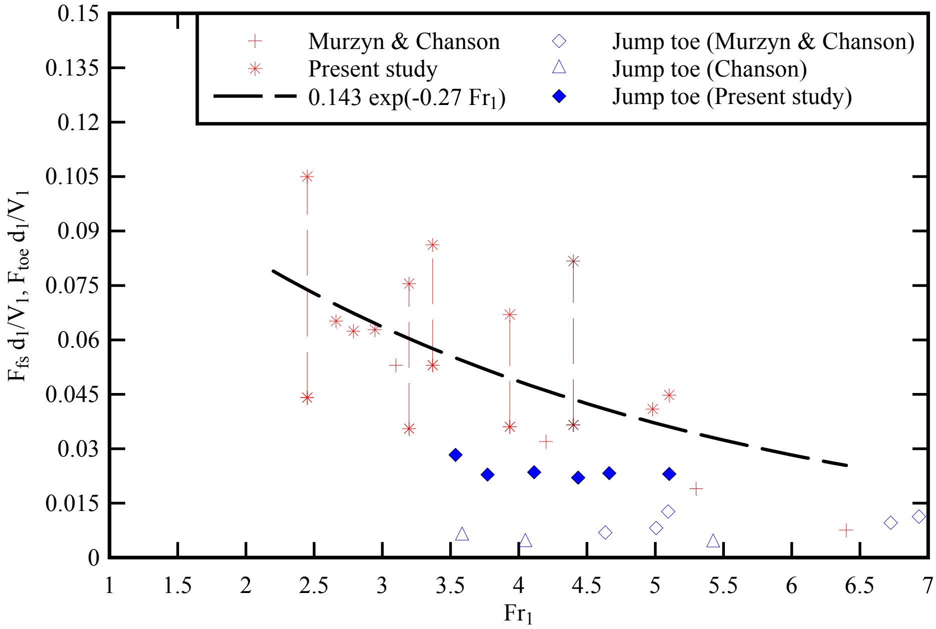


(A) Characteristic free-surface fluctuation frequency as a function of the longitudinal distance from the jump toe



(B) Dimensionless free-surface fluctuation frequency $F_{fs}d_1/V_1$ as a function of the inflow Froude number - Comparison with the experimental data of MURZYN and CHANSON (2009) and Equation (5), and with jump toe fluctuations data (MURZYN and CHANSON 2009, CHANSON 2010, present study)

Fig. 7 - Characteristic frequencies of the free surface fluctuations of hydraulic jumps for Froude numbers between 2.4 and 5.1

3.3 Longitudinal and transverse integral length and time scales

Some simultaneous free surface measurements were performed to characterise the coherent turbulent structures located next to the free surface. In the experimental setup, three acoustic displacement sensors were located above the free surface of the hydraulic jump (Fig. 2), and sampled simultaneously at 50 Hz for 60 s. For a given flow rate Q, the reference sensor was located at a longitudinal distance ($x-x_1$) from the jump toe, and the experiment was repeated for a range of relative position of the two other sensors; these were separated from the reference sensor by the distance Δx and Δz respectively in the longitudinal and transverse directions (Fig. 2). The experiments were conducted for three different flow conditions (Table 4). The distances between the sensors, Δx in the longitudinal direction and Δz in the transverse direction, varied from 41 mm to 230 mm. Table 4 summarises the experimental conditions and Figure 2 illustrates the experimental setup. The cross-correlation function between the signal outputs of the two sensors separated by Δx in the longitudinal direction provided some information on the coherence of the free surface fluctuations in the longitudinal flow direction. The correlation between the outputs of the two sensors separated transversely by Δz yielded a similar information in the transverse direction. The level of correlations characterised the existence of coherent turbulent structures beneath and next to the free surface that could be described by some correlation length and time scales.

Table 4 - Experimental conditions of the length and time scale experiments and simultaneous measurements of free-surface fluctuations and air-water flow properties (Present study)

Q (m^3/s)	B (m)	h (m)	x_1 (m)	d_1 (m)	δ (m)	$x - x_1$ (m)	Δx (mm)	Δz (mm)	Fr_1	Re
0.049	0.50	0.036	1.50	0.0405	0.0055	0.150 0.300 0.450	41 to 230	41 to 230	3.8	9.8×10^4
0.0545	0.50	0.036	1.50	0.0395	0.0184	0.150 0.300 0.450	41 to 230	41 to 230	4.4	1.1×10^5
0.0627	0.50	0.036	1.50	0.0395	0.0156	0.150 0.300 0.450	41 to 230	41 to 230	5.1	1.3×10^5

Note: Q: flow rate; B: channel width; h: gate elevation; x_1: longitudinal position of the jump toe; d_1: water depth immediately upstream of the jump toe; δ: inflow boundary layer thickness (at x = 1.4 m); Δx: longitudinal separation distance between the sensors; Δz: transverse separation distance; Fr_1: upstream Froude number; Re: Reynolds number; x: longitudinal position of the investigated cross-sections (streamwise position).

For each set of inflow conditions, and at every streamwise position $(x-x_1)$ of the reference sensor, the maximum values of the correlation functions $R_{xx',max}$ and $R_{xz,max}$ were recorded. The results showed that the relationships between the maximum cross-correlation coefficients $R_{xx',max}$ and $R_{xz,max}$ and the separation distances exhibited an exponential decay:

$$R_{xx',max} = \exp\left(-1.204 \frac{\Delta x / d_1}{\left(\Delta x / d_1\right)_{30}}\right) \quad (6)$$

$$R_{xz,max} = \exp\left(-1.204 \frac{\Delta z / d_1}{\left(\Delta z / d_1\right)_{30}}\right) \quad (7)$$

where $R_{xx',max}$ is the maximum cross-correlation coefficient in the longitudinal direction, $R_{xz,max}$ is the maximum cross-correlation coefficient in the transverse direction, and $(\Delta x/d_1)_{30}$ and $(\Delta z/d_1)_{30}$ are respectively the dimensionless separation distances for which $R_{xx',max}$ and $R_{xz,max}$ equal 30% of their maximum value. Note that for Δx = Δz = 0, $R_{xx',max} = R_{xz,max} = 1$. Figure 8 presents a comparison between Equations (6) and the experimental results. The normalised correlation coefficient between the data and Equations (6) and (7) were 0.953 and 0.959 respectively.

