# Advanced undergraduate RC circuits: an experimentalist's perspective

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Abstract
In this paper, an advanced undergraduate RC circuit is studied in two different ways. The circuit is a typical series RC circuit with a time-varying voltage source. The temporal profile of the voltage is an isolated, Gaussian shaped pulse. The voltage across the resistor as a function of time is analysed using two different methods: deriving an analytical expression and an analysis in the Laplace domain. An attempt is made to suggest and address common problems that students may have with understanding such circuits. A qualitative physical interpretation of the circuit operation is developed using Green's function.

Keywords: RC circuits, Laplace transform, undergraduate electronics

## 1. Introduction

Series RC circuits are ubiquitous in undergraduate electronics courses at both introductory and advanced levels. However, despite their omnipresence, students' difficulties with predicting the qualitative behaviour [1] and grasping the concepts behind the operation of even the most basic kinds of RC circuits [2] persist. It is for these reasons that further elaboration upon how these circuits behave is required. Consider the following circuit, which represents a typical advanced undergraduate RC circuit:

A time varying voltage source $V_{in}(t)$ is connected across a resistor R and a capacitor C which are in series. While this circuit might seem trivial, a quick scan of popular undergraduate electronics textbooks shows that this problem is only discussed under very specific circumstances. Those circumstances tend to be when special constraints are placed on the nature of $V_{in}(t)$. Similarly, the way in which RC circuits are treated does not equip students

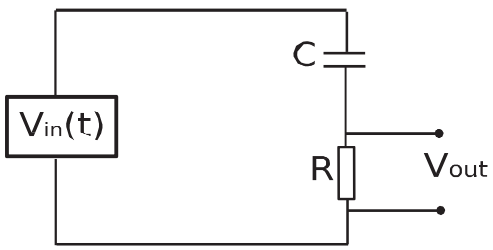


Figure 1. A typical RC circuit with a time varying voltage source.

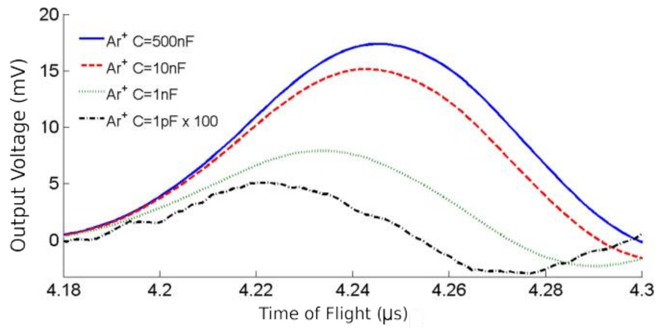


Figure 2. Results from the time of flight experiment. Singly charged argon ($Ar^+$) ions were produced by laser ionization and detected by a Wiley–McLaren time of flight apparatus. Changing the capacitance value of the read-out circuit produced distorted ion signals.

with the necessary conceptual tools to be able to describe how current flows in this kind of circuit and, as is the focus of this paper, how the voltage across the resistor changes as a function of time. In fact, when discussing the voltage across the resistor, only very limited situations are described. Perhaps the most widely used example is that of a periodic square wave voltage source connected across an RC series network. This example appears in countless textbooks [3]. However, when this example is presented, the limiting cases are often emphasized rather than a more complete explanation as to how current flows in general. In the case of an RC circuit with a square wave voltage source, the two limiting cases are when the period of the square wave is much larger than the RC time constant (in which case the voltage across the resistor will be the derivative of the input voltage) and when the period is much smaller than the RC time constant (in which case the voltage across the capacitor will be the integral of the input voltage).

Recently, as part of an experiment, I had reason to consider conceptually what happens in figure 1 in the case where the time varying voltage source varies arbitrarily in time. In fact, the circuit in figure 1 is analogous to what happens in a typical time of flight mass spectrometry experiment. The voltage $V_{in}(t)$ takes the role of packets of ions striking some detector and the measurement of the time varying voltage across R gives the time of flight signal. In order to perform the measurement accurately, the shape of the ion signal should be preserved. I found that depending on the choice of capacitor, the shape of the measured ion signal across R became distorted. The results are shown in figure 2.

