New Methodology for Optimizing Transit Priority at the Network Level

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A new methodology for optimizing transit road space priority at the network level is proposed. Transit vehicles carry large numbers of passengers within congested road space efficiently. This aids justification of transit priority. Almost all studies that have investigated transit priority lanes focus at a link or an arterial road level, and no study has investigated road space allocation for priority from a network perspective. The aim of the proposed approach is to find the optimum combination of exclusive lanes in an existing operational transport network. Mode share is assumed variable, and an assignment is performed for both private and transit traffic. The problem is formulated by using bilevel programming, which minimizes the total travel time. The approach is applied to an example network and the results are discussed. The approach can identify the optimal combination of transit priority lanes and achieve the global optimum of the objective function. Areas for further development are discussed.

Urban traffic congestion is a major challenge facing transport networks in almost all world cities. Transit vehicles carry large numbers of passengers within congested road space efficiently. This is the major justification for provision of exclusive transit priority lanes (*1*, pp. 58–73). While a range of research has concerned provision of transit priority, all of the research has focused on localized projects at a link or arterial corridor level. No research has considered network-based planning of transit priority systems.

This paper presents a new methodology to optimize provision of road transit priority at a network level.

RESEARCH BACKGROUND

Transit priority involves a trade-off between automobile traffic, with low occupancy and high traffic volume, and on-road transit, with high occupancy and low traffic volume. In urban areas with high transit demand, it is relatively easy to justify transit priority. However, in situations of low transit usage and low service frequency, justification is more problematic.

One way to give priority to transit is to provide exclusive lanes for transit vehicles. This is known as road space allocation (RSA) (*2, 3*). RSA in the literature can be classified into two levels or approaches: (*a*) the local level, in which priority is given to transit on one link or

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a series of consecutive links in the form of a corridor or an arterial, and (*b*) the network level, in which priority is given to transit on any link in the network.

A summary of the transit priority studies and their characteristics carried out at the local and network level is presented in Table 1. Bly et al. (*4*) explored transit priority at the local level. An exclusive bus lane was introduced to a link in different conditions, and the associated impacts on the rest of the network were assessed by running a sensitivity analysis. Black (*2*) presented a model for RSA on urban arterials. The model evaluated several predefined scenarios with a performance measure (total user travel time), and all the space allocation was introduced at corridor level. Although a mode choice model was applied to the studied corridor, the secondary network effects of road space change such as route choice were ignored. In another attempt, Jepson and Ferreira (*5*) assessed different road space priority such as bus lane and setbacks on the basis of delays.

Currie et al. (*7*) presented a comprehensive review of the studies on RSA. Having compared the performance measures in the literature, they then proposed an approach to evaluation of transit priority projects. Their approach considered a comprehensive list of impacts such as travel time, travel time variability, and initial and maintenance cost. An analysis method for a long arterial was given by Eichler and Daganzo (*6*) following the concept of intermittent bus lanes (*10, 11*).

All these studies focused on providing priority at the local level. Therefore, the range of priority alternatives was limited and would not necessarily result in the best possible RSA. Furthermore, all of the studies at the local level proposed and utilized an evaluation method that requires the RSA as input to the evaluation framework. Despite this limitation, the local level is accounted for in most of the research carried out on transit priority. That is probably because studies at this level are simpler to carry out and justify at the local level, need less administrative coordination between transportation authorities, and are less costly to implement. At the network level it can be argued that much research on the transit network design problem (*12–15*) would seek the optimal combination of many variables such as bus routes, frequencies, and timetables, and therefore they could be considered as solutions to the RSA. A few researchers have included exclusive lanes in the network design process (*16*). However, since there is already an operational transit network in many cities, rearrangement of the network routes would require public acceptance and entail significant cost.

Waterson et al. (*8*) gave an evaluation approach for a given priority scenario at the network level. The approach considered rerouting, retiming, modal change, and trip suppression. A similar evaluation approach was carried out by using a microsimulation (*9*). However, such approaches can be used as evaluation tools and cannot provide the best combination of priority lanes. No optimization approach has been suggested in the literature for transit network space priority.

