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INTERNAL EROSION OF GRANULAR MATERIALS – IDENTIFICATION OF ERODIBLE FINE PARTICLES AS A BASIS FOR NUMERICAL CALCULATIONS

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Abstract. *In geohydromechanics internal erosion is a process which is still hardly to be quantified both spatially as well as temporally. The transport of fine particles, which is caused by increased hydraulic gradients, is influenced by the pore structure of the coarse grained fabric. The microstructural information of the pore constriction size distribution (CSD) of the solid skeleton has therefore to be taken into account when internal erosion is analyzed either analytically or numerically. The CSD geometrically defines the amount of fine particles, which potentially can be eroded away for a given hydraulic force. The contribution introduces experimental and numerical calculations which aim at the quantification of the amount of erodible fines. Based on this approach a multiphase continuum-based numerical model is used to back calculate the process of internal erosion for one material of the well-known experimental investigation of Skempton & Brogan (1994)*[1].

1 INTRODUCTION

Internal erosion – often also denoted as suffusion – pose a particular threat to water retaining structures like dikes and dams. Approximately 46 % of all dysfunctional embankment dams show problems with internal erosion[2]. It can be assumed that internal erosion is the main cause for instabilities of embankment dams. In 2001 suffusion and piping have caused a reservoir failure of the Malana gravity dam. During this failure the entire content of the reservoir of 200,000 cubic meter water escaped within 20 minutes. The loss of soil in the aquifer has lead to settlements of 5 to 6 meters in an area of about 500 square meters without destruction of the dam[3]. Failures of the levees of New Orleans during Hurricane Katrina are also partly attributed to internal erosion. Sinkholes together with sand boils, the only visual proof of internal erosion which can be seen from the surface, were observed along the embankments[4].

Internal erosion is in most cases a non-visible process taking place within the structure of the soil matrix. Since the finer fraction of the soil does not always contribute to the formation of the stable structure[5], it is located in a more or less loose condition in the voids of the coarse grained fabric. In the case of increased hydraulic gradients fine particles may rearranged and transported through the solid skeleton. The large voids formed by the coarse fraction of the soil are interconnected by pore constrictions (pore throats) which are causative for the blockage or transportation out of the voids. For the case that fine particles are mobilized without blockage suffusion occurs in its original term[6]. If fine particles are partly blocked a localized flow of water may occur and piping-like suffusion starts[1]. The transportation of fine particles causes a reduction of the mean density of the representative elementary volume due to changes of the pore structure of the soil which then results in an increase of the hydraulic conductivity. Both, pore structure and hydraulic conductivity, can be quantified by the grain size distribution (GSD), which is a much more easier measurable property of the soil. The micro-structural information of the pore constriction size distribution (CSD) has therefore to be taken into account in the numerical calculation of internal erosion.

2 CONTINUUM-BASED MULTIPHASE MODELL

A multiphase continuum-based model was developed to describe the internal erosion on an engineering scale considering micro-structural properties of the soil. Consequently, the proposed mathematical model is closely linked to geohydraulic (saturated hydraulic conductivity) and geometrical (CSD) parameters of the hydromechanically unstable soil, which can both be quantified by the GSD. The CSD determines the volume fraction of erosive fines of the soil, which means the proportion of the soil which is geometrically endangered for mobilization and transport. As a consequence of this consideration the matrix of the soil is split into two parts: A primary fabric which is stable and an amount of unstable, i.e. erosive fines (figure 1, left)[6,8].

The porous material is assumed to be fully-saturated with a incompressible pore fluid during the whole process and as an initial condition the total amount of erodible fine particles is in rest leading to a conventional consideration of a 2-phase model (figure 1, middle). With the initiation of erosion a volume fraction of the erodible fine particles φ^a gets fluidized and is transported away which corresponds to a phase transformation since this amount of the solid phase becomes part of the fluid phase. Thus, the fluid phase, consisting of the mobilized part of the erosive fines φ^a and the pore fluid φ^f, can be considered as a dilute biphasic suspension. In contrast, the solid skeleton consists of the volume fraction of erosive fines still in rest φ^{sa} and the volume fraction φ^{sn} of the primary fabric, i.e. the coarse grained particles forming the pore structure. The erosion process continues until the amount φ^{sa} of erodible fine particles is washed out. Consequently, for the mathematical description of the erosion process a 4-phase model has to be considered (figure 1, right).

