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BER Performance of MIMO System Employing Fast Antenna Selection Scheme Under Imperfect Channel State Information

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Abstract—**In this paper, a closed-form expression for Bit Error rate (BER) of a Multiple-Input Multiple-Output (MIMO) system employing the Minimum Mean Square Error MMSE channel estimation method is derived. The numerical results show that when the Channel State Information (CSI) is free of estimation errors BER decreases when the number of receive antennas increases. However under imperfect CSI, BER is getting worse when the number of Rx antennas is increased. In order to improve BER, a fast antenna selection scheme is proposed. The obtained numerical results prove that the proposed antenna selection scheme indeed improves the MIMO system BER performance.**

Keywords-MIMO; CSI; BER, Antenna Selection;

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems are capable of significantly improving the receive signal to noise ratio (SNR) over traditional single-input single-output (SISO) systems. The increased SNR translates into an increased channel capacity and an improved bit error rate (BER) performance. In order to obtain benefits of MIMO techniques, accurate channel state information (CSI) is required at the receiver [1]. If this condition is not met, BER is increased and thus the system capacity is degraded.

BER performance of a MIMO system under the condition of imperfect CSI was investigated in [2]. However, no closed-form expression for BER was provided. MIMO system's BER performance taking into account an imperfect CSI obtained using the minimum mean square error (MMSE) channel estimation method was investigated in [3],[4]. Explicit expressions for BER were derived by including the variance of estimation error matrix. However, in the derived expressions the channel estimation errors were fixed to constants and thus the results were not explicitly related to the channel estimation method.

In this paper, the MMSE method is used to obtain CSI. The corresponding estimation error is built into the closed-form expression for BER when a particular modulation scheme is assumed. The numerical results show that when CSI is free of estimation errors BER decreases when the number of receive antennas increases. However, when the estimation errors are taken into account the opposite happens, BER gets worse as the number of receive antennas increases. To improve the BER performance, selection of an optimal sub-set of antennas using a fast antenna selection scheme is proposed.

The rest of this paper is structured as follows. Section II introduces the system model. The training based MMSE channel estimation is described in section III. Section IV presents derivations for the BER closed-form expression for MIMO system under imperfect CSI. Numerical results are given in section V. Section VI concludes the paper.

II. SYSTEM MODEL

A narrow band block fading MIMO channel is assumed. The numbers of transmit and receive antennas are denoted as M_t and M_r, respectively. The channel is described by the $M_r \times M_t$ complex matrix $\mathbf{H}$ with entries h_{ij} representing the response between the i^{th} receive antenna and the j^{th} transmit antenna. The input-output signal relationship is given by

$$\mathbf{Y} = \mathbf{HS} + \mathbf{n} \tag{1}$$

where $\mathbf{Y} \in \mathrm{C}^{Mr\times 1}$ is the received complex signal vector and $\mathbf{S} \in \mathrm{C}^{Mt\times 1}$ is the transmitted complex signal vector constrained such that its covariance matrix $\mathbf{R}_{SS}=\varepsilon\{\mathbf{SS}^H\}$ satisfies $\mathrm{Tr}(\mathbf{R}_{SS}) = P_{tx}$ is the total average transmit power at the transmitter side over a symbol period, $\mathbf{n}$ represents a zero-mean complex additive white Gaussian noise (AWGN) with covariance matrix $\varepsilon\{\mathbf{nn}^H\}=\sigma_n^2\mathbf{I}$.

III. TRAINING BASED CHANNEL ESTIMATION

The MIMO channel matrix $\mathbf{H}$ is estimated using training sequences, which are known both to the transmitter and receiver. However, when obtained at the receiver they are affected by noise $\mathbf{n}$. This is expressed by the following equation

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$$\mathbf{r}_l = \mathbf{H}\mathbf{p}_l + \mathbf{n}, (1 \le l \le L) \quad (2)$$

where the training signal matrix is given by

$$\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_L] \in C^{Mt \times L}, \ (L \ge M_t) \quad (3)$$

The task of channel estimator is to recover the channel matrix **H** based on the knowledge of **P** and **R**, where

$$\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_L] \in C^{Mr \times L} \quad (4)$$

Assuming that the estimated channel matrix is given by $\hat{\mathbf{H}}$, a given channel estimation method minimizes the estimation error. For the case of the MMSE, which is considered here, the estimation error matrix **e** is minimized by optimizing the received signal processor **A**

$$\begin{aligned} \mathbf{A}_{opt} &= \arg\min_{\mathbf{A}} \left\{ \varepsilon = E\left[\|\mathbf{e}\|_F^2 \right] \right\} \\ &= \arg\min_{\mathbf{A}} \left\{ \varepsilon = E\left[\|\mathbf{H} - \mathbf{R}\mathbf{A}\|_F^2 \right] \right\} \end{aligned} \quad (5)$$