From modelling the time of flight apparatus, it was expected that the peak signal of these particular ions would arrive at $t_o \approx 4.24\ \mu s$ after the acquisition started and be Gaussian shaped with a full width half-maximum (FWHM) of $\sigma_o \approx 0.06$ μs. From this, I was reasonably happy to say that the measurement made with a 500 nF capacitor preserved the ion signal quite well. I then had reason to try to figure out what is happening to the signal shape when the capacitance value changes. Upon researching various materials both on-line and in hard print (both textbooks and published papers), I came to the conclusion that perhaps it can be difficult to try interpret the results in figure 2 using the standard approaches in textbooks. Specifically, as discussed above, substantial emphasis is placed on relating the period of the voltage source to the RC time constant. It might be sensible to ask, then, if the input voltage is not periodic how is that to be done? Another standard example in text books is an isolated square pulse input. The most common approach to that problem is to model the rising edge of the square pulse as a switch closing and the falling edge as a switch opening and treat the voltage source as constant in between. Again, it might then be sensible to ask, how are isolated voltage pulses to be treated when the voltage constantly changes across the pulse profile.

Of course, it is known how to treat these circuits, using the general solution to the differential equation which describes the RC circuit however this rarely appears in electronics textbooks and is usually confined to the realm of engineering mathematics textbooks [4]. In fact, there are many ways to handle these kinds of circuits but the various methods and mathematical techniques seem to exist in their own realms. For example, the general solution to the first order RC circuit is often found in mathematics books but rarely in electronics textbooks. Laplace analysis of RC circuits is often presented in its own section and presented as completely separate from any other type of circuit analysis. It is then left to the reader, student or teacher to bring all these points together in a way that helps a student understand the physical concepts of the circuit's operation. While this may be trivial for the limiting cases described above, it is perhaps not so trivial for arbitrary cases like the one in figure 1. Similarly, a clear physical model that links the mathematical predictions of the various kinds of analysis to the physical concepts that underpin the circuit's operation are often over looked in most textbooks. The role of transient currents [5], for example, or the concept of circuits trying to reach equilibrium [6] is often not presented in textbooks (although it is sometimes discussed [7]). The aim of this paper is to try to help bring various types of analysis together to try develop a complete understanding of how the circuit in figure 1 behaves.

In the following sections I take two different mathematical approaches to describing the circuit in figure 1. The first approach is a purely analytical derivation of the general solution to the differential equation which describes this circuit. The second approach is a Laplace analysis of this particular circuit. A physical model of the operation of the circuit is then presented.

### 1.1. Statement of the problem

The problem can be stated very simply:

'How does the voltage across the resistor in figure 1 change when the input voltage changes arbitrarily in time?'

In the examples and analysis presented in this paper, the voltage ($V_{in}(t)$) has a very specific Gaussian shape with a specific mean value (4.24 μs) and a FWHM of 0.06 μs. The amplitude of the Gaussian is chosen to match the peak voltage of the signal in the experiment (20 mV). These values were chosen in order to match experimental data produced from the time-of-flight mass spectrometry experiment where ions are created at some instant in time by a laser and are detected some time later (in this case 4.24 μs).