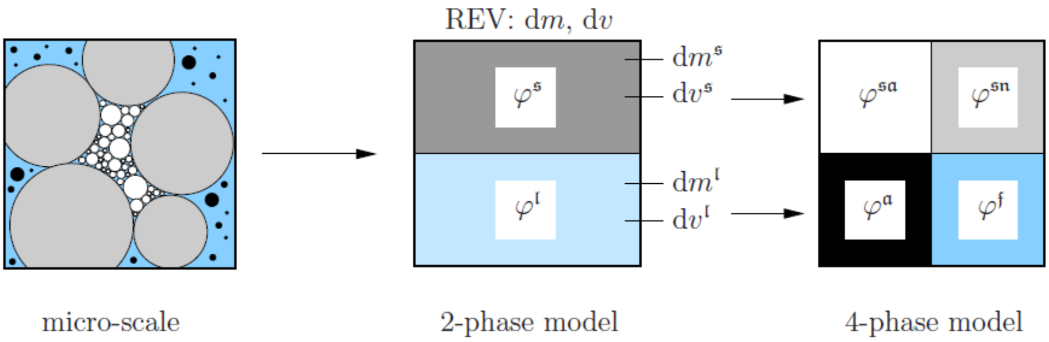

Figure 1: Micro- and macroscopical sketch of a suffusive four-phase mixture

The suffusion process is influenced by several factors: (i) suffusion must be possible with respect to the existing geometric constrictions on the pore scale which is taken into account by introducing the CSD, (ii) the hydraulic forces has to be large enough to initiate a mobilization and maintain transportation of fine particles, (iii) the loss of erodible fines leads to an increase of the overall porosity which results into an increase of the hydraulic conductivity, (iv) deformation of the primary fabric of the soil decreases on the one hand the porosity and the hydraulic conductivity, but on the other hand allows for the mobilization of fines which were not considered as erosive before and finally (v) a change in the CSD may cause blockage of fluidized fines.

The deformation of primary fabric is not included in the present model so far. The rigidity of the primary fabric is motivated with the assumption that the fines are loosely embedded into the pores of the fabric and thus do not contribute to the effective stress of the solid constituent[6]. Likewise, sedimentation and blockage of mobilized fines are neglected during the process. The inclusion of the parameters depending on the micro-structure is implemented based on the GSD. On the one hand, the changes of the hydraulic conductivity is quantified with the approach of Kozeny/Carman[9]. On the other hand, the volume fraction of the erodible fines is quantified as an initial condition with the CSD, which in turn is calculated from the GSD. A comprehensive mathematical description of the model can be found in Steeb & Scheuermann (2010)[10].

3 NUMERICAL APPROACH FOR THE QUANTIFICATION OF ERODIBLE FINES

3.1 Constriction size distribution

For the calculation of the CSD the opening of an ensemble of four adjacent particles forming a pore throat is considered. The particle sizes for an ensemble are calculated by dividing the GSD into sections of equal percentage, and for every section a mean diameter of the particle is calculated. As a result every particle diameter has the same probability of occurrence. The pore constriction is finally defined by a circle which can be constructed in the free area between the particles (cf. figure 2).

Based on this consideration Silveira has developed a method for calculating the CSD for the densest state[11] considering a set of three particles and for the least-dense packing of four particles[12]. For the calculation of the particle sizes he used the GSD finer by weight, which is the result of a conventional sieving analysis. Based on a mixture rule the CSDs of Silveira, which can be considered as a kind of border lines for a true CSD, can be recalculated into a CSD for a medium dense packing using the relative density calculated with the pore ratio[13].

An alternative approach for the calculation of the CSD for a medium dense packing based on the method of Silveira was proposed by Schuler (1996)[14]. However, in contrast to Silveira, Schuler uses the grain size distribution finer by surface (surface distribution) which can be calculated from the GSD (finer by weight) assuming spherical particles with a constant density of the solid for all fractions. By using the surface distribution the weight of the fine fraction is higher, but not too high as it would be if the GSD finer by amount (quantity distribution) is used. Furthermore, Schuler assumes that one particle is shifted in the third dimension (grey colored particle in figure 2). In Figure 2 it is assumed that all particles have the same radius ($r_A = r_B = r_C = r_D$).