In (5), the optimal received signal processor **A** can be found by an analytical differentiation of ε with respect to **A**. As a result, the optimal **A** is given by

$$\mathbf{A}_{opt} = \left[(\mathbf{HP})^H \mathbf{HP} + \sigma_n^2 \right]^{-1} (\mathbf{HP})^H \mathbf{H} \quad (6)$$

Hence, the channel matrix estimated by MMSE can be written as

$$\begin{aligned} \hat{\mathbf{H}} &= \mathbf{R}\mathbf{A}_{opt} \\ &= \mathbf{R}\left[(\mathbf{HP})^H \mathbf{HP} + \sigma_n^2 \right]^{-1} (\mathbf{HP})^H \mathbf{H} \end{aligned} \quad (7)$$

IV. BER Performance under Impererfect Channel State Information

From (5), the estimated channel matrix can be represented by the error free channel matrix plus the channel estimation error matrix as given by

$$\hat{\mathbf{H}} = \mathbf{H} + \mathbf{e} \quad (8)$$

Therefore by replacing **H** in equation (1) by its estimate, one obtains

$$\begin{aligned} \mathbf{Y} &= \hat{\mathbf{H}}\mathbf{S} + \mathbf{n} \\ &= (\mathbf{H} + \mathbf{e})\mathbf{S} + \mathbf{n} \end{aligned} \quad (9)$$

If CSI is available at both receiver and transmitter, SVD operation can be applied to the error free channel matrix **H**

$$\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^H \quad (10)$$

where **U** and **V** are unitary matrices and **D** includes the eigenvalues of $\mathbf{H}\mathbf{H}^H$ as given by

$$\mathbf{D} = diag(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \cdots, \sqrt{\lambda_{\min(Mr, Mt)}}, 0, \cdots, 0) \quad (11)$$

By substituting (10) into (9) one obtains

$$\mathbf{Y} = (\mathbf{U}\mathbf{D}\mathbf{V}^H + \mathbf{e})\mathbf{S} + \mathbf{n} \quad (12)$$

Next, by using

$$\begin{aligned} \tilde{\mathbf{Y}} &= \mathbf{U}^H \mathbf{Y} \\ \tilde{\mathbf{N}} &= \mathbf{U}^H \mathbf{n} \\ \tilde{\mathbf{S}} &= \mathbf{V}^H \mathbf{S} \\ \tilde{\mathbf{e}} &= \mathbf{U}^H \mathbf{e} \mathbf{V} \end{aligned} \quad (13)$$

(12) becomes

$$\tilde{\mathbf{Y}} = (\mathbf{D} + \tilde{\mathbf{e}})\tilde{\mathbf{S}} + \tilde{\mathbf{N}} \quad (14)$$

In this case, the $M_r \times M_t$ MIMO system can be decoupled into $F = min(M_r, M_t)$ SISO sub-channels in which signals are related by the following expressions

$$\tilde{y}_i = (\sqrt{\lambda_i} + \tilde{e}_{ii})\tilde{s}_i + \underbrace{\sum_{j=1, j \ne i}^{F} \tilde{e}_{ij}\tilde{s}_j}_{I_{co}^i} + \tilde{n}_i \quad (15)$$

in which $\tilde{y}_i$, $\tilde{s}_i$, $\tilde{e}_{ij}$ and $\tilde{n}_i$ are the elements in $\tilde{\mathbf{Y}}$, $\tilde{\mathbf{S}}$, $\tilde{\mathbf{e}}$ and $\tilde{\mathbf{N}}$.

One can see from (14) that when the channel estimation errors exist, $\hat{\mathbf{D}} + \tilde{\mathbf{e}}$ is not diagonal. This is equivalent to having co-channel interferences I_{co}^i of the SISO channels.