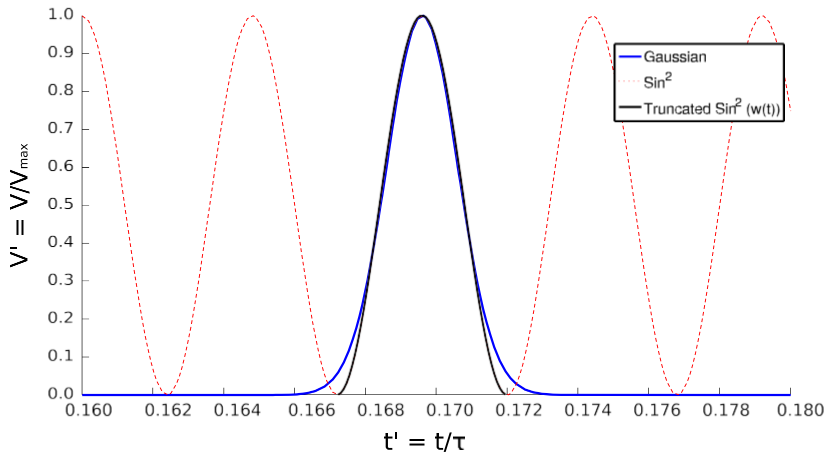


Figure 3. Shown in solid-blue is a modelled Gaussian input function for $V_{in}(t)$ shown in dotted-red is an approximation to that function using a $\sin^2$ function. Plotted in black is the truncated function w(t) as given in equation (1). The x and y axes have been scaled to the RC time constant for a 500 nF capacitor and the maximum voltage of the 500 nF signal respectively.

Mathematically, the temporal profile of the voltage source is modelled as $V_{in}(t) = Ae^{-((t-t_o)^2/2\sigma^2)}$ which is the standard form of the Gaussian function. A is the amplitude, $t_o$ is the mean and σ is the standard deviation. The FWHM of a Gaussian function ($\sigma_o$) can be related to its standard deviation from the relation $\sigma_o = 2\sqrt{2\ln(2)}\,\sigma$. It might be worthwhile to discuss the Gaussian shape a little bit before proceeding. Figure 3 shows (in solid blue) the modelled input Gaussian function that describes $V_{in}(t)$.

The analysis techniques presented further on involve manipulation of the $V_{in}(t)$ function namely: calculating its integral, finding its Laplace transform and differentiating it. Finding the first derivative of a Gaussian function does not present many challenges as it is simply the original Gaussian function multiplied by the first Hermite polynomial. Finding the integral and Laplace transform is perhaps more difficult because the Gaussian function lacks an anti-derivative. Similarly, a Gaussian function only goes to zero at $\pm\infty$ which makes it computationally difficult to handle. For this reason, an approximate function is chosen to represent the Gaussian function. Shown in dotted red in figure 3 is a $\sin^2$ function which is appropriately shifted so that the peak of the Gaussian signal 'lines up' with a maximum of the $\sin^2$ function. The frequency of this $\sin^2$ function will determine the 'broadness' of the peaks and so the frequency can be chosen to closely match the Gaussian function. Shown in black, then, is a truncated $\sin^2$ function. The signal has been truncated such that it goes to zero outside one half period of the function in order to give a representation of a single, isolated pulse. Mathematically, an approximation to the Gaussian function can be made with $\sin^2$ functions with the following equation:

$$w(t) = A\sin^2(\omega(t-\kappa))[u(t-\kappa) - u(t-\beta)], \tag{1}$$

where $\Delta\tau = 4\sigma_o$, $\omega = 2\pi/\Delta\tau$, $\kappa = t_o - \Delta\tau/4$ and $\beta = \kappa + \Delta\tau/2$ and where u(t) is the Heaviside step function.

This 'approximate function' is advantageous for two reasons. Firstly, it makes computations and calculations simpler as the $\sin^2$ function has an anti-derivative and a well known Laplace transform. Secondly, the frequency of the $\sin^2$ pulse is related to the width of the

Gaussian function by $\omega = 2\pi / 4\sigma_o$. Thus, the inverse width of the Gaussian input function can be thought of as a frequency. This perhaps offers a tentative answer to the question that was asked above: 'if the input signal is not periodic, how are we to think of its frequency?' With this scheme, the frequency can be thought of as the inverse width of the input pulse.