For an ensemble of particles as shown in figure 2 spaces with different sized areas are possible. If particles A and B touch each other, the densest packing of the four particles will be obtained and the angle α as well as the area F_{PE} of the space between the particles have their lowest values (cf. Figure 2 b)). As the values for α increase also the area F_{PE} of the space between the particles grows until F_{PE} reaches a maximum (cf. Figure 2 a)). At this condition the ensemble reaches its loosest density. Consequently, the ensemble of particles takes on different densities according to the area of the space between the particles, which can be assigned to the loosest and densest bedding of the particles. In order to take into account the bedding of the soil for the calculation of the pore constrictions, an analogy is introduced between the area F_{PE} of the space between the particles and the porosity n of the soil as shown in equation (1). Finally, the subscription of pores into the space between the particles is a pure geometric problem which can be solved analytically.

A comparison with experimentally determined constriction sizes[15] shows a very good concordance with calculated values. It can be assumed that the introduced procedure for the calculation of the CSD depending on the bedding applies at least for granular soils. The resulting CSD can not only be used to analyze filtration problems[14] or to determine the amount of erodible fines[10], but also for the calculation of hydraulic properties of soils[16].

$$D = \frac{n_{\max} - n}{n_{\max} - n_{\min}} = \frac{F_{PE,\max} - F_{PE}}{F_{PE,\max} - F_{PE,\min}} \tag{1}$$

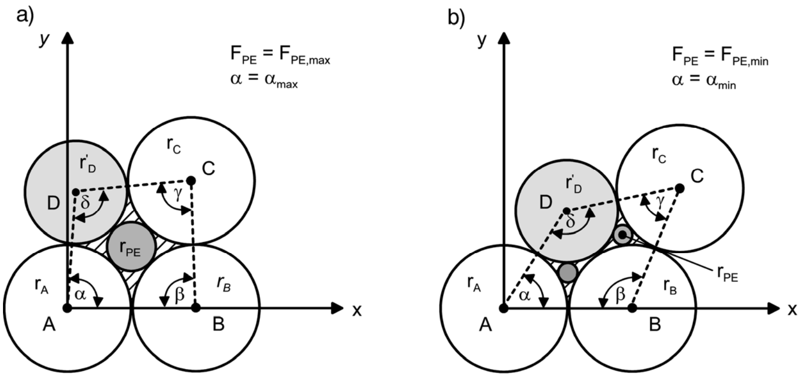

Figure 2: Calculation of pore constrictions within a pore space consisting of four particles a) for the lowest relative density D = 0 and b) for the highest density D = 1

3.2 Calculation of erodible fines

The CSD of the soil is the basis for the calculation of the initial amount α_0 of erodible fines. For the calculation a simplified approach based on a comparison between GSD and CSD of the soil is proposed. The definitions for the procedure are illustrated in figure 3. The calculation bases on three assumptions: (i) the probability of occurrence of a particle with a specific size is described by the SD of the soil, (ii) transport of an erodible particle is only possible via the larger constriction of a pore throat and (iii) the probability of occurrence of a constriction is described by the CSD which is determined by the pore throat area. This assumption includes the fact that the larger the pore throat area - and thus the pore constriction size - the bigger is the probability of occurrence.

For the determination of the erodible fines the SD and the CSD are divided in m same-sized particle or pore classes, respectively. For a given particle class i the probability of occurrence for this class is $P_{SD,i}$. The probability of occurrence of pore constrictions greater than the particles of the class *i* is

$$\sum_{i+1}^{m} P_{CSD,i}. \tag{2}$$

The probability of a coincidence of a given particle class with greater pore constrictions in size correspondingly is

$$P_i = \sum_{i+1}^{m} P_{CSD,i} \cdot P_{SD,i}. \tag{3}$$

In order to determine the initial amount of erodible fines $\alpha_{0,i}$ of class *i* in terms of its mass the probability P_i for a coincidence of particles of class *i* with greater pore constrictions are divided by $P_{SD,i}$ and multiplied with the probability of occurrence $P_{GSD,i}$ of the same particle class but concerning the GSD. The total amount of erodible fines α_0 is then given by

$$\alpha_0 = \sum_{1}^{m} \alpha_{0,i} = \sum_{1}^{m} \frac{P_i}{P_{SD,i}} P_{GSD,i}. \tag{4}$$

$\alpha_{0,i}$ can also be directly calculated in equation (3) using the probability of occurrence $P_{GSD,i}$ based on the GSD instead of $P_{SD,i}$.

