Does Search Boost Efficiency?

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Abstract

Poaching externality, from job-to-job turnovers, which implies planer should allocate less resources on costly job creations. However, these search efforts increase competitions among employers, which could internalize the externality. Whereas the congestion externality requires unit-elastic matching function.

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The seminal work by Burdett and Mortensen (1998) (hence B&M) assumed an exogenous contacting rate, which causes deficiency in understanding of the labour demand. Following Mortensen (2000), the on-the-job search model of B&M is integrated into the matching framework of Pissarides (2000). By allowing different search efforts for employed and unemployed workers, this paper investigates welfare implications of job search.

Mortensen (2000) assumed the same contracting rate for both on-the-job search and off-the-job search. Hence, the cost of employment is zero, and the reservation wage of the unemployed is equal to the value of leisure, thus it is independent from other parameters, such as market tightness and wage-offer distribution. Accordingly, Mortensen (2010) argues that dispersed wage equilibrium is more efficient than the degenerate equilibrium (Diamond 1971). To understand the welfare consequences of job-to-job transactions in general, it is important to understand how these turnovers influence the wage-setting strategy of firms.

Beyond Mortensen (2000), modelling the search problem with different contacting rates allows to split different types of externality which arise in a decentralized market. Specifically, we show that there are two types of externality in the market, congestion externality and poaching externality, with the former causing welfare loss to vacant jobs, and the latter to the filled ones. The poaching externality, i.e. the reason for inefficiency in Mortensen (2000), drives the socially efficient level of vacancies down, but it is internalized in the market, through competition among wage setting firms. Specifically, we show that a higher job-to-job turnover leads to a higher supremum of distribution of wage offers and a stronger employment effect. However, constrained efficiency requires the matching function to be unit elastic to internalize the crowding out externality. Hence, the argument in Mortensen (2010) is not necessarily valid in general. Indeed, on-the-job search drives the numbers of vacancies down in a market equilibrium; however, the socially efficient number of firms is also lower, due to the poaching externality. It is unclear whether a market with dispersed wage is more efficient than one with a single monopoly wage.

1 The Model

Time is continuous. Ex-ante homogeneous firms and workers are infinitely lived, risk neutral, and discount with r. The measure of workers is 1, of which u are unemployed. The measure of employers is determined through free entry. The output of a filled job, y, and the vacancy cost, c are constant.

Workers search with effort s. However, the unemployed search with $s = 1$

but the employed with $0 < s < 1$.[1]There is no searching cost. m is the number of aggregate contacts,

$$m = m(v, u + s(1-u))$$

which satisfies the standard assumptions.[2] $\theta = v/(u + s(1-u))$ denotes the effective market tightness. The average job offer arrival rate for an unemployed worker is

$$\lambda = \frac{m(v, u + s(1-u))}{u + s(1-u)} \equiv \lambda(\theta)$$

The offer arrival rate of an employed worker is instead $s\lambda(\theta)$. The contact rate of a vacancy is $q(\theta) = \frac{\lambda(\theta)}{\theta}$.

2 Search Equilibrium

Following B&M, a job offer is a random draw from a wage offer distribution F on a support of $[\underline{w}, \overline{w}]$. Let J^U denote the present value for an unemployed worker, and $J^E(w)$ for the employed at wage w. The stead-state implies that

$$rJ^U = b + \lambda(\theta) \int_{\underline{w}}^{\overline{w}} \max\left\{J^E(w') - J^U, 0\right\} dF(w') \quad (1)$$

$$(r+\delta)J^E(w) = w + s\lambda(\theta) \int_{\underline{w}}^{\overline{w}} \max\left\{J^E(w') - J^E(w), 0\right\} dF(w') + \delta J^U \quad (2)$$

An unemployed worker obtains unemployment benefits b and receives wage offers at a rate $\lambda(\theta)$, in which case he accepts it if and only if the capital gains are non-negative. An employed worker earns a flow income w, and the job ends at rate δ, leaving the worker unemployed; the worker may get another offer at rate $s\lambda(\theta)$, leading to another job if capital gains are non-negative. Hence, there is a common reservation strategy for all the unemployed, such that $J^E(R) = J^U$, and $\underline{w} = R$ in any equilibrium.

