Towards Quantum Experiments with Human Eyes as Detectors Based on Cloning via Stimulated Emission

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We show theoretically that a large Bell inequality violation can be obtained with human eyes as detectors, in a "micro-macro" experiment where one photon from an entangled pair is greatly amplified via stimulated emission. The violation is robust under photon loss. This leads to an apparent paradox, which we resolve by noting that the violation proves the existence of entanglement before the amplification. The same is true for the micro-macro experiments performed so far with conventional detectors. However, we also prove that there is genuine micro-macro entanglement even for high loss.

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The basic principles of quantum physics such as quantum superpositions and entanglement have already had a major impact on the scientific world view. These phenomena are typically far removed from our everyday experience. It is of interest to explore various ways of bringing quantum phenomena closer to the macroscopic level, and to everyday life. One approach is to ask whether it is possible to perform quantum optics experiments with human eyes as detectors [1]. Quantum cloning of single-photon states via stimulated emission [2–6] has recently allowed the experimental creation of tens of thousands of clones starting from a single-photon qubit [7], which was part of an initial entangled photon pair. Here we show that the characteristics of the human eye are well adapted to the task of distinguishing the resulting multiphoton states. As a consequence, it becomes possible to achieve significant Bell inequality violations in "micro-macro" experiments with human-eye detectors. Motivated by the surprising robustness of these results under photon loss, we furthermore clarify the role of micro-macro entanglement in these experiments. We show that the violation proves the existence of entanglement before (rather than after) the amplification. The same is true for the micro-macro experiments performed so far with conventional detectors [7,8]. On the other hand, we also prove that there is genuine micro-macro entanglement even for high loss. However, revealing it experimentally requires more sophisticated measurements.

The photon detection characteristics of the human eye have been studied in significant detail starting with Ref. [9]. Our results are based on the following theoretical model which describes the experimental evidence very well [10]. The eye is modeled as an ideal threshold detector preceded by very significant losses. More formally, we define the positive operator corresponding to a detection by the eye as $\hat{E}_y = C_L^\dagger \hat{T}_y C_L$, where $\hat{T}_y = \mathbb{1} - \hat{T}_n = \mathbb{1} - \sum_{m=0}^{\theta-1} |m\rangle\langle m|$, with photon number states $|m\rangle$, is the projection operator corresponding to an ideal threshold detector with threshold θ, and $C_L = e^{\gamma(a^\dagger c - ac^\dagger)}|0\rangle_c$ is the loss channel, where a is the mode that we are interested in detecting and c is the initially empty mode whose coupling to a is responsible for the loss. We have introduced the subscript y to mean "yes," corresponding to a successful detection. Analogously, the operator for a nondetection is $\hat{E}_n = C_L^\dagger \hat{T}_n C_L$. Based on Ref. [10] we choose the values $\theta = 7$ for the threshold and $\eta = \cos^2\gamma = 0.08$ for the transmission of the eye. These values provide an excellent fit for the experimental response curve of the eye, which looks like a smoothed out step function, where the step occurs in the vicinity of ca. 100 photons impinging on the eye; cf. Fig. 2 of Ref. [10].

It is *a priori* not easy to design quantum experiments using the eye as a detector. For example, the approach studied in Ref. [1] of observing large numbers of independent entangled pairs does not allow the violation of a Bell inequality if the above realistic eye model is used [11], rather than the more idealized model considered in Ref. [1]. Nevertheless, in the present work we show that quantum experiments with human-eye detectors become a realistic possibility, if detection with the naked eye is combined with cloning via stimulated emission.