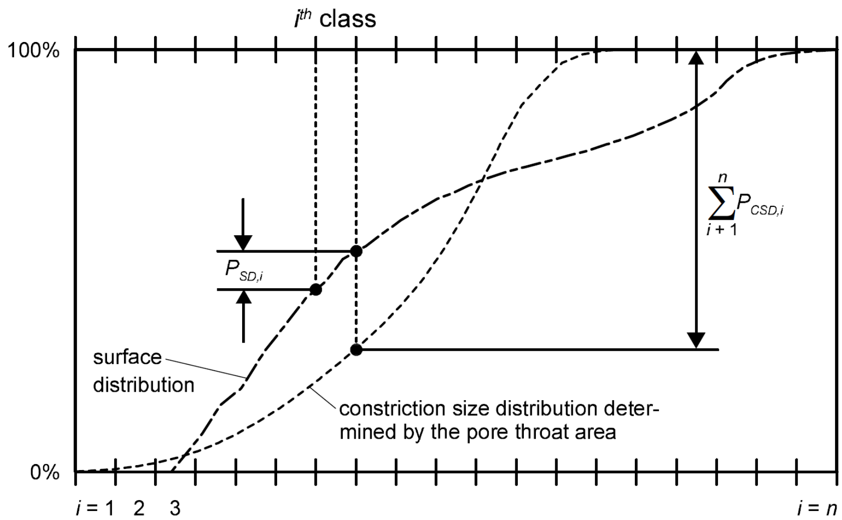

Figure 3: Definitions for the identification of erodible fines[10]

4 EXPERIMENTAL INVESTIGATION ON THE QUANTIFICATION OF ERODIBLE FINES

4.1 Sample properties

Within an experimental study[17] the amount of erodible fine particles α_0 of tri-modal mixtures of glass beads were determined experimentally with column percolation tests. For this purpose, glass beads of sizes 8 mm and 3 mm were used for the primary fabric and glass beads of 0.9 mm size were used as erodible fines. The constriction sizes of a sample consisting of only 8 mm glass beads were too large to block any small glass beads. Only if the fraction of small glass beads gets too high the large particles start to lose contacts and the fine fraction starts to form an own skeleton which means that they contribute on the support of the effective stress. As a rule of thumb the transition is at approximately 30% of the weight content of fine particles. Below this value it can be assumed that the total amount of erodible fines is washed out if it is subject to a hydraulic force. Therefore, 3 mm glass beads are used to create constriction sizes where the fine fraction can be blocked within the sample. Table 1 shows the mixing ratios of the both glass bead sizes 8 mm and 3 mm considered in the study. In the following only the mixing ratio B is considered and discussed exemplarily. Figure 4 shows the four considered GSDs including the fraction of erodible fine particles of 0.9 mm size.

