Quantification of Temporal Fault Trees Based on Fuzzy Set Theory

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Abstract. Fault tree analysis (FTA) has been modified in different ways to make it capable of performing quantitative and qualitative safety analysis with temporal gates, thereby overcoming its limitation in capturing sequential failure behaviour. However, for many systems, it is often very difficult to have exact failure rates of components due to increased complexity of systems, scarcity of necessary statistical data etc. To overcome this problem, this paper presents a methodology based on fuzzy set theory to quantify temporal fault trees. This makes the imprecision in available failure data more explicit and helps to obtain a range of most probable values for the top event probability.

Keywords: Dependability Analysis; Fault Tree Analysis; Fuzzy Logic; Uncertainty analysis; Temporal Fault Trees.

1 Introduction

FTA is a widely used method for evaluating system reliability of safety-critical systems, and supports both qualitative as well as quantitative analysis. Fault trees show logical connections between faults and their causes [1] and thus make it possible to understand how combinations of failures of different components can lead to system failure. After construction of a fault tree, qualitative analysis is performed using Boolean logic by reducing it to minimal cut sets (MCSs), which show the smallest combinations of failure events that are necessary and sufficient to cause the top event. Quantitative analysis of a fault tree can estimate the probability of the top event occurring from the given failure rates of basic failure modes of the system [1].

Even though FTA is a powerful technique widely used in reliability engineering, conventional fault tree analysis has some limitations, e.g. in expressing time- or sequence-dependent dynamic behaviour [2–4] or in handling uncertainties and integrating human error in failure logic [5]. FTA has gone through different modifications to overcome these limitations, e.g. one recent modification is Pandora, which extends fault trees with temporal gates and provides temporal laws to allow qualitative analysis of dynamic systems [6]. Pandora can be used to determine the minimal cut sequences (MCSQs) that cause the top event.

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The outcome of any quantitative analysis largely depends on the accuracy of the failure rates used in the analysis. In conventional FTA, failure rates of components are typically considered to be constant. However, for many complex systems, it is often very difficult to estimate a precise failure rate of components from past occurrences due to lack of knowledge, scarcity of statistical data, and changes in operating environments of the systems etc. [5, 7]. This situation is especially relevant in the early design stages because at that time we may have to consider failure rates of new or undetermined components which have no available quantitative failure data, and thus precise failure rates could not possibly be known. Therefore, human judgment by linguistic expressions, such as 'very low, low, high, very high' can be used to define failure rates. In order to allow the conventional FTA to use linguistic variables and capture uncertainty, different modifications and improvements based on fuzzy logic have been proposed by different researchers [5, 7–11]. Fuzzy Logic is a branch of mathematics that deals with linguistic variables and provides an efficient way to draw conclusions from imprecise and vague information [12].

Recently, attempts have been made to quantify temporal gates in Pandora temporal fault trees [13, 14], but all of them are based on constant failure rates. These approaches do not consider inclusion of a degree of uncertainty in the failure rates of the basic events. However, if uncertainties are left unresolved, then even the most sophisticated and well-defined quantitative model may produce misleading results [5]. Therefore, in this paper, a fuzzy set theory based methodology is introduced to quantify Pandora temporal fault trees, overcoming the limitations in handling uncertainties in failure probabilities and allowing the use of linguistic variables. The failure rates of basic events / components are defined as fuzzy numbers, and then top events probabilities are calculated based on these numbers.

2 Preliminaries on Fuzzy Set Theory

2.1 Fuzzy Numbers and Fuzzy Sets

Fuzzy set theory has been developed to deal with imprecise, vague or partially true information [15]. A fuzzy number A can be thought of as a set of real numbers where each possible value has a weight between 0 and 1 which is referred to as the degree of membership defined by a membership function. Among different forms of fuzzy numbers, triangular fuzzy number (TFN) and trapezoidal fuzzy number (TZFN) are widely used in reliability analysis. Let $x, a, b, c, d \in \mathbb{R}$, and $\mu_A(x): \mathbb{R} \to [0,1]$ represents a membership function. Then, a trapezoidal fuzzy number $A = (a, b, c, d)$ is defined by the membership function as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & for\ a \le x \le b, \\ 1 & for\ b \le x \le c, \\ \frac{x-d}{c-d} & for\ c \le x \le d, \\ 0 & otherwise. \end{cases} \quad (1)$$

where $a < b < c < d$.