The characteristic parameters $(\Delta x/d_1)_{30}$ and $(\Delta z/d_1)_{30}$ were functions of the Froude number Fr_1 and of the longitudinal distance from the jump toe $(x-x_1)/d_1$. The experimental results are presented in Figure 9. Basically, $(\Delta x/d_1)_{30}$ and $(\Delta z/d_1)_{30}$ increased both with an increasing distance from the jump toe $(x-x_1)/d_1$. The results suggested that the flow region with a cross-correlation coefficient of the free surface fluctuations greater than 0.3 enlarged with increasing distance from the jump toe. Further, $(\Delta x/d_1)_{30}$ yielded higher values than $(\Delta z/d_1)_{30}$. That is, the fluctuations of the free surface were better correlated in the longitudinal direction than in the transverse direction.

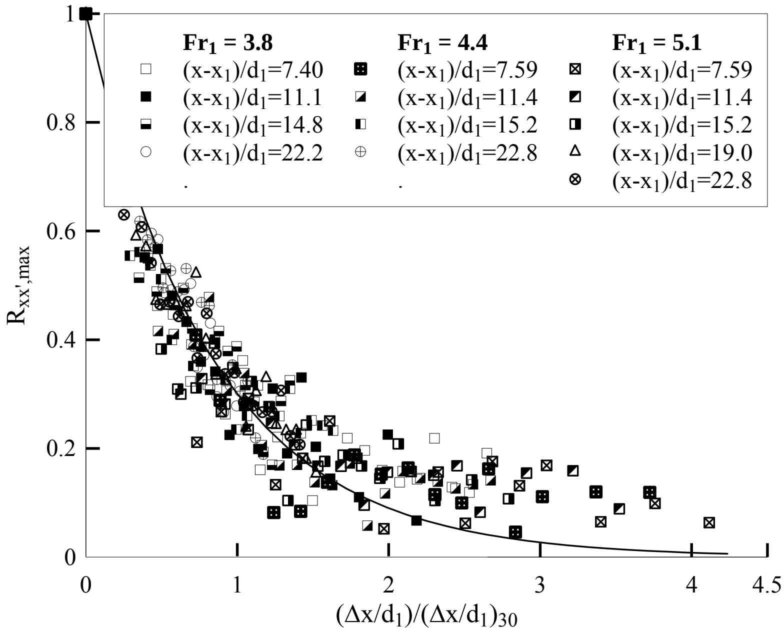


Fig. 8 - Maximum cross-correlation coefficient $R_{xx',max}$ as a function of the longitudinal separation distance Δx - Comparison with Equation (6)

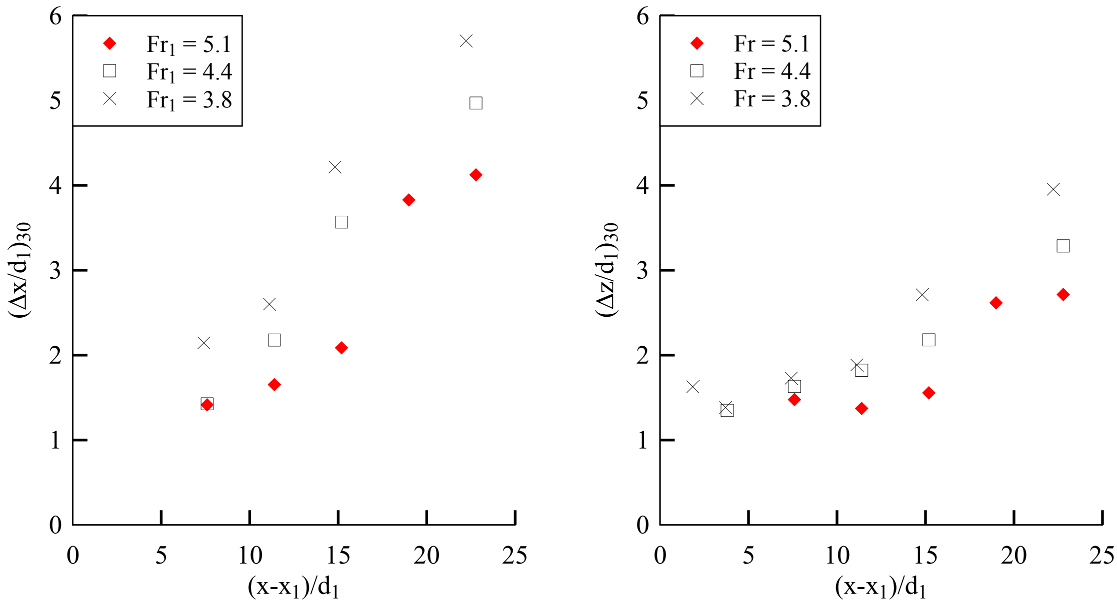


Fig. 9 - Characteristic parameters $(\Delta x/d_1)_{30}$ (Left side) and $(\Delta z/d_1)_{30}$ (Right side) as functions of the dimensionless distance $(x-x_1)/d_1$ from the jump toe

The correlation functions of the free surface fluctuations were linked with some coherence of the large vortical structures interacting with the free surface. Based upon the correlation analyses, some quantitative turbulent properties were derived including integral length and time scales. The maximum cross-correlation coefficient $R_{xx',max}$ and $R_{xz,max}$ results were used to calculate the free surface integral length scales $L_{xx'}$ and $L_{xz.}$ defined as:

$$L_{xx'} = \int_0^{\Delta x_{max}} R_{xx',max}(X)\, dX \qquad (8)$$

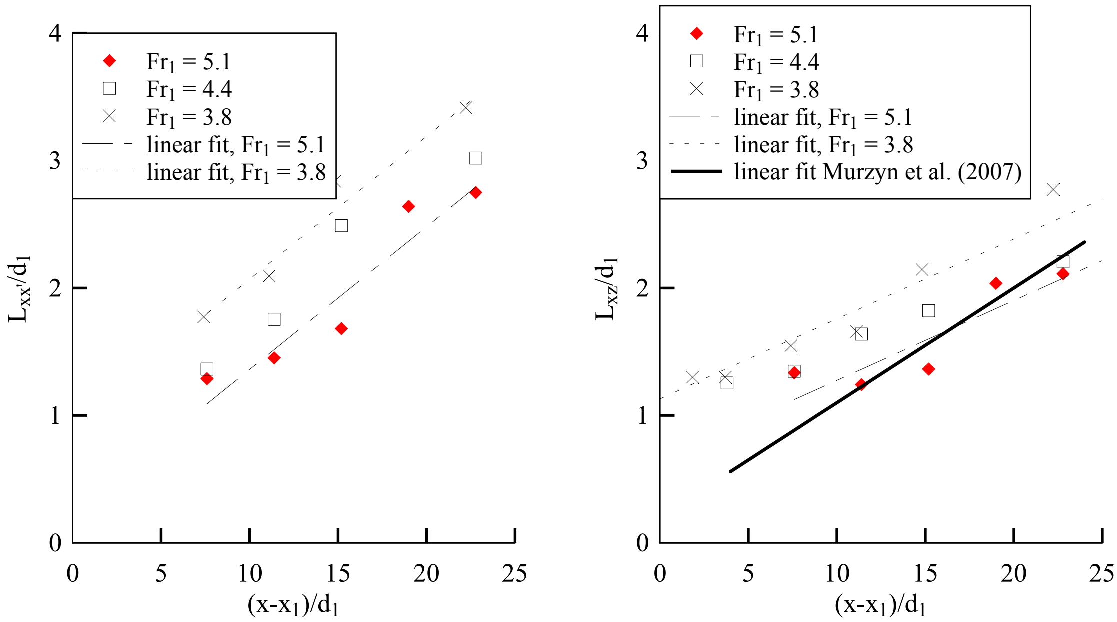
$$L_{xz} = \int_0^{\Delta z_{max}} R_{xz,max}(Z)\, dZ \qquad (9)$$

where X and Z are respectively the longitudinal and transverse separation distances, and Δx_{max} and Δz_{max} represent the upper limit of the sensor separation ($\Delta x_{max} = \Delta z_{max}$ = 230 mm herein, Table 4). Figure 10 presents the integral length scales $L_{xx'}$ and L_{xz} as functions of the distances from the jump toe. The experimental results showed that the free-surface length scales increased with increasing distance from the jump toe. For a streamwise position $(x-x_1)/d_1$ from 7 to 23, the longitudinal length scale $L_{xx'}$ ranged from $1.2d_1$ to $3.5d_1$. The transverse length scale L_{xz} ranged from $1.2d_1$ to $2.6d_1$ for streamwise positions $(x-x_1)/d_1$ between 2 and 23. The results were linked with the inflow Froude number Fr_1 and the present data were best fitted by:

$$\frac{L_{xx'}}{d_1} = 0.112 \frac{x - x_1}{d_1} + (3.02 - 0.545\, Fr_1) \qquad 3.8 < Fr_1 < 5.1 \quad (10)$$

$$\frac{L_{xz}}{d_1} = 0.0627 \frac{x - x_1}{d_1} + (2.54 - 0.371\, Fr_1) \qquad 3.8 < Fr_1 < 5.1 \quad (11)$$

At a given longitudinal location, the longitudinal integral length scale $L_{xx'}$ was slightly larger than the transverse length scale L_{xz}. The result implied that the turbulence was not homogeneous at the free surface of the hydraulic jump. The transverse length scale data were compared to the linear fit of MURZYN et al. (2007) who performed similar free surface measurements with resistive probes (Fig. 10B). Figure 10B, indicates some agreement between the present data and their data, but close to the jump toe: i.e., for $(x-x_1)/d_1$ > 10. In the present study, some larger transverse length scales L_{xz} were observed close to the jump toe ($(x-x_1)/d_1 < 10$).



(A, Left) Longitudinal free surface integral length scales

(B, Right) Transverse free surface integral length scales

Fig. 10 - Longitudinal and transverse free surface integral length scales - Comparison between experimental data and Equations (8) and (9)

The variations of the integral length scales $L_{xx'}$ and L_{xz} were linked to those of $(\Delta x/d_1)_{30}$ and $(\Delta z/d_1)_{30}$ (Eq. (6) & (7)). This is indeed a consequence of the self-similarity of maximum cross-correlation coefficient distributions. Combining Equations (6) and (7) with Equations (8) and (9), the following theoretical relationships may be derived:

$$\frac{L_{xx',theory}}{d_1} = \frac{\left(\Delta x/d_1\right)_{30}}{1.204}\left(1-\exp\left(-1.204\frac{\Delta x_{max}/d_1}{\left(\Delta x/d_1\right)_{30}}\right)\right) \quad (12)$$

$$\frac{L_{xz,theory}}{d_1} = \frac{\left(\Delta z/d_1\right)_{30}}{1.204}\left(1-\exp\left(-1.204\frac{\Delta z_{max}/d_1}{\left(\Delta z/d_1\right)_{30}}\right)\right) \quad (13)$$

The integral length scales $L_{xx'}$ and L_{xz} are proportional to $(\Delta x/d_1)_{30}$ and $(\Delta z/d_1)_{30}$ respectively, notwithstanding for some correction to account for Δx_{max} and Δz_{max} not being infinity. When Δx_{max} and Δz_{max} tend to infinity, Equations (12) and (13) become respectively:

$$\frac{L_{xx',theory}}{d_1} = \frac{1}{1.204}\left(\Delta x/d_1\right)_{30} \qquad \Delta x_{max} \to +\infty \quad (14)$$

$$\frac{L_{xz,theory}}{d_1} = \frac{1}{1.204}\left(\Delta z / d_1\right)_{30} \qquad \Delta z_{max} \to +\infty \quad (15)$$

The evolution of $L_{xx'}$ and L_{xz} would then be proportional to $(\Delta x/d_1)_{30}$ and $(\Delta z/d_1)_{30}$, whose evolutions are presented in Figure 9. A comparison between Figures 9 and 10 shows the close agreement in trends.