## 2. Mathematical analysis of the circuit

### 2.1. Method one: exact analytical expression

The charge on the capacitor plates as a function of time is found by applying Kirchoff's voltage law to the circuit which requires that the sum of the voltages across the capacitor and the resistor is equal to the input voltage:

$$Ri(t) + \frac{q(t)}{C} = V_{\text{in}}(t). \tag{2}$$

Rearranging gives:

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} + \frac{q(t)}{RC} = \frac{V_{\text{in}}(t)}{R}. \tag{3}$$

This equation can be solved by choosing an appropriate integrating factor. Letting $\mu(t) = \mathrm{e}^{t/RC}$ and multiplying both sides by $\mu(t)$ gives:

$$\mu(t)\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{\mu(t)}{RC}q(t) = \frac{\mu(t)V_{\text{in}}(t)}{R}. \tag{4}$$

It can then be noted that the left-hand side equation can be written as:

$$\mu(t)\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{\mathrm{d}\mu(t)}{\mathrm{d}t}q(t) = \frac{\mu(t)V_{\text{in}}(t)}{R}. \tag{5}$$

The product rule is then invoked to give:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mu(t)q(t)) = \frac{\mu(t)V_{\text{in}}(t)}{R}. \tag{6}$$

Direct integration then gives:

$$q(t) = \frac{\mathrm{e}^{-t/RC}}{R}\int_0^t V_{\text{in}}(\tau)\mathrm{e}^{\tau/RC}\mathrm{d}\tau. \tag{7}$$

In its most common form, the equation is written as:

$$q(t) = \frac{1}{R}\int_0^t V_{\text{in}}(\tau)\mathrm{e}^{-(t-\tau)/RC}\mathrm{d}\tau \tag{8}$$

The voltage across the resistor, $V_{\text{out}}$, is found by Ohm's Law:

$$V_{\text{out}}(t) = R\frac{\mathrm{d}q}{\mathrm{d}t}. \tag{9}$$

Substitution of equation (8) into equation (9) and then integration by parts (see appendix A) gives an expression for the output voltage:

$$V_{\mathrm{out}}(t) = \int_0^t \frac{\mathrm{d}V_{\mathrm{in}}(\tau)}{\mathrm{d}\tau} \mathrm{e}^{-(t-\tau)/\mathrm{RC}} \mathrm{d}\tau. \tag{10}$$

Substituting in the expression $w(t)$ for the voltage $V_{in}(t)$ gives:

$$V_{out}(t) = A\omega \int_{\kappa}^{\beta} \sin(2\omega(\tau - \kappa))e^{-(t-\tau)/RC}d\tau, \tag{11}$$

where the limits have changed due to the fact that the step function $u(\tau - \kappa) - u(\tau - \beta)$ ensures that the integrand is zero for $\tau < \kappa$ and $\tau > \beta$ as long as $\beta < t$.

From equation (10) it is seen that the voltage across the resistor ($V_{out}$) is given by the convolution of the derivative of the input voltage and an exponential decay term with a decay constant equal to the time constant of the circuit. It can be inferred from this that if the time constant is very small, the exponential function decays quite fast and $V_{out} = -RC\frac{dV_{in}}{d\tau}$. Similarly if RC is very large, the exponential decay is quite slow and $V_{out} = V_{in}$.

### 2.2. Method two: Laplace analysis

The output voltage across the resistor can be written as follows

$$V_{out}(t) = RC\left[\frac{dV_{in}(t)}{dt} - \frac{dV_{out}(t)}{dt}\right]$$

A solution for $V_{out}(t)$ can then be found in the Laplace domain. On denoting $V(s)$ as the Laplace transform of $V_{out}(t)$ and $X(s)$ as the Laplace transform of $V_{in}(t)$, the output voltage can be found to be

$$V(s) = RC\left[sX(s) - V_{in}(0) - sV(s) + V_{out}(0)\right]$$

which gives:

$$V(s)(1 + RCs) = RCsX(s)$$

upon setting $V_{in}(0) = V_{out}(0) = 0$. This reduces to

$$V(s) = \frac{s}{f + s}X(s), \tag{12}$$

where $f = 1/RC$. $V(s)$. By setting $\frac{1}{f+s} = G(s)$ and $sX(s) = F(s)$, equation (12) can be re-arranged and re-grouped in the following manner

$$V(s) = G(s)F(s) = \left(\frac{1}{f + s}\right) sX(s).$$

The convolution theorem states that the multiplication of two functions in the Laplace domain is the convolution of those two functions in the time domain. That is to say

$$\mathcal{L}^{-1}[G(s)F(s)] = \int_{-\infty}^{\infty} g(t - \tau)f(\tau)d\tau = g(t) \oplus f(t).$$