TABLE 1 Components of Transit Priority Approaches

	Research Reference						
	Priority at the Local Level					Priority at the Network Level	
	Bly et al. (4)	Black (2)	Jepson and Ferreira (5)	Eichler and Daganzo (6)	Currie et al. (7)	Waterson et al. (8)	Liu et al. (9)
Evaluation Criterion							
Travel time impact	✓	✓	✓	✓	✓	✓	✓
Resource impact							
Fuel costs	✓	x	x	x	✓	x	x
Capital costs	x	x	x	x	✓	x	x
Maintenance costs	x	x	x	x	✓	x	x
Transit fleet and crew costs	x	x	x	x	✓	x	x
Construction impacts	x	x	x	x	✓	x	x
Accident cost impacts	x	x	x	x	✓	x	x
Environmental impacts	x	x	x	x	✓	x	x
Public transport reliability impacts	x	x	x	x	✓	x	✓
Modeling Approach							
Traffic flow modeling							
Standard static analytical or mathematical model	✓	✓	—	—	—	✓	—
Dynamic traffic simulation modeling used	—	—	✓	✓	✓	—	✓
Travel behavior modeling							
Speed–flow capacity analysis	✓	✓	✓	✓	✓	✓	✓
Traffic route diversion considered	✓	x	x	x	✓	✓	✓
Mode shift considered	✓	x	x	x	✓	✓	✓
Trip generation considered	x	x	x	x	✓	✓	✓
Disappearing traffic considered	x	x	x	x	x	✓	✓
Scope of the Priority Scheme							
Evaluation of given alternatives	✓	✓	✓	✓	✓	✓	✓
Optimization over the network	x	x	x	x	x	x	x

NOTE: ✓ = included, x = not included, — = not relevant.

Therefore, it is important to develop a methodology to find the optimal combination of exclusive lanes in a given road and transit network.

METHODOLOGY

Because of the vast number of theoretical and practical factors involved in transit priority planning, from demand prediction to detailed appraisal of impacts on users, the problem is introduced with certain simplifying assumptions to demonstrate the applicability of the proposed method. Later it is shown that most of the adopted assumptions can be relaxed without imposing major limitations on the proposed methodology.

Assumptions

The following assumptions are made in this study:

1. The origin–destination (O-D) matrix (total travel demand) is given and remains the same during the analysis period.
2. Only two modes of traffic (buses and private cars) use the network.
3. Road network layout, link characteristics, cost functions, and control details are known.
4. The transit (bus) routes, frequencies, and stop locations are given.

5. Users' travel time is the only performance measure affected by a priority scheme.

Formulation of the Optimization Program

A transport project has many benefits and impacts, which can be defined from different stakeholder perspectives (17, 18). For transit network design, a number of objective functions are introduced in the literature (19, 20). More specifically, for transit priority projects, many of these impacts have been collected in the United Kingdom (21), which were implemented by Currie et al. (7) to evaluate an exclusive bus lane project. The same comprehensive evaluation as given by Currie et al. can be employed here; however, as discussed earlier, for simplicity the total travel time is defined as the key impact of the priority measure from the user perspective.

The network RSA problem is written as a bilevel programming, and the sum of travel time for all users is included in the upper level. With the introduction of an exclusive lane, a mode shift is expected, and a reassignment would have to be carried out for the network. Therefore, a modal split between transit and passenger cars and an assignment model are formulated in this study at the lower level. In other words, the objective function at the upper level is subject to constraints of the modal split and the assignment results.

The objective function is given in Equation 1.

$$\min Z = \sum_{(i,j)\in A} x_{ij}^{c} t_{ij}^{c} + \sum_{(i,j)\in B} x_{ij}^{t} t_{ij}^{t} \tag{1}$$

subject to

$$\phi_{ij} = \begin{cases} 0 & t_{ij}^c = t_{ij}^{t,\mathrm{IVT}} = f_1^c \left(x_{ij}^c + x_{ij}^t \right) \\ 1 & t_{ij}^c = f_2^c \left(x_{ij}^c \right), \ t_{ij}^{t,\mathrm{IVT}} = f_2^t \left(x_{ij}^t \right) \end{cases} \quad \forall (i,j)$$

where Z is the objective function; x_{ij}^c, t_{ij}^c and x_{ij}^t, t_{ij}^t are the passenger flow and travel time on link (i, j) for private car and transit, respectively. The set of all links is regarded as Set A. Set B is a subset of A and includes links that have a transit route. The decision variable ϕ_{ij} is 0 when the link operates with mixed traffic and 1 when one exclusive lane is introduced as a result of a priority scheme. Let Φ be the array of ϕ_{ij} $\forall (i, j)$ showing one feasible combination of exclusive lanes in the network. Functions $f_1^c, f_2^c,$ and f_2^t are cost functions respective to ϕ_{ij} conditions. An example of these functions is given in the numerical example section. The expression $t_{ij}^{t,\mathrm{IVT}}$ represents the in-vehicle travel time on link (i, j).