Integral time scales

The analysis of the longitudinal and transverse cross-correlation functions provided some information on the free surface turbulence time scales. For each separation distance, Δx or Δz, between the sensors, the correlation time scales $T_{xx'}$ and T_{xz} were calculated as:

$$T_{xx'} = \int_{\tau=\tau(R_{xx'}=R_{xx',max})}^{\tau=\tau(R_{xx'}=0)} R_{xx'}(\tau)\, d\tau \qquad (16)$$

$$T_{xz} = \int_{\tau=\tau(R_{xz}=R_{xz,max})}^{\tau=\tau(R_{xz}=0)} R_{xz}(\tau)\, d\tau \qquad (17)$$

where τ is a time lag. $T_{xx'}$ represents a time scale characteristic of the free surface coherent structures on a length span Δx in the longitudinal direction. T_{xz} is a characteristic time of the free surface coherent structures in the transverse direction on a length span Δz.

The correlation time scales were used to estimate the turbulent integral time scales T_X and T_Z:

$$T_X = \frac{1}{L_{xx'}} \int_{X=0}^{X=\Delta x_{max}} R_{xx',max}\, T_{xx'}\, dX \qquad (18)$$

$$T_Z = \frac{1}{L_{xz}} \int_{Z=0}^{Z=\Delta z_{max}} R_{xz,max}\, T_{xz}\, dZ \qquad (19)$$

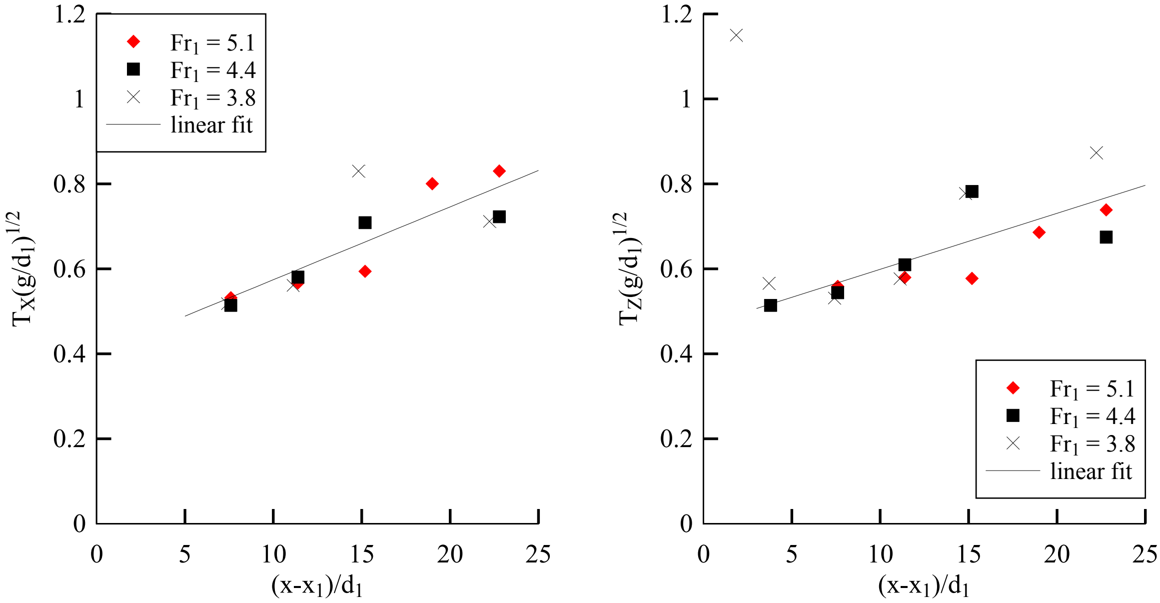
where X and Z are the separation distances respectively in the longitudinal and transverse directions. The turbulent integral time scale T_X represents the time scale of large free surface structures in the longitudinal direction. It integrates in space the correlation time scales at each position Δx, weighted by the maximum cross-correlation coefficient $R_{xx',max}$. T_Z represents the time scale of large free surface structures in the transverse direction.

The experimental data in terms of the integral time scales T_X and T_Z are presented in Figure 11. In Figure 11 the term $(d_1/g)^{1/2}$ represents a characteristic time of the free surface flow. Despite some scatter, the data exhibited a linear increase in integral turbulent time scales with increasing distance from the jump toe. The trend was possibly linked with an increase in large coherent structure sizes and slower convection velocities with increasing distance from the jump toe. The data were further independent of the Froude and Reynolds numbers within the range of the experimental conditions (Table 4). Overall the integral turbulent time scales were best correlated by:

$$T_X \sqrt{\frac{g}{d_1}} = 0.4026 + 0.0172 \frac{x - x_1}{d_1} \tag{20}$$

$$T_Z \sqrt{\frac{g}{d_1}} = 0.4670 + 0.0132 \frac{x - x_1}{d_1} \tag{21}$$

with a normalised correlation coefficient of 0.82 and 0.78 respectively. The integral time scales were observed to be very similar in the longitudinal and transverse directions, although the integral length scale data showed differences between transverse and longitudinal results (Fig. 10). Equations (20) and (21) are compared with the experimental data in Figure 11A and 11B respectively.



(A) Evolution of T_X along the jump (B) Evolution of T_Z along the jump

Fig. 11 - Longitudinal and transverse integral time scales as functions of the longitudinal distances from the jump toe - Comparison with Equations (20) and (21)

4. SIMULTANEOUS FREE SURFACE AND VOID FRACTION MEASUREMENTS

4.1 Presentation

In open channel flows with large free surface fluctuations, the free surface deformations and discontinuity cause some air entrapment. HORNUNG et al. (1995) detailed how this phenomenon generates vorticity. MURZYN and CHANSON (2009) conducted the first simultaneous measurements of free surface and void fraction fluctuations. In the present study, new experiments are presented using the experimental technique of MURZYN and CHANSON (2009), but for different flow conditions (Table 4).

The air-water flow properties and free surface fluctuations were measured simultaneously using an acoustic displacement meter Mic+25/IU/TC and the double-tip conductivity probe. The two instruments were located on the centreline of the channel and vertically aligned: that is, the center of the acoustic displacement meter sampling surface was aligned vertically with the conductivity probe leading tip. For each longitudinal position ($x-x_1$), several records were performed for different vertical elevations of the conductivity probe sensor. The acoustic sensor and the conductivity probe were sampled simultaneously at 5 kHz for 45 s. The same signal processing method was applied to the output signals of the sensors. The raw output signal of the conductivity probe was converted into a binary file of instantaneous void fraction being 0 for water and 1 for air. The signal was then filtered using a band pass 0-25 Hz. The filtering aimed to remove any electrical noise and high-frequency signal component with a frequency greater than the dynamic response of the sensor. The low-pass filtered signal was averaged over 100 points. The output was then linearly interpolated using a constant interval time 0.02 s to facilitate the correlation analysis. (The technique was identical to that developed by MURZYN and CHANSON (2009), but it was applied herein to the 45 s long signals, instead of 12 s long signals.). Figure 12 shows an example of filtered output signals of the conductivity probe and the acoustic sensor.