The inverse Laplace transform for $F(s)$ is given by $\frac{dV_{in}(t)}{dt}u(t)$ and the inverse Laplace transform of G(s) is given by $e^{-t/RC}u(t)$. Putting it together gives

$$V_{out}(t) = \int_{-\infty}^{\infty} \frac{dV_{in}(\tau)}{d\tau} u(\tau) e^{-(t-\tau)/RC} u(t-\tau)\, d\tau. \tag{13}$$

The function $u(\tau)$ ensures that the integrand is zero for all $\tau < 0$ and the function $u(t-\tau)$ ensures the integrand is zero for $\tau > t$ and so the following change in limits is used:

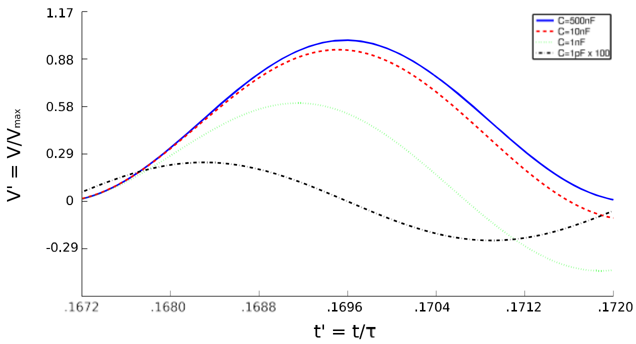


Figure 4. Output voltage calculated by the solution given by the direct integration method. The same solution is given by the Laplace method.

$$V_{out}(t) = \int_0^t \frac{dV_{in}(\tau)}{d\tau} e^{-(t-\tau)/RC} d\tau. \quad (14)$$

The result from the Laplace analysis then re-affirms the result found from the integration method. The benefit of this analysis is that it requires fewer steps and much less calculus than the integration method which relies on advanced calculus methods such as integration by parts and the Leibniz rule of integration, as seen in appendix A. Figure 4 shows the calculated output from the result of the integration and Laplace method.

In figure 4, a $\sin^2$ profile was generated digitally with the parameters described in section one ($t_o$ = 4.24 μs and FWHM = 0.06 μs). The voltage across the resistor is then plotted for different capacitances. Figure 4 shows that changing the capacitance will change the profile of the output. Figure 4 also shows that as the capacitance is decreased, the peak of the signal shifts to earlier times and eventually, if the capacitance is sufficiently low, the output approximates the derivative of the input. These trends are qualitatively identical to the experimental results presented in figure 2. In figure 4 the x-axis is plotted in 'scaled time' which is $t' = t/\tau$, where *tau* = $RC_{500\,nF} = 50 \times 500 \times 10^{-9}$ = 25 μs and the y-axis is plotted in 'scaled voltage' where $V' = V/V_{max}$, where $V_{max}$ is the peak voltage of the 500 nF signal.

## 2.3. A physical interpretation of the circuit

Both the integration and Laplace methods of analysis show that the voltage across the resistor for an arbitrary input voltage will be given by the convolution of the derivative of that voltage with an exponential decay function whose decay time is the RC time constant. It might be helpful to try develop a physical interpretation as to why the solutions appear the way that they do. The impulse response of the circuit can be calculated by setting $V_{in}(t) = \delta(t)$ and solving equation (2). In fact, by doing this, the impulse response turns out to be $h(t) = e^{-t/RC}u(t)$. Then, by using Green's function analysis, the 'forced' response of the system to some arbitrary input voltage $V_{in}(t)$ is written as $q(t) = \int_0^t V_{in}(\tau)e^{-(t-\tau)/RC}d\tau$. The output voltage is found in the same way as before to be $V_{out}(t) = \int_0^t \frac{dV_{in}(\tau)}{d\tau} e^{-(t-\tau)/RC} d\tau$. In this way, the behaviour of the circuit, and the output voltage can be thought of as arising due to competition between the current induced by the voltage source 'forcing' a change in voltage on the capacitor plates and the circuit then trying to decay back to some equilibrium. Each 'instantaneous jump' in voltage forced by the voltage source will cause the capacitor to try and return to equilibrium. Thus, the current through the capacitor can be thought of as the sliding weighted average of the rate of change of the voltage from the supply and the exponential decay constant. This is qualitatively equivalent to the integral solutions derived in this section.

