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Explorations in Digital Cartography

DISCUSSION PAPER 5

Bloch's function D for Weighted Effective Areas : Impact of tuning parameters

by

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ABSTRACT

When Visvalingam's algorithm was first presented, it was noted that it could be driven with any metric to suit different purposes, types of features and degrees of generalisation. It was illustrated with the concept of the effective area (EA). The intelligent application of metrics with and without weighting requires the prior segmentation of polylines with heterogeneous shapes into geometrically similar sections. Automatic line segmentation remains a research challenge. Meanwhile, different weighting functions have been used with different types of lines as noted in this paper. EA has a tendency to output spiky lines. Attempts have been made to give less weight to acute angles and favour obtuse angles. This yields visually more pleasing lines but has a tendency to chop elongated features, such as streams. Bloch's function D (which implements the *Visvalingam/ weighted area* option in recent versions of Mapshaper) was chosen to investigate whether the parameter values could be tuned to improve the output. The tweaking of parameter values can give unexpected and unpredictable results. The results from a systematic approach to tweaking the parameters are presented here. These suggest that the chopping effect may be exacerbated by the use of inappropriate parameter values. By fine tuning Bloch's function D, it was possible to derive pleasing smaller scale representations of convoluted coastlines with a complex network of creeks using under 5 percent of the original points. Although the output is not identical to manual generalisations of the same coastline, they may be acceptable (at this stage of research) for this type of coastline. Function D is not useful for generalising unsegmented coastlines, such as the 1: 50000 coastline of Humberside.

CONTENTS

1. Introduction

As pointed out by Visvalingam and Williamson (1995), Visvalingam's algorithm for line generalisation has an inherent tendency to cut curves when the effective area (EA) of a point is used as the measure of its importance (see Visvalingam, 2016). Visvalingam and Whyatt (1993) suggested that EA could be weighted to change the rank order of points, for example to take account of shape but noted that the intelligent application of weights and rules awaits algorithms for prior segmentation of a line into its constituent line types. Some of the weights used with EA are briefly described below. There is a need to identify the types of lines which most benefit from particular weighting functions and parameter values. This project investigated the utility of Bloch's function D (Bloch, 2014b), described below, with a manually segmented stretch of coastline, referred to here as The Scalp data, which depicts wetlands drained by a network of creeks. The choice of parameter values affects not only how the creeks are depicted but also the order in which different creeks are eliminated/selected and extended. Suitable parameters have to be found by interactive exploration and the systematic approach adopted after going around in circles is presented here, to demonstrate how parameter values affect line generalisation.

With appropriate parameters and filter values, Bloch's function D gives very pleasing results forThe Scalp. As expected, it is not as effective as the unweighted EA for generalising the coastline of Humberside which incorporates different types of geomorphic features. The paper notes some unintended consquences of function D and suggests some areas for furher research.

2. Background

Visvalingam's algorithm was first reported in Whyatt (1991) and Visvalingam and Whyatt (1993). With the growing use of the algorithm, Visvalingam (2015a) provided the necessary background to the algorithm with examples of its application to explain why her specification was not overly prescriptive and why variations in implementation are inevitable and sometimes necessary. The algorithm can be used with suitable metrics as illustrated by Visvalingam and Brown (1999). As stated in Visvalingam and Whyatt, Visvalingam's effective area (EA) could be weighted and its sign could be used since it is indicative of convexity and concavity. EA has a unit weight of 1. For ease of reference, Visvalingam and Whelan (2014) referred to it as v1_EA. Despite successful use in varied projects undertaken within the now disbanded Cartographic Information Systems Research Group of the University of Hull (See Visvalingam, 2015), v1_EA is not always the best heuristic for all purposes and types of data.

Visvalingam and Whyatt (p 46) had stressed that there was a need for a segmentation of lines into a hierarchy of features so that rules and weights could be applied intelligently. Buttenfield (1986, 1987) suggested a set of statistical measures as a first step towards selecting scale-dependent tolerance values and for splitting a line into elements which could be generalised intelligently. However as Zhou (2014) noted the automatic segmentation of lines into meaningful parts remains a research challenge. In the meantime, others have explored some weighted effective areas; abbreviated to WEA by Zhou and Jones (2004). These are briefly reviewed below.

2.1 Zhou and Jones (2004), Zhou (2014)

Zhou and Jones (2004) included terms for skewed, tall/flat and convex/concave shape filters within their complex weighting scheme. Each of these filters was controlled by multiple parameters. This complex formula was designed to enable users to adopt a suck-and see approach to the selection of parameter values, since their effects are difficult to predict. It must be stressed that their weighting scheme can be configured to revert to no more than the use of the original unweighted EA (v1_EA). Also, the tall/flat weight can be parameterised separately. So, a weighting for only the tall triangles need not change the original v1_EA of flat ones. Please see Zhou (2014) in the list of references below for the link to Zhou's source code for his original demonstrator program.

Ordnance Survey (OS) adopted the Zhou and Jones weighting scheme for the generalisation of coastlines in their research prototype (see Revell et al, 2011). The OS VectorMap® District Data (VMD data) production system uses this scheme for generalising coastlines. The filter used for VMD was called MHF02-Gen. According to Sheng Zhou (July 2014, personal communication) 'this filter used different pairings of parameter values to weight the acute and obtuse angles respectively'; the parameter values were not disclosed for commercial reasons.

Zhou's (2014) use of a simpler weighting scheme, WEA = v1_EA * width/height of triangle, without reference to parameters, was discussed by Visvalingam and Whelan (2014). Although this strand of work has inspired others, Zhou's approach was not pursued in this paper given the lack of transparency and Zhou's own dissatisfaction with the current state of development of his WEA (Zhou, 2014). He was particularly concerned with its tendency to chop convoluted, elongated features. Visvalingam and Whelan (2014).suggested that v1_EA's occasional truncation of streams and spits could be exacerbated by his weighting function.

2.2 The National Historical Geographic Information System (NHGIS)

Some sample implementations of Visvalingam's algorithm were listed in Visvalingam (2015b). The list included the use of Visvalingam's algorithm with extensions for automatically deriving a comprehensive multiscale database for the free to use National Historical Geographic Information System data (NHGIS, 2011) derived from the US Bureau of Census TIGER data (McMaster et al, 2005; Schroeder and McMaster, 2007).

Jonathan Schroeder (personal communication, July 2015) explained the specific reasons for the extensions mentioned in Schroeder (2010). These extensions included the following function to achieve a smoother appearance of boundaries and for eliminating long narrow protrusions.

NHGIS_EA = v1_EA * (1 + smoothness * -cos (angle)) * [1]
(length of short side / length of long side) $^{\text{balance}}$ [2]
Where: smoothness = 0.9; balance = 0.25 and the lengths only refer to the sides of the input line.

Schroeder found the weighting function and parameter values by trial-and-error and felt that the utility of the term in line 2 was questionable. At the point when the NHGIS project ended, he favoured the weighting of NHGIS_EA = v1_EA * (1 + 0.9 * -cos (angle)). Schroeder (2010) also included a number of extensions to Visvalingam's algorithm to deal with special cases.

2.3 Bloch (2014a -c)

Different approaches to shape weighting can give different results. But if they are specifically designed to underweight acute triangles and favour obtuse ones, they will have similar, even if not identical, effects depending on the nature of the line and the parameters used.

Bloch (2014a) provided the option ***Visvalingam : weighted area*** in Mapshaper v 0.2.0 to weight only the acute triangles with WEA = v1_EA * (1 – cosine (angle)). Unlike the Zhou and Jones' WEA, this option did not require user-specified parameters. The v1_EA for obtuse angles remained unchanged. Bloch (2014b) refers to this as function A.

Visvalingam and Whelan (2014) found that function A was not suitable for caricatural generalisation and was prone to chopping off linear features with angular cuts as found by Zhou

(2014). However, it produced very pleasing simplifications and typifications which were amenable to post processing. Bloch switched to using function D from Mapshaper version 0.2.3 onwards (Bloch, 2014c). Bloch (August 2014, personal communication) wrote that function D reflected his approach to generalisation since it favours moderately oblique angles, "*so islands maintain a rounder shape at high simplification. As simplification increases, instead of degenerating into small triangular and quadrilateral shapes, islands tend to collapse completely, causing them to disappear from the map*."