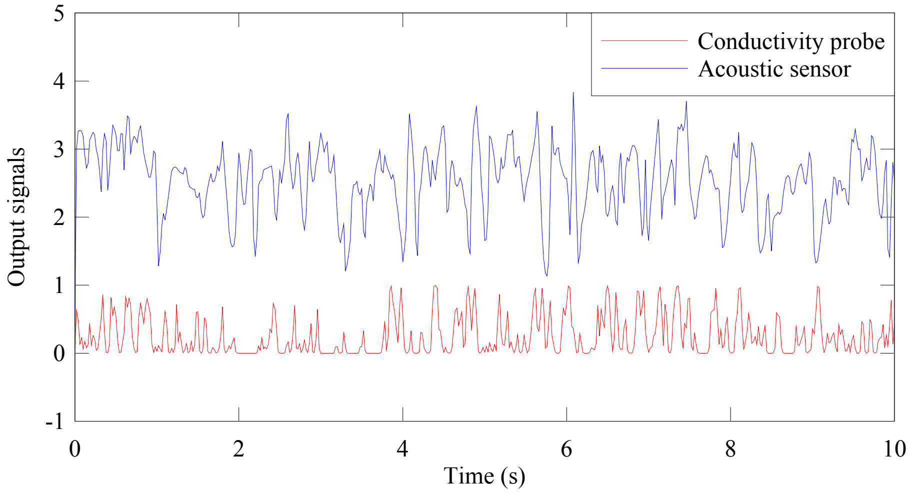


Fig. 12 - Output signals of the conductivity probe (lower) and acoustic displacement meter (upper) after filtering and processing - Flow conditions: d_1 = 0.0405 m, x_1 = 1.50 m, Fr_1 = 3.8, Re = 8.9×10^4

4.2 Results

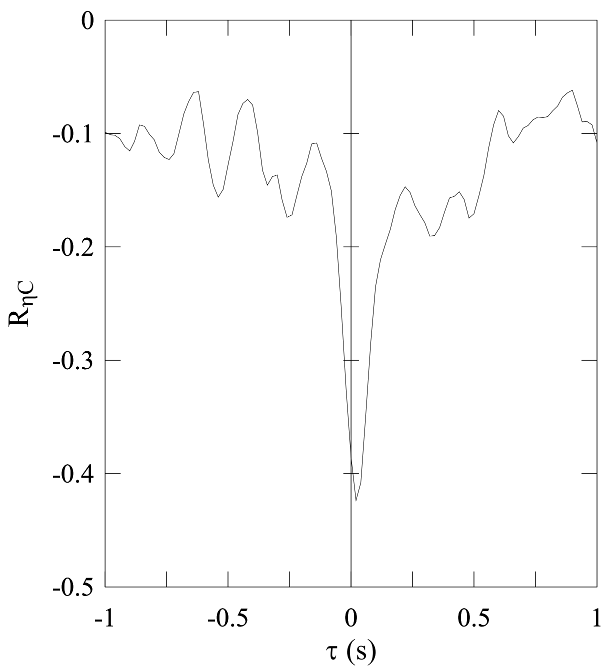
The analysis of the cross-correlation functions between the instantaneous void fraction C_{inst} and instantaneous vertical elevation η_{inst} exhibited two different trends depending upon the vertical elevation y of the conductivity probe sensor. When the conductivity probe sensor was located in the upper part of the flow ($y/d_1 > 2$), the peak in cross-correlation for zero time lag was negative (Fig. 13A), while this peak was positive for $y/d_1 < 2$ (Fig. 13B). Figure 13A presents a typical cross-correlation function between the probe output (leading tip) and the displacement sensor output, when the conductivity probe sensor was located in the upper part of the flow: i.e., the peak for zero time lag was negative. At a vertical location above the "mean" free surface elevation measured by the acoustic sensor, an increase in instantaneous void fraction C_{inst} characterised a void and was associated with a decrease in instantaneous free surface elevation η_{inst}, hence the negative correlation. The result was observed systematically for $3.8 < Fr_1 < 5.1$.

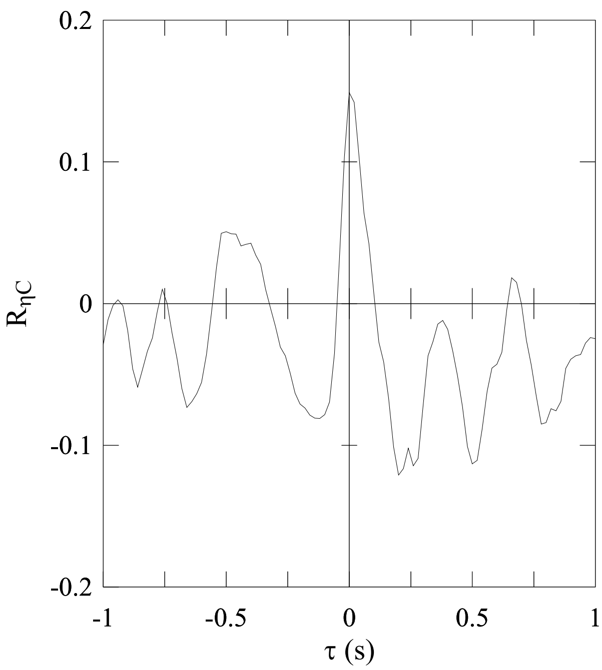
Figure 13B presents a typical cross-correlation function when the conductivity probe sensor was located in the lower flow region ($y/d_1 < 2$). Note the positive peak in the correlation function for zero time lag. An increase in instantaneous void fraction was linked with an increase in free surface elevation. The cross-correlation function between free surface elevation and instantaneous void fraction thus exhibited a positive peak. Note that the results exhibited significant cross-correlation functions at low elevations, only at the closest locations to the jump toe. These locations were: $(x-x_1)/d_1$ = 3.70 for Fr_1 = 3.8, $(x-x_1)/d_1$ = 3.80 for Fr_1

= 4.4 and $(x-x_1)/d_1$ = 3.80 for Fr_1 = 5.1. These regions of positive correlations corresponded to the developing shear layer in the breaking jump. Note further that the cross-correlation functions exhibited some quasi-periodic patterns with increasing time lag as observed by MURZYN and CHANSON (2009).