The lower level is, first, a mode choice model and, second, a minimization of the network equilibrium problem with fixed demand (22). In this study, a logit model for mode choice is used. The model estimates the disutility of each mode on the basis of factors such as travel time, convenience, and cost as follows (23):

$$P^{c/t} = \frac{\exp\left(U^{c/t}\right)}{\exp\left(U^c\right) + \exp\left(U^t\right)}$$

$$U^{c/t} = a_1 * X_1^{c/t} + a_2 * X_2^{c/t} + \cdots + a_n * X_n^{c/t} \tag{2}$$

where $X_1, X_2, \ldots, X_n$ are the attributes of modes car (c) and transit (t) such as travel time and out-of-pocket cost; $a_1, a_2, \ldots, a_n$ are constant coefficients of the model.

After calculation of mode shares, in the second stage of the lower-level analysis traffic demand is assigned to the road network, which accounts for travel time on links and flows. Bus frequencies are constant; therefore it is only necessary to assign private travel demand to the network. This is a minimization problem in itself and requires much computational effort. Equation 3 presents a formulation with a single mode.

$$\min Y = \sum_{(i,j)\in A} \int_0^{x_{ij}} t_{ij}^c(\omega)d\omega \tag{3}$$

subject to

$$\sum_k f_k^{rs} = q_{rs} \quad \forall r,s$$

$$f_k^{rs} \geq 0 \quad \forall k,r,s$$

where f_k^{rs} is the flow on path k connecting origin r to destination s, q_{rs} is the trip rate between r and s, and x_{ij} is given by the following equation, in which $\delta_{k,ij}^{rs}$ is 1 if link (i, j) is on path k for any rs and zero otherwise.

$$x_{ij} = \sum_{rs} \sum_k f_k^{rs} \delta_{k,ij}^{rs} \quad \forall (i,j) \tag{4}$$

The link travel time is acquired when private car demand is assigned to the network. Since in mixed traffic buses travel with cars, bus passengers' in-vehicle travel time on a link is assumed to be equal to car travel time on that link plus access time and waiting time at stops. Transit passenger travel demand can be assigned to the tran-

sit network by using one of the methods explained in the literature (24–27). For example, Spiess and Florian (24) formulate the transit assignment problem as follows:

$$\min W = \sum_{\mathrm{sec}} v_{\mathrm{sec}} t_{\mathrm{sec}} + \sum_{jp} w_{jp} \tag{5}$$

subject to

$$\sum_{\mathrm{sec} \in L_j^+} v_{\mathrm{sec}} + g_j = \sum_{\mathrm{sec} \in L_j^-} v_{\mathrm{sec}} \quad \forall j$$

$$v_{\mathrm{sec}} = \frac{X_{\mathrm{sec}} f_{\mathrm{sec}} V_{jp}}{\sum_{\mathrm{sec} \in S_{jk}} f_{\mathrm{sec}} X_{\mathrm{sec}}} = X_{\mathrm{sec}} f_{\mathrm{sec}} w_{jp}$$

where

S_{jp} = set of line sections directly connecting nodes j and p;

L_j^+ = set of outgoing line sections from node j;

v_{sec}, t_{sec}, and f_{sec} = flow, in-vehicle travel time, and frequency on line section sec, respectively;

g_j = number of trips going to destination node j;

V_{jp} = total flow on route section jp; and

X_{sec} = 1 if line section sec $\in S_{jp}$ is attractive and zero otherwise.

The passenger demand on a section may exceed the capacity of the section in this method. Therefore, the load factor of transit routes (the ratio of the number of passengers to transit vehicle capacity) should be checked for the final answer.

The link and path travel time and flow are determined in traffic assignment by Equations 3 through 5 while they are needed in the utility functions of the modal split model (see Equation 2). In fact, the final values of traffic flows can be found iteratively. In mode choice, some values are assumed for travel time on path (t_k^{rs}), which are corrected in the assignment step. As a result of the assignment, t_k^{rs} is changed, which in turn changes the mode utilities. This cycle continues until the travel times and flows converge to the final values.