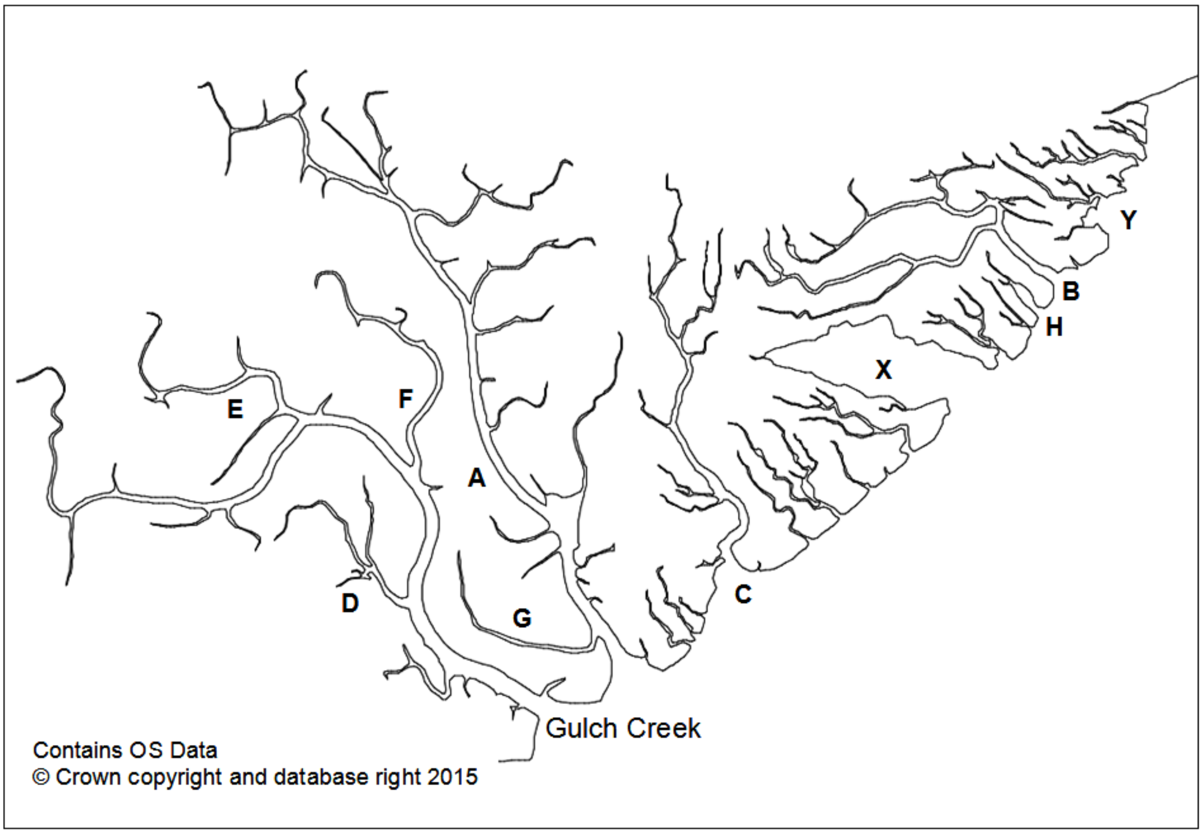

Figure 1: Extract from OS VectorMap® District data

Bloch's function D, v1_EA * (- cos (angle) * 0.7 + 1), which he independently found by trial-and-error, has a similar form to Schroeder's weighting. Schroeder preferred the value of 0.9 for his smoothing factor. This project explores the effect of varying the parameters to Bloch's function D.

From the foregoing, it can be seen that researchers have chosen different weights (and other functions) to solve specific issues. Bloch's functions were designed to extract smooth curves and rounded islands. Zhou (2014) also wanted smooth representations but found that his function had the unfortunate consequence of chopping features with angular cuts. In contrast, McMaster et al (2005) described how they had to post-process the output of Visvalingam's algorithm to retain the rectilinear nature of US census boundaries and the manmade coastline in parts of Florida. Elsewhere, their emphasis has been on deriving smooth boundaries; preserving curves, retaining the legibility of small adjoining polygons, amalgamating islands and preserving topology (Schroeder, 2010). Since weighting functions do not address all such requirements, there was a need for extensions to Visvalingam's algorithm and for post-processing of output.

3. Data

Visvalingam and Whelan (2014), used an extract of the OS VMD data (OS VectorMap® District data, shown in Figure 1) and the SWURCC data of Humberside (illustrated in Figure 10) to evaluate the effectiveness of Bloch's function A (Bloch, 2014b). This study uses the same data to assess the value of Bloch's function D. It assessed whether Bloch's function D, would produce good abstractions of The Scalp data, which describe a different type of fretted coastline to the fjord coastline tested by Bloch (2014d). Visvalingam and Whelan (2014) noted that Bloch's function A could not be used to caricature the Humberside data. So, this study also checks whether the improved function D can now be used with this data.

The page, https://hydra.hull.ac.uk/resources/hull:9040, provides access to both test data with information on the sources, maps and the co-ordinates of the coastlines. The VMD extract was manually segmented by John Whelan. It maps an area of wetlands drained by creeks referred to as The Scalp. The User Guide (OS, 2015) states "the nominal viewing scale is1:25 000, with a recommended viewing scale range of 1:15 000 to 1:30 000". This data is free on the terms of the OS OpenData License (2013).

Only the name of Gulch Creek could be found on free, readily available source maps. In the following text, tables and maps, the other creeks and tributaries are referred to by the letter labels on Figure 1. The manually derived 1:50000 Landranger® map of The Scalp can be viewed at http://www.streetmap.co.uk/map.srf?X=538125&Y=338425&A=Y&Z=120 It s not the aim of this project to replicate the cartographer's depiction of The Scalp at 1:50000. The latter exhibits the outcome of other generalisation processes, such as the widening of the upper reaches of the Gulch Creek and Creek A, and the amalgamation of some mudflats into the wetlands. The VMD coastline shown in Figure 1 does not include the islands at X and H. Similarly, the coast at Y looks different to that on the Landranger® map because it does not show the islands in the bay. The cartographer has also selected only Gulch Creek and creeks A and B for depiction as part of the coastline and omits creek C. This study did not attempt to replicate the manual typification since creeks C and B seem to have comparable visual weights. Tributaries E and F are slightly narrower, but the other creeks and tributaries appear either shorter or thinner in comparison.

4. Aims

As demonstrated by Visvalingam and Whelan, v1_EA can be used to derive simplifications of The Scalp but the output is not very pleasing, as illustrated in Figure 2. This project explored a) whether Bloch's function D could be tuned to filter out only the larger creeks; and if so b) whether the resulting shapes would be more pleasing than those output by v1_EA; and, c) if function D was suitable for caricaturing the Humberside data.

5. Mapshaper option : *Visvalingam / weighted area*

Visvalingam's algorithm may be filtered on rank or effective area (Visvalingam, 2015a). If we start with just the two end points, and add more points by lowering the filter value, the first feature to be added would be that marked X on Figure 1, which as we noted above contains a large island above the high water mark. This is such a large intrusion that all weights add this feature after the points depicting the rough outline of the coast. When filtering on v1_EA, the minor creeks and tributaries, including creek D, are only included after the desired ones, namely Gulch, A, B and possibly C. The numbered labels on Figure 2 (and subsequent figures) show the order in which features are added or extended. The only exception is a single point at the confluence of F and Gulch Creek (at label 4 on Figure 2c). So, the required features can be segmented for retention and post-processing with less than 1.2% of points. However, the output is not pleasing and the preferred extension of Gulch Creek (1.58% points) will also include creek D and spurs for creeks E and F.