## 3. Summary and conclusion

A first order RC circuit with a variable voltage source is discussed in this paper. An attempt is made to try an address common problems that students may have in understanding and analyzing these kinds of circuits when the voltage source is neither periodic or square shaped but is arbitrary in nature. As an example, the specific case of a Gaussian shaped input voltage pulse is discussed as it matches experimental data from a time of flight mass spectrometry experiment. The question of how to relate the period of the input voltage to the RC time constant (as is the approach taken by many textbooks) is tackled by substituting the Gaussian function with a $\sin^2$ function tailored to match the shape of the Gaussian. The period of this function is then related to the width of the Gaussian function. Another problem that is identified is the lack of a coherent framework between different mathematical techniques. To that end, the general solution to the differential equation is found using two different approaches and they are shown to all give the same results. The first, using an integrating factor to solve the equation, shows that the voltage across the resistor will be given by the convolution of the derivative of the input voltage with an exponential decay term whose decay time is the RC time constant. The operation of this circuit as a differentiator is then shown to be a limiting case of this solution. The equation is also solved in the Laplace domain and shown to be equivalent to the integration method. The results from these methods are plotted and shown to agree with the experimental data. Lastly, an attempt is made to develop a conceptual model of what happens in the circuit to give rise to the convolution result. Green's function analysis is used to suggest that the resulting voltage across the resistor can be thought of as a competition between the forced change on the capacitor from the voltage source and the circuit then trying to return to equilibrium. The relative time scales of these processes are then suggested to perhaps be a more general way of analyzing the circuits than simply relating the period of the input to the RC time constant.

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## Appendix A. Calculating the output voltage from the charge in the analytical method

For the sake of completeness, in this section I will discuss how to go from equation (8) to equation (10). Starting with equation (8) which is repeated here:

$$q(t) = \frac{1}{R}\int_0^t V_{\text{in}}(\tau)\,\mathrm{e}^{-(t-\tau)/RC}\,\mathrm{d}\tau \qquad (15)$$

This is subbed into equation (9) to give:

$$V_{out}(t) = \frac{d}{dt}\int_0^t V_{in}(\tau)e^{-(t-\tau)/RC}d\tau \tag{16}$$

The right-hand side of equation (16) is then differentiated using the Leibniz method of integration which states:

$$\frac{\partial}{\partial t}\int_{a(t)}^{b(t)} f(\tau, t)d\tau = \int_{a(t)}^{b(t)} \frac{\partial f}{\partial t}d\tau + f(b(t), t)\frac{db}{dt} - f(a(t), t)\frac{da}{dz}.$$

Taking the equation term by term...

$$\int_{a(t)}^{b(t)} \frac{\partial f}{\partial t}d\tau = -\frac{1}{RC}\int_0^t V_{in}(\tau)e^{-(t-\tau)/RC}d\tau$$

$$f(b(t), t)\frac{db}{dt} = V_{in}(t)$$

and

$$f(a(t), t)\frac{da}{dt} = 0.$$

This then gives:

$$V_{out}(t) = V_{in}(t) - \frac{1}{RC}\int_0^t V_{in}(\tau)e^{-(t-\tau)/RC}d\tau. \tag{17}$$

The second term on the rhs of equation (17) can be integrated by parts by letting $u = V(\tau)$ and $dv = e^{-(t-\tau)/RC}d\tau$ to give:

$$\int_0^t V_{in}(\tau)e^{-(t-\tau)/RC}d\tau = RC\left[V_{in}(t) - V(0)e^{-t/RC}\right] - \int_0^t \frac{dV_{in}(\tau)}{d\tau}e^{-(t-\tau)/RC}d\tau \tag{18}$$

which when subbed into equation (17) gives:

$$V_{out}(t) = V(0)e^{-t/RC} + \int_0^t \frac{dV_{in}(\tau)}{d\tau}e^{-(t-\tau)/RC}d\tau \tag{19}$$

which is equivalent to equation (10) upon $V(0) = 0$.