Cloning by stimulated emission was originally introduced [2,3] in the context of universal cloning [13], i.e., in a setting where all input states are treated equally. Here we focus instead on phase-covariant cloning [14] by stimulated emission [6], in order to stay close to the experiments of Refs. [7,8]. A phase-covariant cloner makes good copies only of input states that lie on a great circle of the Bloch sphere. Considering qubits realized by the polarization states of single photons in a spatial mode **a**, a phase-covariant cloner can be realized based on stimulated collinear type-II parametric down-conversion [6], where the appropriate Hamiltonian for the down-conversion process is $H = i\chi a_H^\dagger a_V^\dagger + \text{H.c.}$, where χ is proportional to the nonlinear susceptibility of the crystal and to the pump amplitude, and a_H and a_V are the horizontal and vertical polarization modes corresponding to the spatial mode **a**. Identifying a_H and a_V with the north and south poles of the Bloch sphere, one can introduce a basis of "equatorial" modes a_ϕ and $a_{\phi\perp}$ via the relations $a_H = \frac{1}{\sqrt{2}} e^{i\phi}(a_\phi + i a_{\phi\perp})$, $a_V = \frac{1}{\sqrt{2}} e^{-i\phi}(a_\phi - i a_{\phi\perp})$. Different choices of the phase ϕ correspond to different bases. Rewriting H in terms of a_ϕ and $a_{\phi\perp}$ gives $H = \frac{i\chi}{2}(a_\phi^{\dagger 2} + a_{\phi\perp}^{\dagger 2}) + \mathrm{H.c.}$; H has the same form for any choice of equatorial basis. This is why the cloning process is phase covariant. We will assume that a choice of basis has been made and denote the corresponding equatorial modes by a and $a_\perp$ for compactness of notation.

We now show that the multiphoton states obtained by cloning single-photon qubits via stimulated emission can be distinguished with the naked eye with a high probability for a conclusive result and high fidelity. Consider cloning the two orthogonal single-photon qubit states $a^\dagger|0,0\rangle = |1,0\rangle$ and $a_\perp^\dagger|0,0\rangle = |0,1\rangle$. The time evolution operator for the cloning process is $e^{-iHt} = UU_\perp$ with $U = e^{(g/2)(a^{\dagger 2} - a^2)}$, $U_\perp = e^{(g/2)(a_\perp^{\dagger 2} - a_\perp^2)}$, where we have defined the amplification gain $g = \chi t$, with t the interaction time for the down-conversion process. After the amplification, the qubit states become $|\Phi\rangle = UU_\perp|1,0\rangle = |A_1\rangle|A_0\rangle_\perp$ and $|\Phi_\perp\rangle = UU_\perp|0,1\rangle = |A_0\rangle|A_1\rangle_\perp$, where we have introduced the notation $|A_1\rangle = U|1\rangle$, $|A_0\rangle = U|0\rangle$, and analogously for the perpendicular modes. It is easy to show, e.g., by integrating the equations of motion in the Heisenberg picture, that $U^\dagger a^\dagger U = \cosh(g) a^\dagger + \sinh(g) a$, which allows one to calculate the mean photon numbers in the two states $|A_0\rangle$ and $|A_1\rangle$, $\langle A_1|a^\dagger a|A_1\rangle = 3\sinh^2(g) + 1$, and $\langle A_0|a^\dagger a|A_0\rangle = \sinh^2(g)$. This shows that stimulating the down-conversion process with a single photon leads to an approximate tripling of the resulting output photon number compared to a vacuum input (for large g).

Our proposal for distinguishing $|\Phi\rangle$ and $|\Phi_\perp\rangle$ using human eyes as detectors, which is illustrated in Fig. 1, is based on this significant difference in typical photon numbers between the states $|A_1\rangle$ and $|A_0\rangle$, in combination with the fact that the eye is a (smooth) threshold detector. The amplification gain g can be adjusted such that $|A_1\rangle$ will give a detection by the eye with high probability (it is "above the threshold"), whereas $|A_0\rangle$ will not (it is "below the threshold"). Under these conditions, separating the two