Other constraints could be considered for the transit priority problem introduced in Equation 1, such as minimum length of an exclusive lane or maximum allowed change in an O-D pair travel time. They can easily be included in the proposed model; however, in this paper they are assumed to be of any value.

Traffic Assignment

When a lane on a link is changed from mixed traffic flow to an exclusive lane for transit (e.g., buses), the capacity to accommodate private cars is reduced. The reduction in capacity may substantially increase travel time for private cars. In this study the cost function (flow–travel time function) of a link is based on the function introduced by the Bureau of Public Roads (28):

$$t = t_0 \left(1 + \alpha \left(\frac{x}{C} \right)^\beta \right) \tag{6}$$

where t, t_0, x, and C are travel time, free-flow travel time, flow, and capacity of the link, respectively, and α and β are constant coefficients. When three traffic lanes are converted to two lanes for mixed traffic and one for transit, the link loses one-third of its mixed traffic

capacity, and with an initial $V/C = 0.8$ (and $\alpha = 0.15$, $\beta = 4$), travel time would increase by 24%. Furthermore, the V/C ratio would increase to an oversaturation state, which would cause significant queues. The rough estimation mentioned above shows that the traffic pattern would change to alleviate the capacity reduction as the result of an exclusive lane. Therefore, if user travel time is included in the performance measure, running a traffic assignment model is inevitable to assess the impact of a space priority scheme.

Method Summary

The outline of a general solution approach to the problem of transit space priority based on the formulation presented in the previous sections is shown in Figure 1. The algorithm starts from a feasible combination of exclusive lanes in the network (i.e., Φ^n, $n = 0$). This combination should comply with the practical constraint in Equation 1. It is always possible to start with the alternative RSAs recom-

mended by road authority experts. This initial feasible answer is then used as an input to the lower-level analysis (box shown by a broken line in Figure 1). In the lower level, as described earlier, a modal split would divide the total demand into private and transit demand by using Equation 2. Then the private demand matrix is assigned by solving the optimization program in Equation 3. The third and last step in the lower-level analysis is the solution of a transit passenger assignment demonstrated in Equation 5. This iteration is shown in the lower-level box of Figure 1, with a return arrow from the decision box to the mode choice. The decision box checks the difference between the assumed travel time and the computed one and checks it against a predefined value.

