Is the Relationship Between Prices and Exchange Rates Homogeneous?[1]

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Abstract

Empirical tests of purchasing power parity (PPP) are implicitly based on the conditions of symmetry and proportionality of the price coefficients. We investigate a separate condition, which we term homogeneity. Specifically, while there may be factors that drive a wedge between prices and exchange rates, when these factors are held constant we would expect a change in exchange rates to be associated with a proportional, or homogeneous, change in prices. To test for the existence of homogeneity in prices, we conduct two experiments. First, we apply a time-varying-coefficient procedure to nine euro-area countries as well as the euro area as a whole during the (monthly) sample period, 1999: M1 to 2011:M3. Second we apply the same procedure to the same group of countries, plus Canada, Japan and Mexico, over the longer period, 1957:M4 to 2011:M3. We find that averages of the price coefficients, corrected for specification biases, are uniformly homogeneous in the long run, providing strong support for PPP.

Keywords*:* Purchasing power parity, symmetry, proportionality, homogeneity, generalized cointegration, time-varying coefficients.

JEL codes: C32; F31

[1] We thank the editor, Joshua Aizenman, and two referees for their very constructive comments.
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1. Introduction

Empirical tests of purchasing power parity (PPP) are implicitly based on the conditions of symmetry and proportionality of the coefficients of the exchange rate-price relationship: symmetry is said to apply if the coefficients on the domestic and foreign price levels are identical (in absolute value); proportionality applies if the coefficients on both price levels are equal to unity (in absolute value). Most empirical studies, however, do not directly test for the existence of these two conditions; those that do carry out such tests typically fail to find evidence in support of the conditions, although, typically, symmetry is more easily accepted than proportionality (e.g., Cheung and Lai (1993), Moosa (1994), Edison, Gagnon and Melick (1997), Li (1999), Moon and Perron (2004), Cerrato and Serantis, (2008)). Yet, as pointed out by Li (1999, p. 410, original italics), "their validity is a *maintained hypothesis* in unit root tests for the long-run PPP using real exchange rates".[1] In this connection, unit root -- as well as co-integration -- tests have often generated results supportive of long run PPP, although the results are subject to considerable uncertainty, reflecting structural changes, varying degrees of volatility related to different exchange-rate regimes, and other possible misspecifications that may arise using long sample periods. At the same time, as pointed out by Taylor and Taylor (2004) in their survey of the PPP literature, most empirical studies tend to reject short run PPP.[2]

PPP, however, is a fundamental building block of international economics and finance theory; it is, therefore, surprising that the support for this basic concept is so weak. In this paper we investigate the possibility that, while the basic PPP relationship does hold, attempts to estimate that relationship have typically been based on an overly-restrictive specification that tends to yield biased estimates. Specifically, we argue that factors other than prices that affect the exchange rate need to be taken into account. These factors could include short run volatility effects, as in Dornbusch overshooting, and medium term structural effects, such as divergences in wage costs or productivity levels. We show that omitting such effects would lead to biased coefficient estimates and, under certain circumstances, these omitted variables would

[1] Examples of such unit root tests on PPP include Lothian and Taylor (1996, 2000) and Enders (2009, pp. 382-84)

[2] By short run PPP we mean that a change in prices is rapidly reflected in nominal exchange rates with little or no process of adjustment. Long run PPP is defined as a situation where a change in prices is eventually fully reflected in exchange rates but only after a period of adjustment.

also be expected to generate symmetric, but not proportional, results. A key issue from the point of view of the theory of PPP is the following: Are the coefficients of prices and exchange rates in an exchange rate-price relationship homogeneous of degree one (henceforth, homogeneous)? If the answer is yes, then changes in the exchange rate result in proportionate changes in prices. There may be other factors affecting PPP that are important in the data, but are not important for the comparative-static analysis of PPP. If this is the case, then the standard textbook cases obtain. However, if the price effects themselves are non-homogenous, then standard models are simply built on a false premise.

In what follows, we conduct two experiments. First, we investigate the homogeneity condition for nine euro-area countries as well as for the euro area as a whole. We use monthly data over the sample period 1999:M1 to 2011:M3, a period corresponding to euro-area membership of the nine countries considered. Second, we extend the experiment to include three additional countries -- Canada, Japan, and Mexico -- over a longer sample period, 1957:M1 to 2011:M3. Our reference currency in both experiments is the U.S. dollar. We apply a generalized cointegration technique (Hall, Swamy and Tavlas (2012), Hall, Kenjegaliev, Swamy and Tavlas (2013a)) based on a time-varying-coefficient regression. The underlying idea of this approach is that the coefficients in a PPP relationship are homogeneous if their bias-free components take unit values even if this relationship may involve omitted and/or variables subject to measurement errors.[3] Therefore, if we find that the bias-free components of the coefficients -- we explain below what we mean by bias-free components -- of a PPP relationship are homogeneous, then the prices and the exchange rate are cointegrated. Moreover, an appealing feature of this technique is that it is possible to obtain estimates of the time-varying coefficients and to reproduce them in visual form to assess the proportionality and symmetry conditions. There is, of course, a "little pinch" of uncertainty in accurate estimation of bias-free components. This could potentially distort the results of the time-varying coefficients and give misleading deductions. We address this issue in this paper.

The rest of the paper is organised as follows. Section 2 presents the time-varying PPP relationship and the time-varying-coefficients' methodology used to estimate that relationship. Section 3 discusses the concept of generalized cointegration

[3] Here bias-free components are those that are free of incorrect-functional form, omitted-variable and measurement-error biases (see the Appendix below).

and its relationship to time-varying-coefficient estimation. Section 4 presents the data and estimation results. Section 5 concludes.

2. Time-varying purchasing power parity

The law of one price states that the prices of identical goods in two countries converted into equivalent currency units will be the same to preclude arbitrage opportunities in both economies. Formally, the standard representation of the law of one price is

$$P_i = S \times FP_i, \quad i = 1, ..., N \tag{1}$$

where S is the nominal exchange rate (defined as the domestic price of one unit of foreign currency), P_i is a domestic price of good *i* and FP_i is a foreign price of that good. The main assumptions here are that the good is tradable and that the market is frictionless.

The general price level in the home economy at time *t* can be computed by taking a weighted average of all the individual prices (Chen and Engel (2005), Sarno and Taylor (2002)),

$$\overline{P_t} = \theta_1 P_{1t} + \theta_2 P_{2t} + ... + \theta_N P_{Nt} = \sum_{i=1}^{N} \theta_i P_{it} \tag{2}$$

The general price level in the foreign economy is

$$\overline{FP_t} = F\theta_1 FP_{1t} + F\theta_2 FP_{2t} + ... + F\theta_N FP_{Nt} = \sum_{i=1}^{N} \theta_i FP_{it} \tag{3}$$

where $\sum_{i=1}^{N} F\theta_i = 1$, and it is assumed that for *i* = 1, ...,N, $\theta_i = F\theta_i$ in both economies and correspond to the weights of the good *i* in the consumption bundle. Moreover, $\overline{P_t}$ and $\overline{FP_t}$ in Eqs. (2) and (3) are usually index numbers representing the price level in the economy. Using Eqs. (2) and (3) the absolute version of PPP is expressed as

$$\theta_1 P_{1t} + ... + \theta_N P_{Nt} = S_t (\theta_1 FP_{1t} + ... + \theta_N FP_{Nt})$$

$$= \sum_{i=1}^{N} \theta_i P_{it} = S_t \sum_{i=1}^{N} \theta_i FP_{it} \tag{4}$$

or, alternatively,

$$\overline{P_t} = S_t \times \overline{FP_t} \quad t = 1, ..., k \tag{5}$$

Eq. (5) is an approximation to PPP. Empirical studies have typically found that in its strict form Eq. (5) does not hold.

Due to the violation of the law of one price, domestic and foreign prices may have different impacts on the formation of the exchange rate (MacDonald (1993)) so that their shares can potentially change in each period. In addition, it is possible that other effects may distort the PPP relationship in the short or medium term. There are almost certainly short run volatility effects on the exchange rate since, typically, the volatility of the logged exchange rate in the short run is much larger than the volatility of prices. It is also possible that medium-term effects, such as a Balassa-Samuelson effect, might affect the price measures. In this circumstance, Eq. (5) can be specified as

$$S_t = A_t \times \overline{P}_t^{b_{1t}} \times \overline{FP}_t^{-b_{2t}} \tag{6a}$$

where S_t is the nominal exchange rate (domestic price of a foreign currency unit) at time t, A_t is a term that captures effects not correlated with prices (such as, for example, speculative activity in the exchange market), and $\overline{P}_t$ and $\overline{FP}_t$ are (again) domestic and foreign prices, respectively. The time-dependent coefficients of prices (b_{1t} and b_{2t}) will reflect a range of omitted factors, such as the effects of transportation costs, tariffs and non-tariff barriers, and, possibly, other misspecifications that may well vary over time (see Swamy, Tavlas, Hall, and Hondroyiannis (2010)).

Eq. (6a) is one possible view of the world suggesting that the basic relationship between prices and exchange rates is non-homogeneous. However a different view of the world would be one in which homogeneity is maintained, but other factors enter the exchange rate-price relationship. Thus, let us assume that there are factors, such as labor productivity, that enter the PPP relationship. Then, the following relationship may hold:

$$S_t = A_t \times \overline{P}_t \times \overline{FP}_t^{-1} \times Z_t^{b} \tag{6b}$$

where the variable Z may create a wedge between the exchange-rate-price relationship over time, but the exchange-rate-price part of relationship (6b) is nevertheless homogeneous. Consequently, Eq. (6b) would be consistent with the comparative static properties of a standard international macro model. Note that Eq. (6b) implies that, in the presence of such factors as differences in productivity levels, the coefficients on the two price variables in (6a) would not be expected to be unity; effectively, such factors act as omitted variables, impacting on the coefficients of the price variables in

(6a). Eq. (6b), in contrast, implies that explicitly taking these factors into account yields homogeneity of prices.

Either of these two possibilities can be nested within a version of Eq. (6a) with time-varying coefficients. In logarithmic terms, the two possibilities can both be written as

$$s_t = b_{0t} + b_{1t} p_t - b_{2t} fp_t \tag{7}$$

where lower case letters represent logs of the respective variables of Eq. (6a). Relative PPP implies that there is a proportionate effect on the exchange rate from the two prices.

Eq. (7) can be used to demonstrate the bias that occurs in a standard OLS regression with fixed coefficients. If we take Eq. (7) and rewrite it in the following way

$$s_t = b_0 + b_1 p_t - b_2 fp_t + (b_{0t} - b_0 + (b_{1t} - b_1) p_t - (b_{2t} - b_2) fp_t),$$

it is clear that in this regression the error term is the entire last term in parenthesis. For OLS to provide consistent parameter estimates, we would require that the two prices be uncorrelated with the error term, and this is almost surely not the case as prices also comprise part of the error term. So, if there are any omitted variables, or if the weights of the two prices vary for any reason, standard techniques are not appropriate for investigating PPP.

The standard definition of PPP implies that Eq. (7) will have equal coefficients for all *t*. Homogeneity will be obtained when $b_{1t} = b_{2t} = 1$ for all *t*, either on average in the long run or at every point in time. However, if the coefficients do not satisfy these homogeneity conditions and are not constant, then relative PPP no longer holds, at least in its conventionally accepted form. To show this consider the following

$$(s_t - s_{t-1}) = (b_{0t} - b_{0t-1}) + (b_{1t} p_t - b_{1t-1} p_{t-1}) - (b_{2t} fp_t - b_{2t-1} fp_{t-1}) \tag{8}$$

In Eq. (8) the exchange rate at time *t-1* is simply subtracted from the current rate at time *t*. This can be rewritten in more compact way as

$$\Delta s_t = (b_{0t} - b_{0t-1}) + (b_{1t} \Delta p_t + \Delta b_{1t} p_{t-1}) - (b_{2t} \Delta fp_t + \Delta b_{2t} fp_{t-1}) \tag{9}$$

Rearranging, Eq. (9) we get

$$\Delta s_t = b_{0t} + b_{1t} \Delta p_t - b_{2t} \Delta fp_t - (b_{0t-1} - \Delta b_{1t} p_{t-1} + \Delta b_{2t} fp_{t-1}) \tag{10}$$

Typically, only the first three terms on the right hand side of Eq. (10) are used to represent relative PPP. However, from Eq. (10) it is clear that, if the coefficients of prices are time-varying, then not only current price changes impact Δs_t, but also the previous period's prices. Moreover, the coefficients on past prices and current changes in prices are inversely related if $\Delta b_{jt} < 0 \quad j = 1,2$; only if $\Delta b_{jt} > 0 \quad j = 1,2$ will the direction of the impact be the same as the direction of the current changes in prices. The effect of past prices diminishes if $-1 < \Delta b_{jt} < 1 \quad j = 1,2$. It should be noted that Δb_{1t} and Δb_{2t} depend on the changes in all relevant variables on which b_{1t} and b_{2t} depend. The lower the absolute change in prices compared to the change in the exchange rate, the higher the values of coefficient difference, and, therefore, past prices dominate in the exchange rate change.

As mentioned, our aim is to test for the existence of homogeneity in Eq. (7); homogeneity requires both coefficients to be equal to unity (although with opposite signs). An attractive element of the time-varying-coefficient regression is that it allows us to display the dynamic evolution of the individual coefficients, which can be analytically evaluated against the null hypotheses without the need of sophisticated statistical tests. As a result, it is convenient to present and examine the evolution of the coefficients in visual form.