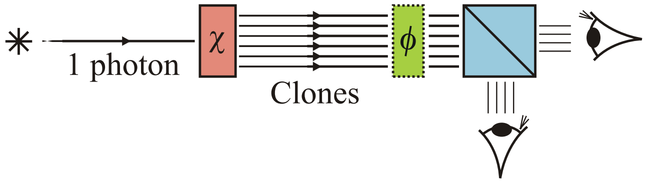

FIG. 1 (color online). A single-photon qubit is amplified through cloning via stimulated emission in a nonlinear crystal (χ). The clones are split into two orthogonal polarization modes, and each mode is detected by a naked human eye. The polarization basis can be varied with the help of a wave plate (ϕ).

modes a and $a_\perp$ and directing each of them to one eye [15], $|\Phi\rangle$ will mostly give rise to detections in the eye exposed to mode a, whereas $|\Phi_\perp\rangle$ will mostly give rise to detections in the eye exposed to mode $a_\perp$.

Since the eye is not a perfect threshold detector, and since the photon number distributions in the two states $|A_0\rangle$ and $|A_1\rangle$ have large variances, there will also be events where both eyes detect something, where none of the eyes detect anything, or even where only the "wrong" eye responds. Introducing the notation $p(y, n|\Phi)$ for the probability of a detection ("yes") in mode a and no detection ("no") in mode $a_\perp$, given the state $|\Phi\rangle$, etc., one can then define the probability for a conclusive measurement, corresponding to a detection in only one eye, as $\varepsilon = p(y,n|\Phi) + p(n,y|\Phi) = p(y,n|\Phi_\perp) + p(n,y|\Phi_\perp)$, where ε stands for "efficiency." The accuracy of the measurement can be quantified via the visibility V, defined as $V = \frac{p(y,n|\Phi) - p(n,y|\Phi)}{p(y,n|\Phi) + p(n,y|\Phi)}$.

Based on the above model of the eye as a photon detector, the probabilities can be expressed as $p(y,n|\Phi) = \langle A_1|\hat{E}_y|A_1\rangle\langle A_0|\hat{E}_n|A_0\rangle$, etc. In order to evaluate the expectation values of $\hat{E}_y$ and $\hat{E}_n$, one has to evaluate general terms of the form $P^{|m\rangle}_{|A_0\rangle} = \langle A_0|C_L^\dagger|m\rangle\langle m|C_L|A_0\rangle$ and $P^{|m\rangle}_{|A_1\rangle} = \langle A_1|C_L^\dagger|m\rangle\langle m|C_L|A_1\rangle$. The projector on a Fock state $|m\rangle$ can be written as $|m\rangle\langle m| = \delta_{a^\dagger a, m} = \frac{1}{2\pi}\int_0^{2\pi} dk e^{-ik(a^\dagger a - m)}$. Using operator ordering techniques following Ref. [16], one can show that $U^\dagger C_L^\dagger e^{-ika^\dagger a} C_L U = U^\dagger e^{-\ln(X_0) a^\dagger a} U = Y^{-(1/2)} e^{-(1/2)\ln X a^\dagger a} e^{(1/2) Z a^{\dagger 2}} e^{(1/2) Z a^2} e^{-(1/2)\ln X a^\dagger a}$, with $X_0 = (1 - \eta + \eta e^{-ik})^{-1}$, $X = X_0 \cosh^2 g - \frac{\sinh^2 g}{X_0}$, $Y = \frac{X}{X_0}$, and $Z = \frac{1}{2}\partial_g X$. This gives $\langle A_0|C_L^\dagger e^{-ika^\dagger a} C_L|A_0\rangle = Y^{-(1/2)}$ and $\langle A_1|C_L^\dagger e^{-ika^\dagger a} C_L|A_1\rangle = Y^{-(1/2)} X^{-1}$, which implies $P^{|m\rangle}_{|A_0\rangle} = \frac{1}{2\pi}\int dk e^{ikm} Y^{-(1/2)}$ and $P^{|m\rangle}_{|A_1\rangle} = \frac{1}{2\pi} \times \int dk e^{ikm} Y^{-(1/2)} X^{-1}$. Using the Cauchy integral formula, one finds $P^{|m\rangle}_{|A_0\rangle} = \frac{1}{m!}\partial_z^m Y^{-(1/2)}|_{z=0}$, and $P^{|m\rangle}_{|A_0\rangle} = \frac{1}{m!}\partial_z^m (Y^{-(1/2)} X^{-1})|_{z=0}$, with $z = e^{-ik}$.