A fuzzy set $\tilde{A}$ of a fuzzy number A is defined by ordered pairs, in a binary relation:

$$\tilde{A} = \{(x, \mu_A(x)) \mid x \in A,\ \mu_A(x) \in [0,1]\} \quad (2)$$

where the membership function $\mu_A(x)$ specifies the degree to which any element x in A satisfies the predefined property P. Large values of $\mu_A(x)$ indicate a higher degree of membership.

Different methods are available to generate fuzzy numbers when no statistical data are available to estimate exact failure rates of components, e.g. 3σσ expression or expert knowledge elicitation [16]. The principle of the 3σσ method is described in [5]. The expert elicitation method has two basic forms: linguistic variables and interval values. The concept of linguistic variables is useful when little statistical data are available to estimate the failure rates of components of a system. The values of linguistic variables are words or sentences in natural languages. For example, we can consider "failure rate of component" as a linguistic variable consisting of fuzzy sets like *very low, low, fairly low, medium, fairly high, high, very high.* Linguistic variables play an important role in dealing with situations which are too complex or vague in nature, i.e., very difficult to describe using conventional quantitative expressions. Basic events can be assessed in a natural way and failure rates of the events can be estimated by suitable membership functions e.g., triangular or trapezoidal membership functions. Lower and upper bounds of a membership function can be obtained either from the point median value and an error factor or by direct assignment based on expert opinion.

2.2 Fuzzy Operators for Boolean Gates

Analogous to conventional FTA, the following fuzzy operators can be defined for the AND and OR gates of the temporal fault tree analysis (TFTA) [11].

AND gate fuzzy operator:

In TFTA, for all statistically independent events, the AND gate fuzzy operator is $P_{AND} = \prod_{i=1}^{n} P_i(t)$, where $P_i(t)(i = 1,2,3 \ldots n)$ is the failure probability of event *i* at time *t.* If the failure probability of event *i* is presented by a fuzzy number as $P_i(t) = (a_i(t), b_i(t), c_i(t), d_i(t))$, then the AND gate fuzzy operator is:

$$P_{AND} = \prod_{i=1}^{n} P_i(t) = \left(\prod_{i=1}^{n} a_i(t), \prod_{i=1}^{n} b_i(t), \prod_{i=1}^{n} c_i(t), \prod_{i=1}^{n} d_i(t)\right) \quad (3)$$

OR gate fuzzy operator:

In TFTA, for all statistically independent events, the OR gate fuzzy operator is $P_{OR} = 1 - \prod_{i=1}^{n}(1 - P_i(t))$, where $P_i(t)(i = 1,2,3 \ldots n)$ is the failure probability of event *i* at time *t.* If the failure probability of event *i* is presented by a fuzzy number as $P_i(t) = (a_i(t), b_i(t), c_i(t), d_i(t))$, then the OR gate fuzzy operator is:

$$P_{OR} = 1 - \prod_{i=1}^{n}(1 - P_i\,(t))$$

$$= (\, 1 - \prod_{i=1}^{n}(1 - a_i(t))\,,1\, - \prod_{i=1}^{n}(1 - b_i(t))\,,1 - \prod_{i=1}^{n}(1 - c_i(t))\,,1 - \prod_{i=1}^{n}(1 - d_i(t))) \quad (4)$$

3 Pandora Temporal Fault Tree Analysis

3.1 Pandora Temporal Gates and Logic

Pandora defines three temporal gates: Priority-AND, Priority-OR, and Simultaneous-AND [13, 14]. These gates allow analysts to represent sequences or simultaneous occurrence of events as part of a fault tree.

The Priority-AND (PAND) gate is used to represent a particular sequence of events and is defined as being true only if: 1) all input events occur; 2) input events occur in sequence from left to right; and 3) no input events occur simultaneously. The symbol '< ' is used to represent the PAND gate in logical expressions, i.e. X < Y means (X PAND Y).