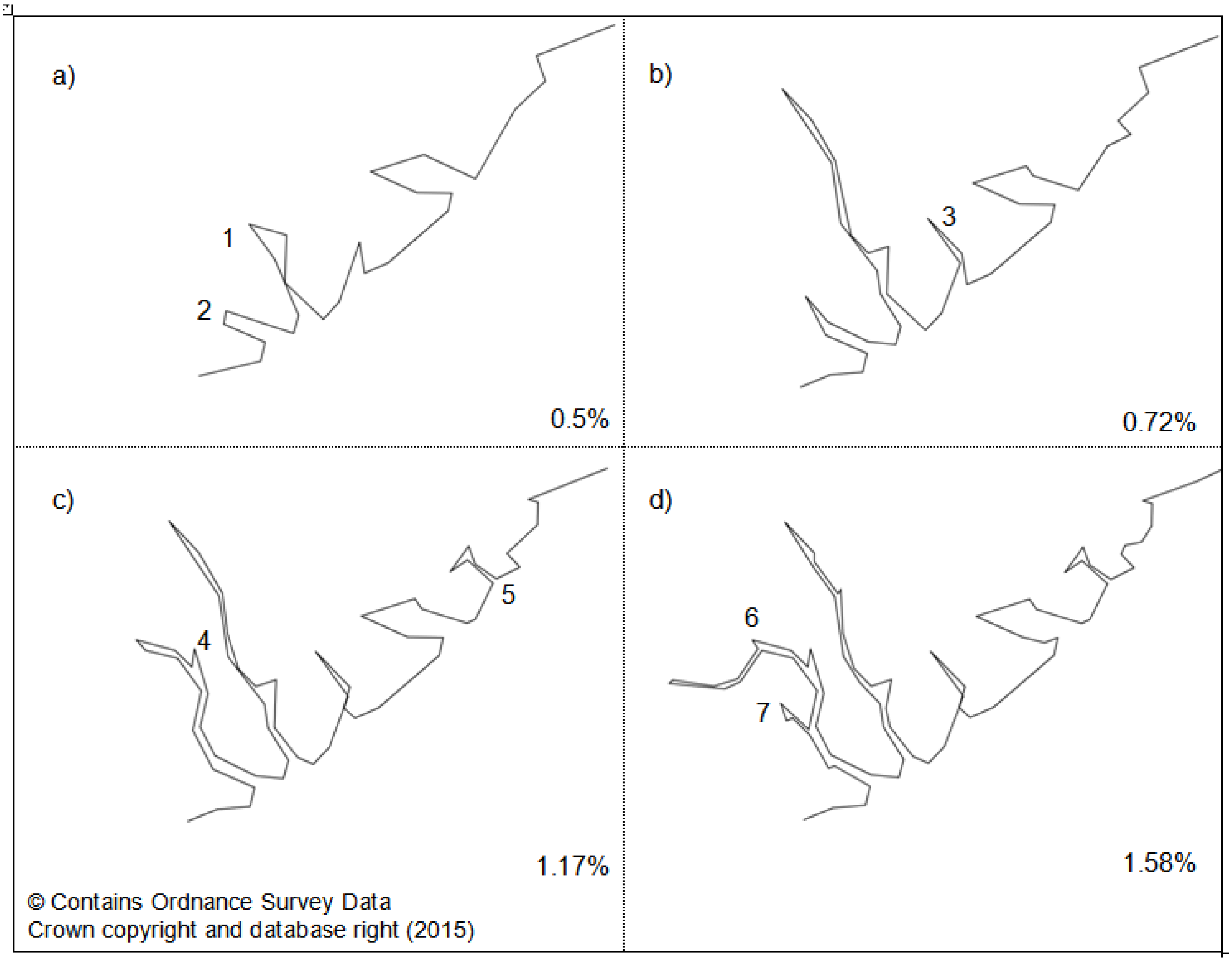

Figure 2: Order of feature inclusion by v1_EA

Bloch has used function D for the *Visvalingam / weighted area* option in Mapshaper 0.2.3 and subsequent versions. The author modified Bloch's function D (Bloch, 2014b) to parameterise the constants in his function.

```
function blochD (x1, y1, x2, y2, x3, y3, scale, translate) {
var cos = cosine (x1, y1, x2, y2, x3, y3);      // see Bloch (2014b)
    var weight = -cos * scale + translate;      // Bloch used scale = 0.7; translate = 1.0
    return weight
} // blochD
```

The EA weighted by the blochD function will be referred to in this paper as bD_EA. It consists of:
bD_EA = v1_EA * blochD (x1, y1, x2, y2, x3, y3, scale, translate)

As noted earlier, Bloch's function D is function blochD with default scale = 0.7 and translate = +1. It is convenient for the following reasons:

- It uses one continuous function for acute and obtuse angles instead of the step functions used in previous versions of Mapshaper.
- It is thus less complex for smoothing than the weighting scheme proposed by Zhou and Jones (2004), which has two parameters each for the differently treated acute and obtuse angles.
- It is fully transparent and thus open to investigation.

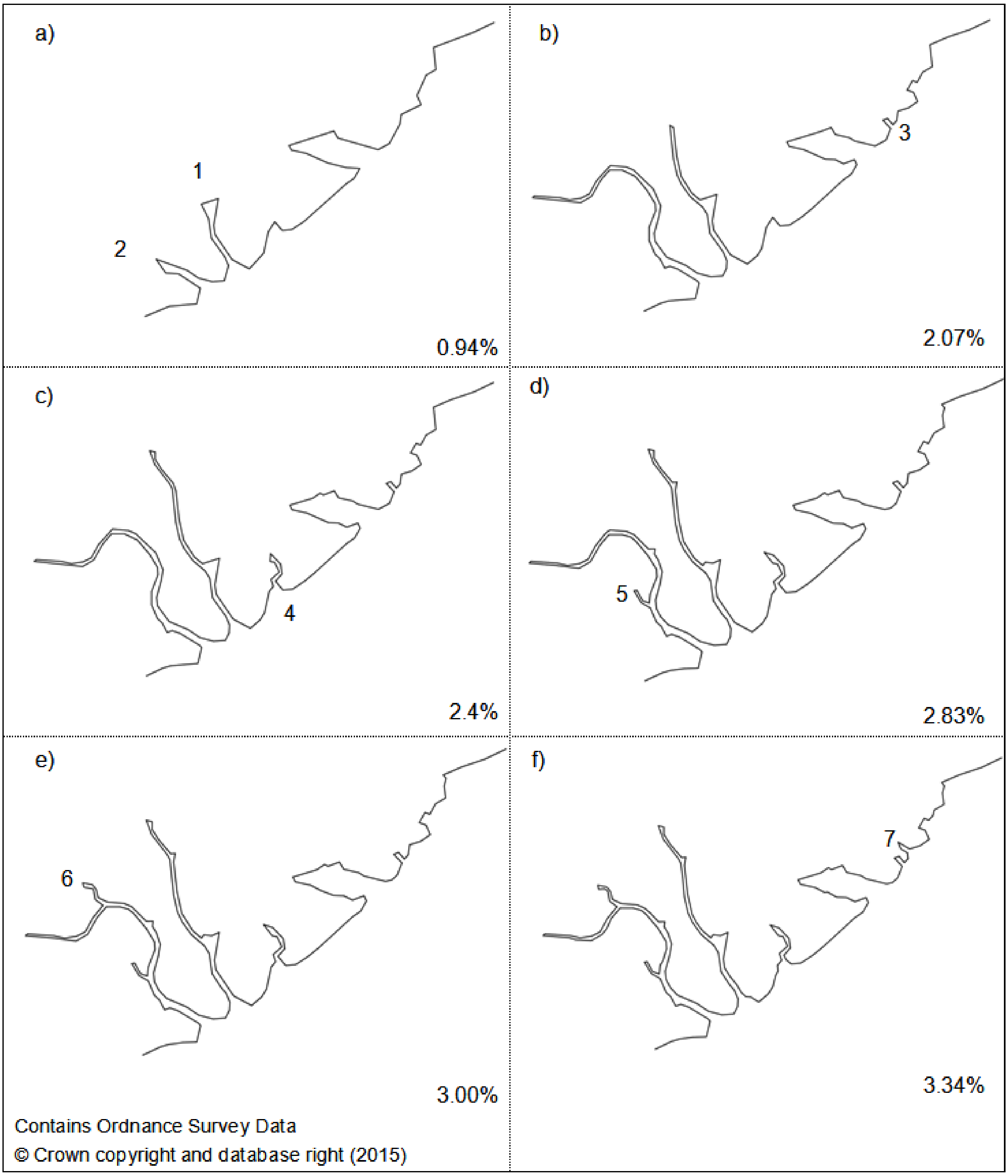

Figure 3: Order of feature inclusion with bD_EA = (v1_EA * - cos * 0.7 + 1)

Figure 3, based on Bloch's default parameter values, shows that creeks A and Gulch appear first, followed by H, C, D and E before B in Figure 3f. The order of feature inclusion is not optimal and some creeks look chopped. Like function A, function D produces pleasing output albeit using more points.

6. The Scalp data

Zhou (2014) stated that '*how filter parameter values affects the generalisation effects is still hard to quantify*'. Schroeder (personal communication, July 2015) and Bloch (personal communication, 2014) also found the best constants for their data by trial-and-error. It was difficult to discern any pattern for The Scalp at first since even a minor adjustment to parameter values seemed to have a dramatic impact at times on the order in which features were included and/or extended. So, a more systematic approach was adopted studying the effects of each parameter in turn. Bloch's favoured parameter values provide a good starting point.