These results make it possible to calculate the detection probabilities $p(y,n|\Phi)$, etc., and thus the visibility V and the efficiency ε, as a function of the gain g, which directly determines the mean photon number after amplification, summed over both polarization modes, $\langle N_{\mathbf{a}}\rangle = 4\sinh^2(g) + 1$; cf. above. The results are shown in Fig. 2. One sees that ε has a maximum for ca. 300 photons. Despite a dip in the region of high efficiency, V always stays greater than $\frac{1}{\sqrt{2}}$, which is an important bound for Bell experiments; cf. below. We thus see that the states $|\Phi\rangle$ and $|\Phi_\perp\rangle$ can be distinguished with high efficiency and accuracy at the same time [17]. This result is true for any equatorial basis. Figure 2 also shows the effect of other losses after the amplification in addition to the unavoidable losses in the eye. Since the model of the eye used is an ideal threshold detector preceded by losses, this can be done simply by varying the value of η. One sees that the effect of

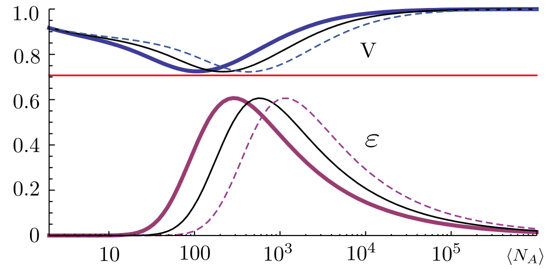

FIG. 2 (color online). Efficiency ε and visibility V, as defined in the text, of the human-eye detection method for amplified single-photon qubits, as a function of the mean photon number after amplification $\langle N_{\mathbf{a}} \rangle$ (thick lines). The efficiency has a maximum of $\varepsilon = 0.61$ for $\langle N_{\mathbf{a}} \rangle = 288$. The visibility never drops below $\frac{1}{\sqrt{2}}$, which is relevant for Bell experiments in the micro-macro setting of Refs. [7,8]; cf. text and Fig. 3. We also show V and ε for the case of additional losses after the amplification, corresponding to overall transmission factors $\frac{\eta}{2}$ (thin lines) and $\frac{\eta}{4}$ (dashed lines).

losses can be completely compensated by increasing the gain.

We now apply these results to the micro-macro scenario of Refs. [7,8]; see also Fig. 3. In these experiments, a first low-gain down-conversion process creates an entangled photon pair into the two distinct spatial modes **a** and **b** in a polarization singlet state, $|\psi_-\rangle = \frac{1}{\sqrt{2}}(a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger)|0, 0, 0, 0\rangle$, where $|0, 0, 0, 0\rangle$ denotes the vacuum for all participating modes. Because of the rotational invariance of the singlet, this can be rewritten in an equatorial mode basis as $|\psi_-\rangle = \frac{1}{\sqrt{2}}(a^\dagger b_\perp^\dagger - a_\perp^\dagger b^\dagger)|0, 0, 0, 0\rangle$. The photon in the **b** spatial mode is detected directly, whereas the photon in the **a** mode is greatly amplified with the phase-covariant cloning process described above, leading to a micro-macro entangled state $|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|\Phi\rangle_{\mathbf{a}}|0, 1\rangle_{\mathbf{b}} - |\Phi_\perp\rangle_{\mathbf{a}}|1, 0\rangle_{\mathbf{b}})$ (still written in the equatorial basis for both spatial modes). The capability of human-eye detectors to distinguish the two states $|\Phi\rangle$ and $|\Phi_\perp\rangle$ with high visibility implies the possibility of observing a violation of the Clauser-Horne-Shimony-Holt (CHSH) Bell inequality with the same visibility for this entangled state, provided that the detection of the unamplified photon in mode **b** does not introduce any errors. Note that measurements in two different equatorial bases for both systems **a** and **b** are sufficient for testing the CHSH inequality. The detection of the unamplified photon also serves as a trigger, signaling that a pair has indeed been produced in the low-gain down-conversion.