The Priority-OR (POR) gate is used to indicate that one input event has priority and must occur first for the POR to be true, but does not require all other input events to occur as well. It can be used to represent trigger conditions where the occurrence of the priority event means that subsequent events may have no effect. The POR is true only if: 1) its left-most (priority) input event occurs; 2) no other input event occurs before the left-most input event; and 3) no other input event occurs at the same time as the left-most input event. The symbol ' | ' is used to represent the POR gate in logical expressions, thus X|Y means (X POR Y).

The Simultaneous-AND (SAND) gate is used to define situations where an outcome is only triggered if two or more events occur approximately simultaneously, e.g. because of a common cause, or because the events have a different effect if they occur approximately simultaneously as opposed to in a sequence. It is true only if: 1) all input events occur; and 2) all events occur at the same time. The symbol '&' is used to represent the SAND gate in logical expressions.

Note that the priority of the gates is as follows: SAND is highest, then PAND, POR, AND, and OR. Thus e.g. A+B&C<D is equivalent to A + ((B&C) <D). '+' is used here to represent OR and '.' is used to represent AND. It is also important to note that in Pandora it is not possible for an event to occur both at the same time and before/after another event, as this would be a contradiction; therefore, PAND and SAND are mutually exclusive, as are POR and SAND. Thus if X.Y is true, then exactly one of X<Y, X&Y, and Y<X must also be true. Furthermore, the structure function of a Pandora fault tree is monotonic, i.e. no event or gate can ever go from an occurred to non-occurred state [6]. In this paper, events are assumed to be non-repairable, to be statistically independent, and to have failure rates with exponential distributions — all common assumptions in FTA. Under these assumptions, the probability of two events occurring exactly at the same time is 0, therefore any MCSQs

containing SAND gates will not be considered for evaluation (as the full MCSQ would also evaluate to 0).

3.2 Fuzzy Probabilities of Temporal Gates

Fuzzy operators for PAND and POR can be derived from formulae in [13] and [17].

1. Fuzzy probability of the PAND gate

In a minimal cut sequence (MCSQ), if there are *N* statistically independent input events in a PAND gate and they occur sequentially, i.e., event 1 occurs first, then event 2,… *N*–1, and finally event *N*, then the probability of that PAND gate can be defined as:

$$P_{PAND} = \prod_{i=1}^{N} \lambda_i \sum_{k=0}^{N} \left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j \neq k}}^{N} (u_k - u_j)} \right] \tag{5}$$

where $u_0 = 0$ and $u_m = -\sum_{j=1}^{m} \lambda_i$ for $m > 0$.

If the failure rate of event *i* is represented by a fuzzy number as $\lambda_i = (a_i, b_i, c_i, d_i)$, then the fuzzy probability of the PAND gate expression is:

$$P_{PAND} = \left[\prod_{i=1}^{N} a_i \sum_{k=0}^{N} \left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j \neq k}}^{N} (u_k - u_j)} \right], \prod_{i=1}^{N} b_i \sum_{k=0}^{N} \left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j \neq k}}^{N} (u_k - u_j)} \right], \right.$$
$$\left. \prod_{i=1}^{N} c_i \sum_{k=0}^{N} \left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j \neq k}}^{N} (u_k - u_j)} \right], \prod_{i=1}^{N} d_i \sum_{k=0}^{N} \left[\frac{e^{(u_k t)}}{\prod_{\substack{j=0 \\ j \neq k}}^{N} (u_k - u_j)} \right] \right] \tag{6}$$

If there are 2 input events in the PAND gate, then according to [17], equation (6) reduces to:

$$P_{PAND} = \frac{\lambda_2}{(\lambda_1 + \lambda_2)} - e^{(-\lambda_1 t)} + \frac{\lambda_1}{(\lambda_1 + \lambda_2)} e^{[-(\lambda_1 + \lambda_2)t]} \tag{7}$$