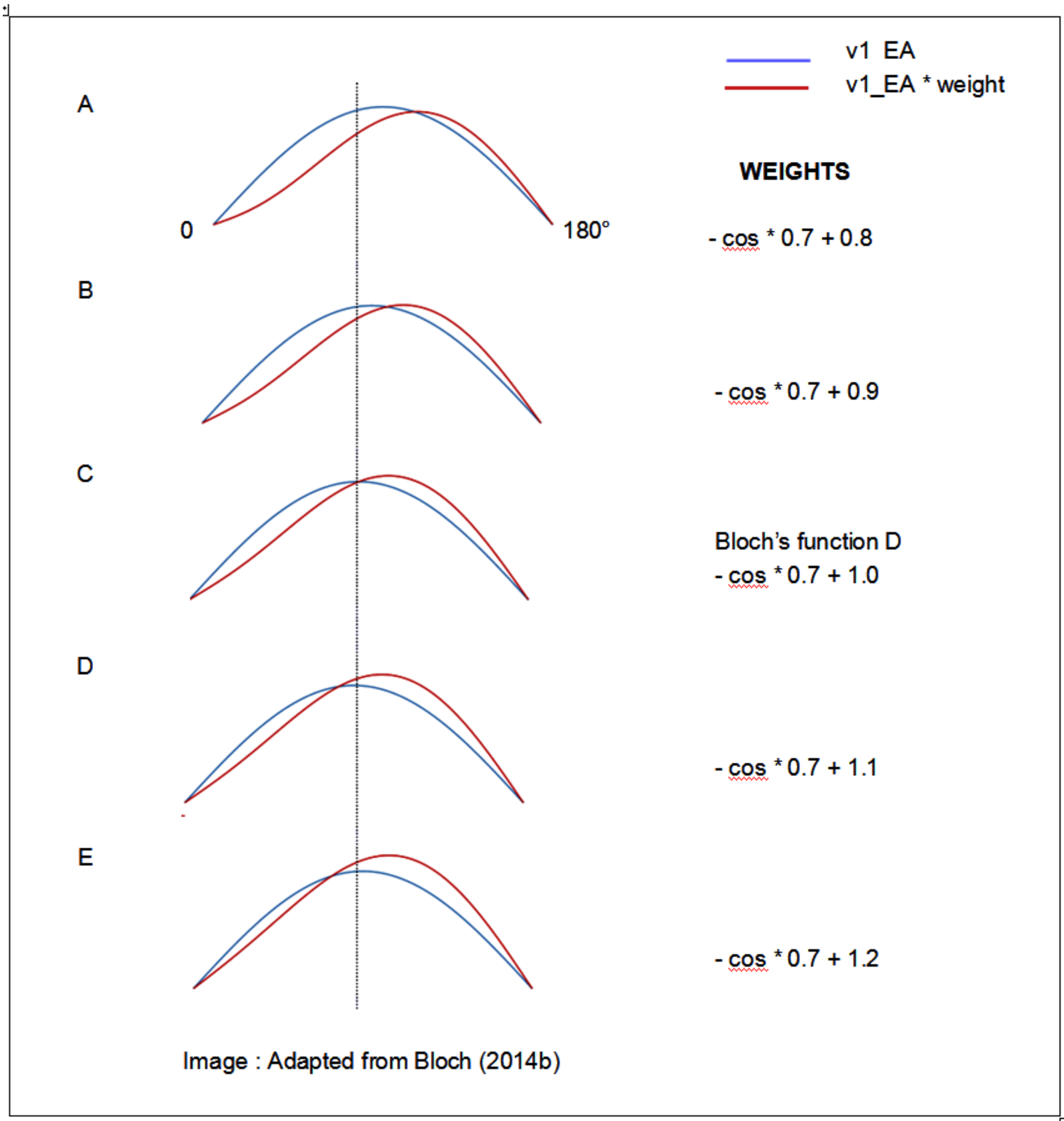

Figure 4: Impact of the translate parameter of function blochD on the computed area

Bloch (2014b) provided Javascript code for graphically comparing how the weighting functions vary in the range 0 to 180 degrees for isosceles triangles with unit length sides. He stated that in figures (such as Figure 4) "*The unweighted (blue) curve peaks at 90 degrees, suggesting that the effective area metric favors angles close to 90 degrees".* This is incorrect. The blue curve describes the area for the isosceles triangles in the range 0 to 180 degrees. There is no weighting since v1_EA uses the standard weight of 1 for all triangles, even those which are scalene. The red curve graphs bD_EA, namely the area of the isosceles triangles after weighting. The scale and translate values can be tuned to gain a better understanding of the effect of the parameter values. In this paper, only a selection of graphs and maps are included as examples.

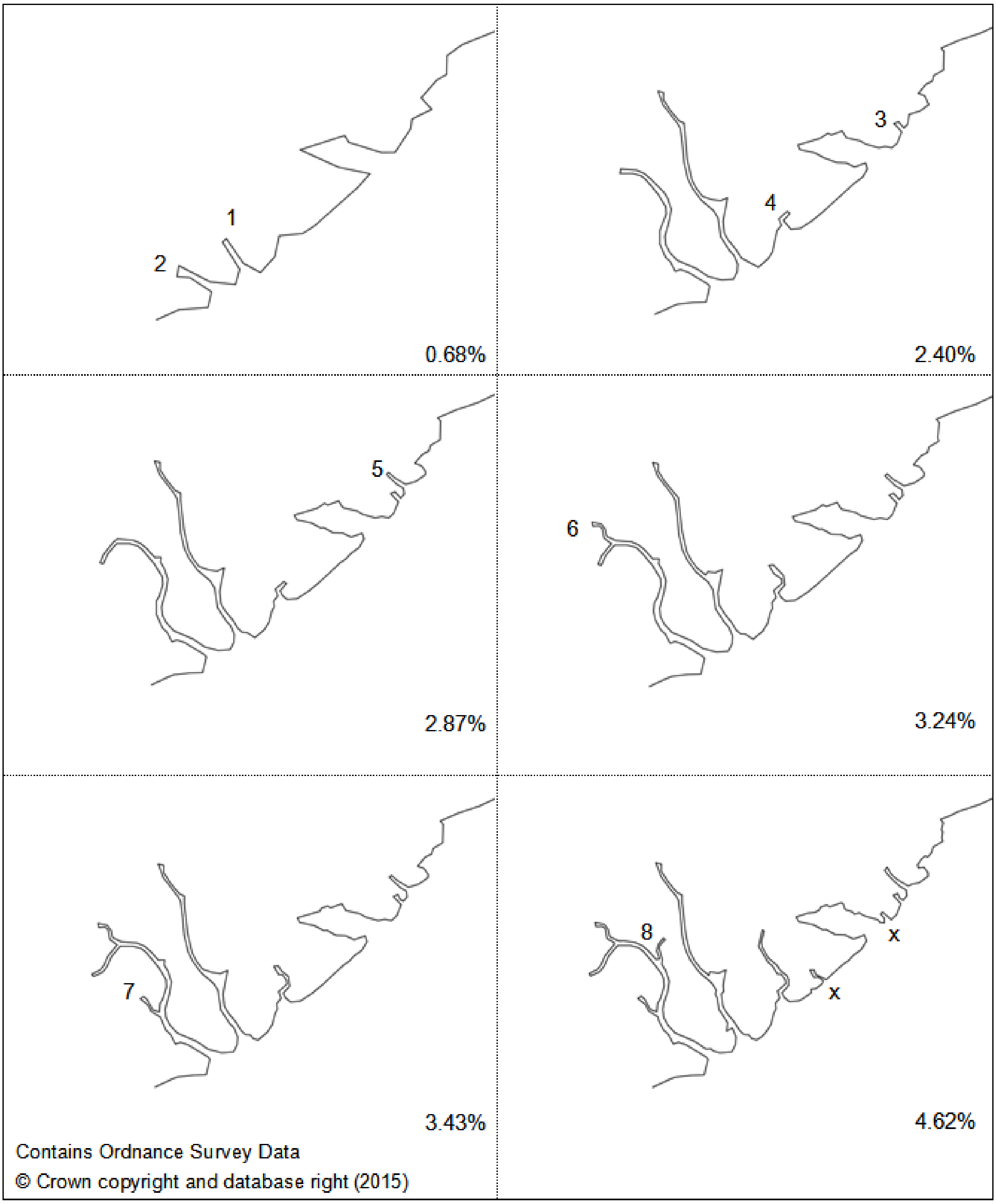

Figure 5: Order of feature inclusion with bD_EA = (v1_EA * - cos * 0.7 + 0.8)

6.1 Varying the translate parameter

Figure 4 shows that the translate parameter a) alters the height of the bD_EA profile, and also b) changes the point at which it crosses the v1_EA curve. In Figure 4C, the graph for Bloch's function D crosses at 90 degrees; this has the effect of underweighting acute angles and favouring obtuse ones, especially those around 120 degrees.