3. Generalized cointegration

The econometric approach we use is provided in detail in (Hall, Swamy and Tavlas (2012, 2014), Hall, Kenjegaliev, Swamy and Tavlas (2013a)); however, as it is a relatively novel approach, we will provide an intuitive account of the ideas used; we also provide references to a formal exposition of the TVC approach used in Appendix A. The approach uses the concepts of generalized cointegration (Hall, Swamy and Tavlas (2012)), and time-varying-coefficient (TVC) estimation (Swamy, Tavlas, Hall and Hondroyiannis (2010)); this approach allows for the consistent estimation of models in the presence of an unknown true functional form, omitted variables, and measurement errors.

Both generalised cointegration and TVC estimation proceed from an important theorem first established by Swamy and Mehta (1975), which was subsequently confirmed by Granger (2008). This theorem states that any nonlinear function can be exactly represented by a model that is linear in variables but which has time-varying coefficients. The implication of this result is that, even if we do not know the correct functional form of a relationship, we can still represent this relationship as a time-varying-coefficient relationship and, hence, estimate it.

This theorem underlies the idea of the concept called generalised cointegration (Hall, Swamy and Tavlas (2012)), which relaxes some of the stringent assumptions of standard cointegration analysis. The particular version of generalized cointegration implemented here does two things. First, it allows for the possibility that we may have important omitted variables. Second, it solves the unknown functional-form problem. That is, under generalized cointegration we are able to estimate bias-free relationships among a set of variables even (i) if we do not know the true, underlying functional form and (ii) even if there are missing variables. Specifically, generalized cointegration works by correcting a relationship for specification errors (such as omitted-variable biases).

Underlying generalized cointegration is a new way of thinking about, and testing for, cointegration that emphasises the properties of the real world rather than a particular model. If, in the real world, a causal cointegrating vector exists which determines a variable, say, the real exchange rate, then, obviously, if one of the explanatory variables (say X) in that relationship changes, the real exchange rate will also change. This circumstance implies that the partial derivative of the real exchange rate with respect to X is non-zero. Thus, if we had a way of obtaining consistent estimate of this partial derivative and testing to see if it is significantly different from zero, this would give us a way of testing for the presence of cointegration in the real world (rather than just among an arbitrary set of variables). So, we might be able to assert that there is a stable relationship between, say, two variables in the real world, even though we do not know its exact functional form and/or all the variables that comprise that relationship. This would still be a very useful statement to make from a policy perspective, although, obviously, not as useful as knowing the complete form of that relationship.

Of course, this may appear as asking a great deal of an estimation technique. However, that is precisely what TVC estimation aims to provide (Swamy, Tavlas, Hall and Hondroyiannis (2010)). This technique builds from the Swamy and Mehta theorem, mentioned above, where it turns out that the time-varying coefficients in a model without omitted variables or measurement error are the partial derivatives of the unknown non-linear functional form. So, in the absence of other misspecification, testing the significance of the time-varying coefficients would be equivalent to testing for generalised cointegration.[4]

Swamy, Tavlas, Hall and Hondroyiannis (2010) show exactly what happens to the TVCs as other forms of misspecification are added to the model. If we allow for the presence of some omitted variables from the model, then each of the true time-varying coefficients gets contaminated by a term that involves the relationship between the omitted and included regressors. Also, if we allow for measurement error, then each of the TVCs is further contaminated by a term that involves measurement errors. . Thus, as one might expect, the estimated TVC is no longer a consistent estimate of the true partial derivatives of the non-linear function, but is instead biased due to the effects of omitted variables and measurement errors. In what follows, we call these biased coefficients the “total effect” coefficients. There are exact mathematical proofs provided for our statements up to this point.

Some parametric assumptions are needed to make TVC estimation fully operational.[5] We make two key assumptions. First, we assume that the time-varying coefficients themselves are determined by a set of stochastic linear equations which make them a function of a set of variables that we call driver (or coefficient-driver) variables (In what follows, we call these coefficients the “total effects” coefficients). This is a relatively uncontroversial assumption. Second, we assume that some of these drivers are correlated with the misspecification in the model and some of them are correlated with the time-variation coming from the non-linear (true) functional form. With this assumption, we can then simply remove the biases from the TVCs by removing the effect of the set of coefficient drivers which are correlated with the misspecification. Effectively, the coefficient drivers absorb omitted-variable and measurement-error biases. This procedure, then, yields a consistent set of estimates of -- what we call “bias-free” coefficients -- the true partial derivatives of the unknown nonlinear function, which may then be tested by constructing t-tests in the usual way. An important difference between coefficient drivers and instrumental variables is that a valid instrument requires a relevant variable which is uncorrelated with the

[4] For recent applications of our technique, see Hall, Kenjegaliev, Swamy and Tavlas (2013a, 2013b).

[5] Formal derivations of the ideas discussed in this paper are provided in Swamy and Tavlas (2005, 2007). These derivations are discussed in the Appendix.

misspecification (disturbance), which often proves hard to find. Additionally, such an instrument is also required to be correlated with the relevant independent variable. For a valid driver we need variables that are correlated with the misspecification and we would expect that this is much easier to achieve.

Swamy, Tavlas, Hall and Hondroyiannis (2010) provide the distribution theory for conducting statistical inference and constructing confidence intervals for the TVC estimation technique; those authors assumed that the coefficients have normal distributions.[6] This proof extends to the case of generalised cointegration even when we are testing under the null of no cointegration. This at first appears surprising and so it is worth explaining why this happens. In the standard OLS testing framework such as the Dickey-Fuller test, the non-standard distribution comes from the fact that under the null of nonstationarity, the error term in the regressions may be non-stationary while under the alternative, it is stationary. This does not occur with TVC estimation. The basic TVC model, such as Eq. (7) contains a unique error term in the form of a term of the intercept. In addition, the errors appear in the state equations (A4 in the appendix). The key assumption to deriving normal inference is that these errors are stationary. However, this is easy to achieve as the coefficient drivers may contain lags of all the variables in the model and hence can always achieve a stationary error process by a sufficiently large number of lags that explain most of the variation in the coefficients.

These consistent (or bias-free) estimates may then be used to test for generalised cointegration, even in the presence of omitted variables. It is important to stress what is being claimed here -- as well as what is not being claimed. This test aims to tell us whether or not there is cointegration in the real world, that is, whether there actually exists a stable function determining the dependent variable of interest. It does not, however, tell us the complete form of that relationship or what the missing variables might be.

4. Data and results

[6] The nonnormal distributions of these coefficients substantially complicate these inference procedures (Swamy et al. (2010, pp. 16-20)).

We provide estimates of the price-exchange rate relationship (Eq. (7)) under two experiments, using monthly data.[7] First, we use as our data sample a group of euro-zone countries that shared the same currency over the estimation period, 1999:M1 to 2011:M3. For this experiment, the nine euro-area countries are Austria, Belgium, France, Germany, Greece, Italy, Netherlands, Portugal and Spain. Additionally, we examine the relationship for the overall euro area. For our second experiment, we extend the group of countries to also include Canada, Japan and Mexico and the sample period to 1957:M4 to 2011:M3. Under both experiments, the United States is assumed to be the foreign country; the bilateral nominal exchange rate is defined as the number of domestic currency units per U.S. dollar (IFS series, ... AG.ZF). All data are from IMF International Financial Statistics (ESDS database). Prices in each country are represented by consumer price indices (CPI) (IFS series, 64…ZF; for EU 64H…ZF). Within the first exercise, with the exception of Greece, the data consist of monthly observations starting from the introduction of euro in 1999:M1 through 2011:M3; for Greece, the data start from 2001:M1, the year (and month) in which that country joined the euro zone. All variables are specified in logs. The coefficient drivers used are given in Section 4.2.

It is worth emphasizing that the estimation period for our first experiment is such that the countries considered were all in the euro area, and, therefore, had their relative nominal exchange rates locked together. Therefore, one might have expected that all these countries would have the same domestic price level, as this is the implication of the strongest form of PPP. Such an expectation, of course, did not turn out to be true. For example over the period of Greek membership the Greek price level grew by 20 per cent more than the German price level. Hence, PPP could not have held for those two countries. This implies that either we abandon the idea of testing for PPP altogether, or we think of PPP as holding in such a way that allows for other things to be going on at the same time. For example, there may be other variables in the PPP relationship, the exclusion of which would bias the coefficients on the price variables. Thus, while it may be true that in an experiment with changes in the price level, there is a proportionate change in the exchange rate, it may also be true that changing productivity levels and labor market (and other) conditions might

[7] In conventional estimation, it is often better to use low-frequency data in estimation because OLS tends to be biased as a result of the missing dynamics. TVC estimation is not subject to this bias. Therefore, in TVC estimation it is best to use the largest data set available -- i.e., the data set with the highest frequency.

drive a wedge in the PPP relationship for an extended period. The generalised cointegration approach outlined above allows for precisely this type of omitted-variable effects.

We begin by estimating the time-varying-coefficient regressions for each country and we test whether the symmetry ($b_1 = -b_2$) and strict proportionality conditions (b_1 = 1 and b_2 = -1) are satisfied using the total-effect coefficients. However, one or both of these conditions could fail because of misspecifications and inherent biases present in PPP relationship. In such a case, if we are able to remove the biases, then we may possibly obtain the true coefficients, which will establish cointegration within the context of the generalized cointegration framework and which should then allow us to assess homogeneity. Therefore, we subsequently estimate bias-free coefficients to investigate the symmetry and proportionality restrictions.

4.1 Total effects

Table 1 reports the average total-effect coefficients for our first experiment. Recall, these coefficients contain specification biases. Several features of these results are noteworthy. First, in the cases of Austria, Belgium, Germany and Greece, the results indicate that the exchange rate (units of euros per unit of the U.S. dollar) is inversely related to domestic prices and positively related to foreign prices. That is, rises in domestic prices and/or falls in foreign prices lead to a nominal appreciation of the domestic currency (i.e., a decrease in the number of units of euros per U. S. dollar) -- the opposite of what is predicted by homogeneity. Second, in the cases of France, Italy, Portugal and Spain, as well as for the euro area as a whole, the coefficients on the domestic price level and the foreign price level are correctly signed; an oddity arises, however, from the magnitude of the price coefficients. Although homogeneity states that the price coefficients should be unity (in absolute value), the coefficients for those four countries, as well as for the euro area, are generally very different from unity. For example, in the case of France the coefficient on home prices is 7.8 while the coefficient on foreign prices is -3.7, both of which deviate sharply from unity. Third, both price coefficients have the same (positive) signs in the case of only one country -- that of the Netherlands. Again, however, the price coefficients for the Netherlands are very different from each other and different from unity.

How can we explain these results? Our conjecture is that the causes of the violation of homogeneity are the biases and the misspecifications inherent in the model and the biases that are reflected in the total-effect coefficients.

To shed light on this issue, consider the (monthly) total-effect TVCs presented in Figures 1, 2 and 3. Consistent with the average TVCs reported in Table 1, for Austria, Belgium, Germany, and Greece, the betas of home prices are mostly negative while for foreign prices they are mainly positive. For France, Italy, Spain, Portugal and the euro zone, most of the TVCs are of the correct sign. For the Netherlands, the home-price TVCs generally cluster around zero, while the foreign-price TVCs are generally positive.

Inspection of these figures reveals an interesting pattern. Movements in the monthly coefficients of domestic and foreign prices mirror each other in all ten cases we considered -- that is, domestic and foreign price coefficients tend to move in opposite directions. Table 2 reports correlations between the coefficient estimates of domestic and foreign prices. These correlations are very close to minus unity in all ten cases. Thus, it appears that a kind of the symmetry condition -- though not the one required by PPP -- holds during the sample period. In other words, this symmetry is not of the type implying that $b_{1t} = -b_{2t}$; instead, the *movements* of the coefficients are symmetrical.

We conjecture that the coefficients of PPP could possibly be uninterpretable and biased. This circumstance could occur if, in the short run, there is little movement in prices, but the exchange rate moves by a significant amount. In the very short run, price changes are almost certainly much smoother (and stickier) than movements in nominal exchange rates, so that the relationship between the exchange rate and prices is hard to pin-down empirically. In the limit, if prices did not move at all, and the exchange rate changes, then the coefficients will not be identified. To explain, rewrite Eq. (7) as follows:

$$s_t = b_{0t} + b_{1t} p_t - b_{2t} fp_t \tag{11}$$

Now suppose the following conditions apply:

$$p_t = fp_t \text{ and } \Delta p_t = \Delta fp_t \textit{ and } \Delta s_t = \eta \tag{12}$$

Then, Eq. (11) reduces to $s_{t-1} + \eta = b_{0t} + (b_{1t} - b_{2t})\, p_t$ (or $s_{t-1} + \eta = b_{0t} + (b_{1t} - b_{2t})\, fp_t$) and b_{1t}, b_{2t} are not identified. When $b_{1t} = b_{2t} = k_t$, then $b_{1t} - b_{2t} = 0$. Now as $\Delta p_t = \Delta fp_t \to 0$, then $k_t \to \infty$. So we tend to see symmetry because, in general, over short periods of time changes in prices tend to be much smaller than changes in nominal exchange rates.

What we, therefore, observe is the symmetry of the total-effect coefficient, which has not been corrected for specification biases; this symmetry gives the odd behavior of the coefficients[8] shown in Figures 1 through 3. We believe that this combination of biased coefficients and lack of identification provides an explanation of the common finding of symmetry of the coefficients in PPP tests.