Overall, the cross-correlation function exhibited a tendency to be negative. It was assumed to be caused by the slow longitudinal fluctuations of the position of the jump toe. If the toe moved forward, the free surface elevation at a position downstream of the toe increased, and the void fraction at a given vertical elevation decreased, hence a negative correlation. This phenomenon happened at a low frequency: it did not affect the instantaneous fluctuations of the cross-correlation function but added a negative constant to it.

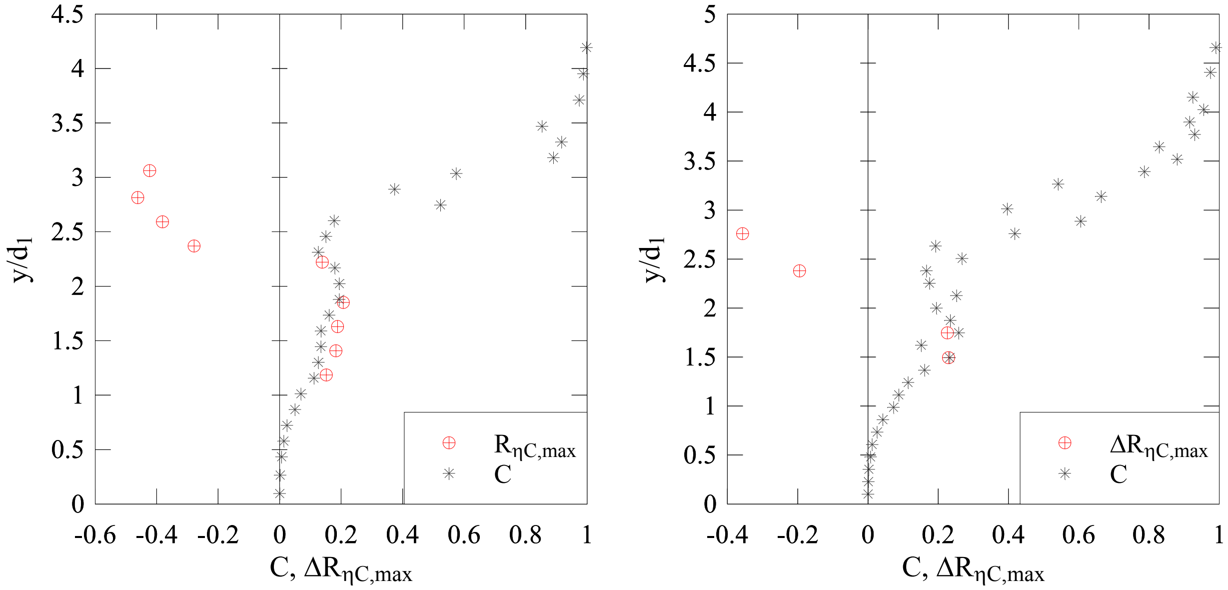
The maximum values $R_{\eta C,max}$ of the cross-correlation functions between the instantaneous void fraction C_{inst} and free-surface elevations η_{inst} were recorded. These gave some measure of the peak amplitude. For example, $R_{\eta C,max}$ = -0.42 in Figure 13A and $R_{\eta C,max}$ = +0.15 in Figure 13B. Figure 14 presents the vertical distribution of the maximum cross-correlation difference $\Delta R_{\eta C,max}$ at a longitudinal position close to the jump toe for different Froude numbers. $\Delta R_{\eta C,max}$ represents the jump in maximum cross-correlation function from its otherwise average value: for example, $\Delta R_{\eta C,max}$ = -0.32 in Figure 13A and $\Delta R_{\eta C,max}$ = +0.25 in Figure 13B. In Figure 14, the time-averaged void fraction data are plotted also on each graph. Figure 14 shows clearly the positive values of $\Delta R_{\eta C,max}$, in the lower elevations (i.e. air-water shear layer) and the negative values in the upper flow region (upper free-surface region).





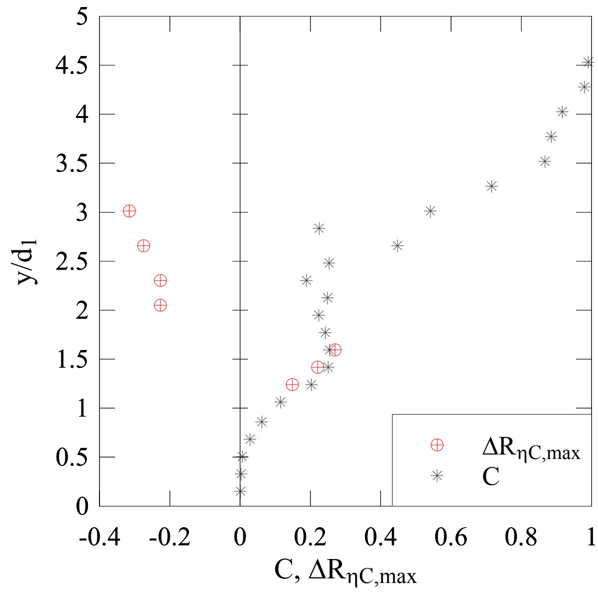
(A) $y/d_1 = 2.94$ (B) $y/d_1 = 1.56$

Fig. 13 - Cross-correlation functions $R_{\eta C}$,: (A) in upper part of the flow (foam and splashing region) and (B) in the lower part of the flow (i.e. developing shear layer) - Flow conditions: $d_1 = 0.0405$ m, $x_1 = 1.50$ m, $(x-x_1)/d_1 = 3.70$, $Fr_1 = 3.8$, $Re = 8.9\times10^4$



(A, Left) $d_1 = 0.0405$ m, $x_1 = 1.50$ m, $(x-x_1)/d_1 = 3.70$, $Fr_1 = 3.8$, $Re = 9.8\times10^4$

(B, Right) $d_1 = 0.0395$ m, $x_1 = 1.50$ m, $(x-x_1)/d_1 = 3.80$, $Fr_1 = 4.4$, $Re = 1.1\times10^5$



(C) $d_1 = 0.0395$ m, $x_1 = 1.50$ m, $(x-x_1)/d_1 = 3.80$, $Fr_1 = 5.1$, $Re = 1.3\times10^5$

Fig. 14 - Vertical distributions of $\Delta R_{\eta C,max}$ and void fraction in hydraulic jump rollers

4.3 Discussion

A key query is the physical significance of the time-averaged free-surface elevation η measured by the acoustic displacement meters. While the acoustic displacement meter technique was robust, simple and non-intrusive, the sensors were not designed to detect a highly aerated, dynamic free-surface. MURZYN and CHANSON (2009) argued that the time-averaged free-surface elevation η characterised the upper free-surface region (η > y*, Fig. 15) that was typically a thin layer where the void fraction was basically larger than 20%, increasing monotonically towards unity.