Figures 4a and 5 show the impact of setting translate to 0.8, which limits the weights to a range between 0.1 and 1.5 (for angles ranging from zero to 180 degrees). The main creeks remain truncated even after the inclusion of D (at number 7), F (at number 8) and other minor features (marked x).

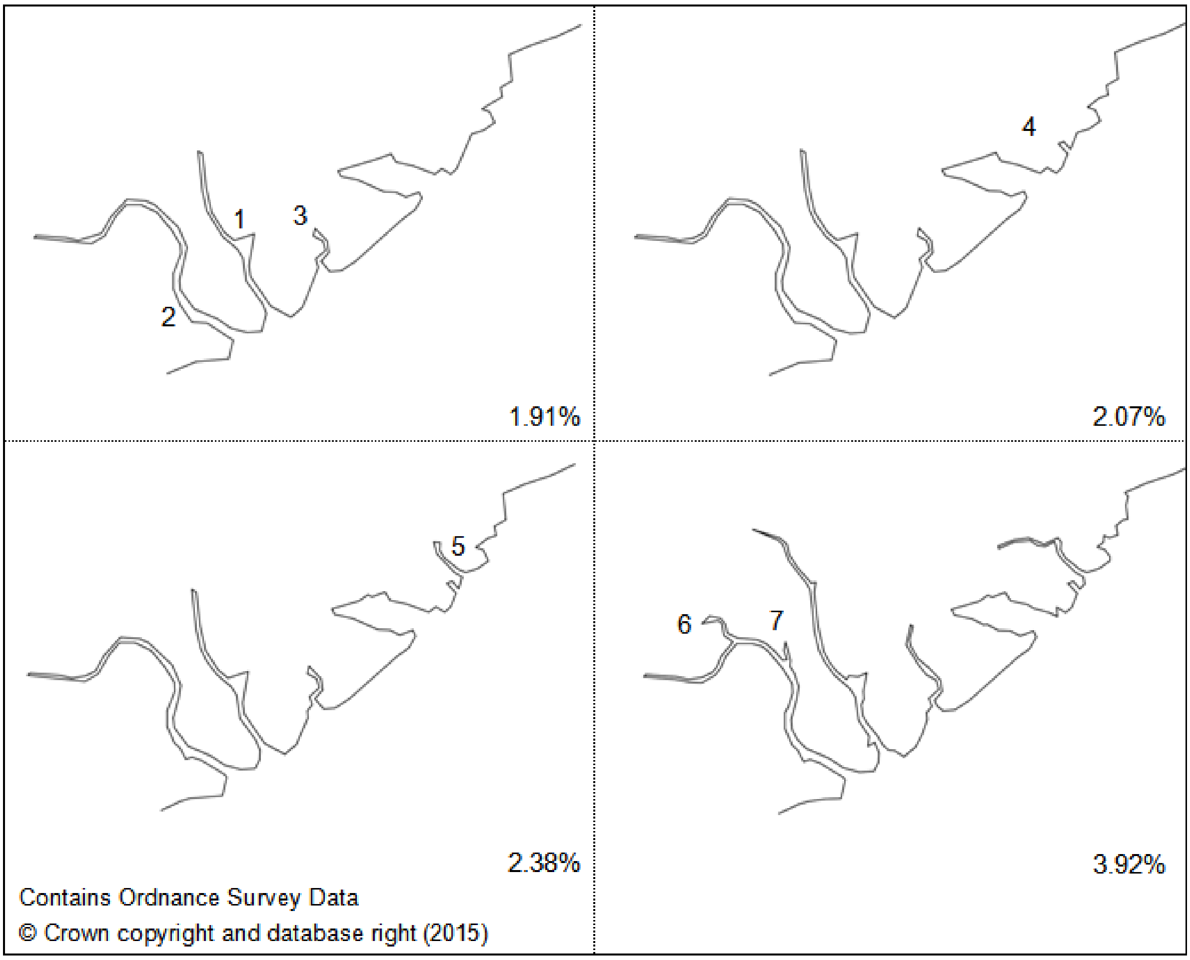

Figure 6: Order of feature inclusion with bD_EA = (v1_EA * - cos * 0.7 + 1.2)

Figures 4e and 6 show the effect of increasing translate to 1.2, which yields weights between 0.5 and 1.9. Apart from feature H (at position 4), the features appear as expected. Also, the tributary of B selected by this weight corresponds to that selected by the cartographer. Unfortunately, the tributaries of Gulch Creek are included before B & C are extended. The results seem more appropriate than when translate was set to 1 in function D; even with under 4% of points, the shapes are not unduly chopped.

6.2 Varying the scale parameter

The scale parameter also changes the range of values for the weight. Scale = 0.7 will limit weights to the range 0.3 to 1.7. The scaling factor does not change the point of intersection of the curves for v1_EA and bD_EA in Figure 4. It only changes the amplitude of the curve for bD_EA. So, changing the scale will still favour obtuse angles.

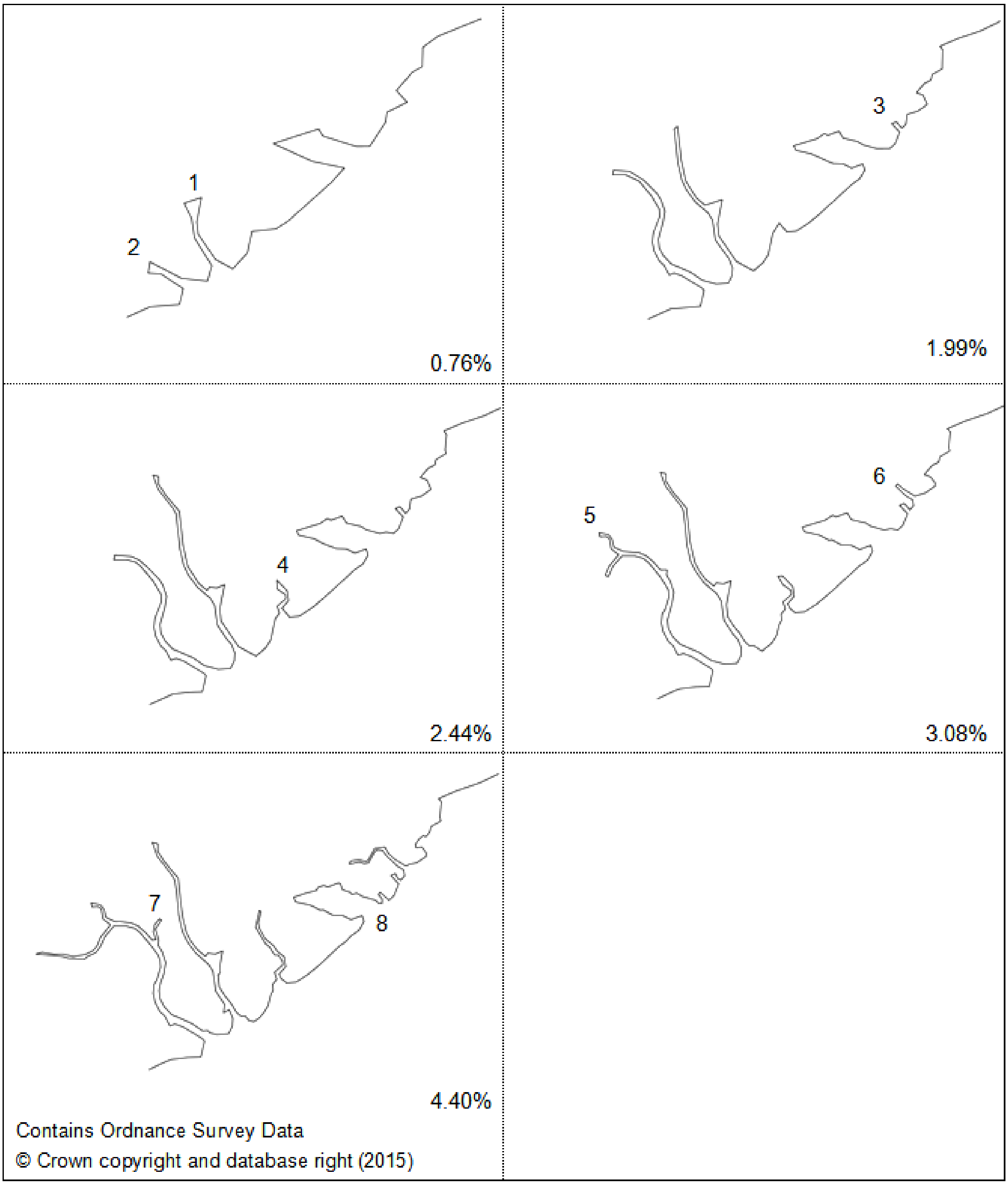

Figure 7: Order of feature inclusion with bD_EA = (v1_EA * - cos * 0.8 + 1)