Table 3 shows the results (for the total effects' coefficients) for our second experiment, which uses both the extended country sample and the longer sample period. Again, the picture is very mixed, with little obvious support for PPP. Clearly, the non-euro-zone countries perform no better than those of the euro zone, with many coefficients far from their expected values. Figures 4, 5 and 6 show the movements in the total coefficients over the extended period. Interestingly, with the possible exception of Canada, here the coefficients do not seem to exhibit the same symmetric movements that prevailed in the previous experiment, and appear to be somewhat more stable over the longer period. There are some indications of a small change in some euro-zone countries around the time of the formation of the euro -- e.g. Austria, Belgium, France, Greece, Italy, Portugal and Spain. However, the coefficients remain far from their expected values. In the case of Mexico, the coefficients seem to exhibit changes in (i) the early 1980s, corresponding to the break-out of the Latin American debt crisis, (ii) 1987, the year in which Mexico adopted a crawling-peg exchange-rate regime, with the exchange rate used as a nominal anchor to bring down inflation, (iii) 1994, the year of the attack on the Mexican peso and the collapse of the crawling peg, and (iv) 2007, the year of the eruption of the global financial crisis. The coefficients for Canada also display changes during the time of the eruption of the 2007 global financial crisis.

4.2 Bias-free effects

We now extract the biases from the total coefficients. As described above, this is done by dividing the set of coefficient drivers into two subsets and removing the

[8] However, we also cannot rule out the possibility that, in some instances, the observed symmetry of coefficients could also brake down, despite the actual symmetry of domestic and foreign prices.

effect of the group associated with the bias. The coefficient drivers used in each regression are three lags of both the domestic and foreign inflation rates.[9]

Table 4 shows the *averages* of the bias-free coefficients for our first experiment over the euro-zone period. (The t-ratios in Table 4 pertain to the null hypothesis that the betas equal unity.) In all ten cases, the averages of the true coefficients satisfy homogeneity almost exactly; in each case the home-price coefficients are close to unity, while the foreign-price coefficients are close to minus unity. Most of the t-ratios for the coefficients of the domestic prices are close to zero, implying that the coefficient estimates are not significantly different from unity. The standard errors are, however, quite large; this is partly due to the large change in the coefficients in moving from the total effect to the bias free ones.

However, an inspection of monthly bias-free coefficients shown in Figures 7, 8 and 9, shows that the monthly variation in the deviations of coefficients from unity is very high, leading to the rejection of homogeneity in the short run, even though the averages of the bias-free estimates are close to unity in absolute values. Moreover, the odd kind of symmetry that was observed in the total effect coefficients no longer holds for the bias-free coefficients. As noted above, the empirical literature typically rejects PPP over the short run, but sometimes supports PPP in the long-run. Our results can be interpreted as strongly supporting long-run homogeneity, while in the short run that condition is clearly violated.

There are some interesting patterns in the behavior of the monthly coefficients of the bias-free effect. Table 5 shows the correlation between the two price coefficients in each equation. Most countries have fairly low correlations between the two coefficients. So, the symmetry effect, which was evident in the total coefficients, is much less pronounced in the bias-free coefficients; the main exceptions are Greece, where the correlation is almost exactly -1, and Germany and the euro zone, for which in both cases it is around -0.8. However, as reported in Table 4, averaging these estimates provides coefficients that are almost exactly proportional.

Table 6 presents the average estimates of the bias-free coefficients for our second experiment over the longer period. The results of this experiment confirm the results of first experiment; thus, they present a picture that strongly supports PPP.

[9] In general, lags are preferable to differences of variables since the lags capture any non-stationarity in the variables, whereas differences are always restricted versions of the lags; such restrictions may be inappropriate.

(Again, the t-ratios repeated in Table 6 pertain to the null hypothesis that the betas equal unity.) All the coefficients are of the correct sign and very close to unity in absolute value.

Consider, for example, the coefficients of the three countries -- Canada, Mexico and Japan -- that have been added in the second experiment. These three countries have followed very different exchange-rate regimes during the period from the late-1950s until early-2011 and have undergone a variety of external and internal shocks. In terms of exchange-rate regimes, the Canadian dollar, in contrast to most other currencies during the Bretton-Woods period, followed a floating exchange-rate regime until 1962 when it moved to an adjustable-peg regime. With the collapse of the Bretton-Woods system of fixed exchange rates in the early 1970s, both the Canadian dollar and the Japanese yen moved to flexible exchange-rate regimes. Mexico followed several types of pegged exchange-rate regimes until the mid-1990s, at which time the peso was allowed to float against the U.S. dollar. In terms of asymmetric shocks among these three countries, such shocks including the bursting of the asset price bubble in Japan in the late-1980s and early-1980s, followed a prolonged period of stagnation and deflation. As noted above, Mexico was hit by the Latin American debt crisis in the early 1980s and the attack on the Mexican peso -- beginning in December 1994 -- which led to a 50 per cent depreciation of the peso against the U.S. dollar (the reference currency under the crawling-peg exchange-rate regime) by March 1995. Yet, despite these (and other shocks), the bias-free coefficients for Canada, Mexico and Japan are all near unity in absolute value, significant, and correctly signed.

These findings clearly mean that a standard analysis based on OLS would yield highly biased results because of the omission of these developments. Indeed many of the events would be very difficult, if not impossible, to capture in a formal model in a fully satisfactory way. TVC estimation is designed specifically to deal with the situation where such biases arise in OLS estimation because TVC estimation is capable of yielding unbiased estimates of the coefficients of interest even in the presence of these confounding effects. It is, therefore, not surprising that we do indeed find that the average coefficients display homogeneity even when such profound structural changes have taken place.

Figures 10, 11 and 12 trace the bias-free, time-varying coefficients over the estimation period 1957:M4 to 2011:M3; again, over the longer period the evidence of symmetric movements seems to have disappeared, and the coefficients are much more stable around their expected values.

We argue that these results of average bias-free coefficients support long-run homogeneity. It is important to recall that the technique that we have employed gives consistent estimates of the bias-free coefficients, even if some important variables are omitted or the variables are measured with errors. Thus, while there may be underlying missing trends or problems with the measurement of the price indices, the estimation technique we used accounts for such misspecifications.

5. Conclusions

Previous empirical studies have typically rejected the proportionality -- or, homogeneity -- condition underlying PPP, while providing some support for the symmetry condition. However, the symmetry condition implies that the basic homogeneity of prices and exchange rates inherent in the PPP relationship does not hold. Effectively, symmetry implies that, over time, a change in the nominal exchange rate will not be fully reflected in domestic prices -- for example, a 50 per cent depreciation of the nominal exchange rate will not produce a proportional rise in the domestic price level. To be clear, our argument is that symmetry does not imply proportionality.

In this paper, we test for the existence of homogeneity, which we defined as proportionality in the presence of omitted variables, using two sets of experiments, one using data from the euro-zone period and one using data over a much longer period. Using a time-varying-coefficient technique, we found that the (total effect) coefficients, which are not corrected for specification biases, exhibit symmetry, but do not exhibit proportionality. Over the shorter euro-zone period we observe an odd proportional movement in the coefficients -- when one coefficient raises the other tends to fall, offsetting the rise in the former coefficient; this effect largely disappears over the longer period. Our interpretation of this finding is that the coefficients are not properly identified. Correcting for specification biases and allowing for missing variables, we find strong support for homogeneity -- that is, proportionality in a PPP specification that accounts for the effects of other variables -- thus, also providing strong support for PPP.

Table 1 Averages of total effect coefficients (1999:M1 to 2011:M3)

Country	$\bar{b}_0$		$\bar{b}_1$	$\bar{b}_2$
	(1)		**(2)**	**(3)**
Austria	*-2.251* *[-2.471]*		*-0.989* *[-0.006]*	*2.151* *[0.007]*
Belgium	*-1.998* *[-3.721]*		*-3.903* *[-0.005]*	*4.941* *[0.007]*
France	*-8.189* *[-6.493]*		*7.843* *[0.005]*	*-3.704* *[-0.007]*
Germany	*-1.625* *[-1.116]*		*-2.149* *[-0.002]*	*3.002* *[0.003]*
Greece	*-3.635* *[-7.304]*		*-0.267* *[-0.009]*	*2.123* *[0.002]*
Italy	*-3.819* *[-6.985]*		*2.668* *[0.001]*	*-0.718* *[-0.003]*
Netherlands	*-3.400* *[-5.446]*		*0.362* *[0.002]*	*1.377* *[0.001]*
Portugal	*-3.282* *[-11.811]*		*1.949* *[0.002]*	*-0.266* *[-0.003]*
Spain	*-2.034* *[-4.976]*		*3.867* *[0.005]*	*-2.810* *[-0.006]*
Euro zone	*-4.520* *[-6.289]*		*3.524* *[0.002]*	*-1.225* *[-0.002]*

Notes. Monthly data starting from 1999:M1 ending in 2011:M3. For each country the number of observations is 144, for Greece the number of observations is 120. Time-varying coefficient equation is given as $s_t = b_{0t} + b_{1t} p_t + b_{2t} fp_t$. All coefficients are averages of total effect coefficients; modified t-ratios are in brackets. The null hypotheses are $H_0 : \bar{b}_0 = 0$, $H_0 : \bar{b}_1 = 1$ and $H_0 : \bar{b}_2 = -1$. The standard errors are computed using the full set of coefficient drivers.

Table 2 Correlation between total effect coefficients of home and foreign prices (1999:M1 to 2011:M3)

Foreign prices	Domestic prices			
	(1)	**(2)**	**(3)**	**(4)**
	Austria	**Belgium**	**Germany**	**Greece**
US	*-0.998*	*-0.999*	*-0.998*	*-0.999*
	France	**Italy**	**Portugal**	**Spain**
US	*-0.996*	*-0.993*	*-0.991*	*-0.998*
	Netherlands	**Euro zone**		
US	*-0.948*	*-0.992*		

Table 3 Averages of total effect coefficients (1957:M4 to 2011:M3)

Country	$\overline{b}_0$	$\overline{b}_1$	$\overline{b}_2$
	(1)	**(2)**	**(3)**
Austria	*2.608* *[3.865]*	*0.572* *[0.029]*	*-1.606* *[-0.052]*
Belgium	*3.355* *[3.753]*	*-0.818* *[-0.027]*	*-0.443* *[-0.014]*
Canada	*-0.077* *[-0.795]*	*0.538* *[0.040]*	*-0.451* *[-0.100]*
France	*1.722* *[3.577]*	*1.605* *[0.034]*	*-2.308* *[-0.105]*
Germany	*1.188* *[4.389]*	*0.239* *[0.114]*	*-0.812* *[-0.293]*
Greece	*1.642* *[3.208]*	*0.098* *[0.245]*	*-0.139* *[-0.024]*
Italy	*5.991* *[2.887]*	*1.378* *[0.006]*	*-3.430* *[-0.026]*
Japan	*3.193* *[22.728]*	*0.145* *[0.508]*	*-0.710* *[-0.458]*
Mexico	*2.153* *[4.563]*	*1.127* *[0.332]*	*-1.661* *[-0.315]*
Netherlands	*1.195* *[4.374]*	*0.056* *[0.203]*	*-0.594* *[-0.138]*
Portugal	*2.174* *[2.793]*	*0.228* *[0.092]*	*-0.627* *[-0.020]*
Spain	*3.669* *[2.676]*	*0.883* *[0.006]*	*-2.040* *[-0.029]*

Notes. Monthly data starting from 1957:M04 ending in 2011:M03. For each country the number of observations is 648. Time-varying coefficient equation is given as $s_t = b_{0t} + b_{1t} p_t + b_{2t} fp_t$. All coefficients are averages of total effect coefficients; modified t-ratios are given in brackets. The null hypotheses are $H_0 : \overline{b}_0 = 0, \quad H_0 : \overline{b}_1 = 1, \quad H_0 : \overline{b}_2 = -1$. The standard errors are computed using the full set of coefficient drivers.

Table 4 Averages of bias-free coefficients (1999:M1 to 2011:M3)

Country	$\bar{b}_1$	$\bar{b}_2$
	(1)	**(2)**
Austria	*1.040* *[0.000]*	*-0.997* *[-0.012]*
Belgium	*1.002* *[0.000]*	*-1.034* *[-2.748]*
France	*1.014* *[0.012]*	*-1.044* *[-2.034]*
Germany	*1.007* *[0.008]*	*-1.051* *[-1.954]*
Greece	*0.998* *[0.000]*	*-1.012* *[-1.633]*
Italy	*1.095* *[0.068]*	*-0.987* *[-3.084]*
Netherlands	*1.031* *[0.041]*	*-0.978* *[-0.006]*
Portugal	*1.066* *[0.097]*	*-1.009* *[-0.005]*
Spain	*0.990* *[0.015]*	*-0.984* *[-1.982]*
Euro zone	*1.110* *[0.091]*	*-1.013* *[-1.996]*

Notes. Monthly data starting from 1999:M1 and ending in 2011:M3. For each country the number of observations is 144, for Greece the number of observations is 120. The coefficients are bias-free coefficients of the equation $s_t = b_{0t} + b_{1t} p_t + b_{2t} fp_t$. Averages of constant terms are not presented in the table; modified t-ratios are in brackets. The null hypotheses are $H_0 : \bar{b}_0 = 0$, $H_0 : \bar{b}_1 = 1$ and $H_0 : \bar{b}_2 = -1$. The standard errors are computed using the square root of the variance formula of $\sum_{d \in A_{1jt}} \hat{\pi}_{jd} z_{dt}$ where A_{1jt} is an appropriate subset of coefficient drivers (Appendix).