mixing ratio	8 mm / 3 mm
A	32/3
B	8
C	4

Table 1: Mixing rations of the primary fabric considered in the study

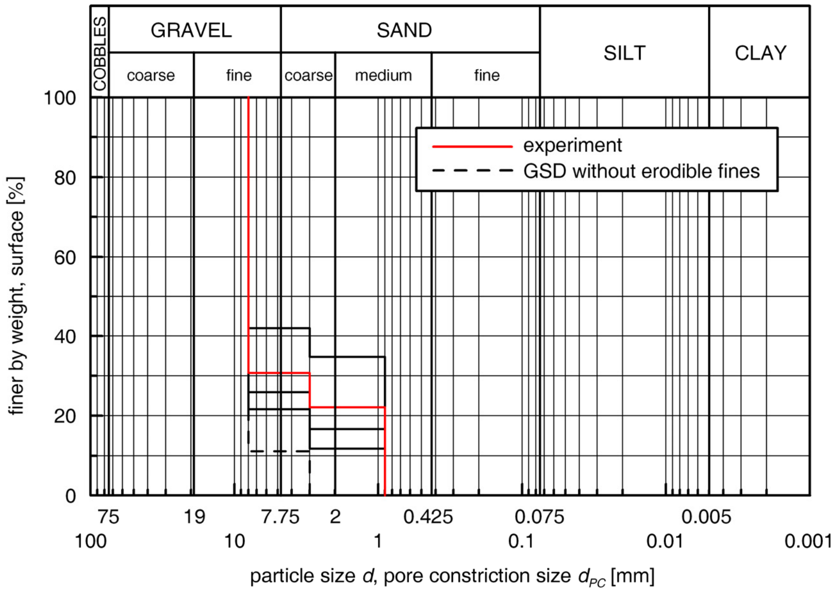

Figure 4: GSD of glass bead mixtures for mixing ratio B including fines [17]

4.2 Tests

A percolation test for the GSD highlighted with a red line in figure 4 was carried out in an experimental set-up which is shown in figure 5, left. The set-up includes a tube with a diameter of 14 cm and a turnbuckle with which the sample can be fixed between two perforated plates. At the lower end of the sample a funnel is installed to collect small glass beads fallen out of the sample. The whole construction is fixed with a sieving machine to vibrate the sample. For the preparation of the sample with a height of 16.5 cm the glass beads forming the primary fabric (8 mm and 3 mm) are filled layer-wise into the tube. The sample were then fixed with the turnbuckle and vibrated with low amplitude to force the sample to settle. Afterwards the turnbuckle was retightened, and the procedure was repeated until the densest condition was reached. In this condition only few 3 mm particles were able to move within the primary fabric.

For the percolation test itself, the erodible fines (0.9 mm) was added in pre-defined, fix portions to the sample from above during vibrating excitation. For this, an amplitude was chosen with which no movement of 3 mm particles could be observed (6.8 instrument setting). The adding of erodible fines was continued until fines were collected at the lower end in the same amount as they were added on the top, which means until a kind of steady state condition was reached. Although, an abort criterion of 1% of the added amount of fines was considered, the percolation test has lasted more than 1000 h.

Subsequently, the amplitude of the sieving machine was increased step-wise and the additionally mobilized and transported amount of fines was collected. Figure 5, right, shows the evolution of erodible fines α_0 for different amplitudes adjusted at the sieving machine. For the calculation of the erodible fines it was necessary to get the maximum total weight of the whole sample including the maximum amount of potentially erodible fines, which can be stored in the pore system of the primary fabric. To determine this value the lower end of the sample was closed with a non-perforated plate and 0.9 mm glass beads were added to the sample at constant vibration until the whole pore space was filled with erodible fines.

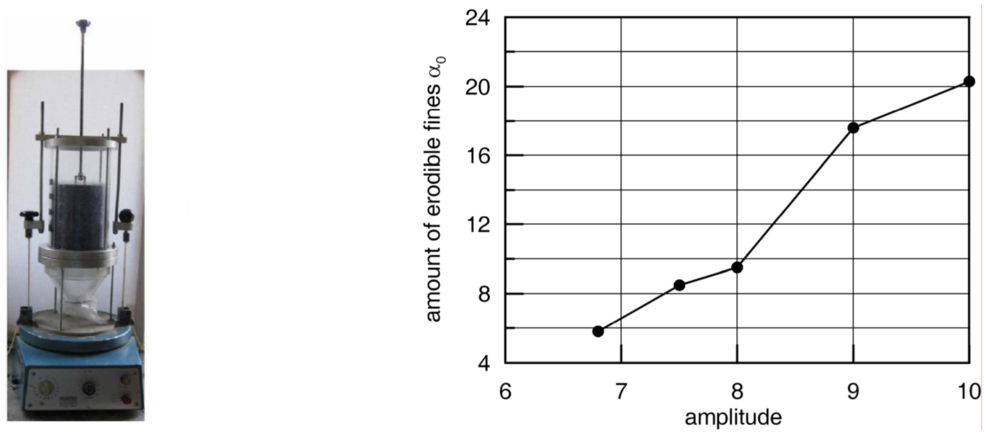

Figure 5: Left: Set-up for percolation tests. Right: Variation of erodible fines α_0 due to seismic excitation

As can be seen on figure 5 a noteworthy amount of fines α_0 was mobilized with increasing seismic energy. A similar behavior could be also observed for the other mixtures A and C. The seismic energy posed to the sample with the sieving machine can be considered as a substitute for a hydraulic gradient which acts on the sample. In fact, in the past hydraulic experiments aimed at the investigation of erosion were conducted in combination with a seismic excitation by vibration to mobilize the whole amount of erodible fines[6].