Figure 7 shows the order in which features are included if the scale factor is changed from 0.7 to 0.8. This gives a larger range of weights from 0.2 to 1.8. B only appears after H and E. The results are not as useful as those in Figure 6. Schroeder's function with smoothness = 0.9 also truncates the features. However, as noted earlier, Schroeder wanted to eliminate long elongated features. Gulch Creek is only extended to its signature form when 5.75% points are included, and well after the inclusion of H, D, G and other unlabelled minor creeks. With scale = 0.8 and 0.9, the creeks are abstracted with angular cuts even at fairly low levels of simplification.

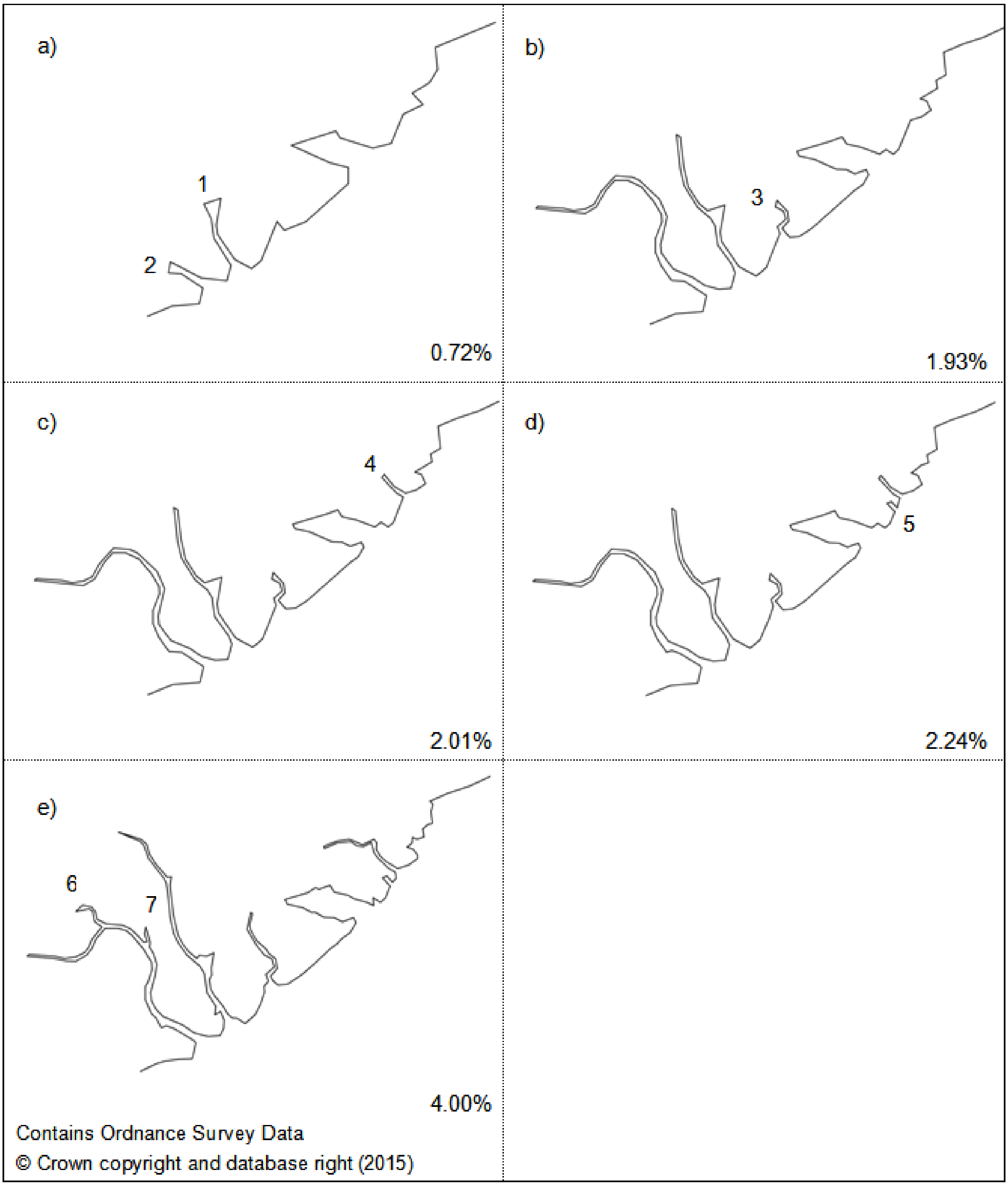

Figure 8: Order of feature inclusion with bD_EA = (v1_EA * - cos * 0.6 + 1)

Figure 8 shows the effect of reducing the scale from 0.7 to 0.6, which gives weights in the range 0.4 to 1.6. The output of Figures 6 and 8 are very similar. However, in Figure 6, H appears before B, whereas in Figure 8, B appears before H. With both weights, E and F appear before creeks A, B and C are extended to give a better, even if slightly over-extended, indication of the shape of the creeks. Also, the extension of A in Figure 8e is some way towards the cartographer's depiction of A in the 1:50000 Landranger® map and the tributary of B selected by the weight corresponds to that selected by the cartographer, who has widened the main creeks and trimmed the tributaries and spurs. These operations are outside the scope of this paper and will be considered elsewhere.