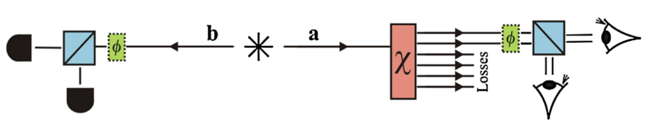

FIG. 3 (color online). We consider the micro-macro entanglement scenario of Refs. [7,8], but with human-eye detectors for the macro system.

The robustness of the visibility with respect to losses shown in Fig. 2 means that a strong Bell inequality violation can be achieved for arbitrarily high losses, provided that the amplification is sufficiently strong. This seems paradoxical, since losses are clearly going to affect the micro-macro entanglement, as information about the macro state ($|\Phi\rangle$ or $|\Phi_\perp\rangle$) leaks into the environment. Even in the case where there are only the losses intrinsic to the eye, i.e., for $\eta = 0.08$, most of the photons are lost, such that the environment contains almost all the available information, which means that the remaining micro-macro entanglement must be quite small. So how can the visibility of the Bell violation remain so high?

This apparent paradox can be resolved by realizing that, while the efficiency of the proposed detection method is quite high, it is always significantly smaller than 1, such that the measurement is nevertheless postselective. Moreover, whereas in the lossless case the macro system lives in a two-dimensional Hilbert space spanned by $|\Phi\rangle$ and $|\Phi_\perp\rangle$, in the presence of losses it lives in a much larger space. Together these two facts open up an important "loophole." Conclusive [i.e., (y, n) or (n, y)] results for different equatorial bases correspond to different, almost orthogonal, subspaces of the high-dimensional Hilbert space. One can construct separable multiphoton states that exploit this loophole to achieve the same visibility as in Fig. 2 [18]. The experimental observation of such a visibility by itself therefore allows no conclusion about the existence of micro-macro entanglement. This is true also for the "orthogonality filter" measurements of Refs. [7,8].

Nevertheless, the same measurements do allow one to prove the entanglement of the original entangled pair before amplification. From this perspective, the amplification and losses can be simply seen as part of the detection process for the original single photon. The Hilbert space of the original photon is only two-dimensional, so there is no risk of different subspaces being detected for different choices of measurement basis. Moreover, the detection efficiency is independent of the choice of equatorial basis because of the phase covariance of the amplification. For proving nonlocality (as opposed to just entanglement), there is still the usual detection loophole due to the limited measurement efficiency. However, it is no more severe than for any other detection method that has comparable efficiency.