In the next step, the value of the objective function is calculated by using Equation 1. The next step determines whether all feasible answers are checked. If the answer is yes, the exclusive lane combination corresponding to the lowest objective function (Z) is identified as the optimal answer. If not, the procedure is repeated with another feasible combination.
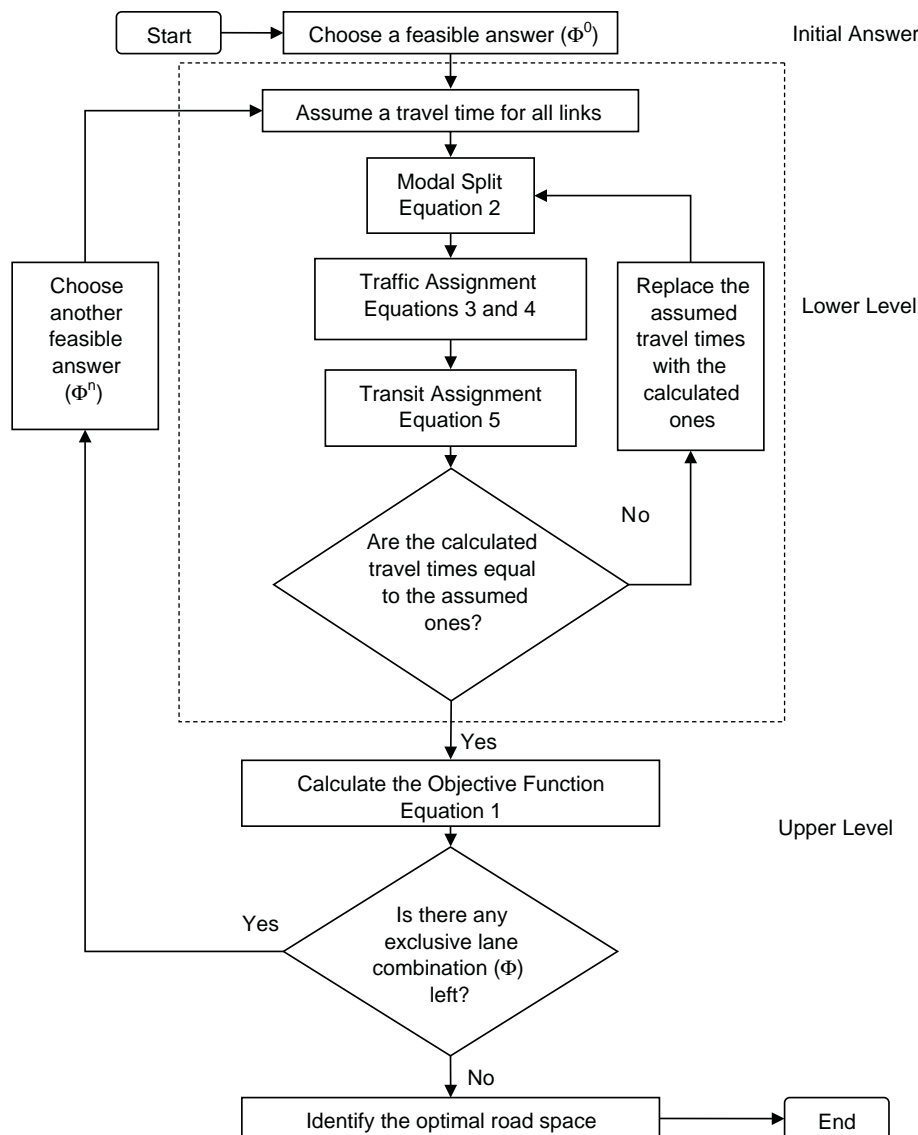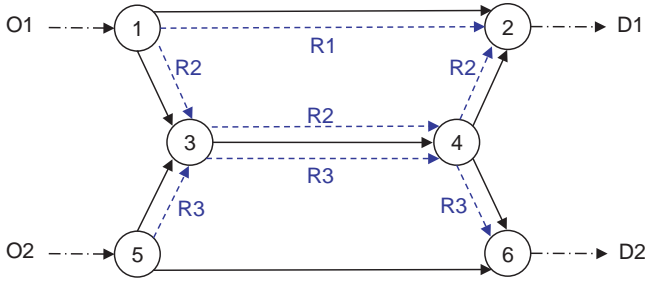

FIGURE 1   Outline of proposed methodology.

FIGURE 2 Example network layout.

TABLE 2 Free-Flow Travel Time on Links

Link	Initial Travel Time (t_0) (min)
(1,2)	9
(1,3)	3
(3,4)	3
(4,2)	3
(4,6)	3
(5,3)	3
(5,6)	9

The method delineated above is applicable to small networks. However, the formulated problem is nonlinear integer programming in nature. Although, the objective function (Z) is linear with the values of flow and travel time, the lower level consists of a user equilibrium and a transit assignment problem, which are nonlinear constraints for the upper level. The upper level also includes the integer decision variable (ϕ_{ij}), which makes the problem an integer programming problem. For each link (i, j) in the network, ϕ_{ij} may take two values (0 or 1) and there are n links in the network, so in general there are 2^n combinations of exclusive transit lanes in the network. This problem is similar to network design problems in general and is an NP-hard problem ($14, 19$). Therefore, finding the optimal answer for a large network by using deterministic algorithms is a tedious task; however, it is possible to approximate the global optima acceptably through heuristic algorithms (20).

NUMERICAL EXAMPLE

In this section, the proposed approach to the optimal RSA problem is applied to an example network. The layout of the network, which consists of six nodes and seven directed links and the operational bus routes (R1, R2, and R3), is shown in Figure 2. Travel demand is from Nodes 1 and 5 (origins) to Nodes 2 and 6 (destinations), respectively. The total demand matrix in passengers per hour in the peak hour of the day is given as follows:

$$D = \begin{bmatrix} q_{12} & q_{16} \\ q_{52} & q_{56} \end{bmatrix} = \begin{bmatrix} 5{,}000 & 0 \\ 0 & 4{,}500 \end{bmatrix} \quad (7)$$