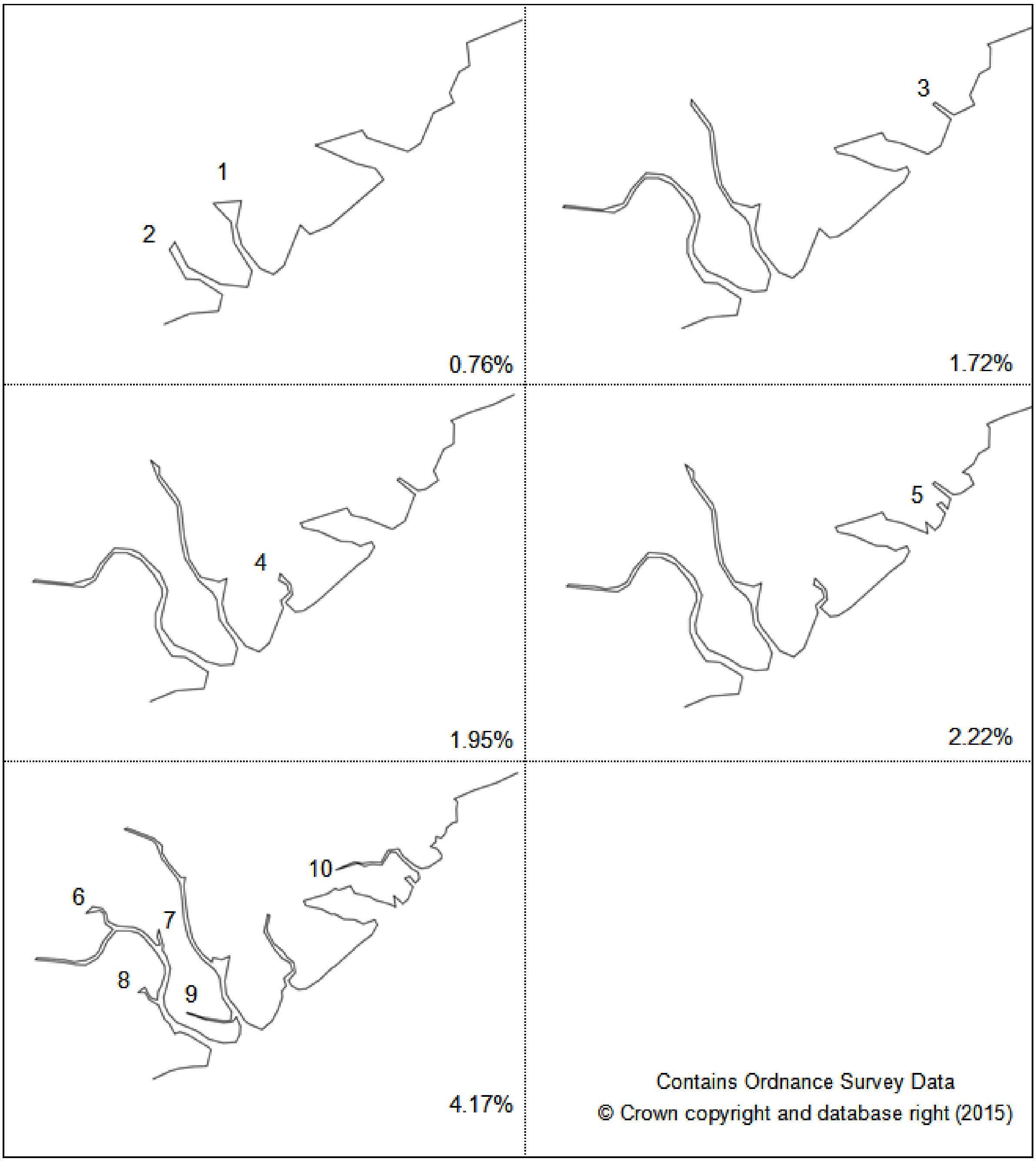

Figure 9: Order of feature inclusion with bD_EA = (v1_EA * - cos * 0.5 + 1)

When scale is set to 0.5, the order of inclusion of desired features is initially better with B (the cartographer's choice) appearing before C (Figure 9). However, it then includes some smaller creeks before extending B. Increasing scale to 0.55, has the effect of including G before D. Also, both 0.5 and 0.55 pick the left tributary of B. Switching to scale = 0.56, introduces yet another minor feature before extending B. Yet, from 0.57 to 0.59, the results are very similar to 0.6.

bD_EA = - cos * 0.6 + 1.2 (igure not shown), makes features appear in the order A, Gulch, B, then C at 2%. However, H, D, E, F, and G appear in that order before the creeks are extended, which is unhelpful. Curiously, when scale is set to 0.61 (figure not shown), the order of feature appearance is similar to scale = 0.6 but B appears before C. This shows that very small changes to parameter values can make a dramatic difference to feature order and extension and the optimal parameters can be difficult to find as Zhou (2014) found with his more complex weighting scheme.

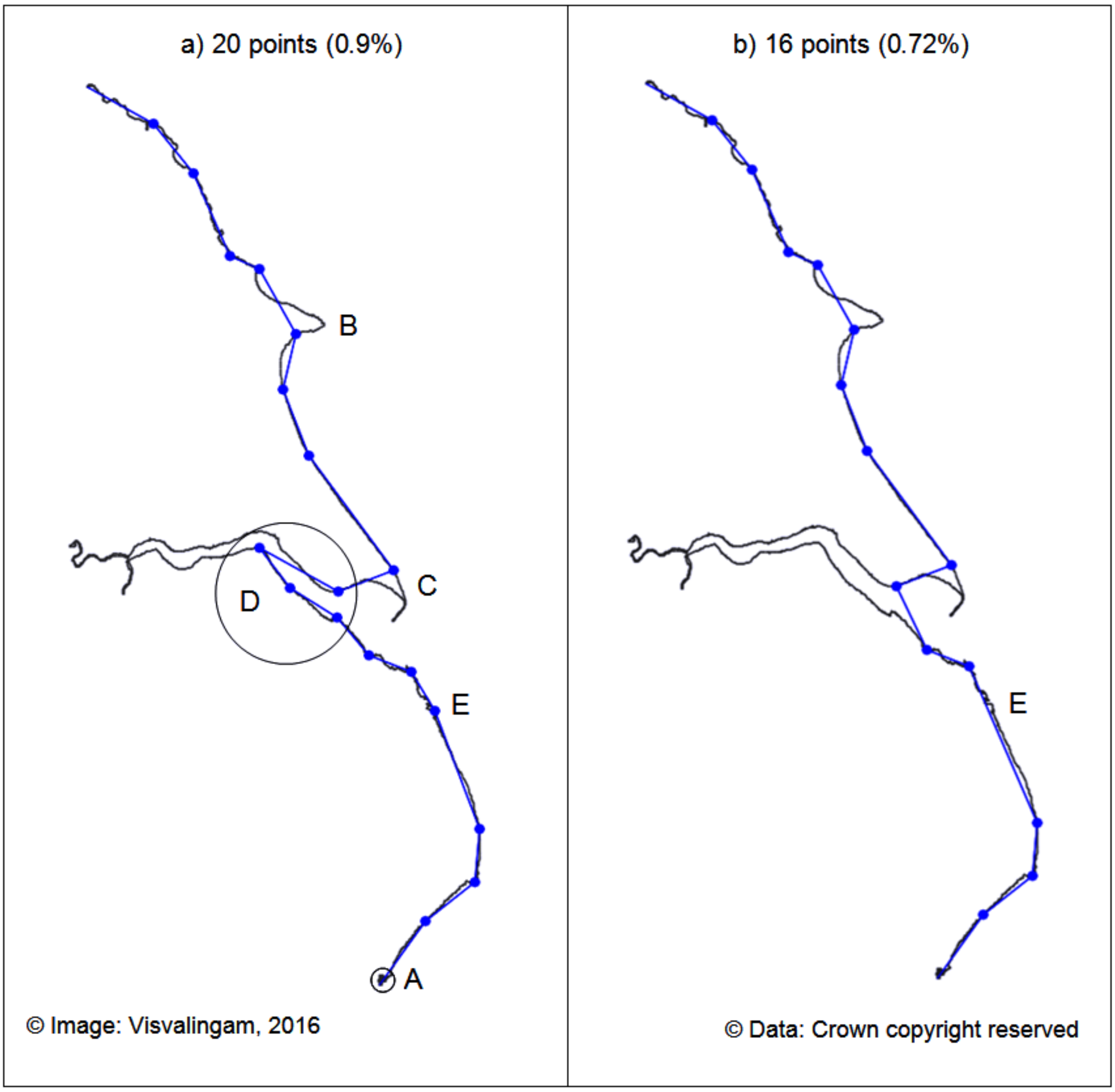

Figure 10: Output for Humberside data using Bloch's function D with scale = 0.6

7. The Humberside data

The Scalp is the blotch, encircled and labelled A at the southern extreme of Figure 10a. Visvalingam and Whelan (2014) used Bloch's function A with the Humberside data to demonstrate that while weighting can produce more pleasing typifications of The Scalp, it adversely affects the derivation of caricatural generalisations of complex forms (see their Figure 7). They noted that Bloch's aim was simplification, not caricatural generalisation but showed that his function A would truncate linear features at all levels of filtering; this effect is just not noticeable at low levels of simplification unless viewed at larger scales than intended.

As already noted, Function D is much better than function v1_EA for The Scalp. With some parameter values, such as scale = 0.6, it produces more pleasing typifications of the main creeks. Function D was then tested with the more complex Humberside data. Unfortunately, function D was little better than function A for this data as illustrated in Figure 10; the equivalent output of function A can be found in Figure 7a(iii) in Visvalingam Whelan (2014). bD_EA truncates the promontories of Flamborough at B and Spurn at C and also the Humber Estuary at D, before the removal of point E, which does not cause much areal displacement.