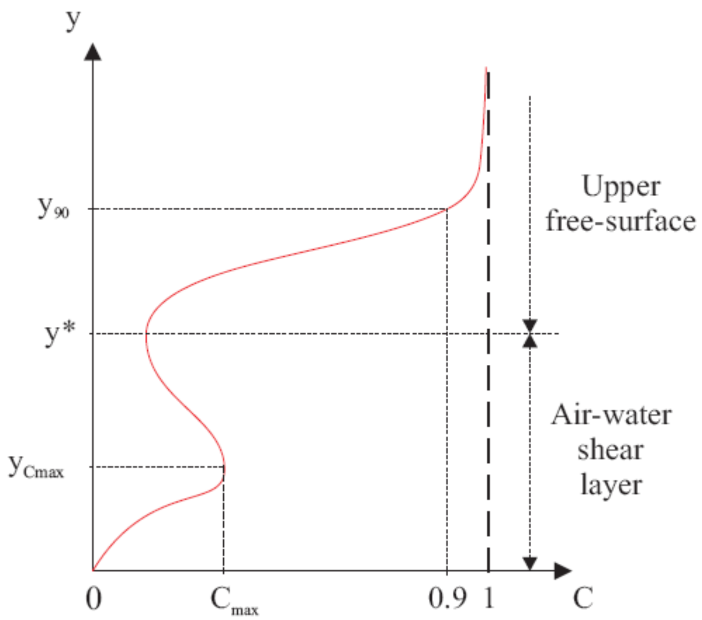


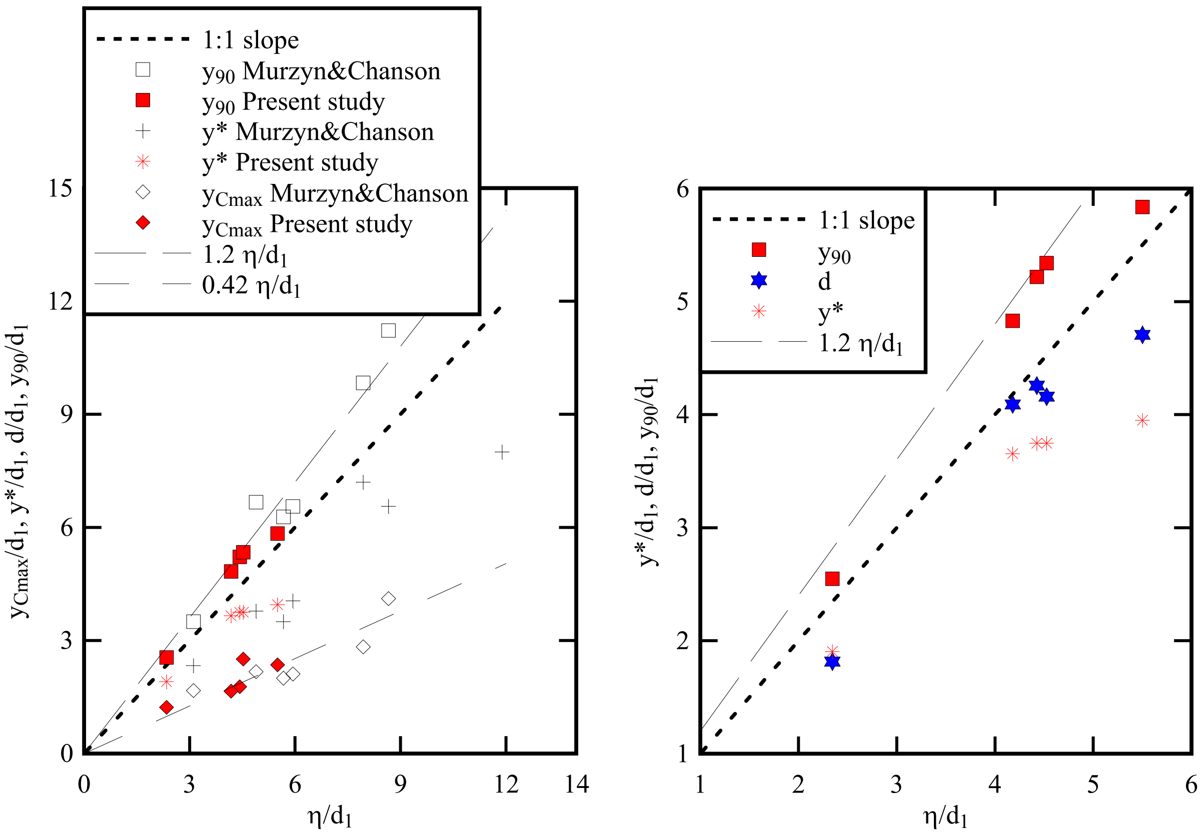
Fig. 15 - Sketch of the vertical distribution of void fraction in the hydraulic jump roller