Table 5 Correlation between bias-free effect coefficients of home and foreign prices (1999:M1 to 2011:M3)

Foreign prices	Domestic prices			
	(1)	(2)	(3)	(4)
	Austria	**Belgium**	**Germany**	**Greece**
US	*0.583*	*0.604*	*-0.824*	*-0.975*
	France	**Italy**	**Portugal**	**Spain**
US	*-0.234*	*-0.051*	*-0.558*	*-0.602*
	Netherlands	**Euro zone**		
US	*-0.382*	*-0.870*		

Table 6 Averages of bias-free coefficients (1957:M4 to 2011:M3)

Country	$\bar{b}_1$	$\bar{b}_2$
	(1)	**(2)**
Austria	*1.011* [0.001]	*-1.067* [-0.066]
Belgium	*1.080* [0.001]	*-1.056* [-0.019]
Canada	*1.058* [0.002]	*-1.070* [-0.079]
France	*1.014* [0.001]	*-1.056* [-0.137]
Germany	*1.020* [0.001]	*-1.037* [-0.254]
Greece	*1.042* [0.001]	*-1.014* [-0.010]
Italy	*1.052* [0.001]	*-1.076* [-0.063]
Japan	*0.948* [0.015]	*-1.038* [-0.764]
Mexico	*1.029* [0.081]	*-1.044* [-0.387]
Netherlands	*1.056* [0.001]	*-1.010* [-0.103]
Portugal	*0.987* [0.001]	*-1.066* [-0.023]
Spain	*1.056* [0.002]	*-1.010* [-0.048]

Notes Monthly data starting from 1957:M4 and ending in 2011:M3. For each country the number of observations is 648. The coefficients are the bias-free components of the coefficients of equation $s_t = b_{0t} + b_{1t} p_t + b_{2t} fp_t$. Averages of the constant terms of the time-varying coefficients model are not presented in the table. Modified t-ratios are in brackets. The null hypotheses are $H_0 : b_1 = 1,\ \ H_0 : b_2 = -1$. The standard errors are computed using the square root of the variance formula of $\sum_{d \in A_{1jt}} \hat{\pi}_{jd} z_{dt}$ where A_{1jt} is an appropriate subset of coefficient drivers (Appendix).

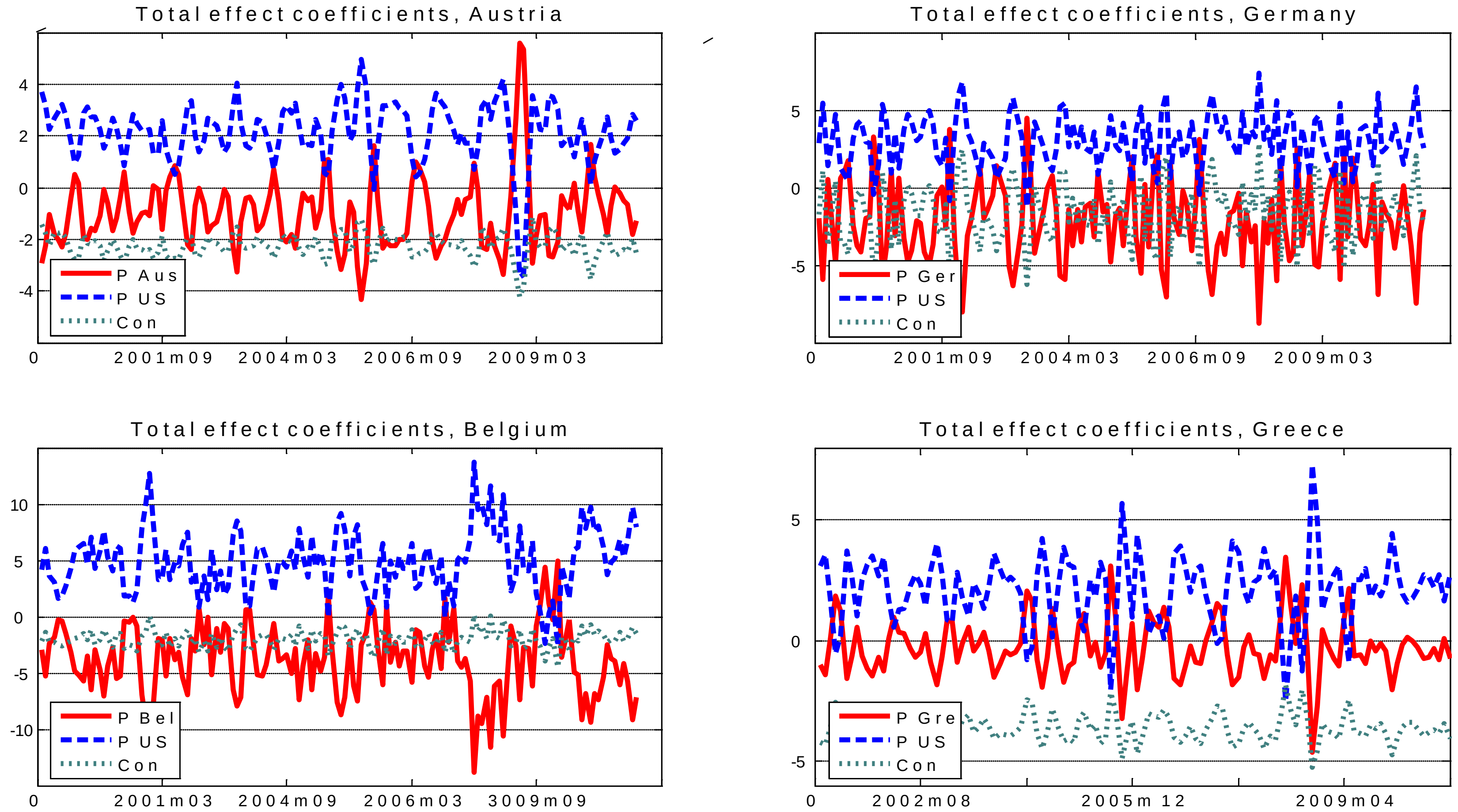


Figure 1 Total effect coefficients for Austria, Belgium, Germany and Greece (1999:M1 to 2011:M3). Time-varying coefficients of domestic prices: P Aus (Austria), P Bel (Belgium), P Ger (Germany), P Gre (Greece). Time-varying coefficients of foreign prices (USA): P US. Time-varying "constant" term: Con

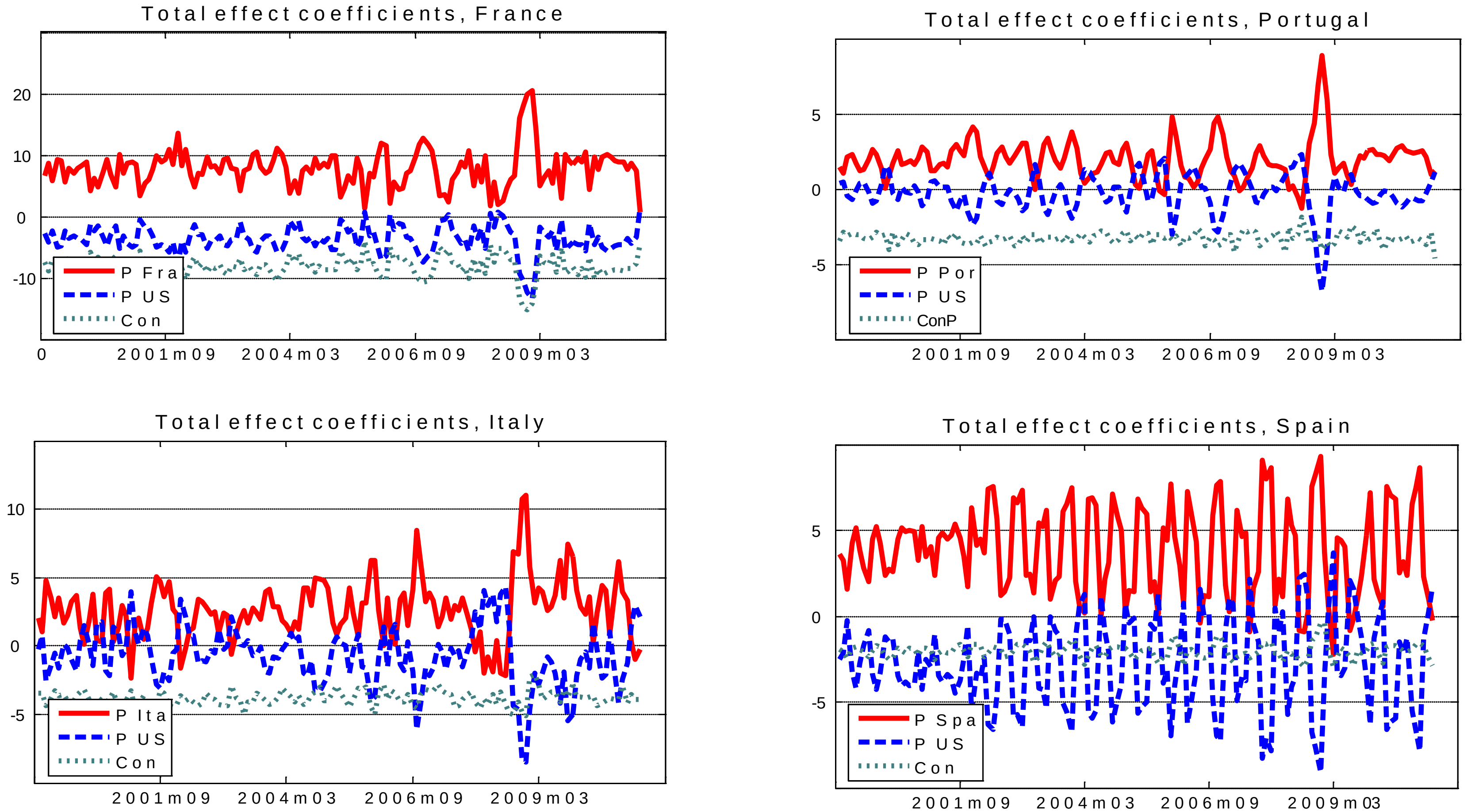
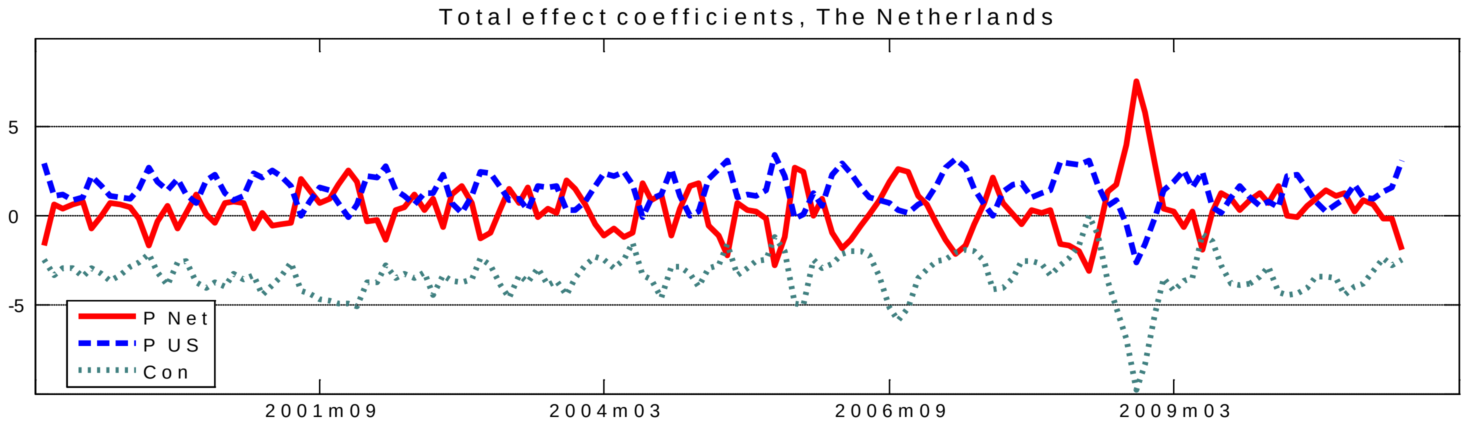


Figure 2 Total effect coefficients for France, Italy, Portugal and Spain (1999:M1 to 2011:M3). Time-varying coefficients of domestic prices: P Fra (France), P Ita (Italy), P Por (Portugal), P Spa (Spain). Time-varying coefficients of foreign prices (USA): P US. Time-varying "constant" term: Con.