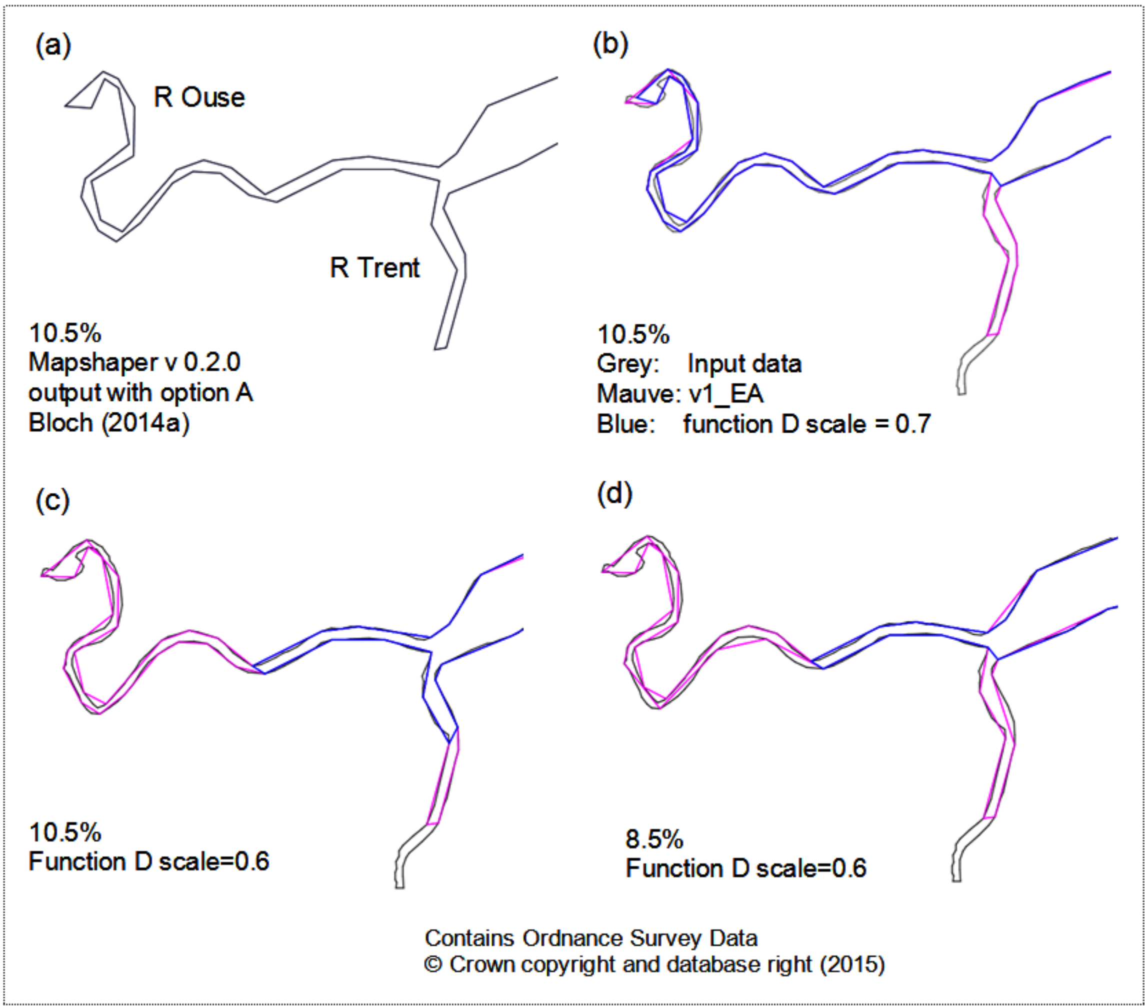

Figure 11: Comparison of output using v1_EA and with Bloch's functions A and function D

Figure 11 contains an enlargement of the map when the data for Humberside (Figure 10) is filtered at different levels. Function D is not as good as function A (Figure 11 a) or v1_EA (mauve lines); it produces unbalanced simplifications of the rivers draining into the Humber with both scale = 0.7 and 0.6, although scale = 0.6 produces an indicative simplification, which can be post-processed. Please see Visvalingam and Whyatt (1993) and Visvalingam and Whelan (2016) for discussion on the representation of such convoluted lines.

Schroeder's smoothness = 0.9 does not truncate Flamborough (B) but it does truncate C and D in Figure 10a. It also favours points like E in Figure 10a and its output is very similar to the blue line in Figure 11b (for function D with scale = 0.7).

8. Summary and Discussion

The above observations suggest the following:

a. **Function D is useful for simplifying The Scalp data.** Like function A it eliminates the smaller creeks and tributaries by down weighting acute angles. Unlike function A, it favours oblique angles. By doing so, it preserves the meandering curves of the larger creeks. Unfortunately, this weighting strategy was found to be unsuitable for the Humberside coastline.

b. **The precise effects of varying the parameters of function D are difficult to predict,** especially if both translate and scale are tuned simultaneously. So, it is not surprising that Schroeder, Bloch and Zhou had to select parameter values by trial and error. The Zhou and Jones (2004) weighting scheme, has many more parameters than Bloch's function D. Even this study involved a lot of tedious exploration of parameters before a pattern was discerned. The best results for The Scalp data were obtained with v1_EA * (-cos * 0.6 + 1).

c. **Parameter values can exacerbate inappropriate angular truncation of linear features.** This study has provided guidance on how to find parameter values for deriving pointed, or if so desired, angular terminations to elongated features, such as creeks. Specifically:
    - Angular truncation can be obtained with translate < 1 or scale > 0.7. Schroeder's scaling by 0.9 (see above) will produce chopped creeks as in Figure 7 - but this would be appropriate for retaining the rectilinear man-made coastlines as in Florida. It should be possible to automatically detect man-made angular coastlines.
    - Pointed ends may be derived with translate > 1 or scale in the range 0.57 to 0.61.

    This pattern of output may not apply to other line configurations.

d. **With appropriate parameter values, it is possible to derive typifications that are better** than those output by v1_EA and the digitised version of The Scalp coastline in the 1:50000 SWURCC data for Humberside; for an enlarged view of the latter see Figure 2a in Visvalingam and Whelan (2014). The map of The Scalp at 1:250,000 scale in OS (2016) was drawn by professional cartographers. This project did not consider the amalgamation of islands, such as at H, X and Y.

e. **Weighting functions which under weight acute and over weight obtuse angles appear to have a limited range of applicability**. With the Humberside data, this truncates significant landmarks before eliminating points, like E in Figure 10. As noted earlier, with scale = 0.6, function D returns weights in the range 0.4 to 1.6. Bloch's (2014b) visualisation of the effect of varying the parameters of function D (see Figure 4) is useful, but it can be misleading as noted in Section 6. Other figures in Bloch (2014b) show that function D with scale = 0.6 will return the greatest weight (approaching 1.6) for an almost straight line; points subtending 120 degrees only have a weight of 1.3. Point E with a weight of 1.5977 assumes undue importance relative to the more acute triangular features. The areas for the acute angled points on the promontories and estuary will be reduced by about half or even more. For example, the last point to be eliminated at B on Figure 10a only has a weight of 0.5757 for a near 45 degree angle. Yet, this triangular feature would have been deemed a critical point, like those on his cat's ears, by Attneave (1954), whose paper has inspired a great deal of research in digital cartography and psychology.

There is a need for further research on:

- The scope of applicability of function D which looks promising. Is it possible to find suitable parameter values for different types of candidate lines using machine intelligence?
- Whether bD_EA thresholds be selected automatically for given scales? Visvalingam (2016) suggested how Tobler's rule-of-thumb could be used to make a first stab at scale-related thresholds for v1_EA. This rule-of-thumb is difficult to apply when using function D (or similar weighting schemes) since a bD_EA value can result from triangles with different areas and shapes.

- Elegant adaptations of function D to take account of the above observation that the perception of angles is not entirely consistent with the cosine values as used in function D (see Table of cosine values in NASA (undated)), initially ignoring the well known visual illusions which distort the perception of planar angles and distances.

9. Conclusion

This paper provided further illustration of how Visvalingam's algorithm with v1_EA tends to cut curves as noted by Visvalingam and Williamson (1995). It reviewed some weighting functions which have been devised for various purposes and then explored Bloch's function D (Bloch, 2014b) which implements the ***Visvalingam /weighted area*** option in Mapshaper to date. The paper shows that Zhou's (2014) concerns over the angular truncation of elongated curves could result from his use of inappropriate parameters, which remain undisclosed. With suitable parameters, function D provides elegant depictions of the main creeks in The Scalp wetlands of Lincolnshire. These are not comparable with the cartographer's depiction of the main creeks in the 1:50000 Landranger® map. However, function D is better than v1_EA for The Scalp, even if it is unsuitable for filtering more extensive and complex coastlines, such as that of Humberside. The paper also identified a problem with the function D weighting scheme, namely an undue deflation of triangles with visually significant acute angles and over-promotion of visually less significant obtuse angles approaching a straight line. Schroeder's formula is similar to function D but the smoothness (scaling) factor of 0.9 was less useful for the data sets considered here.

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