Briefly relaxing our focus on human eyes as detectors, we now show that proving genuine micro-macro entanglement in the presence of losses is possible using measurements that are not postselective. Following Ref. [19] one can derive a condition which has to be fulfilled for all separable states: $|\langle \vec{J}_{\mathbf{a}} \cdot \vec{J}_{\mathbf{b}} \rangle| \leq \langle N_{\mathbf{a}} N_{\mathbf{b}} \rangle$. Here $\vec{J}_{\mathbf{a}}$ and $\vec{J}_{\mathbf{b}}$ are the Stokes (polarization) vectors corresponding to spatial modes **a** and **b**, and $N_{\mathbf{a}}$ and $N_{\mathbf{b}}$ are the corresponding photon number operators. One can choose a convention where $J_{z\mathbf{a}} = a_H^\dagger a_H - a_V^\dagger a_V$, and $J_{x\mathbf{a}} = a^\dagger a - a_\perp^\dagger a_\perp$; i.e., the x direction is identified with the arbitrary phase choice ϕ that was used to define the modes a and $a_\perp$ above. The dynamics of J_y (in fact, of any Stokes vector component in the $x-y$ plane) is exactly equivalent to that of J_x. For our micro-macro scenario, the state of $\mathbf{b}$ is a single-photon state, leading to the simplified criterion $|\langle \vec{J}_\mathbf{a} \cdot \vec{\sigma}_\mathbf{b} \rangle| \leq \langle N_\mathbf{a} \rangle$, where $\vec{\sigma}_\mathbf{b}$ is the vector of Pauli spin matrices. This means that we have to evaluate $\langle \vec{J}_\mathbf{a} \cdot \vec{\sigma}_\mathbf{b} \rangle = \langle \Psi_- | C_{L\mathbf{a}}^\dagger \vec{J}_\mathbf{a} \cdot \vec{\sigma}_\mathbf{b} C_{L\mathbf{a}} | \Psi_- \rangle$ for the micro-macro state $|\Psi_-\rangle$ defined above. One can show that $\langle \Psi_- | C_{L\mathbf{a}}^\dagger J_{z\mathbf{a}} \sigma_{z\mathbf{b}} C_{L\mathbf{a}} | \Psi_- \rangle = \eta$, whereas for the equatorial components $\langle \Psi_- | C_{L\mathbf{a}}^\dagger J_{x\mathbf{a}} \sigma_{x\mathbf{b}} C_{L\mathbf{a}} | \Psi_- \rangle = \langle \Psi_- | C_{L\mathbf{a}}^\dagger J_{y\mathbf{a}} \sigma_{y\mathbf{b}} C_{L\mathbf{a}} | \Psi_- \rangle = \eta(\langle A_1 | a^\dagger a | A_1 \rangle - \langle A_0 | a^\dagger a | A_0 \rangle) = \eta(2\sinh^2 g + 1)$. On the other hand, $\langle N_\mathbf{a} \rangle = \langle \Psi_- | C_{L\mathbf{a}}^\dagger (a^\dagger a + a_\perp^\dagger a_\perp) C_{L\mathbf{a}} | \Psi_- \rangle = \eta(\langle A_1 | a^\dagger a | A_1 \rangle + \langle A_0 | a^\dagger a | A_0 \rangle) = \eta(4\sinh^2 g + 1)$, which finally yields $|\langle \vec{J}_\mathbf{a} \cdot \vec{\sigma}_\mathbf{b} \rangle| - \langle N_\mathbf{a} \rangle = 2\eta$. One can see that the violation of this genuine micro-macro entanglement criterion is sensitive to photon loss as expected. However, some micro-macro entanglement persists even for high loss. Experimentally demonstrating micro-macro entanglement in this way would require counting large photon numbers with single-photon accuracy.

We have shown that quantum experiments with human eyes as detectors appear possible, based on a realistic model of the eye as a photon detector. There are significant experimental challenges. For example, one has to ensure that the detection of the photon in **b** heralds the presence of the photon in **a** before the amplifier with a high efficiency η_h. The effect of $\eta_h < 1$ is roughly to multiply the visibility V in Fig. 2 by η_h. Values of η_h as high as 0.83 have already been reported [20]. Note also that for proving entanglement (as opposed to violating a Bell inequality) one only needs $V > \frac{1}{2}$ [21]. The temporal multimode character of the amplification can cause additional noise, but good mode matching should be possible even in the high-gain regime [22]. We intend to discuss implementation issues in much greater detail in a future publication [12].

We find the possibility of observing quantum effects directly with our own eyes fascinating. One might ask in what way the proposed experiment would differ from simply detecting a single entangled photon pair with conventional photon detectors and then visually observing the detectors' displays. We would argue that, first, the amplification in the present experiment is of a different nature compared to the amplification that occurs in a conventional photodiode, in that the choice of detection basis can be made after the amplification process, emphasizing the coherent nature of the latter. Second, detection by the eye brings the observer much closer to the quantum phenomenon. Not only can the existence of "micro-micro" entanglement be detected unambiguously with human eyes as detectors, but the directly observed "micro-macro" quantum state would in fact be entangled, even though its entanglement cannot be proved by human-eye-based measurements.

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