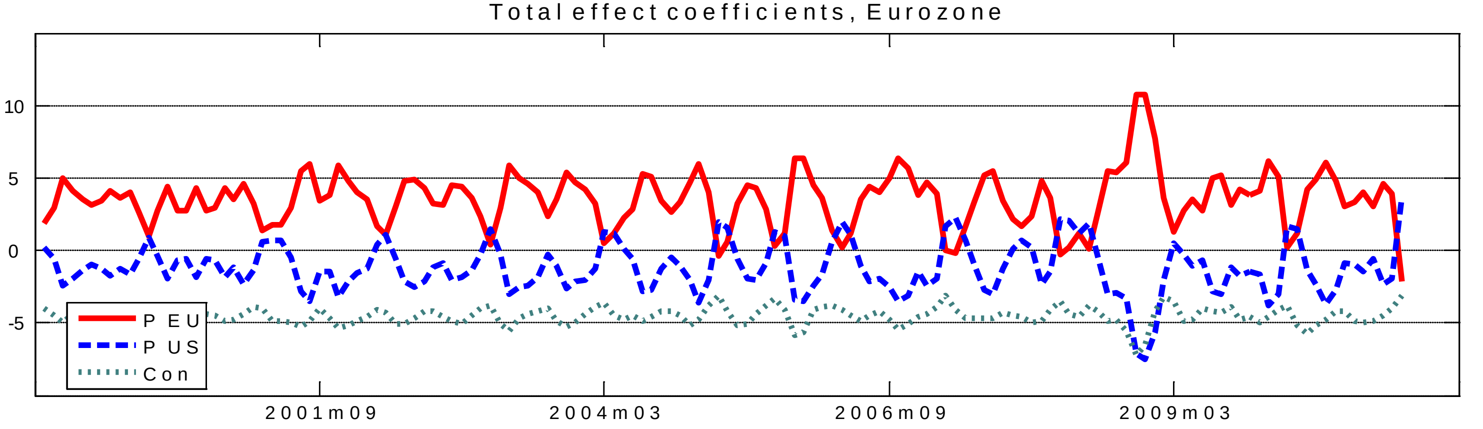


Figure 3 Total effect coefficients for the Netherlands and EU (1999:M1 to 2011:M3). Time-varying coefficients of domestic prices: P Net (The Netherlands), P EU (Euro zone). Time-varying coefficients of foreign prices (USA): P US. Time-varying “constant” term: Con.

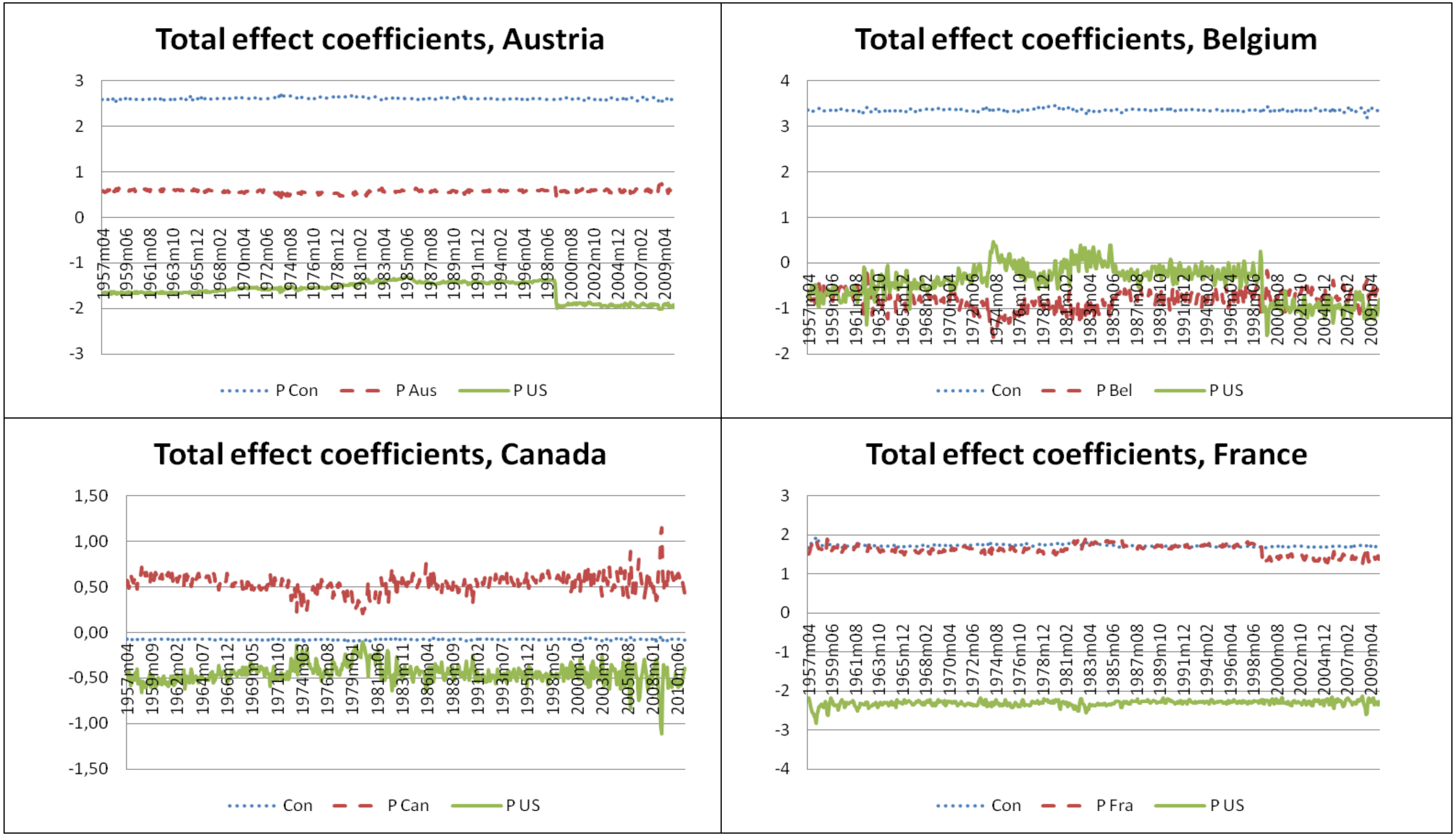


Figure 4 Total effect coefficients for Austria, Belgium, Canada and France (1957:M4 to 2011:M3). Time-varying coefficients of domestic prices: P Aus (Austria), P Bel (Belgium), P Can (Canada), P Fra (France). Time-varying coefficients of foreign prices (USA): P US. Time-varying "constant" term: Con.

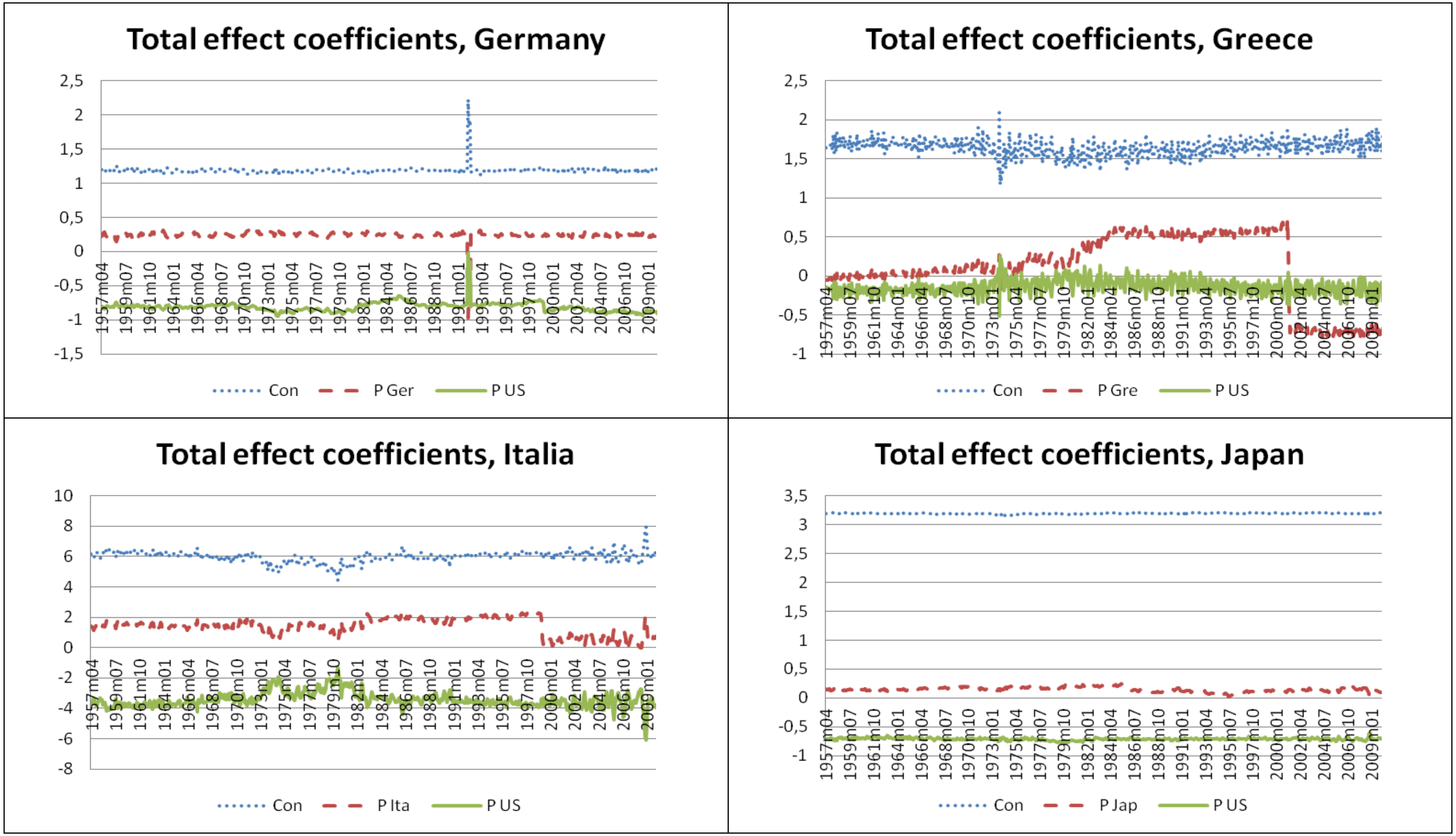


Figure 5 Total effect coefficients for Germany, Greece, Italia and Japan (1957:M4 to 2011:M3). Time-varying coefficients of domestic prices: P Ger (Germany), P Gre (Greece), P Ita (Italia), P Jap (Japan). Time-varying coefficients of foreign prices (USA): P US. Time-varying "constant" term: Con.

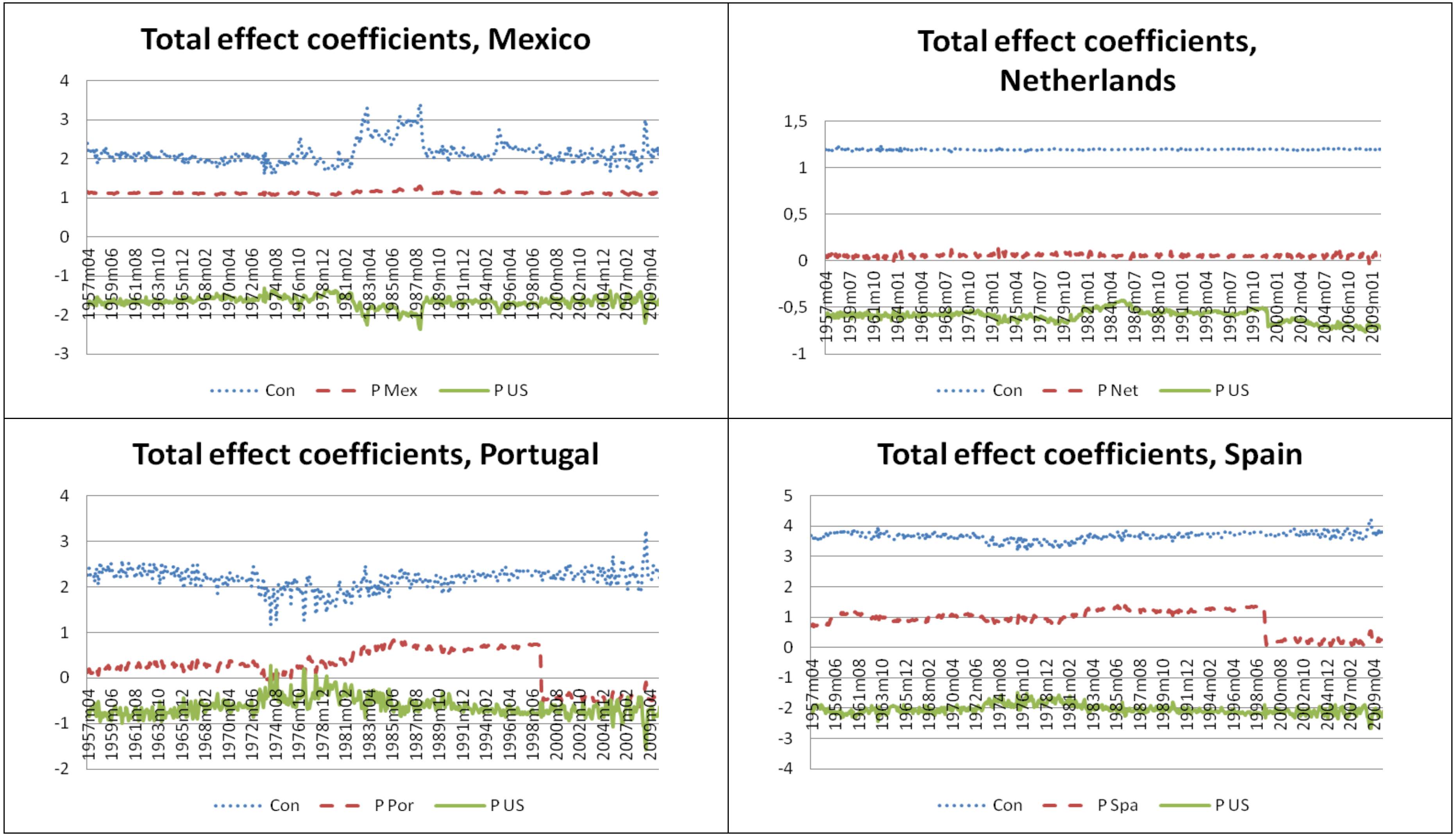


Figure 6 Total effect coefficients for Mexico, Netherlands, Portugal and Spain (1957:M4 to 2011:M3). Time-varying coefficients of domestic prices: P Mex (Mexico), P Net (Netherlands), P Por (Portugal), P Spa (Spain). Time-varying coefficients of foreign prices (USA): P US. Time-varying "constant" term: Con.