According to equation (15), the signal power on the i^{th} sub-channel is given as

$$\begin{aligned} p_s^i &= E\left\{ \left| (\sqrt{\lambda_i} + \tilde{e}_{ii})\tilde{s}_i \right|^2 \right\} \\ &= \left(\lambda_i + \sigma_{e,ii}^2 \right) p_i \end{aligned} \quad (16)$$

The noise power on the i^{th} SISO channel is

$$
\begin{aligned}
p_n^i &= E\left\{ \sum_{j=1, j\neq i}^{M} \left|\tilde{e}_{ij}\tilde{s}_j\right|^2 + \left|\tilde{n}_i\right|^2 \right\} \\
&= \sum_{j=1, j\neq i}^{M} p_j \sigma_{e,ij}^2 + \sigma_n^2
\end{aligned} \tag{17}
$$

In (16) and (17), p_i is the transmit power for the i^{th} SISO channel. $\sigma_{e,ij}^2$ represents the variance of the ij^{th} element of the error matrix **e**. Under the assumption that MIMO operates under Rayleigh *i.i.d* channel conditions [5]

$$\sigma_e^2 = \Delta / F \tag{18}$$

where Δ denotes the mean square error (MSE) of a MIMO channel estimator. It can be calculated from the trace of the channel estimation error correlation matrix as given by

$$
\begin{aligned}
\Delta &= tr\left(E\left[\mathbf{e}\mathbf{e}^H\right]\right) \\
&= tr\left\{\left[\left(\mathbf{H}^H\mathbf{H}\right)^{-1} + \frac{1}{\sigma_n^2 M_r}\mathbf{P}\mathbf{P}^H\right]^{-1}\right\} \\
&= tr\left\{\left[\Lambda^{-1} + \frac{1}{\sigma_n^2 M_r}\mathbf{Q}^H\mathbf{P}\mathbf{P}^H\mathbf{Q}\right]^{-1}\right\}
\end{aligned} \tag{19}
$$

where **Q** is the unitary eigenvector matrix of $\mathbf{H}^H\mathbf{H}$ and Λ is the diagonal matrix with eigenvalues of $\mathbf{H}^H\mathbf{H}$. They can be obtained by the eigenvalue decomposition $\mathbf{H}^H\mathbf{H} = \mathbf{Q}\Lambda Q^H$.

By assuming equal power allocation for each sub-channel, (16) and (17) can be rewritten as

$$p_s^i = \frac{p_t}{F}(\lambda_i + \sigma_{e,ii}^2) \tag{20}$$

$$p_n^i = \frac{p_t}{F}\sum_{j=1, j\neq i}^{F} \sigma_{e,ij}^2 + \sigma_n^2 \tag{21}$$

The SNR on the i^{th} sub-channel can then be expressed as

$$\gamma_i = \frac{p_s^i}{p_n^i} = \frac{\lambda_i + \sigma_{e,ii}^2}{\sum_{j=1, j\neq i}^{F} \sigma_{e,ij}^2 + F\sigma_n^2 / p_t} \tag{22}$$

The obtained expression (22) for SNR can be used to derive a closed-form expression for BER. This is possible when a particular type of modulation scheme is assumed.

Here, the M-QAM modulation is chosen to complete the derivation of closed-form expression for BER under the condition of imperfect CSI. The BER expression for M-QAM square modulation can be written as [6]

$$\rho_e \approx 0.2\exp\left(-\frac{1.6\gamma}{2^l - 1}\right) \tag{23}$$

in which ρ_e is the BER; γ is the channel SNR; l is the modulation level and 2^l=M. In this case, substitution of (22) into (23), leads to the following expression for BER on the i^{th} sub-channel

$$\rho_e^i \approx 0.2\exp\left(-\frac{1.6}{2^l - 1}\frac{\lambda_i + \Delta/F}{(F-1)\Delta/F + F/\rho}\right) \tag{24}$$

Accordingly, the BER for the MIMO system can be written as an average over F SISO sub-channels,

$$
\begin{aligned}
BER &= \frac{1}{F}\sum_{i=1}^{F}\rho_e^i \\
&\approx \frac{1}{5F}\sum_{i=1}^{F}\exp\left(-\frac{1.6}{2^l - 1}\frac{\lambda_i + \Delta/F}{(F-1)\Delta/F + F/\rho}\right)
\end{aligned} \tag{25}
$$

The closed-form expression (25) for BER of MIMO system shows that for a specific modulation, it is a function of singular values, the channel matrix rank and the MSE of the channel estimator.

The derived expression for BER is valid for the case when a fast antenna selection scheme is applied to the MIMO system. In this case, suitable modifications are required. They concern the channel matrix which in the new case is defined with respect to the optimal sub-set of transmit or receive antennas.

V. NUMERICAL RESULTS

To investigate the MIMO BER performance under the condition of imperfect CSI, the MMSE training-based channel estimation and 16-QAM modulation scheme is assumed. To determine the MSE of MMSE method, expressions presented in [7] are used. Figure 1 shows the BER performance versus SNR for 2×2 and 8×2 MIMO systems operating under Rayleigh conditions, as assumed in derivations leading to the BER expression (25).