Let $G(w)$ represents the proportion of workers receiving a wage not greater than w. J^V is the present-discounted value of a vacancy, and $J^F(w)$ the value of a filled job which pays a wage w. Specifically,

$$rJ^V = q(\theta) \max_{w \geq R} \left\{ \frac{u + s(1-u)G(w)}{u + (1-u)s} [J^F(w) - J^V] \right\} - c \quad (3)$$

The wage offer $w \geq R$ will be accepted if the worker is unemployed, with a probability $u/(u + s(1-u))$ or, if he is employed with a wage not greater

[1]For empirical evidence, see Christensen et al. 2005, Rosholm and Svarer 2004.

[2]For details of aggregate matching function see Pissarides and Petrongolo (2001).

than w, with a probability $s(1-u)G(w)/(u+s(1-u))$. Similar arguments define $J^F(w)$,

$$J^F(w) = \frac{y-w}{r+s\lambda(\theta)[1-F(w)]+\delta} \tag{4}$$

With $r \to 0$, the unique solution to the search-and-matching equilibrium is defined recursively through the conditions of reservation wage,

$$R = \frac{[s\lambda(\theta)+\delta]^2 b + (1-s)s\lambda^2(\theta)y}{[s\lambda(\theta)+\delta]^2 + (1-s)s\lambda^2(\theta)} \tag{RC}$$

the job creation condition,

$$\frac{c}{q(\theta)} = \frac{\delta(y-b)}{[s\lambda(\theta)+\delta]^2 + (1-s)s\lambda^2(\theta)} \tag{JC}$$

and the steady-state unemployment,

$$u = \frac{\delta}{\delta+\lambda(\theta)}. \tag{SS}$$

3 Efficiency

Subject to the matching constraint, a utilitarian planner chooses job vacancy level and unemployment to maximize the income flow net of recruiting costs:

$$\max_{u,v}\{(1-u)y + ub - cv\} \tag{5}$$

$$\text{s.t.} \quad \delta(1-u) = \lambda(\theta)[1-F(R)]u$$

By virtues of $\theta = v/[u+s(1-u)]$ and $F(R)=0$, the unique solution to the planner's problem, θ^*, is such that

$$c = \frac{(y-b)\lambda'(\theta^*)\delta}{[\delta+\lambda(\theta^*)][\delta+s\lambda(\theta^*)] - \theta^*(1-s)\delta\lambda'(\theta^*)} \tag{6}$$

To derive the relationship between efficient level of job creation and efforts of on-the-job search by implicitly differentiating the above first-order condition,

$$\frac{d\theta}{ds} = \frac{\lambda'(\theta)[\lambda(\theta)\delta + \lambda(\theta)^2 + \theta\delta\lambda'(\theta)]}{(\delta+\lambda(\theta))[\lambda^2(\theta)\delta + \lambda^2(\theta)s\lambda(\theta) - 2s\lambda'(\theta)^2]} < 0 \tag{7}$$

As $\lambda(.)$ is strictly concave the above expression is negative. This finding leads the following result:

Proposition 1. *A higher on-the-job search effort implies a lower efficient number of job vacancies to be created due to poaching externality.*

Comparing with Pissarides (2000), whereas $s = 0$, on-the-job search creates extra costs, i.e. the poaching externality, for the planner. As the matching is a stochastic rationing, if a vacancy is filled by the unemployed, other unfilled jobs are worse off, but the other unemployed are better off (thick market effect). On the other hand, a job, terminated due to the loss of its employee to another firm which pays higher wages, turns into a vacancy. Opposed to the standard model, these vacancies do not leave the market. Henceforth, the firm poached by other employers has to bear a vacancy and start paying c. Accordingly, by recruiting an employed worker, a firm exercises the poaching externality, thus rendering the filled firms worse off, but leaving unemployed welfare unchanged. In short, filled firms suffer losses due to the workers could transfer from job to job, i.e. the poaching externality.