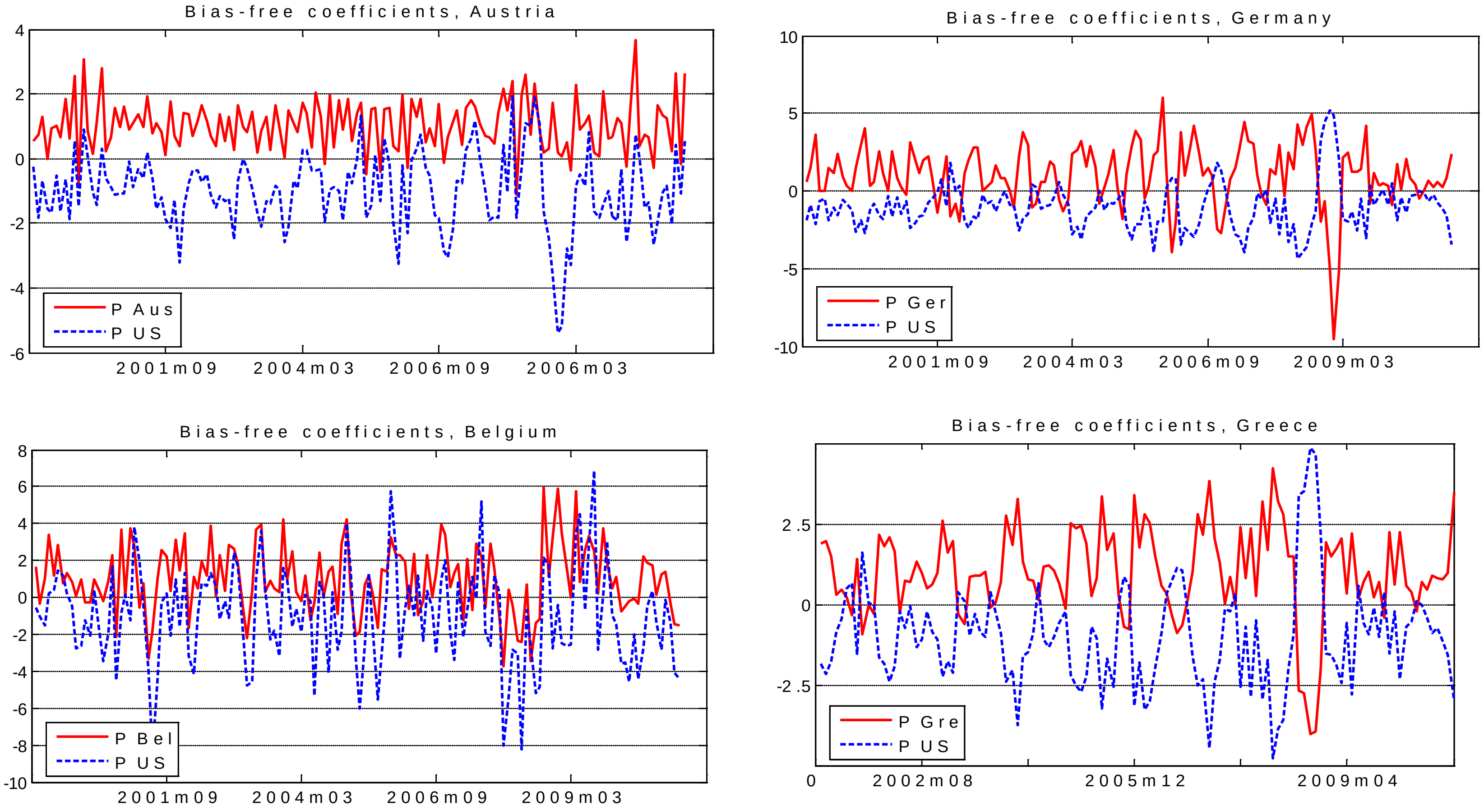


Figure 7 Bias-free coefficients for Austria, Belgium, Germany and Greece (1999:M1 to 2011:M3). Time-varying coefficients of domestic prices: P Aus (Austria), P Bel (Belgium), P Ger (Germany), P Gre (Greece). Time-varying coefficients of foreign prices (USA): P US

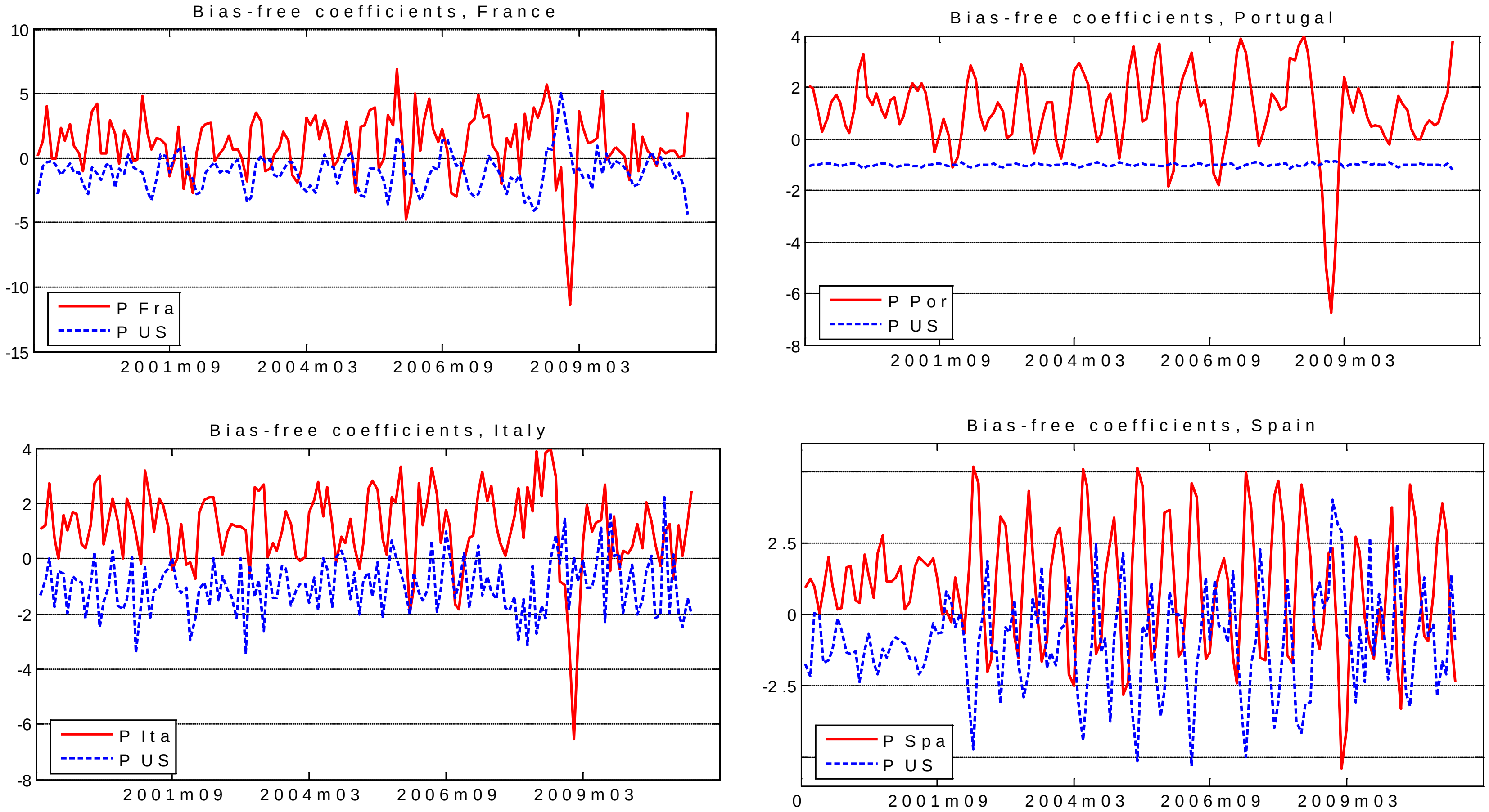


Figure 8 Total effect coefficients for France, Italy, Portugal and Spain (1999:M1 to 2011:M3). Time-varying coefficients of domestic prices: P Fra (France), P Ita (Italy), P Por (Portugal), P Spa (Spain). Time-varying coefficients of foreign prices (USA): P US

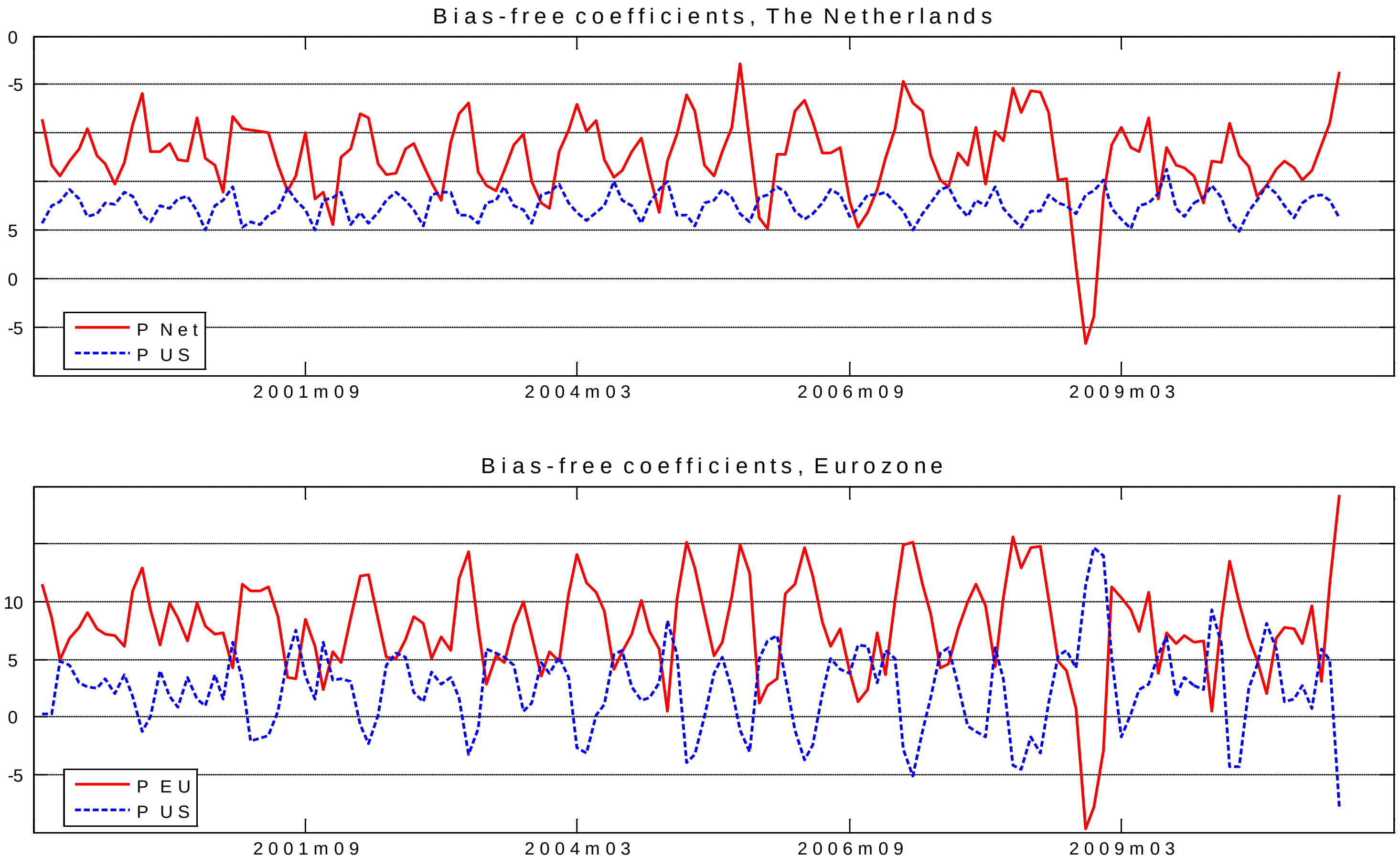


Figure 9 Total effect coefficients for the Netherlands and EU (1999:M1 to 2011:M3). Time-varying coefficients of domestic prices: P Net (The Netherlands), P EU (Euro zone). Time-varying coefficients of foreign prices (USA): P US