One can see that for all of the presented cases, BER decreases as SNR is increased. For a given MIMO system, the BER results obtained under the condition of perfect knowledge of CSI are always better than the ones for imperfect CSI. When performance of the two MIMO systems is compared, the following is found. Under the condition of perfect CSI, the BER performance of the 8x2 MIMO system is better than the one offered by the 2x2 MIMO system.

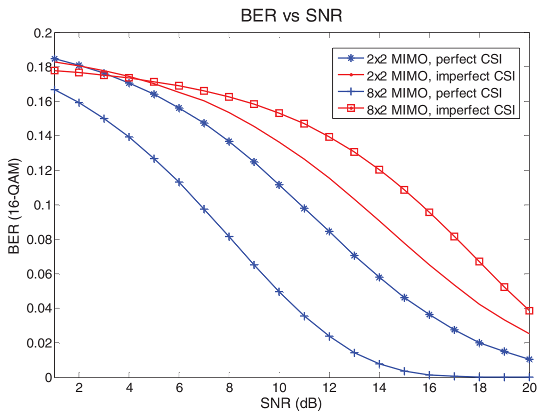

Figure 1. BER performance versus SNR using 16QAM

However, when the imperfect CSI is assumed, the BER of 8x2 MIMO is worse than that of the 2x2 MIMO. This is further demonstrated by results shown in Figure 2.

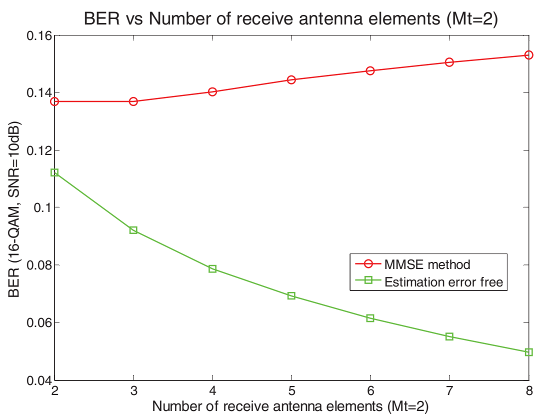

Figure 2. BER performance versus receive antenna number

When the number of transmitting antennas is fixed to 2 and the number of receive antennas increases, the BER including estimation error increases while the one with the error free CSI decreases. This can be explained by the fact that when the number of Rx antennas increases, the accuracy of MMSE channel estimation is getting worse.

This finding triggers the idea to use only a subgroup of Tx antennas. A subgroup of Tx antennas can be chosen using a fast antenna selection (AS) scheme [8]. Here, we propose using the NBS antenna selection scheme, as described in [9]. It is worth mentioning that NBS is not an optimal AS scheme. However its simplicity makes it a popular choice. Figure 3 presents the BER results for the MIMO system with imperfect CSI when the NBS antenna selection is applied. One can see that the antenna selection improves the BER performance (2/8×2 refers to the system where 2 out of 8 Rx antennas are selected). As observed from results in Figure 3, The BER performance becomes comparable to the 2x2 MIMO system.

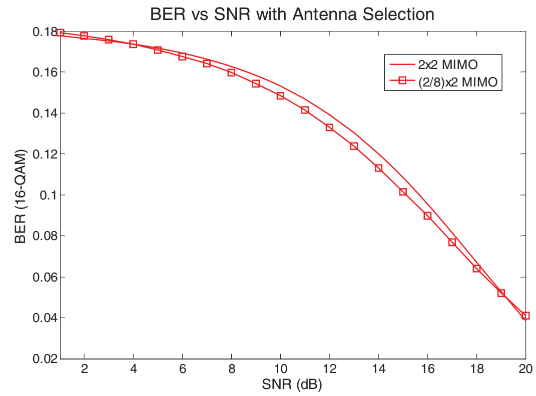

Figure 3. BER performance versus SNR with antenna selection

It shows a considerable improvement when compared with BER results for the 8x2 MIMO system earlier reported in Figure 1.

VI. CONCLUSIONS

In this paper, we have derived a closed-form expression for BER of a MIMO system operating under Rayleigh signal propagation conditions. The derived expression assumes imperfect knowledge of CSI obtained using the MMSE channel estimation method. The simulation results have shown that BER with imperfect CSI is worse than the one with perfect knowledge of CSI. It increases as the number of Rx antennas is increased. To improve it, a fast antenna selection scheme is proposed. The simulations results have demonstrated that the proposed antenna selection scheme improves the BER performance.

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