2. Fuzzy probability of the POR gate

For any minimal cut sequence of *N* statistically independent events in a POR gate with the expression $E_1|E_2|\ldots|E_{N-1}|E_N$, and failure rates $\lambda_1, \lambda_2, \ldots, \lambda_{N-1}, \lambda_N$ respectively, then the probability of the POR gate can be defined as:

$$P_{POR} = \frac{\lambda_1 \left(1 - \left(e^{-(\sum_{i=1}^{N} \lambda_i)t} \right) \right)}{\sum_{i=1}^{N} \lambda_i} \tag{8}$$

If the failure rate of event *i* is represented by a fuzzy number as $\lambda_i = (a_i, b_i, c_i, d_i)$, then the fuzzy probability of that POR gate expression is:

$$P_{POR} = \left[\frac{a_1\left(1-\left(e^{-\left(\sum_{i=1}^{N} a_i\right)t}\right)\right)}{\sum_{i=1}^{N} a_i}, \frac{b_1\left(1-\left(e^{-\left(\sum_{i=1}^{N} b_i\right)t}\right)\right)}{\sum_{i=1}^{N} b_i}, \frac{c_1\left(1-\left(e^{-\left(\sum_{i=1}^{N} c_i\right)t}\right)\right)}{\sum_{i=1}^{N} c_i}, \frac{d_1\left(1-\left(e^{-\left(\sum_{i=1}^{N} d_i\right)t}\right)\right)}{\sum_{i=1}^{N} d_i} \right] \quad (9)$$

3.3 Fuzzy top-event probability and most likely failure probability

Quantitative analysis of a temporal fault tree provides a way to estimate the probability of the top event occurring from the given failure rates of basic components. All the basic event failure rates are considered as fuzzy numbers to minimize error due to vagueness or uncertainty in the data. The fuzzy probability of MCSQs consisting of AND, PAND and POR gates are estimated by using (3), (6) and (9) respectively. On getting the fuzzy probabilities of all MCSQs, the fuzzy top-event probability can be obtained by (4). As failure rates are considered as fuzzy numbers, all results obtained are also fuzzy numbers with a membership function. We can represent fuzzy set of fuzzy top-event probabilities $P_T \triangleq (P_{aT}, P_{bT}, P_{cT}, P_{dT})$ as:

$$\tilde{P}_T \triangleq \{(P_{aT}, \mu(P_{aT})), (P_{bT}, \mu(P_{bT})), (P_{cT}, \mu(P_{cT})), (P_{dT}, \mu(P_{dT}))\}$$

where P_{aT}, P_{bT}, P_{cT} *and* P_{dT} are elements of the fuzzy number P_T and $\mu(P_{aT}), \mu(P_{bT}), \mu(P_{cT})$ and $\mu(P_{dT})$ are membership values of those elements.

This may be one of the intended outcomes of the TFTA, but if required, the most likely top-event (failure) probability can be obtained from a fuzzy top-event probability via defuzzification; a process of mapping values from a fuzzy domain into a crisp domain. Although several other methods also exist, the weighted average method can be used to obtain the most likely top-event probability as follows:

$$M(P_T) = \frac{P_{aT}\times\mu(P_{aT})+P_{bT}\times\mu(P_{bT})+P_{cT}\times \mu(P_{cT})+P_{dT}\times \mu(P_{dT})}{\mu(P_{aT})+\mu(P_{bT})+\mu(P_{cT})+ \mu(P_{dT})} \quad (10)$$

4 Case Study

For the purposes of illustrating the application of fuzzy logic in quantitative temporal analysis, we use the fuel system first presented in [13], shown in Fig.1. The system is a redundant fuel distribution system for a ship. Under ordinary operation, there are two primary fuel flows, one for each engine: Pump1 delivers fuel from Tank1 to Engine1, and Pump2 delivers fuel from Tank2 to Engine2. Flowmeters monitor the rate of flow to each engine and provide sensor information to the Controller.