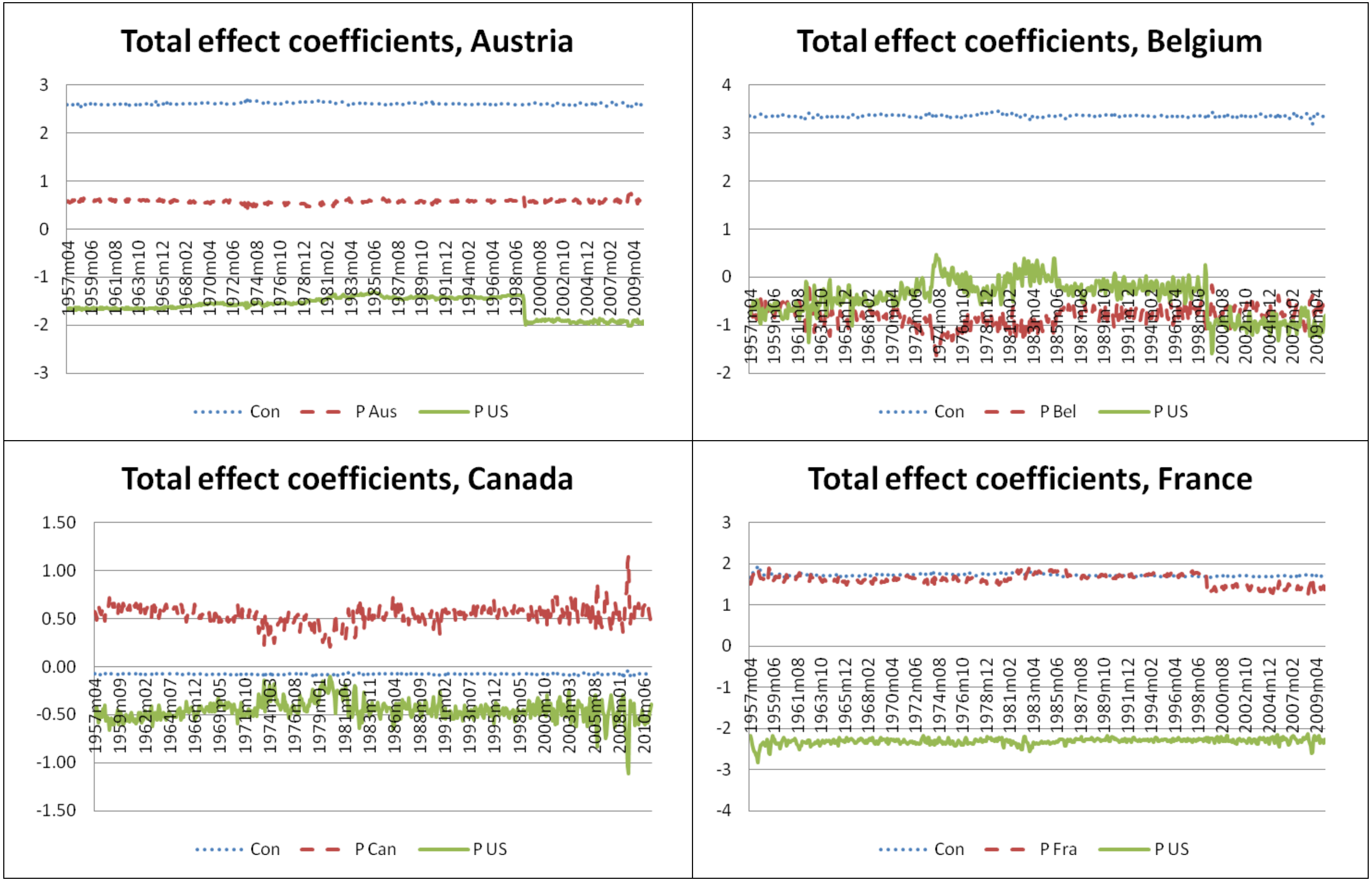


Figure 1 Total effect coefficients for Austria, Belgium, Canada and France. Time-varying coefficients of domestic prices: P Aus (Austria), P Bel (Belgium), P Can (Canada), P Fra (France). Time-varying coefficients of foreign prices (USA): P US. Time-varying "constant" term: Con.

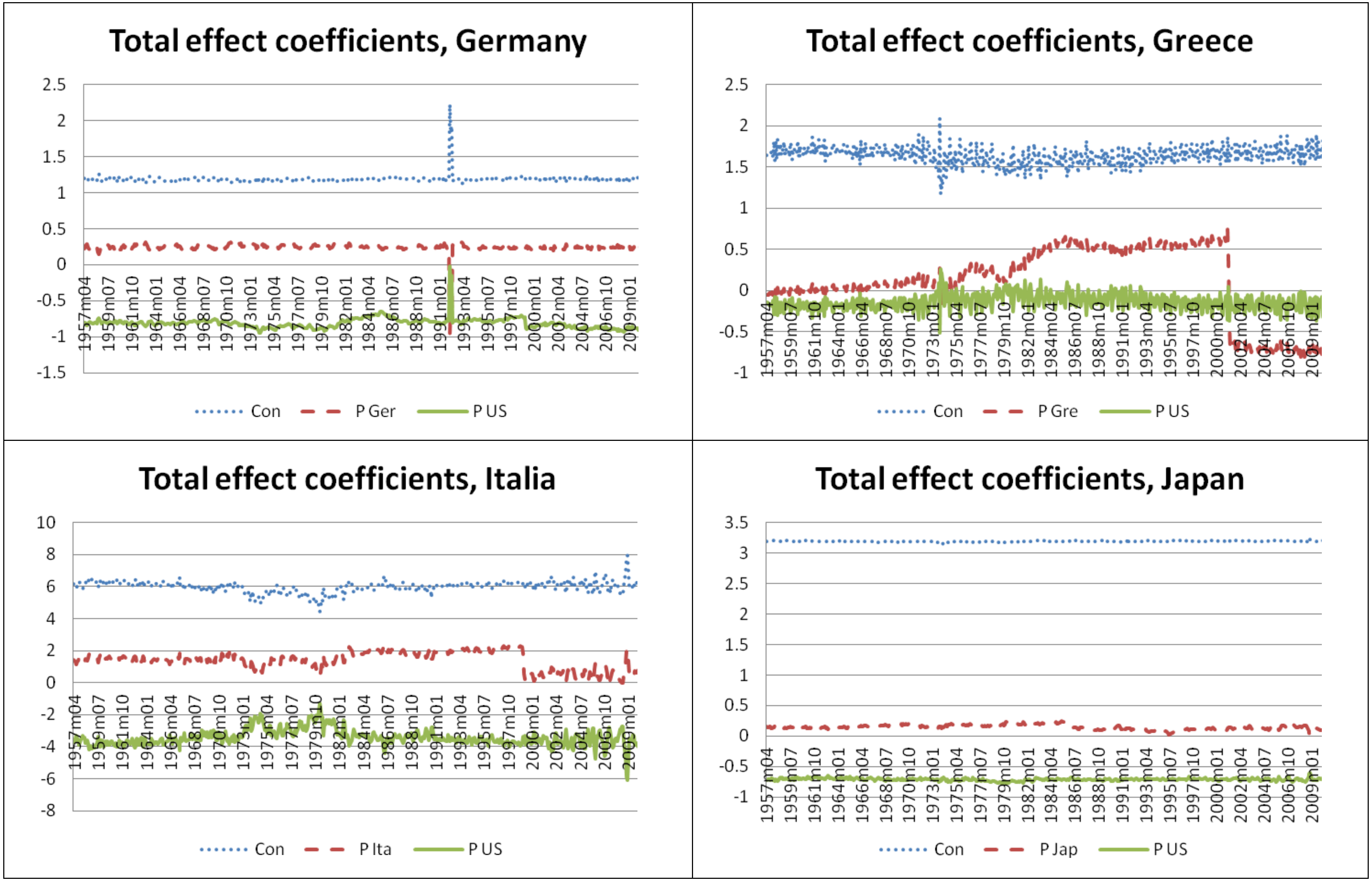


Figure 2 Total effect coefficients for Germany, Greece, Italia and Japan. Time-varying coefficients of domestic prices: P Ger (Germany), P Gre (Greece), P Ita (Italia), P Jap (Japan). Time-varying coefficients of foreign prices (USA): P US. Time-varying “constant” term: Con.

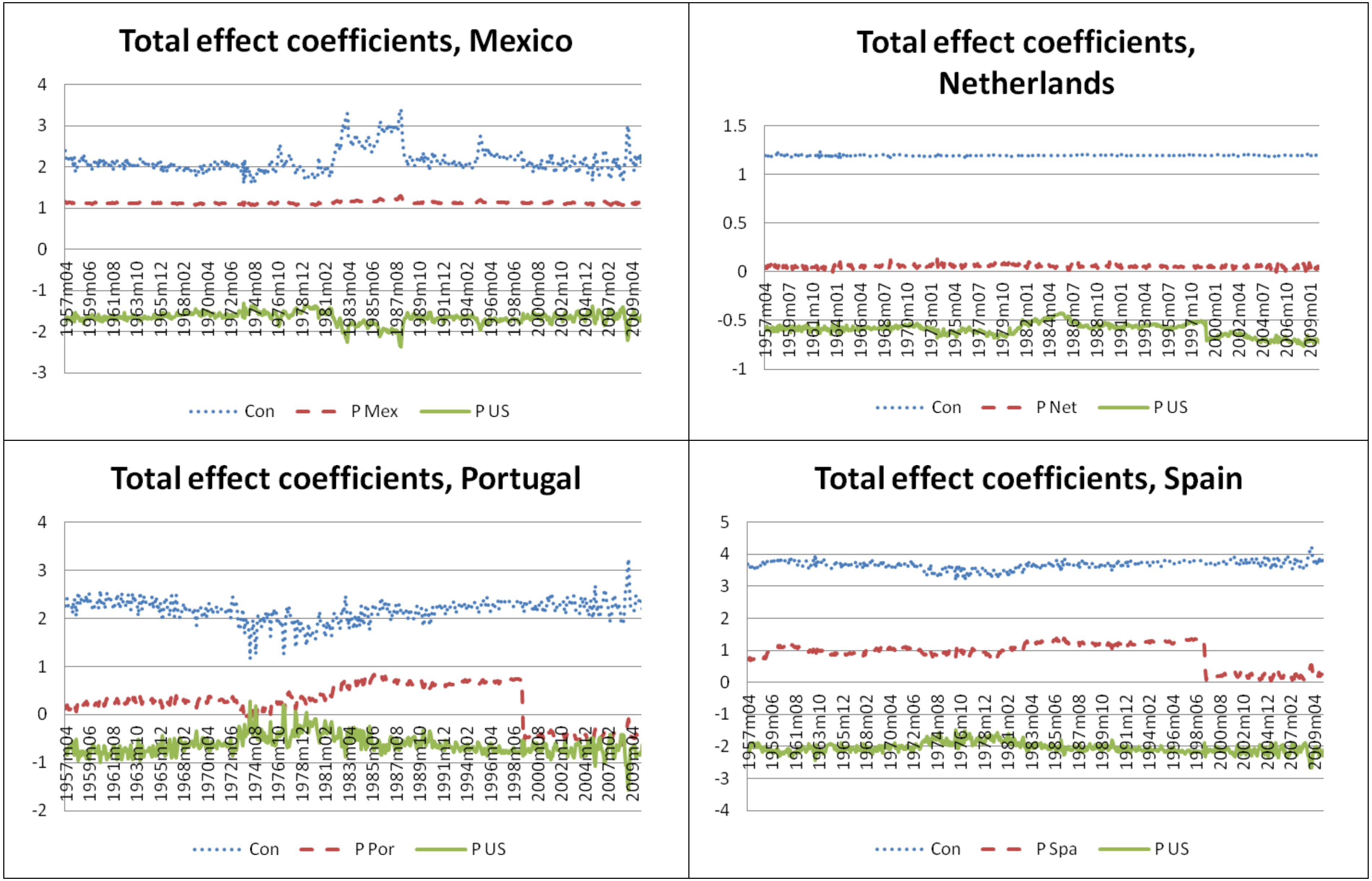


Figure 3 Total effect coefficients for Mexico, Netherlands, Portugal and Spain. Time-varying coefficients of domestic prices: P Mex (Mexico), P Net (Netherlands), P Por (Portugal), P Spa (Spain). Time-varying coefficients of foreign prices (USA): P US. Time-varying "constant" term: Con.

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Appendix: A Technical Exposition of TVC Estimation

When studying the relation of a dependent variable, denoted by y_t^*, to a hypothesized set of $K-1$ determinants, denoted by (x_{1t}^*, ..., $x_{K-1,t}^*$), where these $K-1$ determinants may be only a subset of the complete set of the determinants of y_t^*, a number of problems can arise. Any specific functional form of the relation may be incorrect and may therefore lead to specification errors resulting from the erroneous functional form. Another problem that can arise is that x_{1t}^*, ..., $x_{K-1,t}^*$ do not exhaust the complete list of the determinants of y_t^*, in which case the relation of y_t^* to x_{1t}^*, ..., $x_{K-1,t}^*$ may be subject to omitted-variable biases. In addition to these problems, the available data on y_t^*, x_{1t}^*, ..., $x_{K-1,t}^*$ is be perfect measures of the underlying true variables, causing the errors-in-variables problem. In what follows, we propose the correct interpretations and an appropriate method of estimation of the coefficients of the relationship between y_t^* and x_{1t}^*, ..., $x_{K-1,t}^*$ in the presence of the foregoing problems. We also propose solutions to all these problems.

Suppose that T measurements on y_t^*, x_{1t}^*, ..., $x_{K-1,t}^*$ are made and these measurements are in fact, the sums of "true" values and measurement errors: $y_t = y_t^* + v_{0t}$, $x_{jt} = x_{jt}^* + v_{jt}$, $j = 1, ..., K-1$, $t = 1, ..., T$, where the variables y_t, $x_{1t}, ..., x_{K-1,t}$ without an asterisk are the observable counterparts of the variables with an asterisk which are the unobservable "true" values, and the v's are measurement errors.

It is useful at this point to clarify what we believe is the main objective of econometric estimation. In our view, the objective is to consistently estimate the effect on a dependent variable of changing one of its complete set of determinants holding all of the remaining determinants of this set constant. That is, we aim to find consistent estimators of some of these effects. This interpretation is, of course, standard one usually placed on the coefficients of a typical econometric model, but the validity of this interpretation depends crucially on that of the assumption that the conventional model gives estimates of bias-free coefficients, which is not the case in the presence of model misspecification.