4.3 Results and comparison with calculation

The amount of erodible fines α_0 was calculated in two different ways: (1) α_0 was calculated based on the CSD of the overall material including the fine fraction consisting of the 0.9 mm glass beads. In fact, this approach considers the fine fraction as part of the primary fabric able to block detached fine particles. (2) Only the CSD of the primary fabric is considered to be able to block particles from the fine fraction. In this case, the total amount of fine particles is considered not to be part of the primary fabric. This consideration is valid for the initial condition when contact erosion is considered with a combination of two separate, adjoining layers (filter/base combination).

The results of α_0 from the both approaches represent the borders in between the real amount of erodible fines should lie. A comparison between calculated and experimentally determined α_0 is given in figure 6. As can be seen, the experimental results lie exactly between the calculations covering a wide range of erodible fines. The other two mixtures A and C show similar results with a smaller range of values for α_0.

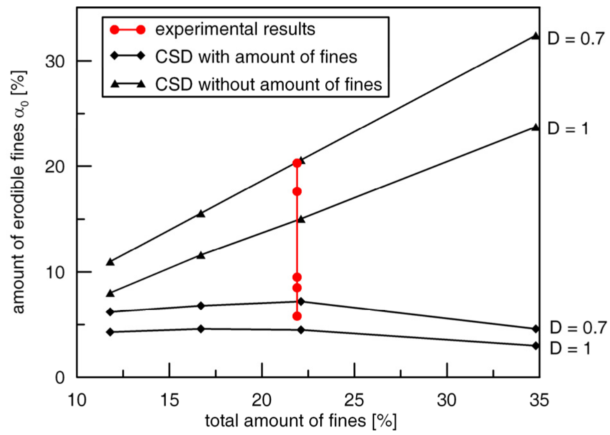

Figure 6: Comparison between calculated and experimentally determined α_0

5 BACK ANALYSIS

The method for calculating the amount of erodible fines α_0 from the CSD was used to quantify the initial conditions for the analysis of the experimental investigation of Skempton & Broagen (1994)[1] using the continuum-based multiphase model introduced before[10]. In the presented paper calculation results are exemplarily presented for material B, which can be considered as susceptible to suffusion based on the criterion of Kenny & Lau (1985)[5]. Figure 7 shows the GSD and the SD of material B with the resulting CSD for a relative density $D(n) = 0.7$. The CSD was calculated with the whole amount of fine particles giving the lower boundary for α_0 which was determined to be α_0 = 9 %. The porosity of the sample in the experiment was $n = 0.37$, and the hydraulic conductivity was determined with $k = 8.6 \cdot 10^{-3}$ m/s.