Depending on the placement of the exclusive lane (Φ^n), the appropriate cost function, f_1^c, f_2^c, or f_2^t, is chosen:

$$\phi_{ij} = \begin{cases} 0 & t_{ij}^c = t_{ij}^t = t_{0,ij}^c \left(1 + \alpha \left(\dfrac{x_{ij}^c + x_{ij}^t}{IC_{ij}^c} \right)^\beta \right) \\ 1 & t_{ij}^c = t_{0,ij}^c \left(1 + \alpha \left(\dfrac{x_{ij}^c}{C_{ij}^c} \right)^\beta \right), \ t_{ij}^t = t_{0,ij}^t \end{cases} \quad \forall (i,j) \quad (8)$$

Parameter C_{ij} represents the link car capacity, and IC_{ij} is its initial value with no priority on the link. $t_{0,ij}$ determines the travel time on the link for both modes. α and β are constant parameters of the functions, and x and t are variables. The following are assumed for all (i, j): $\alpha = 0.2$, $\beta = 4$, $IC_{ij}^c = 2{,}400$ veh/h, $C_{ij}^c = 1{,}600$ veh/h, and $t_{0,ij}^c = t_{0,ij}^t = t_0$ (given in Table 2).

In the mode choice model, it is assumed that mode attributes are average O-D travel time ($\bar{t}^{rs}$) and average out-of-pocket cost ($\bar{m}^{rs}$) and that $a_1 = -1$ (1/s) and $a_2 = -0.6$ (1/cents). The travel times of buses are increased by 40% to take into account delays due to access time and waiting time at stops. Other parameters are as follows: $\bar{m}^{c,rs} = 80$ cents, $\bar{m}^{t,rs} = 75$ cents, average car occupancy = 1 passenger per vehicle, and average bus occupancy = 40 passengers per bus.

Table 3 presents the results for the base case, in which no exclusive lane is introduced. As expected, Link (3, 4) has the highest flow rate in the network. On the basis of the values presented in the last two columns of Table 3, the value of the objective function is 107,992 passenger-minutes in the peak hour.

A similar process and calculations similar to those in Table 3 should be performed for any alternative combination of exclusive lanes in the example network. For simplicity in this example, it is assumed that only two combinations of exclusive lanes are feasible:

TABLE 3 Analysis Results of Base Case

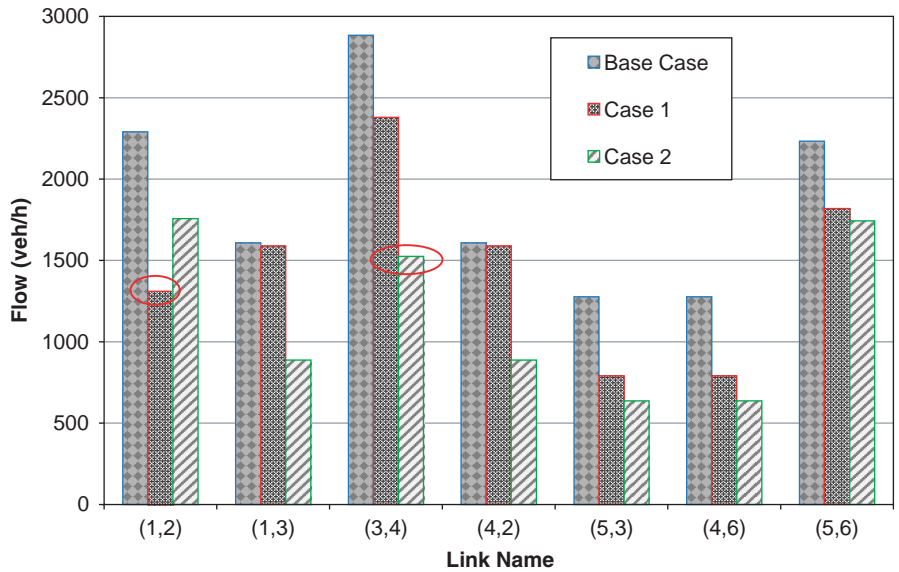| Name	Capacity (veh/h)	t_0 (min)	x_{ij}^c (veh/h)	x_{ij}^t (pass/h)	$t_{ij}^c = t_{ij}^t$ (min)	$x_{ij}^c * t_{ij}^c *$Occup. of Car	$x_{ij}^t * t_{ij}^t$
(1,2)	2,400	9	2,291	869	10.49	24,038	12,859
(1,3)	2,400	3	1,608	232	3.121	5,019	1,016
(3,4)	2,400	3	2,884	1,223	4.251	12,264	7,406
(4,2)	2,400	3	1,608	232	3.121	5,019	1,016
(5,3)	2,400	3	1,276	991	3.048	3,890	4,236
(4,6)	2,400	3	1,276	991	3.048	3,890	4,236
(5,6)	2,400	9	2,233	0	10.35	23,103	0

FIGURE 3   Car flow rate on network links.