To deal with this issue, consider a set of time-varying coefficients that provide a complete explanation of the dependent variable y.

$$y_t = \gamma_{0t} + \gamma_{1t} x_{1t} + ... + \gamma_{K-1,t} x_{K-1,t} \quad \text{(A1)}$$

where $y_t = s_t$, $x_{1t} = p_t$, and $x_{2t} = fp_t$, and $K - 1 = 2$. Let a set, denoted by S_1, contain those regressors of (A1) that take the value zero with probability zero and let another set, denoted by S_2, contain the remaining regressors of (A1) that take the value zero with positive probability. We call (A1) "the time-varying coefficient (TVC) model". (Note that this model is formulated in terms of the observed variables). The derivation of model (A1) given, for example, in Swamy and Tavlas (2007) shows that γ_{0t} is the sum of (i) the relevant intercept, (ii) v_{0t}, and (iii) the error term -- this term being the correct function of certain 'sufficient sets' of omitted regressors is unique -- and for $j > 0$, γ_{jt} is the sum of (i) the partial derivative of y_t^* with respect to one of its complete set of determinants holding all of the remaining determinants of this set constant and (ii) omitted-variable and (iii) measurement-error biases. This derivation does not assume that the true functional-forms of the relationships underlying (A1) are known and shows that the formulas for omitted-variables and measurement-error biases can be derived without knowing what variables are omitted from (A1). This derivation also shows that (A1) is obtained by inserting appropriate measurement errors at the right places in a model of y_t^* with unique coefficients and error term. Here the uniqueness means that the coefficients and the error term are invariant to changes in the relationship between the included regressors, x_{1t}^*, ..., $x_{K-1,t}^*$, and omitted regressors. We call the partial derivative component of γ_{jt} its "bias-free component". Econometric models with nonunique coefficients and error terms are usually employed in the econometrics literature. Any distributional assumption we might make about a nonunique error term is arbitrary. Estimates of nonunique coefficients and predictions of nonunique error terms given by arbitrary error distributions prove nothing. Note, also, that, if the true functional forms are non-linear, the corresponding time-varying partial derivatives may be thought of as the correct representations of the respective data-generating non-linear structures and so they are able to capture any possible function.

It is important to stress that, while we start from a TVC model, its estimation technique is typically referred to in the literature as time-varying-coefficient estimation; the objective here is not to simply estimate a model with changing coefficients. We start from (A1) because this is the correct representation of the underlying data generation process. In the case of the TVC procedure followed in this paper, we extend the standard TVC model typically considered in the literature; specifically, we decompose each of these varying coefficients into two parts, a consistent estimate of the bias-free part and the remaining part, which is due to biases from the various misspecifications in the model. If the data-generating model is linear, we would get back to a constant coefficient model. If the data-generating model is non-linear, the coefficients of the linear-in-variables form in (A1) will vary over time to reflect this circumstance. The key point is that the TVC technique used here produces consistent estimates of the bias-free components of the coefficients of structural relationships in the presence of model misspecification.

For empirical implementation, model (A1) has to be embedded in a stochastic framework. To do so, we need to answer the question: What are the correct stochastic assumptions about the TVC's of (A1)? We believe that the correct answer is: the correct interpretation of the TVC's and the assumptions about them must be based on an understanding of the model misspecification which comes from any (i) omitted variables, (ii) measurement errors, and (iii) misspecification of the functional form. It turns out that misspecification (iii) is easy to avoid. We expand on this argument in what follows.

Notation and Assumptions Let m_t denote the total number of the determinants of y_t^*. The exact value of m_t cannot be known at any time. We assume that m_t is larger than K-1 (that is, the total number of determinants is greater than the determinants for which we have observations) and possibly varies over time.[11] This assumption means that there are determinants of y_t^* that are excluded from (A1) since (A1) includes only K-1 determinants. Let x_{gt}^*, $g = K, \ ,..., \ m_t$, denote these excluded determinants. Some of these determinants represent all relevant pre-existing conditions so that these conditions are controlled. This is a standard way to reduce spurious correlations to zero. Thus, to make these controls knowledge of relevant pre-existing conditions is

[11] That is, the total number of determinants is itself time variant.

not needed. The equations $y_t^* = f_t(x_{1t}^*, ..., x_{m_t t}^*) = \alpha_{0t}^* + \sum_{\ell=1}^{m_t} \alpha_{\ell t}^* x_{\ell t}^*$ are two different but equivalent forms of the relationship between y_t^* and all of its determinants with $\alpha_{0t}^* = y_t^* - \sum_{j=1}^{K-1} \alpha_{jt}^* x_{jt}^* - \sum_{g=K}^{m_t} \alpha_{gt}^* x_{gt}^*$, $\alpha_{jt}^* = \partial y_t^* / \partial x_{jt}^*$ if $x_{jt}^* \in S_1$ and $= \Delta y_t^* / \Delta x_{jt}^*$ with the right sign if $x_{jt}^* \in S_2$ $j = 1, ..., K-1$, and $\alpha_{gt}^* = \partial y_t^* / \partial x_{gt}^*$, $g = K, ,..., m_t$. These partial derivatives are unique as long as x_{jt}^* and x_{gt}^* are continuous. The true functional form of the relationship between y_t^* and x_{1t}^*, …, $x_{m_t t}^*$ determines the time profiles of α^*s. These time profiles are unknown, since the true functional form is unknown. Note that an equation that is linear in variables accurately represents a non-linear equation, provided the coefficients of the former equation are time-varying with time profiles determined by the true functional form of the latter equation. This type of representation of a non-linear equation is convenient, particularly when the true functional form of the non-linear equation is unknown. Such a representation is not subject to the criticism of misspecified functional form. For $g = K, ,..., m_t$, let $x_{gt}^* = \lambda_{0gt}^* + \sum_{j=1}^{K-1} \lambda_{jgt}^* x_{jt}^*$ where $\lambda_{0gt}^* = x_{gt}^* - \sum_{j=1}^{K-1} \lambda_{jgt}^* x_{jt}^*$ denotes the relevant intercept and $\lambda_{jgt}^* = \partial x_{gt}^* / \partial x_{jt}^*$ if $x_{jt}^* \in S_1$ and $= \Delta x_{gt}^* / \Delta x_{jt}^*$ if $x_{jt}^* \in S_2$, $j = 1, ,..., K-1$, denote the other coefficients of the regression of x_{gt}^* on $x_{1t}^*, ..., x_{K-1,t}^*$. The true functional forms of the relationships between x_{gt}^* and x_{1t}^*, …, $x_{K-1,t}^*$ determine the time profiles of λ^*s. The partial derivative λ_{jgt}^* is unique if x_{jt}^* is continuous.

The following theorem gives the correct interpretations of the coefficients of (A1):

Theorem 1 *The intercept of (A1) satisfies the equation,*

$$\gamma_{0t} = \alpha_{0t}^* + \sum_{g=K}^{m_t} \alpha_{gt}^* \lambda_{0gt}^* + v_{0t} - \sum_{j \in S_2} (\alpha_{jt}^* + \sum_{g=K}^{m_t} \alpha_{gt}^* \lambda_{jgt}^*) v_{jt} \tag{A2}$$

and the coefficients of (A1) other than the intercept satisfy the equations,

$$\gamma_{jt} = \alpha_{jt}^* + \sum_{g=K}^{m_t} \alpha_{gt}^* \lambda_{jgt}^* - \left(\alpha_{jt}^* + \sum_{g=K}^{m_t} \alpha_{gt}^* \lambda_{jgt}^* \right) \left(\frac{v_{jt}}{x_{jt}} \right) \text{ if } j \in S_1$$

$$= \alpha_{jt}^* + \sum_{g=K}^{m_t} \alpha_{jt}^* \lambda_{jgt}^* \text{ if } j \in S_2 \tag{A3}$$

Proof: See Swamy and Tavlas (2007).[12] □

Note that the distinction between the dicrete and continuous regressors of (A1) is lost if the term involving v_{jt} times x_{jt} in (A3) is removed from (A3) and added on the right-hand side of (A2). This distinction gives additional information that can be used to estimate omitted-variables bias in γ_{jt} with j > 0. Eq. (7) with K = 3 is a special case of model (A1). Thus, we may interpret the TVC's in terms of the underlying coefficients with the correct functional forms, the observed explanatory variables and measurement errors in the dependent variable and included regressors. The quantity $\left(\sum_{g=K}^{m_t} \alpha_{gt}^* \lambda_{jgt}^*\right)$ measures omitted-variables bias and $\left(-\left(\alpha_{jt}^* + \sum_{g=K}^{m_t} \alpha_{gt}^* \lambda_{jgt}^*\right) \times \left(v_{jt} / x_{jt}\right)\right)$ measures measurement-error bias. The coefficient α_{jt}^* is unique if x_{jt}^* is continuous. Given the set of omitted regressors, the omitted-variable biases are unique.

By assuming that the α^*s and λ^*s are possibly time-varying, we do not *a priori* rule out the possibility that the relationship of y_t^* with all of its determinants and the regressions of the determinants of y_t^* excluded from (A1) on the determinants of y_t^* included in (A1) are non-linear. It should be noted that for ℓ = 0, 1, …, m_t, each $\alpha_{\ell t}^*$ is functionally dependent on the $x_{\ell t}^*$. Also, we can assume that the regressors of (A1) are correlated with their own coefficients.[13]

Theorem 2 *For* $j = 1, \ \ldots, \ K-1$, *the terms without involving the* v_{jt} *in (A2) and (A3) are unique.*

Proof. See Swamy and Tavlas (2001, 2007).

It can be seen from (A3) that the component α_{jt}^* of γ_{jt} is free of omitted-variables bias $\left(= \sum_{g=K}^{m_t} \alpha_{gt}^* \lambda_{jgt}^*\right)$, measurement-error bias $\left(= -\left(\alpha_{jt}^* + \sum_{g=K}^{m_t} \alpha_{gt}^* \lambda_{jgt}^*\right) \times \left(v_{jt} / x_{jt}\right)\right)$, and of functional-form bias, since we allow the

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[12] The differences between (A2) and (A3) and those in Swamy and Tavlas (2007) arise as a direct consequence of our division of the set of the regressors of (A1) into S_1 and S_2 in this paper.

[13] These correlations are typically ignored in the analyses of state-space models. Thus, inexpressive conditions and restrictive functional forms are avoided in arriving at (A2) and (A3) so that Theorem 1 can easily hold; for further discussion and interpretation of the terms in (A2) and (A3), see Swamy and Tavlas (2001).

α^*s and λ^*s to have the correct time profiles. Note that α_{jt}^* is the coefficient of x_{jt}^* in the correctly specified relation of y_t^* to all of its determinants. Hence, α_{jt}^* represents the bias-free component of the coefficient of x_{jt}^*.

The bias-free component α_{jt}^* is constant if the relationship between y_t^* and all of its determinants is linear; alternatively, it is variable if the relationship is non-linear. We often have information from theory as to the right sign of α_{jt}^*. Any observed correlation between y_t and x_{jt} is spurious if $\alpha_{jt}^* = 0$.[14] The term $\left(\sum_{g=K}^{m_t} \alpha_{gt}^* \lambda_{0gt}^*\right)$ is the correct function of the sufficient sets (the λ_{0gt}^*'s) of omitted regressors (the x_{gt}^*'s) and hence is the unique error term of (A1) which is not without an error term.

A key implication of (A2) and (A3) is that, in the presence of a misspecified functional form, omitted variables, and measurement error, the errors in a standard regression will contain the difference between the right-hand side of (A1) and the right-hand side of the standard regression with the errors suppressed. So the errors will contain the included *x* variables.

The time-varying coefficients are decomposed to give consistent estimators of the bias-free components of coefficients in a model which accounts for unknown functional forms, its excluded variables and measurement error. The key to this decomposition is to use a set of observable variables (z), called coefficient drivers, which explain the time variation in the coefficients.

$$\gamma_{jt} = \pi_{j0} z_{ot} + \sum_{d=1}^{p-1} \pi_{jd} z_{dt} + \varepsilon_{jt} \qquad \text{(A4)}$$

It is assumed that the regressors of (A1) are conditionally independent of their coefficients given the coefficient drivers. This set of coefficient drivers can be split into three subsets so that one subset, say the first subset, should be correlated with any true variation in the bias-free component while the other two subsets, say the second and third subsets, should be correlated with the biases that are present. Effectively, the coefficient drivers absorb omitted-variable and measurement-error biases; for further discussion, see Swamy, Tavlas, Hall and Hondroyiannis (2010). Once this is achieved

[14] We use the term spurious in a more general sense than Granger and Newbold's (1974), where it strictly applies to linear models expressed in terms of integrated variables. Here we mean any correlation which is observed between two variables when the true bias-free component of the coefficient in the regression of one on the other is actually zero.

we can estimate the biases, which come from the second and third subsets of coefficient drivers. We remove the estimates of biases from the estimates of total coefficients (simply by subtracting these terms from A4) to obtain a consistent estimator of the underlying bias-free components. These second and third subsets of coefficient drivers act rather like the dual of conventional instruments. The key difference, however, is that some of these drivers should be correlated with the misspecifications rather than uncorrelated with an error term, as in the case of instruments, and this should be much easier to achieve in a real world situation.

Swamy, Tavlas, Hall and Hondroyiannis (2010) then give a formal derivation of the inference procedures and confidence intervals for both the coefficients of (A4) and the corresponding TVC's of (A1).