## Appendix B. Worked example of Laplace method

In this section, the Laplace method is applied directly to the function w(t) presented in the main body of the paper. In the main body of the paper, the solutions were found for a general $V_{in}(t)$ (equation (10) and then applied to the specific example under consideration (equation (11)). Thus, the analysis is what constitutes a deductive learning method. In the following section, I show the same problem but the other way around. The Laplace method is applied directly to the function w(t) and the solution is calculated. The link to the general case is then made at the end. This is meant as a worked example for a teaching tool in an inductive, teaching lesson based on constructivism. Various teaching techniques such as inquiry-based learning, problem based learning and project based learning use inductive based methods whereby specific examples are introduced first and then general solutions and theories are developed from those examples.

Starting with w(t):

$$w(t) = A \sin^2(\omega(t-\kappa))[u(t-\kappa) - u(t-\beta)] \tag{20}$$

and re-stating the differential equation in the Laplace domain:

$$V(s)(1 + \mathrm{RC}s) = \mathrm{RC}sX(s)$$

where V(s) is the Laplace transform of $V_{out}(t)$ and X(s) is the Laplace transform of w(t), X(s) can be found in the following way:

$$X(s) = A\left[\frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+4\omega^2}\right)\right]\left(\mathrm{e}^{-\kappa s} - \mathrm{e}^{-\beta s}\right)$$

V(s) is then written as:

$$V(s) = A\frac{1}{f+s}\left(\frac{2\omega^2}{s\left(s^2+4\omega^2\right)}\right)\left(\mathrm{e}^{-\kappa s} - \mathrm{e}^{-\beta s}\right),$$

where $f = 1/\mathrm{RC}$. The terms are then grouped to give:

$$F(s) = \left(\frac{2\omega^2}{s^2+4\omega^2}\right)\left(\mathrm{e}^{-\kappa s} - \mathrm{e}^{-\beta s}\right)$$

and

$$G(s) = \frac{1}{f+s}.$$

The convolution theorem is invoked to write $F(s)G(s) = \int_{-\infty}^{\infty} g(t-\tau)f(\tau)\mathrm{d}\tau$. The inverse Laplace transform of $G(s)$ is $g(\tau) = e^{-f\tau}u(\tau) = e^{-\tau/RC}u(\tau) \rightarrow g(t-\tau) = e^{(t-\tau)\ RC}u(t-\tau)$ and the inverse Laplace transform of $F(s)$ is $f(\tau) = \omega \sin(2\omega(\tau-\kappa))[u(t-\kappa)-u(t-\beta)]$. This gives:

$$V_{out} = A\omega \int_{-\infty}^{\infty} \sin(2\omega(\tau-\kappa))u(\tau)e^{-(t-/\tau)\ RC}u(t-\tau)d\tau[u(\tau-\kappa)-u(\tau-\beta)].$$

The various step functions that appear ensure that the integrand is zero outside of certain limits. The function $u(\tau)$ ensures that the integrand is zero for $\tau < 0$ and the function $u(t-\tau)$ ensures that the integrand is zero for all $\tau > t$. Similarly the function $u(\tau-\kappa)-u(\tau-\beta)$ ensures the integrand is zero for $\kappa > \tau > \beta$. This with the assumption that $\kappa > 0$ and $\beta < t$,
the following limit change takes place.

$$V_{out}(t) = A\omega \int_{\kappa}^{\beta} \sin(2\omega(\tau-\kappa))e^{-(t-\tau)\ RC}d\tau.$$

2 ω τ − κ

At this point is can be noted that $\frac{dA \sin(\ (\qquad))}{dt} = A\omega \sin(2(\omega\tau-\kappa))$ which gives.

$$V_{out}(t) = \int_{0}^{t} \frac{dV_{in}}{d\tau} e^{-(t-\tau)/\ RC}d\tau.$$

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