A comparative analysis was conducted between the acoustic displacement meter and void fraction data. The time-averaged free-surface elevation recorded with the acoustic displacement meter was compared with the void fraction profile measured with the leading tip of the dual-tip conductivity probe. Some results are presented on Figure 16A where y_{Cmax} is the vertical elevation where $C = C_{max}$, y* is defined as the boundary between the turbulent shear layer and the upper part of the flow dominated by free surface strong fluctuations, and y_{90} is the characteristic elevation where C = 90% (see definition in Fig. 15). The data (Fig. 16A) showed that the free-surface measurement η of the acoustic displacement sensor was slightly above the characteristic location y* for all investigated Froude numbers and that:

$$1 < \frac{y_{C\max}}{d_1} < \frac{y^*}{d_1} < \frac{\eta}{d_1} < \frac{y_{90}}{d_1} \qquad (22)$$

where d_1 is the upstream flow depth. The results showed that the free-surface elevation measured by the acoustic displacement sensor was within the upper free-surface region. This region was typically a thin air-water layer in which the void fraction increased rapidly from 20% to 90%. The present findings complemented the results of MURZYN and CHANSON (2009), and Equation (22) narrowed the physical measure of the free-surface location in hydraulic jump ($y^* < \eta < y_{90}$) (Fig. 16A). Importantly the findings were found to be valid over a wide range of turbulent hydraulic jumps with Froude numbers between 3.1 and 8.5. The time-averaged free-surface elevation data η was further compared with the equivalent clear-water depth d deduced from the void fraction distribution:

$$d = \int_0^{y_{90}} (1 - C)\, dy \qquad (23)$$



(A, Left) Comparison of acoustic displacement sensor and phase-detection conductivity probe data - Data: Present study (Red symbols, 3.1 < Fr_1 < 5.1), MURZYN and CHANSON (2009) (Black symbols, 5.1 < Fr_1 < 8.5)

(B, Right) Comparison of free surface elevation η and equivalent clear water depth d (Present study, 3.1 < Fr_1 < 5.1)

Fig. 16 - Comparison of free surface and void fraction measurements using acoustic displacement sensor and phase-detection conductivity probe respectively

The results are presented in Figure 16B for the present data set. Note that the characteristic elevation y* and y_{90} are also reported in Figure 16B for completeness. The results showed a close agreement between the equivalent clear water depth deduced from the void fraction distribution (Eq. (23)) and the time-averaged free-surface elevation η measured with the acoustic displacement meter. This is illustrated in Figure 17 for two upstream Froude numbers. Overall the approximation η ≈ d was correlated with a normalised coefficient of 0.97 for the entire data set (3.1 < Fr_1 < 5.1, Table 4).

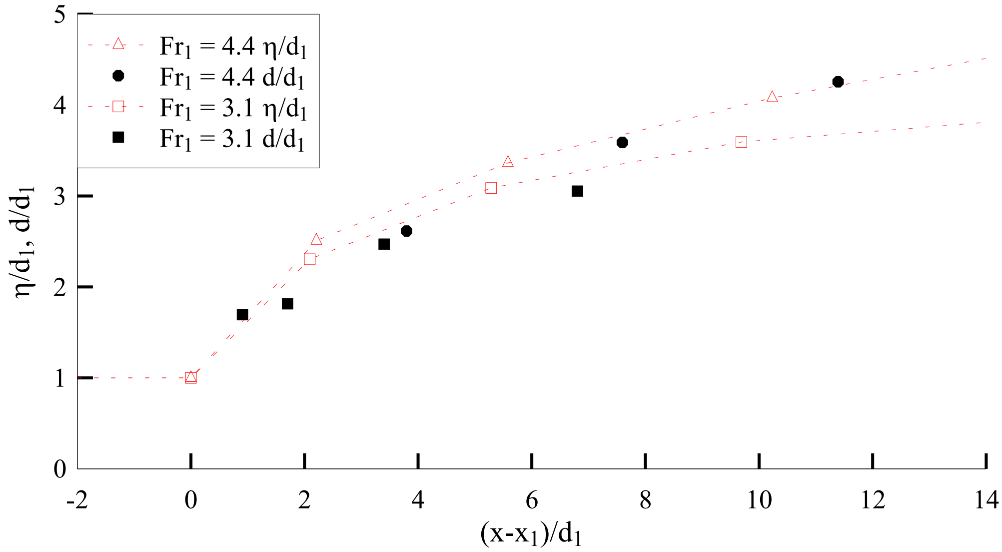


Fig. 17 - Dimensionless longitudinal free-surface profiles of hydraulic jumps: comparison between the time-averaged free surface elevation η/d_1 and equivalent clear water depth d/d_1 for Fr_1 = 3.1 and 4.4

5. CONCLUSION

The present study focused on the free surface fluctuations and air-water properties in hydraulic jumps with relatively small Froude numbers (2.4 < Fr_1 < 5.1) and relatively large Reynolds numbers (6.6×10^4 < Re <

1.3×10^5). The experimental work complemented the earlier physical studies of MURZYN et al. (2007) performed with similar Froude numbers and comparatively lower Reynolds numbers, and of MURZYN and CHANSON (2009) with larger Froude numbers and smaller Reynolds numbers. The dynamic free surface measurements were performed with several non-intrusive sensors, namely acoustic displacement meters, to record the mean and turbulent surface profiles, characteristic frequencies, and integral length and time scales. The air-water flow measurements were conducted with a dual-tip conductivity probe. Some measurements of free surface fluctuations and two-phase properties were conducted simultaneously, and a correlation analysis was conducted.

The free surface data indicated that the shape of the mean free surface profile was well defined and in agreement with visual and photographic observations. The turbulent fluctuation profiles exhibited a maximum in standard deviations in the first part of the hydraulic jump roller. Its amplitude increased monotically with increasing Froude number. The free surface fluctuations exhibited some characteristic frequencies between 1.4 and 4 Hz, with the majority below 3 Hz. Some simultaneous free-surface measurements at a series of two closely located points yielded the free surface length and time scales of free-surface fluctuations in terms of both longitudinal and transverse directions. The maximum cross-correlation coefficient between the free surface fluctuations at two different locations decreased exponentially with increasing distance between the sensors. The integral length and time scales increased with increasing longitudinal distances from the jump toe, and the longitudinal length scales were greater than the transverse length scales. The length scale data seemed to depend upon the inflow Froude number, while the time scale data showed no definite trend.

The simultaneous measurements of instantaneous void fraction and free surface fluctuations exhibited different features depending upon the phase-detection probe sensor location in the different regions of the roller: a positive correlation in the shear layer region, and a negative correlation in the free surface region. The acoustic displacement meters yielded a time-averaged free-surface elevation that corresponded to the upper free-surface region where the void fraction increased rapidly from 0.20 to 0.90, and the quantitative values were very close to the equivalent clear-water depth.

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