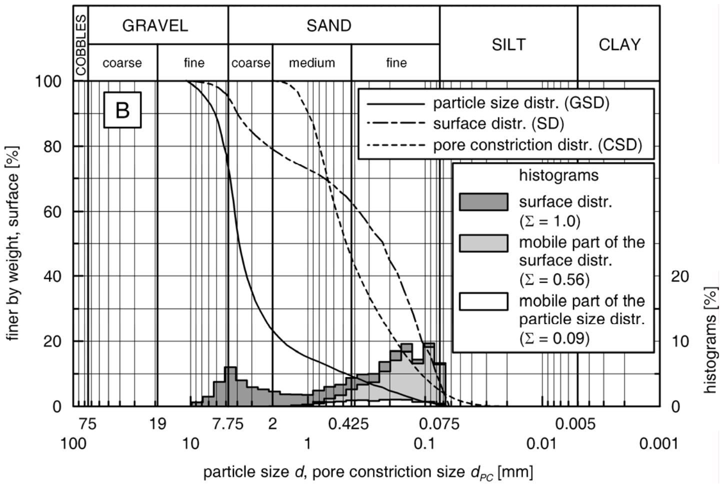

Figure 7: GSD of material B with CSD and calculated amount of erodible fines α_0

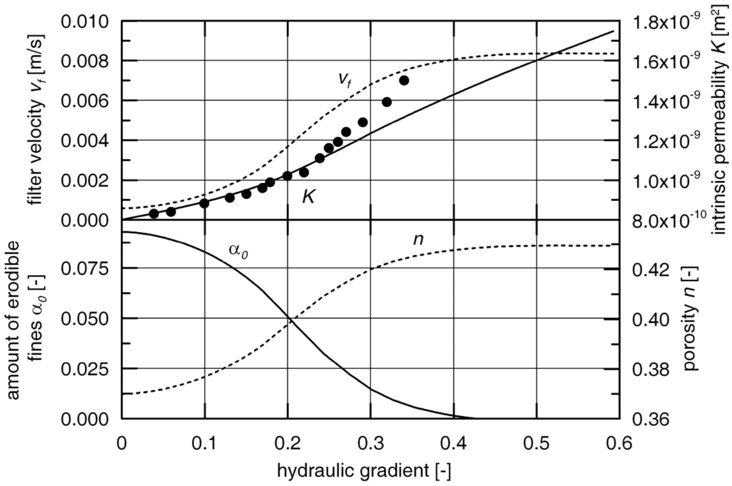

Figure 8: Simulation results: Above: Comparison between experimentally determined (symbols) and calculated (solid line) filter velocities v_f and evolution of intrinsic permeability K (dotted line). Below: Evolution of amount of erodible fines α_0 (solid line) and porosity n (dotted line).

Unfortunately, no information is available about the temporal evolution of the experiment. It is known, that within the experiment the hydraulic gradient was increased step-wise with small increments. Altogether the experiment continued about 90 min. It is assumed that a steady state conditions was awaited before a new gradient was adjusted. However, in the numerical calculation the hydraulic gradient was increased linearly leading to a consideration of a non-equilibrium process. A detailed description of the model and the equations governing the process of erosion are given in Steeb & Scheuermann (2010)[10].

The only available information about the experiments is qualitative descriptions of the observation during the experiment concerning the development of the erosion process and measurements of the filter velocity v_f measured at specific hydraulic gradients. Figure 8 shows a comparison between measured and calculated velocities of the water obtained for an optimized set of erosion parameters. With a gradient of $i > 1.5$ the velocity starts to become non-linear caused by an increasing influence of developing pipes and turbulent flow conditions. With an $i > 2.5$ strong movement of particles over the whole cross-section and piping at the inner tube-wall are reported.

Although, the development of pipes is not included in the model, it was possible to reproduce the development of the flow velocity up to this value of i. However, this result should be regarded with suspicion, since some conditions are not considered in the model, like the fact that eroded material was deposited on the surface of the sample while it is considered to be removed in the calculation. Nevertheless, this result gives an indication, that there might be a kind of transition between first movements of particles in the pore system of the primary fabric and the development of pipes leading to a localized flow of water.

6 CONCLUSION

The presented experimental results show only an extract of the overall study performed to quantify the micro-structural information about the amount of erodible fines α_0 of a soil endangered to suffusion. Due to the fact that the experiments are long-lasting only three tests were performed without repetition and variation of sample length. An all-embracing investigation of this topic has to include a larger number of tests with repetition to reproduce results and to quantify deviations.

A numerical quantification of α_0 based on the consideration of the constriction sizes of the soil is generally possible at least for the extreme border lines. One possibility to include a more realistic evolution of α_0 is the dynamic recalculation of α_0 within the numerical procedure based on a new CSD considering a loss of erodible fines in the GSD. This modification necessitates an improvement of the overall procedure for the calculation of the CSD.

The calculation results of the back analysis presented here as well as the results published earlier[10] show clearly that the presented continuum-based multiphase model enables the calculation and analysis of suffusion processes. However, a further development of the model requires experimental investigations which are especially designed for this purpose. In this connection, the most important parameter to be quantified in the experiment is the porosity of the sample and its temporal evolution during the suffusion process. One possibility to quantitatively observe the porosity during the experiment is Time Domain Reflectometry (TDR) which enables in the modified variation of spatial TDR the record of the spatial variation of the porosity along the sample[18]. In future investigations at the Ruhr-University in Bochum and at The University of Queensland this technique will be used to further investigate suffusion and to further develop the continuum-based multiphase model.

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