Case 1, bus lane on Link (1, 2) [i.e., $\Phi^1 = (1, 0, 0, 0, 0, 0, 0)$]; and Case 2, bus lane on Link (3, 4) [i.e., $\Phi^2 = (0, 0, 1, 0, 0, 0, 0)$].

The same procedure was followed to find the value of the objective function in Cases 1 and 2. Figures 3 and 4 show comparisons of the flow rates for Cases 1 and 2. Figure 3 shows that the link car flow is at the lowest level when an exclusive lane is introduced on that link [see Link (1, 2) in Case 1 or Link (3, 4) in Case 2]. Figure 4 shows that at the same time on a link with an exclusive lane, the number of buses is the highest (see the same links). Because of the passenger diversion, the total number of passengers (car + bus) on the link with an exclusive lane was the highest among the three study cases.

The final result of the objective function and its relative improvement for Cases 1 and 2 are presented in Table 4 and Figure 5. As a result of the bus lane, in both Cases 1 and 2 the bus mode share was

increased and the car share decreased; thus, the total travel time by car is reduced after the introduction of a bus lane. The last column of Table 4 shows the net benefit of the proposed priorities compared with the base case. According to Table 4, Case 2 could improve the objective function by 3.4%. This improvement is highlighted in Figure 5.

DISCUSSION OF RESULTS

The proposed approach provides a decision-making tool for finding the optimal RSA associated with transit priority at the network level. In this paper, the outline of an optimization approach was given. Through the introduction of exclusive transit lanes in the network, the
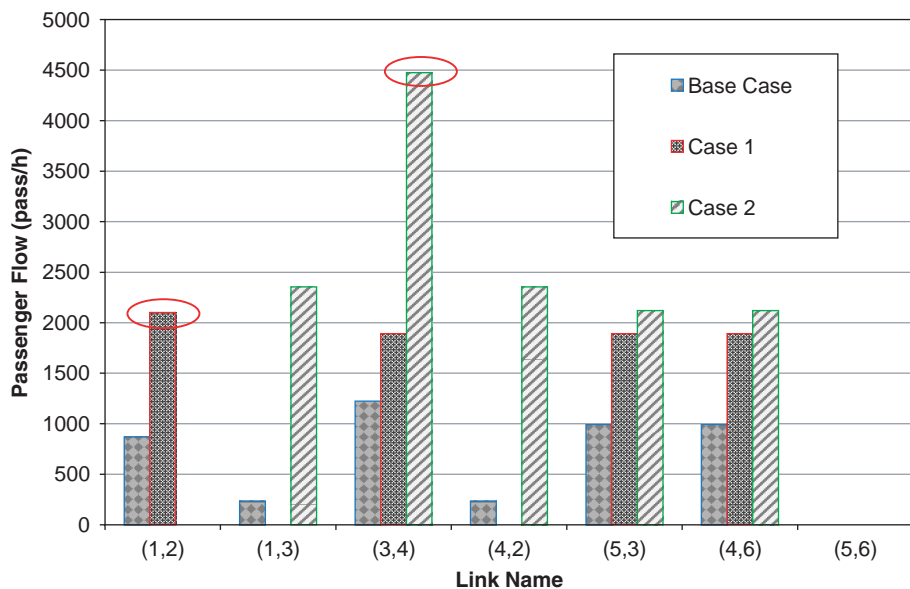

FIGURE 4   Transit passenger flow rate on network links.

TABLE 4   Objective Function Improvement

	Total Travel Time in Car (pass-min)	Total Travel Time on Bus (pass-min)	Total Travel Time All Modes (pass-min)	Difference from the Base Case (%)		
				Car	Bus	All
Base case	77,224	30,769	107,992	0.00	0.00	0.00
Case 1	53,465	52,055	105,520	−30.77	69.18	−2.29
Case 2	47,797	56,502	104,300	−38.11	83.64	−3.42

utility of transit and private modes is changed and network usage patterns are altered for each mode. Therefore, in the proposed methodology, a modal split model is included to determine the new mode shares along with a traffic assignment and a transit assignment model for traffic diversion and passenger diversion, respectively.