The planner values this negative externality in a sense that, as job creation is costly, firms recruiting from amongst the unemployed drive the number of vacancies down. Instead, if a vacancy is filled through poaching, the aggregate number of vacancies in the market is unchanged. In other words, for vacant jobs, the average length of search is longer with job-to-job turnovers. Consequently, the higher the on-the-job search effort, the stronger this externality, and the higher efficiency loss to allocate resources to create vacancies, i.e. a cost to the planner, hence the lower θ^*.

However, by comparing with the (JC) condition, the following theorem argues that the poaching externality is internalised. Instead, the constrained efficiency requires special properties of the matching function

Theorem 1. *The decentralized labour market is constrained efficient if and only if the matching function is unit-elastic.*

Proposition 1 is the Hosios (1990) condition in terms of wage posting. As a monopsony wage setter, a firm has full bargaining power. According to the Hosios condition, the search equilibrium is socially efficient if, and only if, the matching function is homogeneous of degree one and the firm's share of surplus β equals the elasticity of the matching function in regards to the vacancy. In a wage posting game $\beta = 1$, hence the Hosios condition implies theorem 1.

The elasticity of matching function, $\eta(\theta)$,[3] measures the relative effectiveness of the firms in making contacts. Hence, $1 - \eta(\theta)$ measures congestion externality that marginal firms caused to other firms. Following the

[3]Formally, the elasticity of the matching function with respect to vacancy can be derived

$$\eta(\theta) = \frac{\partial m(u + s(1-u), v)/\partial v}{m(u + s(1-u), v)/v} = \frac{\frac{v}{u+s(1-u)} m_v(u + s(1-u), v)}{\frac{m(u+s(1-u),v)}{u+s(1-u)}} = \frac{\theta\lambda'(\theta)}{\lambda(\theta)}.$$

Hosios condition, constrained efficiency requires that the agents who cause congestion externality are punished by grant with a lower bargaining power. Following this principle, wage posting firms have full bargaining power, accordingly the decentralized market is constrained efficiency if and only if the firm's congestion externality is neutralised, i.e. the matching function is unit-elastic. As an implication of the theorem, it is straightforward to see that if $\eta(\theta) < 1$, for example, a Cobb-Douglas function, there will be excess entry of vacancies in a decentralized market, due to the congestion externality, that is not been taken into consideration.

Constrained efficiency only concerns of crowding out externality as s may "internalize" poaching externality through two mechanisms: first, it increases the supremum of wage-offer distribution F; second, it boosts employment effects. As workers could switch to a better job with a higher wage, firms have an incentive to raise their no-quitting premium, so that the supremum of F rises, i.e. $\partial\overline{w}/\partial s > 0$.[4] The *employment effect* states that the worker's wage distribution first-order stochastically dominates the wage offer distribution, i.e. $G(w) < F(w)$. For any wage w, $G(w)$ decreases in s, and this implies that a higher on-the-job search effort boosts the employment effect and shifts G down, so that there will be more workers employed with a higher wage. In short, competition among firms offsets the poaching externality. Following theorem 1, the decentralized market is constrained efficient if, and only if, the congestion externality is "internalized".

4 Conclusion

Market frictions lead to job search, which in turn results in equilibrium wage dispersion. On the other hand, search incurs poaching externality and congestion externality. Due to competitions among employers, poaching externality from job-to-job turnovers is internalized in the market. Hence, a decentralized market is efficient if and only if the matching is unit-elastic.

[4]One can derive this relation by solving equilibrium reservation wage and showing

$$\frac{\partial\overline{w}}{\partial s} = \frac{(y-b)\lambda(\theta)\delta^2(2\delta+\lambda(\theta))}{[2s\lambda(\theta)\delta+\delta^2+s\lambda^2(\theta)]^2} > 0.$$

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