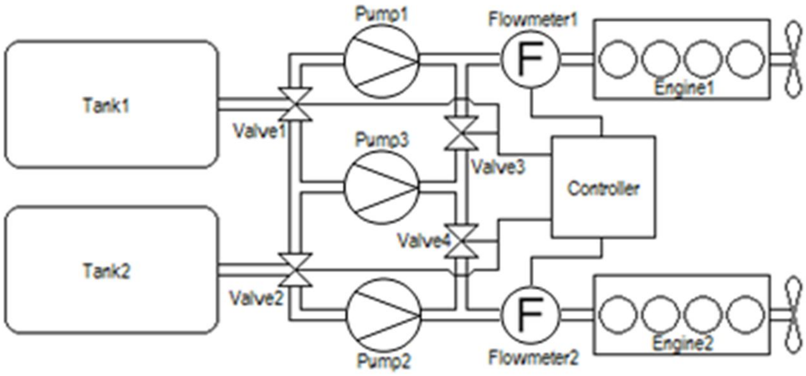

Fig. 1. Fuel distribution system

The Controller introduces dynamic behaviour to this system, allowing it to adapt to possible failures. If either flowmeter detects insufficient flow, the Controller can activate the standby Pump3 and redirects fuel flow accordingly using the valves. For example, if there is a problem with the flow to Engine1, the Controller can switch Valve1 and open Valve3 so that fuel flows from Tank1 to Engine1 via Pump3. However, Pump3 can only be used to replace either Pump1 or Pump2, but not both. A failure of both Pump1 and Pump2 will result in at least one engine being starved of fuel; for example, if Pump1 fails and Pump3 replaces it, Pump3 is then no longer available to replace Pump2 if that pump also fails. This results in degraded propulsion functionality for the vessel, as speed and maneuverability will be reduced.

Temporal gates can be used to model the dynamic behaviour in this scenario and helps to correctly capture the sequences of events that lead to failure. At the top level, the causes of omission of fuel to Engine1 can be expressed using temporal gates as follows (Engine2 is symmetrical, but with the order of events reversed):

```
O-Engine1= ((O-Pump1 | O-Pump2).O-Valve3)
            + (O-Pump2 < O-Pump1)
            + (O-Pump2 & O-Pump1)
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Thus omission of fuel to Engine1 (`O-Engine1`) has three possible causes, depending on the sequence of events:

- If there is no fuel from Pump1 (O-Pump1), then Pump3 replaces it, as long as Pump2 has not failed first; this precondition can be represented using the POR gate. Thus in this situation, an omission of fuel can be caused by omission of fuel from both Pump1 and Pump3 (via Valve3).
- If Pump2 fails first, then Pump3 replaces it and will be unavailable to replace Pump1 if it also fails. Thus sequential failure of Pump2 and then Pump1 will lead to an omission of fuel to Engine1 (represented using the PAND gate).
- If both Pump2 and Pump1 fail at the same time (represented with the SAND gate), then Pump3 can only replace one of them. Behaviour in this situation is non-

deterministic (as Pump3 may replace either Pump1 or Pump2, but not both), and thus as a pessimistic estimation, simultaneous failure of Pump1 and Pump2 is given as a cause of failure for both engines.

After performing a qualitative analysis on this system, the resulting minimal cut sequences are as follows:

```
E1 = (P1|P2).P3 + (P1|P2).V1 + (P1|P2).V3 + (S1<P1)|P2
   + (S1&P1)|P2 + (CF<P1)|P2 + (CF&P1)|P2 + P2<P1 + P1&P2
E2 = (P2|P1).P3 + (P2|P1).V2 + (P2|P1).V4 + (S2<P2)|P1
   + (S2&P2)|P1 + (CF<P2)|P1 + (CF&P2)|P1 + P1<P2 + P1&P2
```

The failure events of MCQS are:

- P1/P2/P3 = Failure of Pump1/2/3 (e.g. blockage or mechanical failure)
- V1/V2/V3/V4 = Failure of Valve 1/2/3/4 (e.g. blockage or stuck closed)
- S1/S2 = Failure of Flowmeter1/2 (e.g. sensor readings stuck high)
- CF = Failure of the Controller

As O-Engine1 and O-Engine2 are caused by the same events in the opposite sequences, the fuzzy probability of these two top events are same. As mentioned earlier, we assume that all events are independent and the probability of two independent events occurring at the same time is effectively 0, therefore we will not consider any MCSQ consisting of SAND gate. Thus for this example system, we will not consider S1&P1|P2, CF&P1|P2 and P1&P2 during quantification of the fuzzy probability of the top event.

In this paper, we have used a trapezoidal membership function to convert basic event failure data to a trapezoidal fuzzy number. Fuzzy failure rate information for the example system is shown in Table 1. Results of the fuzzy quantitative evaluation of each minimal cut sequence of top event are shown in Tables 2 and 3 respectively. The results are obtained by considering that the system is operating at 10000 hours of its life cycle, i.e. t=10000 hours.

Table 1. Fuzzy failure rates of components for fuel system

Component	Failure rate/hour("Around")	Trapezoidal representations			
	(Point median value, λ_p)	λ_A	λ_B	λ_C	λ_D
Tanks	1.5E-5	7.5E-6	1.125E-5	1.875E-5	2.25E-5
Valve1 & Valve2	1E-5	5E-6	7.5E-6	1.25E-5	1.5E-5
Valve3 & Valve4	6E-6	3E-6	4.5E-6	7.5E-6	9E-6
Pump1 & Pump2 & Pump3	3.2E-5	1.6E-5	2.4E-5	4E-5	4.8E-5
Flowmeter Sensor	2.5E-6	1.25E-6	1.875E-6	3.125E-6	3.75E-6
Controller	5E-7	2.5E-7	3.75E-7	6.25E-7	7.5E-7

Table 2. Fuzzy probability of first four MCSQs for top event O-Engine1

Failure Rate	Pr ((P1 | P2).P3)	Pr ((P1 | P2).V1)	Pr ((P1 | P2) .V3)	Pr ((S1 < P1) | P2)
λ_A	2.025E-2	6.677E-3	4.045E-3	8.696E-4
λ_B	4.067E-2	1.377E-2	8.386E-3	1.827E-3
λ_C	9.077E-2	3.235E-2	1.989E-2	4.437E-3
λ_D	1.176E-1	4.299E-2	2.656E-2	5.983E-3

Table 3. Fuzzy probability of remaining two MCSQs for top event O-Engine1

Failure Rate	Pr ((CF < P1) | P2)	Pr (P2 < P1)
λ_A	1.751E-4	1.093E-2
λ_B	3.69E-4	2.276E-2
λ_C	9.02E-4	5.434E-2
λ_D	1.22E-3	7.266E-2

Using (4) the fuzzy probability of the top event is obtained as follows:

P_T = (4.232E–2, 8.518E–2, 1.889E–1, 2.432E–1).

According to the results, the interval [8.518E–2, 1.889E–1] is the most likely range of values for the top event probability, whilst 4.232E–2 and 2.432E–1 are the lower and upper bound of the top event probability respectively. To verify the accuracy of the result the same case study was modelled in Isograph Reliability Workbench 11.0 (IRW) [18] and using the point median value of the failure rate, the top event probability was 1.497E-1, which lies within the range of most likely values obtained by the proposed method. The fuzzy top event probability can also be mapped into a single value by defuzzification using (10); if the fuzzy top event probability is as follows:

$\tilde{P}_T$ = {(4.232E–2, 0.75), (8.518E–2, 1), (1.889E–1, 1), (2.432E–1, 0.75)}.

Then using (10), the most likely top event probability is 1.395E–1 which is relatively close to the value obtained using Isograph Reliability Workbench.

5 Conclusion

In this paper, we showed how uncertainty can be incorporated in TFTA by applying fuzzy set theory to Pandora temporal fault trees. Adopting a fuzzy methodology may help to model situations where limited quantitative information is available and often only with a wide range of uncertainty. The method we present is capable of handling the linguistic variables and the imprecision of the uncertainties associated with the modelling of failures and their dependencies, and can more explicitly highlight areas of uncertainty in the data. This can lead to a more effective quantification of uncertain failure data in dynamic systems, producing more realistic and robust results that help to avoid mistaken assumptions and potential over/under estimations of system reliability. However, it is important to emphasise that the results can only be as reliable as the input data, and the inclusion of fuzzy data cannot create accuracy where none previously existed. In future, we hope to extend this work by looking at how temporal FTA approaches like Pandora could be extended to include fuzzy logic operators, as well as to further develop practices for performing uncertainty analysis.

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