The issue of computational difficulty was raised, and the necessity of introducing an efficient method in real scaled networks was pointed out. In the proposed method, each combination of exclusive transit lanes is input into the methodology; in a real scaled network, the number of these combinations would increase exponentially by the increase in the number of links. The methodology of this paper can be extended to large-scale networks by applying a heuristic algorithm that would reduce the number of combinations analyzed.

Most of the assumptions made for the proposed method can be relaxed, depending on the application. It was assumed that the total demand is given and fixed. If the feedback process from the urban transportation planning process (29) is considered for the trip distribution or trip generation steps, this method could be used for variable demand analysis. Such a model can consider effects such as shift in travel starting time or trip suppression. A combined distribution, modal split, and assignment model can be used where such a calibrated model is available.

The second assumption introduced in this study considered only cars and buses in the network. To take into account other vehicles such as heavy vehicles, appropriate equivalence factors (30) can be used.

An alternative transit mode such as rail, which operates on a dedicated lane, could also be considered in the proposed method. A nested logit model (23) can be applied in this case instead of the logit model used in the proposed method.

The method was applied to a simple network. The example network was also examined with different demand levels and link cost functions. At low demand levels, any exclusive lane in the network would result in higher total travel time, while some alternative combinations can be justified at high demand levels, considering the objective function of Equation 1. That is because travel time for an O-D pair in this network is always shorter by car than by bus at low densities. The same trend is observed for the values of β in Equation 8. When β is small, which means travel time is not sensitive to the flow–capacity ratio (x/C), introduction of a bus lane does not decrease the objective function (Z). With higher values of β, changing one lane to a bus lane can reduce Z in some combinations.

CONCLUSION AND FURTHER WORK

This paper has presented a new methodology for optimizing provision of transit priority at a network level. It indicates that previous approaches to road space priority allocation lack the network view in which all possible combinations of exclusive lanes could be nominated. In addition, the impact of the proposed alternatives on all
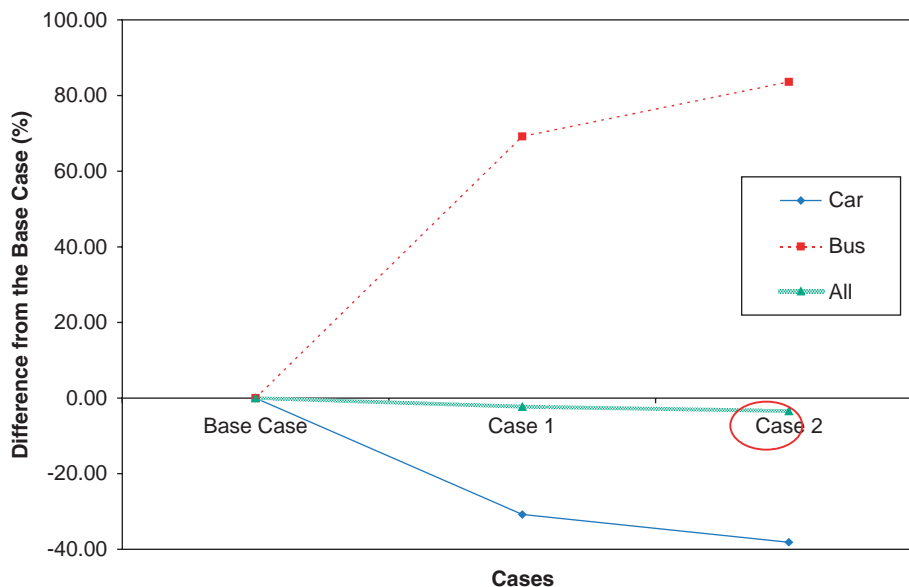

FIGURE 5   Relative difference from the objective function.

network users cannot be considered in the existing local (corridor) approaches. The aim of the new methodology is to find the optimal combination of exclusive lanes without disturbing transit routes or building new roads. The problem was formulated by using bilevel programming. Total travel time was used as the performance measure in the upper level; mode choice, traffic assignment, and transit assignment models were adopted at the lower level.

The approach presented here could be expanded by adding practical constraints to the upper-level formulation as well as more detailed models to the lower level, as indicated in the section on discussion of results. Incorporation of the proposed method with a computationally effective algorithm would be another area for future study. Combining road space priority with time priority at signals is another area that needs to be addressed in future work. The combination of time and space priority could lead to the development of an optimal priority solution for congested urban transport networks.

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